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# A note on the motion of Mars on the second Strasbourg astronomical clock

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## Abstract

The astronomical clock erected in the 1570s in the Strasbourg cathedral by the mathematician Conrad Dasypodius and the Habrecht clockmakers showed the mean motions of the superior planets in a geocentric setting, but the motion of Mars on the clock appears to have been less accurate than those of Jupiter and Saturn. In fact, it is clear that something went wrong, that either a wheel was cut with the wrong number of teeth, or that some calculation error was made. However, up to now, no satisfactory explanation has been given for this discrepancy. This note endeavours to suggest a plausible solution and also relates some of the chosen periods in the clock to the values given in the *Prutenic tables*.

## Keywords

Strasbourg clock, astronomical clock, Mars, Prutenic tables

## Introduction

Günther Oestmann has recently published an English updated translation of his dissertation on the second astronomical clock of the cathedral of Strasbourg, designed and constructed in the 16th century by the mathematician Conrad Dasypodius (1531?-1601?), the clockmakers Isaac Habrecht (1544-1620) and his brother Josias (1552-1575), and a few others (Oestmann (2020)). My purpose here is not to review this work, but to have a closer look at one particular feature of the clock, namely the motion of Mars.

I became interested in that question while preparing a new edition of Ungerer's seminal work on the astronomical clock of the cathedral of Strasbourg (Ungerer and

Ungerer (1922)). It turns out, as a matter of fact, that the motion of Mars was much more inaccurate on that clock than those of Jupiter and Saturn, and also much more inaccurate than it could have been. The mechanisms of the 16th-century clock are almost entirely kept in the *Musée des arts décoratifs* in Strasbourg, and the parts of interest here are the celestial globe and the astrolabe.

## The motion of the Sun

We need first to examine how the Sun moves on the clock, because the motion of the Sun is later used to derive those of the planets. On the celestial globe, the motion of the Sun was displayed with a period of  $T_E = 536550/1469$  days with respect to the globe, that is about 365d 5h 57m 48s, which is close to the variable tropical year derived from the *Prutenic tables* (Reinhold (1551)), based on Copernicus's *De Revolutionibus*. I will not consider here how Dasypodius may have come to  $T_E$ , but I merely recall that in Copernicus's theory (Swerdlow and Neugebauer (1984)), the precession is slowly variable (Swerdlow (1986); Morando and Savoie (1996)), and this causes in turn the tropical year to be variable. Meeus and Savoie (Meeus and Savoie (1992)) gave a value of 365 days 5 hours 55 minutes and 58 seconds for the tropical year in the *Prutenic tables*, and it may refer to their computation for 1551. In any case, this value is much less accurate than the value of the tropical year in the Alphonsine tables, although the mean value of the Prutenic tropical year is close to that of the Alphonsine tables (Swerdlow 1986, p. 110).

So, on the celestial globe, Dasypodius obviously approximated the tropical year from the *Prutenic tables*, although he should in fact have used the sidereal year, the globe representing the actual sky, not the coordinate system (mobile zodiac) linked to the Earth. The sidereal year derived from the *Prutenic tables* is 365 days 6 hours 9 minutes and 39 seconds and the difference between the two (tropical and sidereal) years is quite large. We can also observe that in the *Hypotyposes* published by Dasypodius in 1568 (Peucer 1568, p. 312), Peucer gives the (variable) tropical year for 1559 as 365 days 5 hours 55 minutes 16 seconds 17 thirds. This work was cited by Oestmann in 2020 (Oestmann (2020)), but without relating it to the value of the tropical year used on the clock. Looking back, it is in fact surprising to see that none of the authors who wrote on the clock related the year used on the clock to the *Prutenic tables*. The first to analyze the gears, Jean-Baptiste Schwilgué (1776-1856), the maker of the current astronomical clock in Strasbourg cathedral, only gave the sidereal day, not the sidereal or tropical year (Schwilgué c1845, vol. 2, p. 220). Surprisingly, the seminal book by Ungerer published in 1922 doesn't give any details of that clock (Ungerer and Ungerer (1922)). Then, when Henri Bach (1909-1991) wrote on the globe in 1960 (Bach (1960)), he gave the sidereal year as 365.248468105 days (which should have been 365.24846834. . .) and he suggested that Dasypodius's target value was the sidereal year from the Alphonsine tables. He also suggested that Dasypodius might have taken as target the average between the tropical and sidereal year. In his articles published in 1978-1979 (Bach (1978, 1979)), he also didn't relate the year to the *Prutenic tables*. Neither Pouille in 1983 (Pouille (1983)), nor Bach and Rieb in 1992 (Bach et al. (1992)), gave any explicit value of the (tropical) year used on the clock. Pouille moreover also tried to relate the indications of the clock to the Alphonsine tables, when

in fact the *Prutenic tables* were more relevant, as already observed by Oestmann in 1993 (Oestmann (1993)). In 2005, Scheurenbrand (Scheurenbrand et al. 2005, p. 112) also missed the link with the *Prutenic tables*, even though he gave the value 365.24847 days. And most recently, in 2020, Oestmann (Oestmann 2020, p. 140) still merely gave the (tropical) year used by Dasypodius as 365.2485 days. He only compared this value to the sidereal year.

## The astrolabe dial and the planets

The astrolabe dial of the clock also showed the motion of the Sun and of the known planets, as well as the *rete* for the stars, of course in a geocentric perspective. This rete, however, should be understood as representing the mobile zodiac, and therefore the motion of the Sun with respect to the rete should be that of the tropical year. In any case, the astrolabe did display the same motions as on the globe, namely that the rete rotated with respect to the Sun with the exact same speed as the celestial globe with respect to the Sun, and so did the Sun in a fixed frame, making one turn in 24 hours. On the astrolabe, all the planets had uniform motions. Mercury and Venus, however, did not have any independent motion and were always moving with the Sun. Their actual motion is an oscillation around the Sun, not taken into account in the clock which only displayed mean motions.

The three other planets, namely Mars, Jupiter and Saturn, however had independent motions and Dasypodius tried to have them move with their tropical periods. Dasypodius could not do this with Mercury and Venus, because it would then have caused these planets to be at times opposite the Sun, which does never take place.

In the sequel, I will consider the synodic ( $S$ ) and tropical ( $T$ ) periods of the planets, and subscript them with  $E$  (Earth),  $M$  (Mars),  $J$  (Jupiter) or  $S$  (Saturn). It is easy to show that

$$S_x = \frac{T_x \cdot T_E}{T_x - T_E}$$

where  $x$  denotes one of Mars, Jupiter or Saturn. And we also have

$$\frac{S_x}{T_E} = \frac{T_x}{T_x - T_E} = \frac{1}{1 - \frac{T_E}{T_x}}$$

The ratios  $\frac{S_x}{T_E}$  are important here, because the revolutions of Mars, Jupiter and Saturn are derived from that of the Sun in a mobile reference frame rotating with the Sun, hence the synodic revolutions.

Let us now compare the synodic and tropical revolutions on the clock with the actual ones:

Planet	synodic revolution		tropical revolution	
	clock	real	clock	real
Mars	$\frac{1083600}{1469} \approx 738$ d.	780 d.	$\frac{553719600}{765349} \approx 723$ d.	687 d.
Jupiter	$\frac{6438600}{16159} \approx 398$ d.	399 d.	$\frac{6438600}{1469} \approx 4383$ d.	4331 d.
Saturn	$\frac{16096500}{42601} \approx 378$ d.	378 d.	$\frac{16096500}{1469} \approx 10957$ d.	10747 d.

The equation for  $S_x$  given above can be used to relate the synodic and tropical revolutions in this table. For instance, we have

$$S_M = \frac{1083600}{1469} = \frac{\frac{553719600}{765349} \times \frac{536550}{1469}}{\frac{553719600}{765349} - \frac{536550}{1469}}$$

Some of these values were sometimes given incorrectly in earlier publications. Schwilgué obtained 729 days instead of 723 for the tropical revolution (Schwilgué c1845, vol. 2, p. 221), because he mistakenly thought that the 129 teeth wheel in Mars's gears had only 128 teeth. For the same reason, in 1979 (Bach (1979)), Bach gave the value 732 days instead of 738 for Mars's synodic revolution, and moreover he compared it to a real tropical revolution of 686 days, which does not make sense. When he examined the gears in 1947, he had actually counted 129 teeth for the above-mentioned wheel (Pouille 1983, p. 42), but in 1979 he used Schwilgué's value, probably trusting it more than his own.

Now, as we can see in this table, the synodic revolutions have been well approximated for Jupiter and Saturn, but the approximation for Mars is rather bad. The synodic revolutions are also less sensitive to errors on the sidereal or tropical revolutions, which explains that even with 1 or 2 % of error on the tropical revolutions of Jupiter and Saturn, the displayed synodic revolutions are relatively more accurate.

### How the ratios were chosen for Jupiter and Saturn

From the above results, there are two investigations which should be conducted. First, can we find how Dasypodius (or Habrecht) obtained the revolutions of the planets assuming that of the Sun? And second, why is the period of Mars so bad?

Unfortunately, Dasypodius himself is not very helpful, as his descriptions (Dasypodius (1578, 1580b,a)) of the clock do not enter into details. He does not even give the revolution periods of the planets. It is however easy to reverse engineer some of the work and to see that the tropical revolutions of Jupiter and Saturn were merely equated to 12 and 30 tropical revolutions of the Sun:

$$\begin{aligned} 12 \times \frac{536550}{1469} &= \frac{6438600}{1469} \\ 30 \times \frac{536550}{1469} &= \frac{16096500}{1469} \end{aligned}$$

Using the above relations between  $T_x$  and  $S_x$ , we also obtain the simple ratios between the synodic revolutions and the tropical year of the Sun:

$$\begin{aligned} \frac{6438600}{16159} / \frac{536550}{1469} &= \frac{12}{11} \\ \frac{16096500}{42601} / \frac{536550}{1469} &= \frac{30}{29} \end{aligned}$$

A note is in order here. In his book on the Strasbourg clock (Oestmann 2020, p. 168), Oestmann writes that calculations should only use simple ratios, in order to have the

designer's goals become clear. He then goes on by setting, *arbitrarily*, the ratios of the speeds of the idealized planets as 8/15 (Mars), 1/12 (Jupiter) and 1/30 (Saturn), derives the ratios of the synodic periods, and then checks that the clock gears meet these ratios, except for Mars. I think that this is the wrong way to tackle such a problem. We should first consider the real motions of the planets, as given by the astronomical knowledge of the time, such as given in the *Prutenic tables*. Then, we should identify what were the goals in the construction and how these goals were met. We should not set right away some ratio like 8/15, which may have no reality at all in a construction. In Oestmann's description, there is a confusion between the ratios he *believes* Dasypodius chose (or should have taken) and those he actually *chose*. I do not, for instance, believe that Dasypodius ever chose (and missed) the ratio 8/15 for Mars, as I explain below.

Now, in the above, there can be no doubt that the numbers 12 and 30 somehow lie at the basis of the Jupiter and Saturn gears. (Incidentally, the astrolabe also displays the lunar nodes, which involve the simple ratio 37/2.) This does however not mean that Dasypodius approximated the ratios  $T_x/T_E$  with 12 and 30. If Dasypodius had tried to approximate the actual tropical years of Jupiter and Saturn, he would have come up with more accurate ratios. I therefore believe that Dasypodius tried to approximate the ratios  $S_x/T_E$  with 12/11 and 30/29, and this is consistent with the fact that these ratios are the ones needed for the gears, not the ratios  $T_x/T_E$ . Moreover,  $S_x/T_E$  is closer to 12/11 or 30/29 than  $T_x/T_E$  is to 12 or 30, even relatively:

$$1 - 10747/365.2484/30 = .019\dots$$

$$378/365.2484 - 30/29 = 0.0004\dots$$

It may be the case that Dasypodius saw that the values of  $T_x/T_E$  were close to 12 and 30, but these values were certainly not the actual targets.

## How the motion of Mars may have been obtained

For Mars, things are not so simple. It is likely that Dasypodius started with the ratio of the tropical revolution of Mars to that of the Sun, which is about 1.881. This value could easily be derived from the mean daily motion in longitude of Mars in the *Prutenic tables*, namely  $0^\circ 31' 26'' 30''' \dots$ :

$$\frac{360^\circ}{0^\circ 31' 26'' 30''' \dots} / \frac{536550}{1469} \approx 1.881$$

Of course, at the time of Dasypodius, there weren't yet any decimal fractions and Dasypodius wouldn't have written such a value 1.881, but he may have worked on larger numbers, multiplying them for instance by 1000, or perhaps by 3600 if he was working with sexagesimal values. Dividing by sexagesimal values could for instance be achieved by converting the values to their smallest unit, such as seconds, or thirds, etc., and then manipulating only integers. Eventually, Dasypodius would for instance have obtained 1881/1000 for the above ratio.

But this ratio is not as close to an integer as those for Jupiter and Saturn, and Dasypodius couldn't merely settle for the value 2, for instance. Oestmann

suggested (Oestmann 2020, p. 168) that Dasypodius may have targeted the simple ratio  $15/8 = 1.875$ , and this would have led to  $S_M/T_E = 15/7$  and a synodic revolution of about 783 days on the clock, close to the real value. But on the clock,  $S_M/T_E = 1032/511 = \frac{8}{73} \times \frac{129}{7}$ , and Oestmann did not come up with an explanation for that ratio.

Now, we can also notice that the ratio of the tropical revolution of Mars to that of the Sun on the clock is about 1.981 or more exactly  $1032/521$ . And we can reformulate our question and ask how Dasypodius (or Habrecht) came to obtain  $1032/521$ ? Or how did Dasypodius arrive at  $1032/511$  in case he started with the ratio of the synodic period to the tropical year of the Sun?

One might first consider that the key to the solution lies in the proximity of 1.881 and 1.981. If Dasypodius (or Habrecht) mistakenly read 1.981 (or 1981) for 1.881 (or 1881), the ratio  $1032/521$  might have been derived, then the ratio  $1032/511$  and then the gear pairs

$$\frac{1032}{511} = \frac{8}{73} \times \frac{129}{7}$$

which were used on the clock. The value  $1083600/1469$  given above for the synodic revolution is then obtained by multiplying the ratio  $536550/1469$  by  $1032/511$ .

But of course, we do not know how Dasypodius expressed his ratios. Perhaps he didn't use decimal representations, but sexagesimal ones, as in the astronomical tables such as the *Prutenic tables*. As a matter of fact, using the convenient notation  $a; b, c, \dots$  for  $a + b/60 + c/60^2 + \dots$ , we have:

$$\begin{aligned} 1.881 &= 1 + \frac{52}{60} + \frac{51}{60^2} + \dots = 1; 52, 51, \dots \\ 1.981 &= 1 + \frac{58}{60} + \frac{51}{60^2} + \dots = 1; 58, 51, \dots \end{aligned}$$

In these two expansions, the first can become the second by a mere error of one digit, either in the decimal or in the sexagesimal expansion.

Either way, whether 1.881 was copied as 1.981 or 1; 52,51 was copied as 1; 58,51, it could have led to the ratio  $2377/1200$  and this ratio may have been approximated by  $1032/521$ , although we do not know by which process. There are moreover two paths from 1; 58,51 to  $1032/511$ . It may be that Dasypodius somehow obtained the ratio  $1032/521$ , and from it derived that of  $1032/511$ . Or, it may be that 1; 58,51 was used to obtain the sexagesimal expansion for the ratio  $S_x/T_E$  which is 2; 1,10, and then, from this expansion the ratio  $1032/511$  could be obtained.

Had Dasypodius started with a tropical revolution of Mars of about 687 days, and derived the ratio of the synodic revolution to that of the tropical year of the Sun, he would have obtained about  $2.131 = 2; 7,50$  whereas  $1032/511 = 2.019\dots = 2; 1,10$ . It does not seem clear how  $1032/511$  could be mistakenly derived from 2.131, either in its decimal or in its sexagesimal expression. I therefore consider it more likely that the error occurred on the ratio  $T_x/T_E$  and that this then led to an incorrect ratio  $S_x/T_E$  which was then approximated with  $1032/511$ . This could have been done by a multiplication by 7, then by 73, given that  $511 = 7 \cdot 73$ .

We have therefore a very plausible path by which the ratio  $\frac{1032}{511}$  could have been obtained, and this path rests on a transcription error, either in an (unlikely) decimal expansion of the ratio of tropical years, or in the sexagesimal expansion of this ratio. In other words, 1;52,51 may have been transcribed as 1;58,51, perhaps 2;1,10 was then derived, and this value was recognized as being close to  $1032/511 = \frac{8}{73} \times \frac{129}{7}$ .

Now, whatever the real erroneous process, if the correct value of the ratio of the tropical period of Mars to that of the Sun had been used, it could have been likewise approximated by  $\frac{980}{521}$ . And the ratio of the synodic period of Mars to the tropical year of the Sun would then have been  $\frac{980}{459}$  which could have been written as

$$\frac{980}{459} = \frac{7}{51} \times \frac{140}{9}$$

giving two pairs of gears as replacements to the pairs

$$\frac{8}{73} \times \frac{129}{7}$$

### Comparison with earlier explanations

When Schwilgué examined Dasypodius's gears in the 1830s or 1840s, he did not really elaborate on the motion of Mars being less accurate than those of Jupiter and Saturn. In fact, it is fair to say that he didn't care too much for these old gears. And, as I mentioned above, he erroneously attributed 128 teeth to the Mars wheel (Schwilgué c1845, vol. 2, p. 215) which would have resulted in a synodic revolution of 732 days. Schwilgué's successors, the Ungerers, apparently didn't do any work on Dasypodius's astrolabe, and we have to wait for Bach who analyzed the gears in 1947, probably because he was then working on the Oslo astronomical clock which was completed in 1952. Around 1960, J.-P. Rieb who was in touch with Bach built a partial small replica of Dasypodius's astrolabe work and he took the correct value 129 after examining the wheels (private communication). Yet before 1980, Bach apparently still thought, following Schwilgué, that the Mars wheel had 128 teeth (Bach (1979)), but at some point he accepted the value 129. In 1983 (Pouille 1983, p. 42), Pouille tried to find a solution to the problem of Mars and thought that Mars's 129-teeth wheel actually had 130 teeth (synodic revolution: 743 days) but should have had 136 teeth (synodic revolution: 778 days). This is a reasonable analysis resulting in revolution values close to the theoretical ones. When Bach and Rieb's book was published in 1992 (Bach et al. (1992)), there was no attempt to analyze the error on Mars. The authors only observed that the motion of Mars was the least accurate of the clock. In his book published in 2020 (Oestmann (2020)), Oestmann also made an attempt to derive Dasypodius's gears, but he stopped short from providing a solution. Earlier, in 1993 (Oestmann 1993, p. 119), he had suggested that Mars's 129-teeth wheel should instead have 136 teeth, perhaps following Pouille. In 2000, based on a misunderstanding, he had instead suggested that the wheel should have had 122 teeth (Oestmann 2000, p. 119), and this was repeated in his edition of Dasypodius' *Heron mechanicus* (Dasypodius 2008, p. 9). (I notified the latter error to Oestmann in 2013. The value 122 would have resulted in a catastrophic synodic revolution of 698 days.) And in 2005, Scheurenbrand (Scheurenbrand et al. 2005, p. 113) suggested that Dasypodius should have taken the ratio 117/6 instead of 129/7

(keeping  $8/73$ ), resulting in a synodic revolution of 781 days, or even should have replaced the four gears by the pair  $32/15$  (synodic revolution: 779 days).

## Conclusion

In fact, none of these suggestions are satisfactory. If we assume that the clock has only one incorrect wheel, and if 129 teeth are replaced by 136 teeth, then one obtains a synodic revolution of 778 days (and not 780), and the tropical revolution will be 689 days (not 687). It is better than the current values with 129 teeth, but it does not explain how the ratio  $1032/521$  was obtained. How could 136 have become 129? Even Scheurenbrand's suggestion for changing two gears is not as good as it seems. First, it does not explain how the ratio  $117/6$  could have been found, and second it is still not as good as the pair derived from  $980/459$  which could have been obtained. And the same applies to the ratio  $32/15$ .

It seems much more likely that the correction of Mars's incorrect gearings is not merely to replace one gear by another, or two gears by two others, or four gears by two, but to replace four gears by four other gears. This seems to indicate that during the construction of the gears the errors were not detected, perhaps because of an excessive trust in the computations. But if the computations had been made by Habrecht, wouldn't they have been examined by Dasypodius? This leads me to believe that Dasypodius made the computations, including the factoring, and that no one proofchecked his computations, not even the Habrechts. The error then crept in the gears, although it may have been detected at a later stage.

Of course, there is no written account of the computation errors, but the task of the historian is not only limited to tangible testimonies such as manuscripts. A historian should not only try to unravel the history, using the existing evidence, but he/she should also suggest more or less plausible explanations. A historian is not merely a translator or someone who finds something and makes it clearer to a larger audience, but he/she is also someone who may suggest missing links. In the above case, it is clear that something went wrong in the design of the motion of Mars, but none of the historians up to now have been able to give a satisfactory explanation. Occasionally, it has been suggested to replace one wheel by another, but I believe that such a replacement is not the best solution, and that it is necessary to move back earlier. The error on Mars is not merely that of one or two wheels wrongly cut as has been claimed earlier, but it is certainly that of a ratio wrongly computed or incorrectly copied, and consequently of four wrong wheels. I have suggested what these four wheels could have been, had the computation error not have taken place.

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