Astronomical clocks 1735-1796

A companion to Ludwig Oechslin's *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert* (1996)

(Work in progress)

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version 0.15, 8 September 2025

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History of versions:

25 August 2025: version 0.1 online.
26 August 2025: version 0.11 online.
29 August 2025: version 0.13 online.
8 September 2025: version 0.15 online.

Preface

In the introduction to the chapter on 19th and 20th century astronomical clocks in the recent *General history of horology* [3], I wrote that in the 18th century, a number of remarkable astronomical clocks were made in Europe by Bernardo Facini (1665-1731), Johannes Klein (1684-1762), Johann Georg Neßtfell (1694-1762), Claude-Siméon Passemant (1702-69), Francesco Borghesi (1723-1802), David Ruetschmann (David a Sancto Cajetano, 1726-96), Michael Fras (Aurelius a San Daniele, 1728-82), Philipp Matthäus Hahn (1739-90), and many lesser figures, and this was a way to fill a gap, because in that general work, we unfortunately skipped the 17th and 18th centuries astronomical clocks.

At the time of publication, I was aware of all these clocks, and had seen several of them. I had also published an analysis of one of Passemant's clocks. But I had not been working in depth on most of the clocks I mentioned. The present work gives me an opportunity to have a closer look at these great clocks, those that Klaus Maurice called "*Die Uhrwerke der Theologe*" [1, p. 266-280], using Ludwig Oechslin's work on the "*Priestermechaniker*" as a basis. In fact, in that work Oechslin wrote that

> Die vorliegende Arbeit hofft Grundlagen bereitzustellen, die andere zu weiteren Forschungen anregen können [2, p. 6].

and this is exactly what it did in the present work.

[1] Klaus Maurice. *Die deutsche Räderuhr — Zur Kunst und Technik des mechanischen Zeitmessers im deutschen Sprachraum.* München: C. H. Beck, 1976. [2 volumes].

[2] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[3] Denis Roegel. Clocks as astronomical models: The nineteenth and twentieth centuries. In Anthony Turner, James Nye, and Jonathan Betts, editors, *A general history of horology*, pages 273–287. Oxford: Oxford University Press, 2022.

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Contents

Preface **3**

1 Introduction **15**
1.1 The foundation of this work . . . 15
1.1.1 Oechslin's drawings . . . 15
1.1.2 Oechslin's notations . . . 17
1.1.3 Oechslin's photographs . . . 18
1.2 A companion work . . . 19
1.2.1 The motivation of my work . . . 19
1.2.2 New notations . . . 20
1.3 Types of trains and velocities . . . 24
1.3.1 Simple trains . . . 24
1.3.2 Moving frames using the input and the output . . . 28
1.3.3 Cranks on epicyclic wheels . . . 35
1.3.4 Periods of rotation . . . 40
1.4 The extent and limits of this work . . . 40
1.4.1 Methodology . . . 40
1.4.2 The limits of this work . . . 43
1.4.3 How to use this work . . . 43
1.4.4 Future versions . . . 44
1.5 References . . . 46

2 Adams' orrery in Munich (c1748-1757) [O:11.1] **49**
2.1 Introduction . . . 49
2.2 The base motion . . . 59
2.3 The period of the pendulum . . . 60
2.4 The tropical year . . . 60
2.5 The motions inside cage 15 . . . 61
2.5.1 Tubes 70 and 72 . . . 61
2.5.2 The Earth and the Moon . . . 62
2.6 The Sun, Mercury and Venus . . . 64
2.7 The motions of Mars . . . 66
2.8 The motions of Jupiter and its satellites . . . 68
2.9 The motions of Saturn and its satellites . . . 71
2.10 Conclusion . . . 75

See at the beginning the rights associated to this document.

2.11 References 75

3 Pater Aurelius's clock in Munich (1769) [O:5.1] 77
3.1 Introduction 77
3.2 Description of the clock 78
3.2.1 The going work 78
3.2.2 The main dial 82
3.2.3 The lunar sphere 86
3.2.4 The celestial sphere 86
3.2.5 The other dials 89
3.3 Conclusion 93
3.4 References 94

4 Frater David's first clock in Vienna (1769) [O:6.1] 97
4.1 Introduction 97
4.2 The going work 99
4.3 The rear side 104
4.3.1 The day of the month 105
4.3.2 The time dial 105
4.4 The front side 106
4.4.1 The yearly motions 109
4.4.2 The time of the day and the day of the week 112
4.4.3 The motion of the planets 114
4.4.4 The line of apsides 118
4.4.5 The motion of the Moon 119
4.4.6 The main dial 121
4.4.7 The upper dial 125
4.5 References 129

5 Frater David's Schwarzenberg clock (1793) [O:6.2] 135
5.1 Introduction 135
5.2 The going work 136
5.3 The main dial 136
5.3.1 The motion of the Sun 140
5.3.2 The motion of the line of apsides 141
5.3.3 The mean motion of the Moon 142
5.3.4 The line of the nodes (dragon) 143
5.3.5 The corrected motion of the Moon 144
5.4 The other dials 144
5.4.1 The italic hours 144
5.4.2 The phase of the Moon 145
5.4.3 The synodic month 145
5.4.4 The draconic month 146
5.4.5 The day of the week 146
5.4.6 The calendar dial 146

See at the beginning the rights associated to this document.

5.5 References 147

6 Frater Fridericus's mechanism in Bamberg (1772) [O:4.1] 151
6.1 Introduction 151
6.2 The going work 153
6.3 The main driving motion 153
6.4 The motion of the Earth around the Sun 153
6.5 The rotation of the Earth 156
6.6 The shadowline on Earth 157
6.7 The Mercury system 157
6.8 The lunar system 159
6.8.1 The mean motion of the Moon 159
6.8.2 The lunar nodes 160
6.8.3 The line of apsides 160
6.8.4 The rotation of the Earth 161
6.9 References 161

7 Hahn's *Weltmaschine* in Stuttgart (1769) [O:8.1] 163
7.1 Introduction 163
7.2 History and general description 169
7.3 The celestial globe 175
7.3.1 The motions of the Moon 181
7.3.2 The motions of the Sun, Mercury and Venus 182
7.3.3 The motions of Mars, Jupiter and Saturn 185
7.4 The planetary system or orrery 188
7.4.1 The Sun 190
7.4.2 The fixed cage 190
7.4.3 The mean motions of Mercury, Venus, the Moon, the Earth and Mars 190
7.4.4 The upper part of the orrery 194
7.4.5 The lower part of the orrery 196
7.5 The calendar part (summary) 209
7.6 References 215

8 Hahn's globe clock in Stuttgart (1770) [O:8.5] 219
8.1 Introduction 219
8.2 The clockwork 220
8.3 The orrery 221
8.3.1 The mean motions of the planets 221
8.3.2 The motion of the Moon 223
8.3.3 The anomalies of the planets 224
8.4 The tellurium 224
8.4.1 The motion of the Earth 225
8.4.2 The motion of the Moon 225
8.5 The celestial globe and the lunar globe 226

See at the beginning the rights associated to this document.

8.5.1 The celestial globe . . . 226
8.5.2 The lunar globe . . . 229
8.6 References . . . 230

9 Hahn's orrery in Furtwangen (1774) [O:8.12] 233
9.1 Introduction . . . 233
9.2 The going work . . . 234
9.3 The orrery . . . 234
9.3.1 The first planets and the Moon . . . 234
9.3.2 Jupiter and its satellites . . . 236
9.3.3 Saturn and its satellites . . . 239
9.4 References . . . 244

10 Hahn's astronomical clock in Basel (1775) [O:8.10] 245
10.1 Introduction . . . 245
10.2 The going work and the indication of time . . . 246
10.3 The lunar dial . . . 248
10.4 The solar dial . . . 250
10.5 The calendar . . . 250
10.6 The celestial map . . . 251
10.7 References . . . 251

11 Hahn's globe clock in Aschaffenburg (1776/1777) [O:8.4] 253
11.1 Introduction . . . 253
11.2 The base motion . . . 257
11.3 The orrery . . . 258
11.3.1 The mean motions of the planets . . . 258
11.3.2 The motion of the Moon . . . 260
11.3.3 The anomalies of the planets . . . 261
11.4 The tellurium . . . 262
11.5 The celestial globe . . . 263
11.5.1 The motions of the Moon and the Sun . . . 263
11.5.2 The motions of the planets . . . 266
11.6 References . . . 271

12 Hahn's globe clock in Villingen (1770-1783) [O:8.9] 273
12.1 Introduction . . . 273
12.2 The geographical globe . . . 273
12.3 The celestial globe . . . 275
12.4 References . . . 276

13 Hahn's *Weltmaschine* in Gotha (1780) [O:8.3] 277
13.1 Introduction . . . 277
13.2 The going work . . . 281
13.3 The tellurium . . . 281
13.4 The orrery . . . 284

See at the beginning the rights associated to this document.

13.4.1 The mean motions . . . 284
13.4.2 Taking the excentricities into account . . . 286
13.4.3 The satellites . . . 287
13.5 The celestial globe . . . 292
13.5.1 The motions of the Moon and the Sun . . . 292
13.5.2 The motions of the planets . . . 295
13.6 References . . . 300

14 Hahn's tellurium in Winterthur (1780) [O:8.11] 303
14.1 Introduction . . . 303
14.2 The driving mechanism . . . 305
14.3 The motion of the main support arm . . . 306
14.4 The orientation of Earth's axis . . . 307
14.5 The motion of the lunar nodes . . . 307
14.6 The motion of the Moon . . . 308
14.7 The motion of the Earth around its axis . . . 308
14.8 The rotation of the Sun . . . 309
14.9 Conclusion . . . 309
14.10References . . . 309

15 Hahn's globe clock in Darmstadt (1785) [O:8.6] 311
15.1 Introduction . . . 311
15.2 The going work . . . 311
15.3 The tellurium . . . 313
15.4 The celestial globe . . . 316
15.4.1 The lunar sphere at the top . . . 316
15.4.2 The inner workings of the celestial sphere . . . 317
15.5 References . . . 320

16 Hahn's globe clock in Karlsruhe (1785) [O:8.8] 321
16.1 Introduction . . . 321
16.2 The going work . . . 321
16.3 The geographic sphere . . . 322
16.4 The celestial sphere . . . 323
16.5 References . . . 324

17 Hahn's globe clock in Zurich (1785) [O:8.7] 327
17.1 Introduction . . . 327
17.2 The going work . . . 327
17.3 The Earth globe . . . 335
17.4 The celestial globe . . . 336
17.4.1 The general motion of the globe . . . 336
17.4.2 The motion of the Moon . . . 337
17.4.3 The motion of the Sun . . . 337
17.4.4 The motion of the lunar globe . . . 338
17.5 References . . . 338

See at the beginning the rights associated to this document.

18 Hahn's *Weltmaschine* in Nuremberg (c1770-1790) [O:8.2] 339
18.1 Introduction . . . 339
18.2 The clockwork . . . 345
18.3 The calendar work . . . 347
18.4 The orrery . . . 349
18.4.1 The mean motions of the planets and the Moon . . . 350
18.4.2 The motion of the Moon . . . 352
18.4.3 The eccentric motions of the planets . . . 353
18.5 The celestial globe . . . 354
18.5.1 The input motion . . . 355
18.5.2 The tropical orbital periods of Mercury and Venus . . . 356
18.5.3 The motion of the Sun . . . 356
18.5.4 The lunar nodes . . . 357
18.5.5 The motion of the Moon . . . 358
18.5.6 The corrected motions of Mercury and Venus . . . 358
18.5.7 The motion of Mars . . . 360
18.5.8 Input motions for Jupiter and Saturn . . . 361
18.5.9 The motion of Jupiter . . . 362
18.5.10 The motion of Saturn . . . 364
18.5.11 The motion of Uranus . . . 365
18.6 The satellite systems . . . 366
18.6.1 The tellurium . . . 367
18.6.2 The system of Jupiter . . . 371
18.6.3 The system of Saturn . . . 377
18.6.4 The system of Uranus . . . 381
18.6.5 The possible source of the problems . . . 386
18.7 Hahn's description of the clock . . . 388
18.8 References . . . 404

19 Johann's globe clock in Mainz (1796?) [O:9.1] 407
19.1 Introduction . . . 407
19.2 Description of the clock . . . 410
19.2.1 The going work . . . 410
19.2.2 The two orreries . . . 411
19.2.3 The tellurium . . . 414
19.2.4 The celestial sphere . . . 419
19.3 References . . . 426

20 Klein's geographic clock in Prague (c1732-1737) [O:2.2] 429
20.1 Introduction . . . 429
20.2 Description of the clock . . . 430
20.2.1 The time side of the clock . . . 433
20.2.2 The geographic side of the clock . . . 437
20.3 References . . . 441

See at the beginning the rights associated to this document.

21 Klein's geographic clock in Dresden (1738) [O:2.1] 445
21.1 Introduction . . . 445
21.2 The time side of the clock . . . 450
21.3 The geographic side of the clock . . . 451
21.3.1 The rotating motion of the shell and the Sun . . . 451
21.3.2 The motion of the ecliptic and the oscillation of the shell 451
21.4 References . . . 453

22 Klein's Tychonic clock in Prague (1751) [O:2.3] 457
22.1 Introduction . . . 457
22.2 The going work . . . 459
22.3 The front side of the clock . . . 459
22.4 The Tychonic system . . . 465
22.4.1 The general diurnal motion . . . 465
22.4.2 The motion of the Earth . . . 469
22.4.3 The motions of the planets . . . 471
22.4.4 The motion of the Moon . . . 476
22.4.5 The motion of the ecliptic . . . 476
22.4.6 Summary of the motions . . . 476
22.5 References . . . 477

23 Klein's Copernican clock in Prague (1752) [O:2.4] 479
23.1 Introduction . . . 479
23.2 Copernican system side . . . 481
23.2.1 The motion of the planets around the Sun . . . 481
23.2.2 The motions within the Earth-Moon system . . . 486
23.3 Time side . . . 488
23.4 Conclusion . . . 492
23.5 References . . . 492

24 Neßtfell's *Planetenmaschine* in Vienna (1745-1753) [O:3.1] 495
24.1 Introduction . . . 495
24.2 History and general description of the machine . . . 497
24.3 The going work . . . 504
24.4 The orrery . . . 505
24.4.1 The mean motions and rotations of the planets . . . 506
24.4.2 The motion of the Sun . . . 516
24.4.3 The satellites of Jupiter and Saturn . . . 517
24.4.4 The motion of the Moon . . . 524
24.5 The Earth-Mercury system . . . 528
24.5.1 The mean orbital motion of the Earth . . . 534
24.5.2 The rotation of the Earth around its axis . . . 535
24.5.3 The line of apsides . . . 535
24.5.4 Two more front gears . . . 536
24.5.5 The motion of Mercury . . . 536

See at the beginning the rights associated to this document.

24.5.6 The calendar part . . . 538
24.6 Conclusion . . . 539
24.7 References . . . 539

25 Neßtfell's *Planetenmaschine* in Munich (c1755-1761) [O:3.2] 543
25.1 Introduction . . . 543
25.2 The going work . . . 551
25.3 The Ptolemaic allegories . . . 552
25.4 The orrery . . . 552
25.4.1 The mean motions and rotations of the planets . . . 553
25.4.2 The motion of the Sun . . . 563
25.4.3 The satellites of Jupiter and Saturn . . . 564
25.4.4 The motion of the Moon . . . 567
25.5 The Earth-Mercury system . . . 570
25.5.1 The mean orbital motion of the Earth . . . 570
25.5.2 The rotation of the Earth around its axis . . . 571
25.5.3 The line of apsides . . . 572
25.5.4 A front gear . . . 572
25.5.5 The motion of Mercury . . . 573
25.5.6 The calendar part . . . 575
25.6 Conclusion . . . 576
25.7 References . . . 576

26 Rinderle's clock in Furtwangen (1787) [O:7.1] 579
26.1 Introduction . . . 579
26.2 Oechslin's description . . . 582
26.2.1 The base motions and assumptions . . . 582
26.2.2 The going work . . . 583
26.2.3 The motion of the planisphere . . . 584
26.2.4 The motion of the Moon . . . 584
26.2.5 The motion of the Sun . . . 585
26.2.6 The motion of the lunar nodes . . . 585
26.2.7 The day/night separation . . . 585
26.3 Wenzel's analysis . . . 586
26.3.1 The motion of the Moon . . . 586
26.3.2 The motion of the Sun . . . 587
26.3.3 The motion of the lunar nodes . . . 587
26.4 Summary on the errors and interpretation of the clock . . . 587
26.5 References . . . 588

27 Seige's metal orrery fragment in Prague (c1781?) [O:10.3] 591
27.1 Introduction . . . 591
27.2 The base . . . 592
27.2.1 The going work . . . 592
27.2.2 The outputs of the going work . . . 593

See at the beginning the rights associated to this document.

27.2.3 The base system of the Sun and the Moon 595
27.3 The four corner systems 597
27.3.1 The Sun/Moon daily system 598
27.3.2 The satellites of Jupiter 599
27.3.3 The satellites of Saturn 599
27.3.4 The Sun/Moon annual system 600
27.4 The central orrery 602
27.5 References 604

28 Seige's wood orrery fragment in Prague (c1785?) [O:10.2] 607
28.1 Introduction 607
28.2 The base period 607
28.3 The planets of the upper orrery 608
28.4 References 609

29 Seige's *Weltmaschine* in Prague (1791) [O:10.1] 611
29.1 Introduction 611
29.2 The going work 615
29.3 The four sides of the base 615
29.3.1 The annual dial 615
29.3.2 The daily Earth shadow system 619
29.3.3 The side of the yearly Earth shadow system 621
29.3.4 The side of the Sun/Earth/Moon daily system 634
29.4 The upper part of the machine 647
29.4.1 The Sun/Moon annual system 649
29.4.2 The Sun/Moon daily system 651
29.4.3 The satellites of Jupiter 653
29.4.4 The satellites of Saturn 655
29.4.5 The central orrery 657
29.5 References 660

30 Stuart's clock in Salzburg (1735) [O:1.1] 661
30.1 Introduction 661
30.2 Description and analysis of the clock 663
30.2.1 The going work 666
30.2.2 The three dials and the globe 666
30.3 References 672

Conclusion 675

General bibliography 677

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CONTENTS

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Chapter 1

Introduction

This introduction is a work in progress and is not yet finalized. Newer versions will be put online from time to time.

1.1 The foundation of this work

In 1996, Ludwig Oechslin published a remarkable work on the structure of 28 astronomical clocks or astronomical mechanisms made by "*Priestermechaniker*," that is priests who were also mechanicians, in Germany, Austria and elsewhere [19].[1] A 29th mechanism described by Oechslin is George Adams's orrery. Oechslin studied almost every aspect of these clocks, including their history, and he also disassembled and restored most of them.[2] This was done to a large extent as part of the preparation of the Hahn exhibitions in 1989/1990 [21, 22]. Oechslin counted the teeth of every gear, measured them, drew their layout and computed the gear ratios.

In addition to a general volume synthesizing the examination of all the clocks, Oechslin's work contains detailed drawings of each clock, summaries of gear ratios and calculation of periods, and some photographs.

1.1.1 Oechslin's drawings

Oechslin draws the wheels, arbors and tubes of the clocks all from the side, but only names the arbors and tubes with numbers. The wheels themselves are not named[3] but their teeth numbers are shown. These drawings were probably made with the AutoCAD software, and one can only be impressed by them, as Oechslin managed to fit a lot of information in a very rational way.

[1] On the overlap between theology and technology, see in particular Warnke's essay [23]. He lists all the *Priestermechaniker* mentioned here, except Frater David and Seige.

[2] The exceptions are Hahn's Gotha *Weltmaschine*, and the clocks of Klein and Rinderle, see [19, p. 5].

[3] To be correct, Oechslin gave names to wheels in only three cases, that is for Klein's clocks in Prague, because he added the letters used in Boehm's drawings [2].

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For instance, Oechslin draws the wheels in such a way that one can easily follow their rotation along a train. An example is given in figure 1.1 which corresponds to the system of Jupiter's satellites in Seige's 1791 clock. A number of arbors, tubes and wheels are shown. Usually, wheels that mesh touch each other, but in some cases Oechslin had to separate them in the layout and to use dotted lines to connect them. Moreover, each wheel is drawn as a rectangle with a missing side. Seen from that side, the wheel is assumed to rotate clockwise. In general, meshing wheels are drawn with the missing sides in alternating positions. This convention works for wheels with parallel axes or at other angles. Although Oechslin in general does not draw wheels from the front, it could be done, and we could then for instance add arrows.

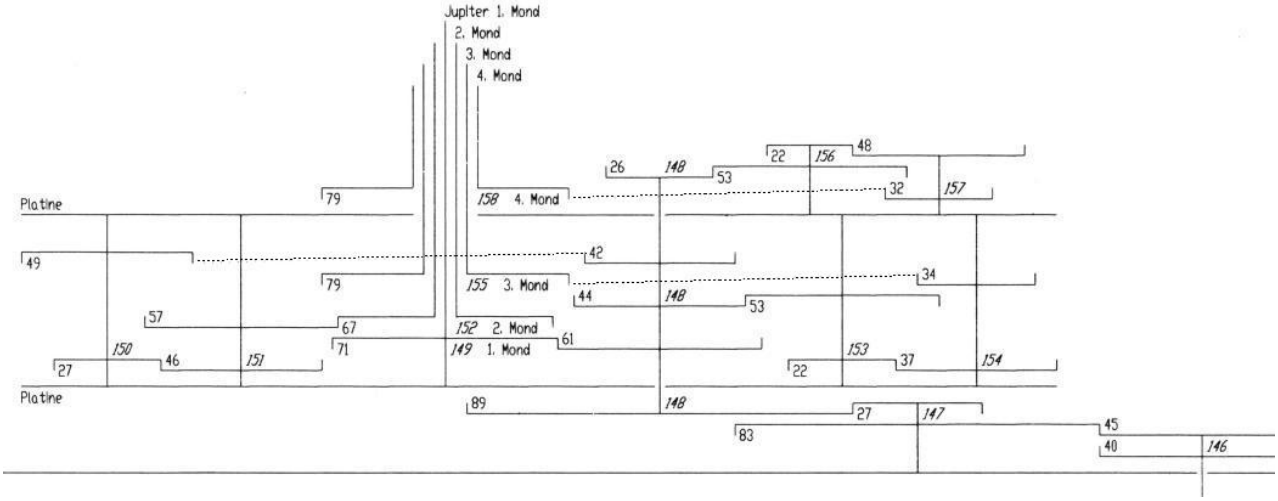


Figure 1.1: Excerpt of Oechslin's drawing for Seige's 1791 machine (Oechslin 10.1).

Oechslin's notation for worms is shown in figure 1.2. The vertical arbor on the right carries a worm (with one thread marked "1w") and this worm meshes with a 34-teeth wheel on arbor 30. Another worm on that arbor meshes with another 30-teeth wheel on the vertical arbor on the left, and so on. The convention for the rotation also applies to the worms.

Oechslin's convention for the wheels does not imply a vantage point, but it does imply that time increases. This is a reasonable assumption for a clock.

Oechslin also names reference frames. There is the absolute still frame, but any part is in fact a reference frame. Usually, a reference frame is some part that will serve as a reference, such as a an arbor, a tube, an arm, a globe, some plate, etc., and it is useful to name these parts, because some computations will make reference to them.

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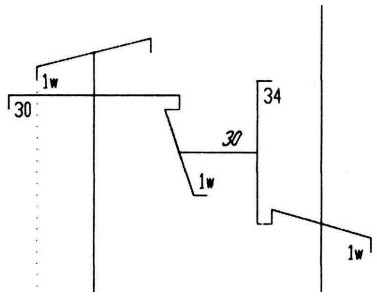


Figure 1.2: Excerpt of Oechslin's drawing for Neßtfell's machine (Oechslin 3.1).

1.1.2 Oechslin's notations

An important task when studying and describing a clock is to compute the motions of its parts. Different wheels rotate at different speeds.

Oechslin's idea is to count rotations, or more exactly ratios of rotations. For instance, for



Oechslin would consider that when arbor 151 makes a turn, arbor 150 makes $-\frac{46}{27}$ turns. This is the basic relation of two meshing wheels. In that case, Oechslin provides a negative ratio. His idea is that the two wheels are seen from the same direction, and when one rotates clockwise, the other one rotates counterclockwise.[4] Of course, this is not always true, depending how the wheels mesh, and also depending on the vantage point. For instance, the motion of arbor 151 (which turns clockwise from above) could be measured from above, and the motion of arbor 150 (which turns clockwise from below) could be measured from below. In that case, the ratio between the two motions should be positive, and not negative.

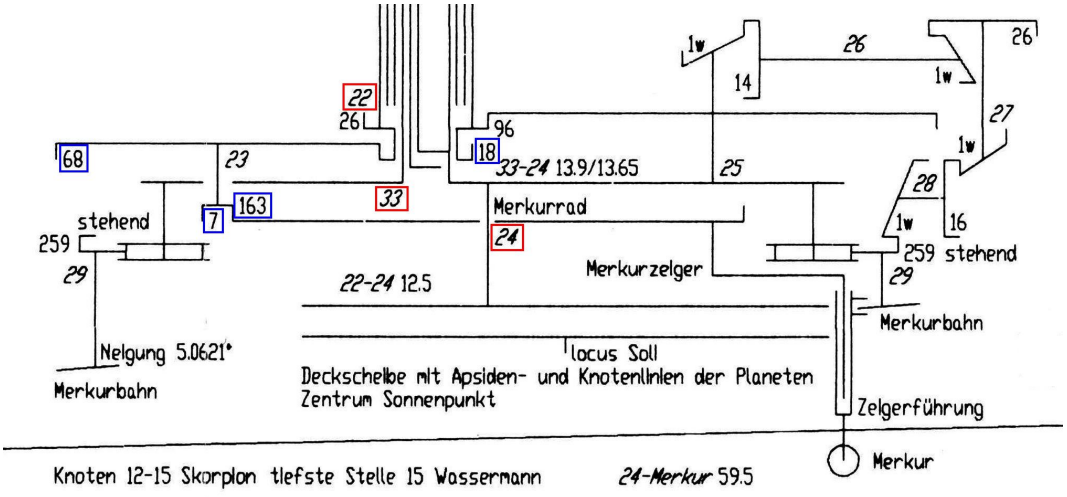
Most of the time, though, the arbors are vertical and one can agree on a simple viewpoint, such as from above. In some particular cases, though, we need to specify the vantage point.

Oechslin's notation for the above is U^c_{a-b} (U for *Umdrehungen*, or rotations) for the number of rotations of arbor a with respect to arbor b in the reference frame c. In other words, this expression gives the number of rotations of a when b makes one turn in the reference frame c. In the simple case above, if the reference frame is 0, we have $U^0_{150-151} = -\frac{46}{27}$. In figure 1.3, U^{33}_{22-24} is the number of turns of tube 22 when 24 makes one turn, with respect to frame

[4]This is not true if one wheel meshes inside another. In that case, Oechslin gave negative numbers to the wheel with interior teeth. This is a possibility, but it is practical only for parallel axes and if the point of view remains the same. Usually, this is the case. But if the point of view changes, the signs cannot be used blindly. In any case, every time there was an interior gearing, I have used a positive ratio and not a negative one.

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33. In the figure, the reference frames are marked in red (and italics) and the teeth numbers in blue.



$U_{13-15} = U_{13-20}\,U_{20-15}$			-0.997270	23h 56m 04.13, synodische zu siderischer Erdrotation, Sterntag
U^{20}_{15-22}	$\frac{179}{32}\,\frac{32}{170}$	1		1d, Verteiler
U^{33}_{22-24}	$\frac{68}{18}\,\frac{163}{7}$		87.968254	87d 23h 14m 17s, Merkur auf 33
U^{33}_{22-29}	$\frac{96}{26\pm1}\,\frac{14}{\pm1}\,\frac{26}{\pm1}\,\frac{16}{\pm1}\,\frac{259}{\pm1}$		5569536.000000	5569536d, 15248.8780a, Merkur Knoten auf 33

Figure 1.3: Excerpt of Oechslin's drawing (above) and calculations (below) for Neßtfell's machine (Oechslin 3.1). (Note that the expression U^{20}_{15-22} has a typo.)

1.1.3 Oechslin's photographs

Oechslin's book contains some photographs, but their small number is rather disappointing. Moreover, all the photographs, except those on the dust covers, are in black and white. This is probably explained by the need to reduce the publisher's expenses. However, there are certainly many more photographs of each of the clocks. In fact, when Oechslin worked on Bernardo Facini's clock located in the Vatican museums in 1982 [16], and perhaps already before, he developed the habit of carefully documenting the clocks, taking photographs of each part on graphpaper, taking measurements, etc. I know of no other horologist who does the same thing, but all horologists should actually do so. Only the restorers at the MIH (*Musée International d'Horlogerie*) seem to perpetuate this tradition, even to this day.

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1.2 A companion work

1.2.1 The motivation of my work

Unfortunately, although I consider Oechslin's work exceptional, I believe that it hasn't received the attention it deserves.

This is due to the very detailed nature of the work, to its small run (550 copies), to its cost, to its language (German), to its black and white iconography, to the small print of the plates, but also to the way the gear computations have been presented. Since the 1980s, Oechslin has been using his own notation illustrated above, which he masters, but which is almost a cypher for others.

Oechslin's notation is in fact twofold. There are drawing conventions, and Oechslin came up with a convention for side drawings that shows on which side of a wheel this wheel moves clockwise (in normal conditions). This is a very interesting innovation, and although it should also be supplemented by face drawings, I don't have something better to offer at the moment. In Oechslin's book, the gear trains are only shown from the side, at least on the plates. The worms could perhaps have be drawn differently, without sacrificing Oechslin's principles. In a number of cases, an additional view from the front (or from above) would also have been useful for the understanding. More photographs of details — and nowadays partial 3D models — would also have been useful for the understanding. Another possible improvement to Oechslin's drawings is the need to better distinguish the various tubes which are one in each other in many orreries. It is often difficult to follow the lines visually, and it would be better to use at least two different types of lines.

The convention for side drawings of the trains must have been elaborated around 1985-1989, because Oechslin doesn't use it in his work on Facini's clock [17], but he does in his brochure on Hahn's trains [18].[5] In fact, in 1985, Oechslin still only used front views of the trains, although he had already come up with his notations for ratios.

The other part of Oechslin's notation is that for the calculation of ratios, and these notations are very cryptic. He seems to have first used them in print in 1985 [17]. They often involve three indices such as U^c_{a-b}, which is further complicating the understanding. Moreover, Oechslin's periods are often negative (for example for Hahn's *Weltmaschine* in Nuremberg), because he measured the motions positively clockwise, when in fact most astronomical motions are counterclockwise.

As a testimony to how difficult Oechlin's book is, I want to mention that I own a (signed) copy of Oechslin's book since 2003, and I have also studied many clocks, some of them astronomical, but I have only started to go through Oechslin's computations in 2024, in part because the book seemed so daunting. It seemed such a precious book, the plates were so difficult to manipulate, the

[5] However, in that brochure, Oechslin numbered the wheels and not the arbors, but this is equivalent, since an arbor corresponds to a wheel. The only difference is that several numbers can then correspond to a same arbor.

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notations were so terse, that I never really made any use of Oechslin's work. For 20 years I have even kept Oechslin's book in its original packaging!

In fact, Oechslin seems to be the only one to use his notations. I am not aware of another work seriously making use of them.[6] And he seems to be faithful to his notations. For instance Ducommun's orrery was described by Oechslin in 2018 [20] with the same notations as in his 1996 book.

As mentioned above, Oechslin numbers arbors and frames, and not wheels. This could perhaps be improved, because it is often difficult to locate the arbor numbers in a complex drawing. The wheels could be named with letters, for instance and these numbers used instead of the arbors. On the other hand, I have been able to analyze all the clocks, even though I never had names for the wheels. Sometimes, however, when I needed to speak of this or that wheel, I had to write things like "the 72-teeth wheel on arbor such and such." Next to the wheels, the number of teeth are also given. Oechslin distinguishes the arbors by writing their number in italics. This is not always clear, a confusion with the teeth numbers may arise. More than once, I have mistakenly used an arbor number instead of the teeth number, and this is a good enough reason to banish numbers for arbors. Perhaps some additional marker should have been used, perhaps a line over the arbor number, or a bracket. But the best solution would probably be not to use numbers.

1.2.2 New notations

I had been considering the use of new notations for the description of gear trains around 2003 when I first saw the Türler clock in Zurich and Oechslin's 1985 book [17] which describes his notations, but I never pursued this project. In 2024, when I was studying Ducommun's orrery at the MIH in La Chaux-de-Fonds [20], I finally began to work out new notations and to see if it was possible to improve upon those of Oechslin.

In the meantime, I have often been struck by bad notations, cryptic notations that only horologists are accustomed to, or notations that are too language-dependent. For instance, Oechslin uses U for rotations (*Umdrehungen* in German), and other people sometimes use Z for teeth numbers (*Zahl* in German). It is true that some international standards such as ISO 701 define z as the number of teeth, but it is ridiculous to use such a notation when only teeth numbers are used, and not more advanced gear parameters. Moreover, some people use the z notation although they are not forced to do so. In the current work, the only gear parameter I use is the number of teeth. I am not

[6] The only short use of Oechslin's graphical representation of train I know of is in Marini's *Imago Cosmi* [14, p. 411]. Marini gives a figure for a tellurium using an *approximation* of Oechslin's conventions, but without respecting the rules for the rotation of the gears, which seems to indicate that Marini didn't understand Oechslin's conventions. In his book, Marini has also used some of the numerical periods given by Oechslin, but without actually describing how the clocks are working, and comparing the values with modern values, which is meaningless.

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even using the radius. So, if I had to name the number of teeth of a wheel, there would be no ambiguity to use something like N, perhaps subscripted by the wheel number. I encourage everyone who describes clocks with only radius and teeth numbers to use only R and N, at least in English, and not some unnecessary complicated notation, even if some standard prescribes it. Of course, if you are writing in German, using Z might be a good idea. I am not opposing the standard, but some people seem to believe that standards apply everywhere.

Now, regarding the computation of gear ratios, I believe that it is simpler to manipulate the velocity in a given frame of reference, and to make the units implicit, for instance the number of turns per day. In 2024, I have been able to redo all of Oechslin's computations for Ducommun's orrery, and this was an encouraging step. Shortly after my study of Ducommun's orrery, I had another look at Oechslin's 1996 book.

It then occurred to me that it is possible and useful to go again through all the computations first in order to check them (which had probably not yet been done for the 29 shown mechanisms), and then in order to present them in a more intelligible way, and also to make them accessible to a wider audience. This work has now been completed, and I believe that it demonstrates the feasibility of my new approach. It is in fact a lot simpler to use velocities instead of revolutions.[7] There are less indices, it is less prone to errors, and there are basically no special formulas to remember. It is also simpler to measure velocities in astronomical clocks positively counterclockwise, which is what I have done.

My notation (which is not entirely new) is the following: V_a^b is the velocity of arbor a with respect to frame b, measured from some conventional vantage point, and this velocity will always be measured in turns per day, a convenient unit in astronomical clocks. Moreover, the positive direction will be the mathematical "direct" one, which is counterclockwise. Almost all astronomical motions are counterclockwise, and this seems to be the best choice, to avoid carrying unnecessary negative signs.

It should be noted that the correspondence between a drawing and the computation of a ratio depends on the point of view. Oechslin's drawings do not specify the vantage point for the computation, only the vantage point corresponding to a clockwise motion. In general, unless otherwise mentioned, I am measuring the velocities on Oechslin's drawings from the top and from the right.

I will almost always only use simple relations such as

$$\mathrm{V}_a^b = -\mathrm{V}_b^a \tag{1.1}$$

[7] I am actually not the first one to use velocities. For instance, in 1993 Oestmann did use them in his book on the Strasbourg astronomical clock, but his notations are cumbersome and the derivations lack clarity. And Oechslin also defined it, but without really using it.

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and

$$\mathrm{V}_a^c = \mathrm{V}_a^b + \mathrm{V}_b^c \qquad \text{(Chasles' relation)} \tag{1.2}$$

Frame 0 is always the absolute frame and I have decided to make it always explicit, even if no frame moves. Making frame 0 implicit simplifies the notations, but breaks the symmetry of Chasles' relation.

Although the V notation is sufficient, a number of dials are time-related, and the hands have the usual clockwise motion. In order to avoid using negative values in this case, I am sometimes using a different velocity symbol for such indications. V will stand for an astronomical velocity, and T for the "time" motion.

Incidentally, in a few isolated cases, Oechslin uses notations such as U_{16}^{22} to denote the velocity of arbor 16 with respect to frame 22, and this is basically the same notation as mine, but Oechslin doesn't use it extensively.

1.2.2.1 Oechslin's notation redefined

Oechslin's notation U_{b-a}^c can be defined in terms of velocities. The number of turns of arbor b when arbor a makes a turn is the ratio of the velocities of b and a, or

$$U_{b-a}^c = \frac{\mathrm{V}_b^c}{\mathrm{V}_a^c} \tag{1.3}$$

When I first worked through Oechslin's calculations for Ducommun's orrery, I made a modified version of Oechslin's notation, and instead of U_{b-a}^c, I used $\mathrm{O}_{a\to b}^c$, where O stands for Oechslin, and where the arrow highlights the value represented, namely a number of turns of arbor b.

Within the same reference frame R, we have relations such as

$$U_{a-b}^R = U_{c-b}^R \times U_{a-c}^R \tag{1.4}$$

$$U_{a-b}^R = \frac{1}{U_{b-a}^R} \tag{1.5}$$

but the first relation would have been much clearer if Oechslin had used the modified notation:

$$\mathrm{O}_{b\to a}^R = \mathrm{O}_{b\to c}^R \times \mathrm{O}_{c\to a}^R \tag{1.6}$$

because it is similar to Chasles' relation for vectors.

We can also change reference frames. With the modified notation, we have for instance

$$\mathrm{O}_{b\to a}^c + \mathrm{O}_{c\to a}^b = \frac{\mathrm{V}_a^c}{\mathrm{V}_b^c} + \frac{\mathrm{V}_a^b}{\mathrm{V}_c^b} = \frac{\mathrm{V}_a^c}{\mathrm{V}_b^c} - \frac{\mathrm{V}_a^b}{\mathrm{V}_b^c} \tag{1.7}$$

$$= \frac{\mathrm{V}_a^c - \mathrm{V}_a^b}{\mathrm{V}_b^c} = \frac{\mathrm{V}_a^c + \mathrm{V}_b^a}{\mathrm{V}_b^c} = \frac{\mathrm{V}_b^c}{\mathrm{V}_b^c} = 1 \tag{1.8}$$

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and therefore

$$\mathrm{O}^c_{b\to a} = 1 - \mathrm{O}^b_{c\to a} \tag{1.9}$$

which is Oechslin's

$$U^c_{a-b} = 1 - U^b_{a-c} \tag{1.10}$$

From this equation, we can derive another one:

$$\begin{aligned}
\mathrm{O}^c_{b\to a} &= 1 - \mathrm{O}^b_{c\to a} \\
&= 1 - \mathrm{O}^b_{c\to d} \times \mathrm{O}^b_{d\to a} \\
&= 1 - \mathrm{O}^b_{d\to a} + \left(1 - \mathrm{O}^b_{c\to d}\right) \times \mathrm{O}^b_{d\to a} \\
&= 1 - \mathrm{O}^b_{d\to a} + \mathrm{O}^c_{b\to d} \times \mathrm{O}^b_{d\to a}
\end{aligned} \tag{1.11}$$

which is Oechslin's

$$U^c_{a-b} = 1 - U^b_{a-d} + U^c_{d-b} \times U^b_{a-d} \tag{1.12}$$

The latter is often used by Oechslin in his computations, but it is obviously a complex formula and using it is error-prone. It is best to avoid using it.[8]

1.2.2.2 The use of the new notation

In my opinion, using velocities instead of Oechslin's notation is simpler and more intuitive. The notation itself is simpler, as we always assume the same time basis. If we obtain a period in days, we can always convert it in years later. The numerical results are entirely equivalent.

Moreover, whereas Oechslin *multiplies* numbers of turns, we only have to *add* velocities.

[8] In the small guide he wrote for the 1989 Hahn exhibition [18], Oechslin considers a number of gear trains with increasing complexity. These trains can all be analyzed with the velocity approach, without having to memorize special formulas. I am drawing the attention of the reader to the fact that some of Oechslin's drawings in this brochure are ambigous. For instance, on page 9, one should stress that the wheel 2 is fixed. On page 10, the wheel 4 is fixed and not tied to wheel 1, contrary to the appearances. In this drawing, the wheels 2 and 3 are tied together. On page 11, the wheels 1 and 4 have separate motions, but the wheels 2 and 3 are tied together. The wheels 4 and 8 are also tied together. Once these observations are made, the computations of the velocities are straightforward. Oechslin gave general formulas for each case he described, and these formulas could easily be derived from a velocity analysis. There is however little point using these general formulas, as they lead to a very abstract perspective.

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1.3 Types of trains and velocities

This section enumerates the various types of trains and shows how the velocities are computed, using actual examples from the mechanisms described in this book. The velocities are always relative to a frame. This frame will very often be the absolute frame, which is that of the environment of the clock or mechanism. But it can also be that of a moving part of the mechanism.

I will first consider simple trains, with an input and an output, and the train may itself be put in motion from the outside. I will then look at frames whose motion is derived from the input and the output, this time taken as input. This will bring us to the topic of complex ratios and how they were obtained, in particular by Frater David.[9] Finally, I will consider the case of cranks on epicyclic wheels which play an important role for the uneven motions in the orreries and even more in the geocentric globes.

1.3.1 Simple trains

In this section, I am looking at a train from the point of view of its input, or globally. We can thus consider either a fixed train with an input, or a moving train with two inputs, the usual one, and that causing the motion of the entire frame.

I will not consider extensively how trains have been designed, as this is outside the scope of this book. For more on this topic, see Oechslin [19, v.1, p. 80-99], in particular regarding Hahn's methods and the sources in Wolff's *Anfangs-Gründe aller mathematischen Wissenschaften* [24] and Hahn's second *Werkstattbuch* [13]. Finding appropriate ratios sometimes took days to Hahn [19, v.1, p. 87].

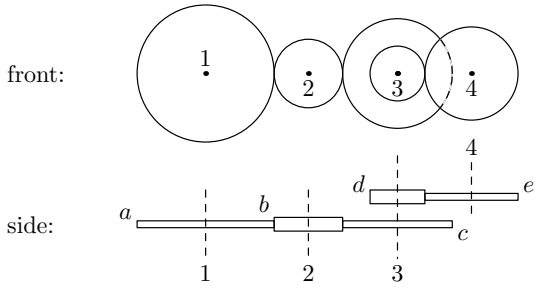
1.3.1.1 The case of a fixed train in a frame

The simplest trains are those with arbors on fixed axes. Each arbor carries one or more wheels and each wheel meshes with a wheel on another arbor. There is usually a first and a last wheel, but we could actually have several "output" wheels.

An example is given below. The wheels are in a frame F. The frame itself may be in motion, but this will not concern us here, as all the velocities will be relative to F.

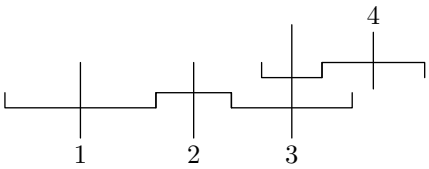
[9] On the use of this name, when others call him Cajetano, see section 1.4.3.

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The arbors are numbered 1, 2, 3, and 4, as Oechslin does. The teeth counts are a, b, c, d, e.

Oechslin never shows the wheels from the front. Instead, he might have presented the previous example as



This assumes that the arbor 1 rotates clockwise as seen from above.

Arbor 1 turns at a certain speed V_1^F (number of turns/unit of time). This is an angular speed.

The ratio of speeds are merely equal to the inverse ratio of teeth counts.

(*) $\dfrac{\mathrm{V}_2^F}{\mathrm{V}_1^F} = \dfrac{a}{b}$: when arbor 1 makes one turn, arbor 2 makes $\frac{a}{b}$ turns;

(**) We can also write $\mathrm{V}_2^F = \mathrm{V}_1^F \times \dfrac{a}{b}$

(***) Or $\mathrm{V}_4^F = \mathrm{V}_1^F \times \dfrac{a}{b} \times \dfrac{b}{c} \times \dfrac{d}{e}$

When there is no ambiguity, the reference frame need not be given.

Since we also want to distinguish the direction of rotation, we introduce signs. We choose some *conventional* positive direction. Often it will be positive counterclockwise, because almost all astronomical motions are counterclockwise in the solar system.

When two wheels mesh, the direction changes. So, instead of (*), (**), and (***), we will write

$$\frac{\mathrm{V}_2^F}{\mathrm{V}_1^F} = -\frac{a}{b} \tag{1.13}$$

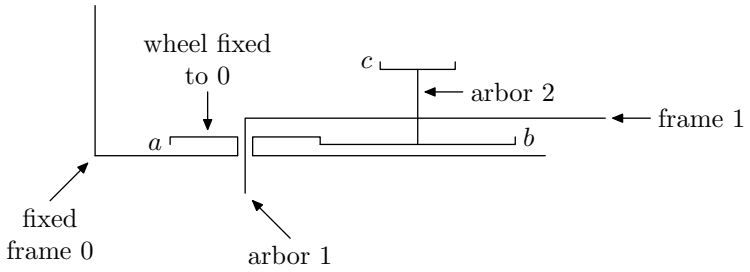
$$\mathrm{V}_2^F = \mathrm{V}_1^F \times \left(-\frac{a}{b}\right) \tag{1.14}$$

$$\mathrm{V}_4^F = \mathrm{V}_1^F \times \left(-\frac{a}{b}\right) \times \left(-\frac{b}{c}\right) \times \left(-\frac{d}{e}\right) \tag{1.15}$$

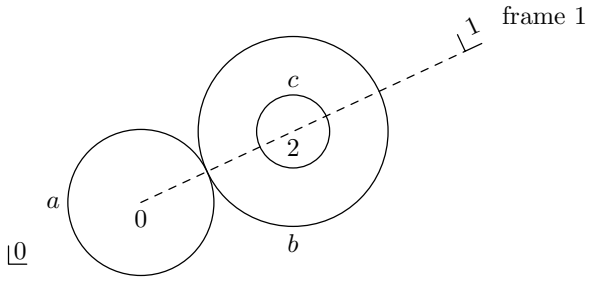
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1.3.1.2 The case of a train on a frame which is in motion with respect to another frame

When one frame rotates around the other one, we have different speeds for a same wheel. For instance, consider this from Hahn's globe in the Nuremberg *Weltmaschine* (Oechslin 8.2):



Arbor 2 carries the two wheels b and c and is pivoting on frame 1. Frame 1 moves with arbor 1. When the arbor 1 rotates, it also rotates frame 1 and causes arbor 2 to turn, because of the fixed wheel on frame 0. From above, we have:



Notice here that the two axes are numbered 0 and 2, because the wheel a is on frame 0, and the two other wheels are on arbor 2.

If we assume that we are in frame 1, we have as in case 1:

$$\mathrm{V}_2^1 = \mathrm{V}_0^1 \times \left(-\frac{a}{b}\right) \tag{1.16}$$

In frame 0, the velocity of the frame 1 is V_1^0.

The angle by which arbor 2 has rotated with respect to frame 0 is in fact the sum of the angle of frame 1 with respect to frame 0 and of arbor 2 with respect to frame 1. Consequently, the velocities add up: $\mathrm{V}_2^0 = \mathrm{V}_2^1 + \mathrm{V}_1^0$.

These simple formulas suffice for all the mechanisms in this book. There is no need to remember anything more complex.

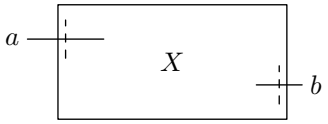
1.3.1.3 The more general case of the moving frame

In the previous example, the motion of arbor 1 (or frame 1) causes the motion of the arbor 2. More exactly, the motion within frame 1 is due to the motion of frame 1 with respect to frame 0, and to the relative motion of the wheel fixed to frame 0. That wheel might also be moving.

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1.3.1.4 Generalization of the previous cases

It is easy to find many different cases of trains, and to derive formulas for each of these trains, but we should step back and try to find common patterns. In fact, as well as the basic (but not sole!) connection between two wheels is to have them meshing at a constant distance, the basic building block of a train is a series of meshing wheels on fixed arbors:



Such a train is represented as a box X. It has an input wheel a and an output wheel b, whose axes (dashed lines) are fixed within X. These axes may be parallel (as shown) or not, but when a turns, so does b. How the wheel b turns as a function of a depends on what is inside the box X. With a simple gear train, the motion of b is proportional to the motion of a.

With this simple train, we can have different configurations with b as the output:

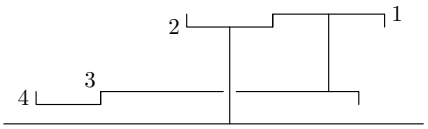
- X can be fixed and a turns;
- a can be fixed and X may turn;
- both a and X can turn.

Moreover, X can be mobile in translation or in rotation. The wheel a could for instant mesh with a rack.

The output wheel b may or may not mesh with another wheel. b could for instance only carry a hand or some other indicator.

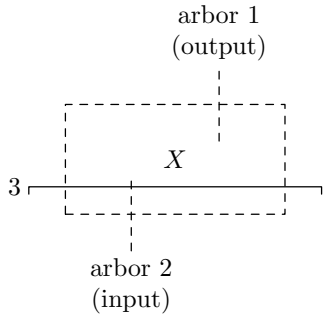
However, if X rotates around a fixed point, it is necessary to have at least one input (a or b) mesh with a wheel with a fixed axis.

The two cases shown earlier are merely particular cases of this general case. For instance, the third case of Oechslin in his classification of gears [18] is the following:



Here the arbor 1 is at a fixed position on the wheel 3 which rotates. We can consider that the wheel 3 is a fixed element of the box X, and that the input is arbor 2 and the output is arbor 1:

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The box X rotates around the same axis as arbor 2, but this is a particular case. In general, X may rotate around a different axis as the input.

Oechslin's cases 4 and 5 can also be reinterpreted in terms of the train-box. Oechslin's case 5 is merely that where both X and the input are mobile. It doesn't seem necessary to call such a construction a "differo-epicyclic train."

1.3.2 Moving frames using the input and the output

Usually, either the box X is fixed and the input wheel a is moving, or both are moving, producing some motion for wheel b.

It is however also possible to have a and b as inputs, and this will cause the frame X to move. It is not a different construction, it is just that the inputs are not the same as usual.

Such constructions can be used to obtain motions which are difficult to obtain using the above methods. They have been used by Neßtfell in his two planetary machines (Oechslin 3.1 and 3.2) and by Frater David in his clock in the Palais Schwarzenberg (Oechslin 6.2), albeit not for the same reasons.

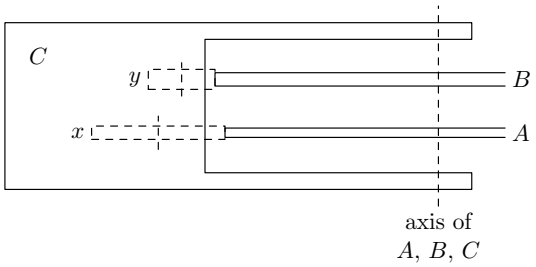
1.3.2.1 General principles

Of course, there is nothing really magic here, we merely have a function of two parameters $z = f(w, r)$, where w is the motion of some wheel, r the motion of some frame and z the motion of some output (which, admittedly, needs to be carefully defined). With certains trains, there exists a function g such that $r = g(z, w)$, in other words the motion of the frame r is determined by the motion of w and z.

However, in order to make this well-defined, we need to clarify what is z. In the example above, with arbor 2, z might be thought of as both the relative motion of arbor 2 with respect to frame 1, and the absolute position of the arbor 2. It is simplest to combine these two by introducing a new wheel having the same axis as the rotation axis of the frame and meshing with the output. Likewise, we introduce a new wheel having the same axis as the rotation axis of the frame and meshing with the input. This will define the two input motions, and the output motion which is that of the frame.

The most general case is thus illustrated here:

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C is the rotating frame and the wheels A and B are the input motions. They all rotate around the same axis. The motions can be provided through, say, a central arbor for A and a tube for B, and the output can likewise be transferred through a tube fixed on C. It is thus possible to see this entire construction as a box having two inputs, A and B, and one output C. Such a box can be reused in other contexts.

It should also be noted that A and B may have interior gearings (as is in Neßtfell's constructions), but they are shown here with normal outside teeths. The two wheels x and y are on fixed axes on C, but usually not on the same one. The velocity of y with respect to C is a function of the velocity of x. In general, we have a linear rational function:

$$\mathrm{V}_y^C = \frac{p}{q} \times \mathrm{V}_x^C \text{ where } p, q \in \mathbb{Z}^* \tag{1.17}$$

We define $\alpha = \frac{n_A}{n_x}$ and $\beta = \frac{n_B}{n_y}$ where n_A, etc., are the number of teeth of A, etc. Some of these numbers may be negative in case of interior gearings. We also set $\gamma = \alpha/\beta$.

Let $a = \mathrm{V}_A^0$, $b = \mathrm{V}_B^0$ and $c = \mathrm{V}_C^0$ be the absolute velocities of A, B and C. Computing the motion of C is then straightforward. In the reference frame of C, we have

$$\mathrm{V}_B^C = (\mathrm{V}_A^C \times \alpha) \times \frac{p}{q} \times \frac{1}{\beta} = \mathrm{V}_A^C \times \frac{p}{q} \times \gamma \tag{1.18}$$

$$\mathrm{V}_B^C = \mathrm{V}_B^0 - \mathrm{V}_C^0 = b - c \tag{1.19}$$

$$\mathrm{V}_A^C = \mathrm{V}_A^0 - \mathrm{V}_C^0 = a - c \tag{1.20}$$

Therefore

$$b - c = (a - c) \times \frac{p}{q}\gamma \tag{1.21}$$

and

$$\frac{p}{q}\gamma = \frac{b-c}{a-c} = \frac{b/a - c/a}{1 - c/a} \tag{1.22}$$

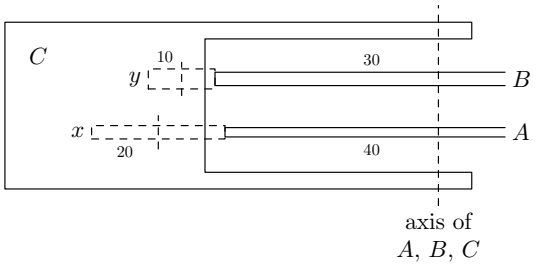
We also have

$$c = \frac{b - a\frac{p}{q}\gamma}{1 - \frac{p}{q}\gamma} \tag{1.23}$$

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Given the details of the train, we can compute the velocity of C. Incidentally, it is clear that this construction can be used to add two velocities (take $\frac{p}{q}\gamma = -1$ and multiply by 2) and in fact to obtain any linear combination of two velocities.

As an application, consider for instance the train



with $p/q = 25/17$, $a = 1$ and $b = 3$. We obtain $c = 103$.

Conversely, with $a = 1$, $b = 3$ and $c = 103$, we find

$$\frac{p}{q}\gamma = \frac{3-103}{1-103} = \frac{50}{51} \tag{1.24}$$

and we can choose for instance $p/q = 25/17$ and $\gamma = 2/3$. Other solutions are possible.

In general, given a value of $r = c/a$, we can choose b/a such that $\frac{b-c}{a-c}$ only contains small enough prime factors, in which case we can easily design the train within C (ratio p/q) and choose the ratios for x and y. But one should not take any random value for a and b. For instance, in the previous example, with $a = 2$ and $b = 6$, we still end up with primes 97 and 101.

1.3.2.2 The reduction of primes

If we start with an input motion of (say) one turn per day, can we obtain any rational output period using only small primes? For instance, can we get a wheel making a turn in 991/613 days, where both numerator and denominator are prime, using, say, only wheels with less than 50 teeth? An answer to this question may not seem obvious at first.

This question was certainly considered by many authors. Oechslin cites for instance the case of Hahn who gave an (incorrect) train to obtain a period of 179 days (179 is prime), without using a 179-teeth wheel.[10]

1.3.2.2.1 A theoretical answer

It is however easy to give a theoretical answer. We can first observe that if we can get a motion in 991 days and another one in 613 days, these two motions can be combined to obtain the fraction 991/613. This follows from

[10] See [19, p. 92-95].

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the fact that a train corresponding to a multiplication by 613 also corresponds to a division by 613 when reversed, that is when we exchange the input and the output. Chaining a train for 991 and another one for 1/613, produces a train for the ratio 991/613. This is certainly not the best way to obtain this ratio, but it is a theoretical construction. It shows that in a theoretical approach, we only need to consider integer velocities. In the above, we can also assume $a = 1$ and $b, c \in \mathbb{N}$. In that case, we have

$$\frac{p}{q}\gamma = \frac{b-c}{1-c} \tag{1.25}$$

If c is a prime number, taking $b > 1$ always results in a ratio with smaller prime factors. For instance, if $c = 991$ and $b = 2$, we have

$$\frac{b-c}{1-c} = \frac{991-2}{990} = \frac{23 \times 43}{2 \times 3^2 \times 5 \times 11} \tag{1.26}$$

Even if we take $c = 3$ and $b = 2$, we have

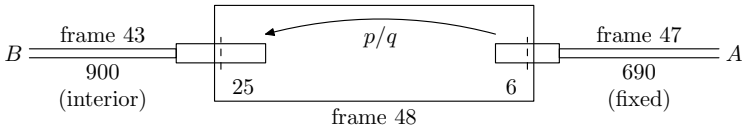
$$\frac{b-c}{1-c} = \frac{1}{2} \tag{1.27}$$

and this shows that in fact we can even get rid of the factor 3. It is possible to implement any ratio using only the prime factor 2. We could build an astronomical clock with whatever complex motion using only wheels with say 16 and 32 teeth. This wouldn't be efficient, but it would be possible.

1.3.2.2.2 Neßtfell's construction

Neßtfell and Frater David used configurations such as the above, but their aims were not the same. I will first consider Neßtfell's constructions which he implemented in his two planetary machines starting in 1753 (Oechslin 3.1 and 3.2).

Here is for instance his configuration for the revolution of Saturn:



Saturn is held by a carriage (frame 48) which is stuck between a fixed wheel A and a moving interior wheel B. The 900-teeth wheel on frame 43 rotates with the velocity $\mathrm{V}_{43}^{0} = \frac{25}{897}$ and meshes with a 25-teeth pinion in the carriage. A 6-teeth pinion of the carriage meshes with the fixed 690-teeth wheel (frame 47). Finally, the velocity of the 25-teeth pinion is $p/q = 11227/120$ times that of the 6-leaves pinion. This ratio only involves primes up to 109.

With this configuration, we have $a = 0$, $b = 25/897$, $\gamma = -\frac{115}{36}$ and

$$c = \frac{b - a\frac{p}{q}\gamma}{1 - \frac{p}{q}\gamma} = \frac{25/897}{1 + 11227/120 \times 115/36} = \frac{1440}{15493283} \tag{1.28}$$

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and the period is

$$\mathrm{P}_{48}^{0} = \frac{15493283}{1440} = 10759.2243\ldots \text{ days} \tag{1.29}$$

Now, although the numerator 15493283 contains the large prime 51817, the above construction led to this period without using a wheel with that many teeth.

In the light of the later work of Frater David, it is tempting to consider that Neßtfell also tried to reach exact ratios with epicyclic gears. But this assumption is difficult to defend, first because the periods implemented by Neßtfell are not exactly those that he gave in his description of the machines [15], and second because the implemented ratios do not derive from simple periods in seconds or other units. For instance, for Jupiter, Neßtfell produces a revolution in 1070139/247 days, and this value does not have a simple expression in seconds. This applies to all the planets, except Saturn.

Oechslin did no try to analyze Neßtfell's methods, but I believe that the epicyclic constructions used by Neßtfell do have another purpose. His aim was clearly to have the planets move along rails, and for this to work, Neßtfell decided to use large gears, two of which are moving, and the third one being fixed. For instance, for Saturn, the fixed wheel has 690 teeth and the moving wheels have 897 and 900 teeth. Neßtfell clearly understood that he could set the planets on moving carriages located between these large gears.

Neßtfell obviously computed the motions of the planets, but his purpose was not to obtain the exact values given in his description of the clock. Perhaps he would have done so if he knew that he could reach these values, but I believe this was not clear to him.

I rather think that Neßtfell arranged each train so that the revolution period of the planet was sufficiently close to the one he gave in his description, and so that the train obtained could be constructed. However, in no case has he obtained exactly the value that he thought was the most accurate.

1.3.2.2.3 Frater David's method

In his clock in the Palais Schwarzenberg (Oechslin 6.2) constructed around 1793, Frater David used a more systematic method, always expressing the periods sought in integer numbers of seconds, minutes, hours or days. He explained his technique in his books [4, 5, 6] and a series of articles published shortly before his death [7, 8, 9, 10, 11]. This technique was summarized by Oechslin [19, p. 95-99, 153-154]. Frater David may have been inspired by Neßtfell's planetary machine in Vienna, which he restored at about the same time,[11] but his purpose was different. Whereas Neßtfell certainly used epicyclic

[11] See [19, p. 227-228]. This connection was not put forward by Bertele when he wrote about this clock, and he still viewed the origin of epicyclic gears in Frater David's clock as a mystery we would never be able to elucidate [1, p. 24].

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gears because the planets were located on moving carriages, Frater David used epicyclic gears for the specific purpose of getting some definite ratios.

Moreover, Frater David's also wanted to use only wheels with less than 100 teeth, but as Oechslin observed [19, p. 176], this was not that efficient, because it came at the expense of a greater number of wheels and pinions.

The figure 1.4 shows the arrangement of Frater David's gears. A central arbor carries a fixed wheel B, a moving wheel C and a rotating frame A. This frame carries three wheels, two which are also named A, and one idle wheel W (for *Wechselrad* in German).

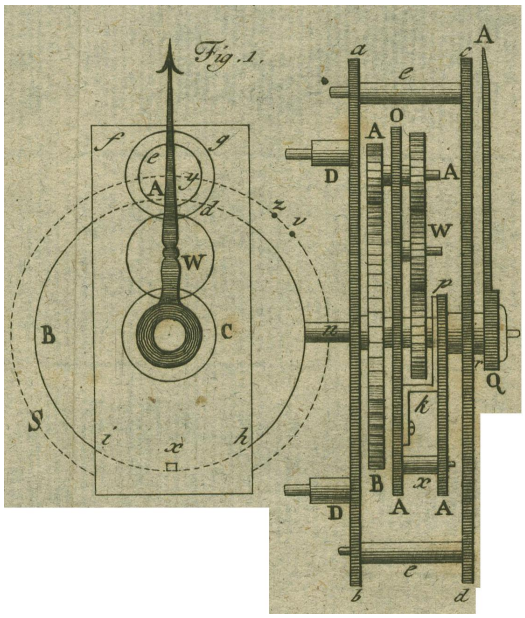


Figure 1.4: Frater David's epicyclic box as illustrated in 1791 [4]. The figure was redrawn in the 1793 edition, but the structure is similar [5].

Although the notations are a bit clumsy, the idea is to have an input motion on wheel C, and that this input motion causes the frame A to rotate at a certain speed. This is in fact exactly a case of the general structure given earlier, but with different names.

But what is interesting is that Frater David explained how he obtained the trains. He does not manipulate velocities, but the periods of rotation (the inverses of the velocities). Let p be the period of rotation of frame A, say in hours. And let r be the period of rotation of the input wheel C. And let s be the ratio between the velocity of B and that of C with respect to the frame A. That is, we have

$$\mathrm{V}_B^A = s \times \mathrm{V}_C^A \tag{1.30}$$

In general, p is what we are trying to achieve, and for that purpose we can carefully choose the input period r and the train inside the epicyclic box, with the velocity ratio s.

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We have

$$\mathrm{V}_B^A = \mathrm{V}_C^A \times s = -\mathrm{V}_A^B = -\mathrm{V}_A^0 \tag{1.31}$$

$$= \left(\mathrm{V}_C^0 + \mathrm{V}_0^A\right) \times s \tag{1.32}$$

Hence

$$\mathrm{V}_0^A(s+1) = -\mathrm{V}_C^0 s \tag{1.33}$$

$$\frac{\mathrm{V}_A^0}{\mathrm{V}_C^0} = \frac{s}{s+1} \tag{1.34}$$

and

$$\frac{\mathrm{P}_A^0}{\mathrm{P}_C^0} = \frac{p}{r} = \frac{s+1}{s} \tag{1.35}$$

$$\frac{1}{s} = \frac{p-r}{r} \tag{1.36}$$

For instance, Frater David considers the case where the input wheel C makes a turn in an hour ($r = 1$) and the frame A makes a turn in 23 hours ($p = 23$) [4, p. 24]. He then finds $1/s = 22$ and he chooses the train $1/s = \frac{8}{36} \times \frac{9}{44}$. Instead of the prime 23, Frater David now only has the prime 11.

In another example [4, p. 25], he takes $p = 101$ and $r = 1$ and finds $1/s = 100$.

Frater David also considers the case where the frame A and the input wheel C turn in opposite directions, but it suffices here to add signs to the periods.

In a further example [4, p. 32], Frater David considers the case of $p = 23$ and $r = 6$. The idea is always to choose r such that $\frac{p-r}{r}$ contains only small primes. If p is a prime, any positive integer value of r will lead to a smaller prime.

In another more realistic example [4, p. 100], Frater David considers the tropical revolution of the Sun. Taking $p = 31556928$ seconds, he first chooses $r = 32928$ and obtains a first train whose greatest prime is 71. He also considered $r = 63072$ and then the greatest prime is 73. The first solution is the one he took for the Palais Schwarzenberg clock. The period of arbor 42 in that clock is exactly r.

Frater David also explains how he obtained the tropical motion of the Moon in the Palais Schwarzenberg clock ($p = 2360584$ and $r = 10584$) [4, p. 185].

And finally, Frater David has also used this construction for the motion of the apsides and the lunar nodes in the Palais Schwarzenberg clock. The details are given in his *Praktische Anleitung für Künstler* [6, p. 40 and 59]. In these two trains, he took p and r to be periods expressed in minutes (as he does in some of his examples), namely $p = 4670432$ and $r = 42432$ for the apsides, and $p = 9789403$ and $r = 29403$ for the nodes.

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1.3.3 Cranks on epicyclic wheels

A number of constructions have cranks on epicyclic wheels, that is, cranks on wheels whose center is rotating. These cranks are used to produce oscillatory motions and not merely rotating motions. These motions are not particularly difficult to compute and usually all we need to compute is the velocity of the arbor supporting the crank.

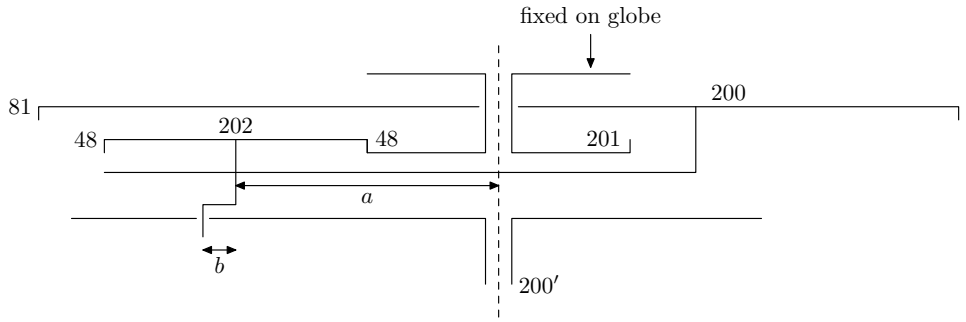
The oscillatory motions are mainly used to account for the uneven motion of the planets around the Sun, and for the retrogradations of the planets in geocentric representations. A number of constructions have been used in the clocks described here, and I will review the main ones.

1.3.3.1 Elliptical motions

The first category of oscillatory motions are meant to accelerate or slow down the motions of the planets or the Moon to account for the main anomaly in their motion, due to their non circular orbit.

1.3.3.1.1 The motion of the Sun in the Nuremberg *Weltmaschine*

For instance, in Hahn's Nuremberg *Weltmaschine* (Oechslin 8.2), we have the following configuration:



A 81-teeth wheel on tube (frame) 200 rotates on a fixed tube 201 tied to globe G and carrying a 48-teeth wheel. The tube 200 also carries a support on which arbor 202 pivots. This arbor carries a 48-teeth wheel meshing with the previously mentioned 48-teeth wheel, and a crank which moves tube 200′ which carries the Sun. Tube 200′ has the same mean motion as tube 200, but because of the crank, it is sometimes ahead and sometimes behind 200.

Tube 200 makes one turn counterclockwise in one tropical year and corresponds to the mean motion of the Sun. Its velocity is V_{200}^{G} where $G = 201$ is the globe.

The question is: what is the motion of tube 200′? We could actually compute it very precisely using the offsets a and b which are given by Oechslin. But we are mainly interested here in the period of oscillation and we want to know if Hahn's construction is correct.

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If $b = 0$, then tubes 200 and 200′ would have the same motion. It is also clear that if $b > 0$, tube 200′ has an oscillating motion around a mean position which is that of tube 200.

In order to compute this oscillation, we should imagine that we are on frame 200 and look at the motion of arbor 202. We can therefore compute

$$\mathrm{V}_{202}^{200} = \mathrm{V}_{G}^{200} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{G}^{200} = \mathrm{V}_{200}^{G} \tag{1.37}$$

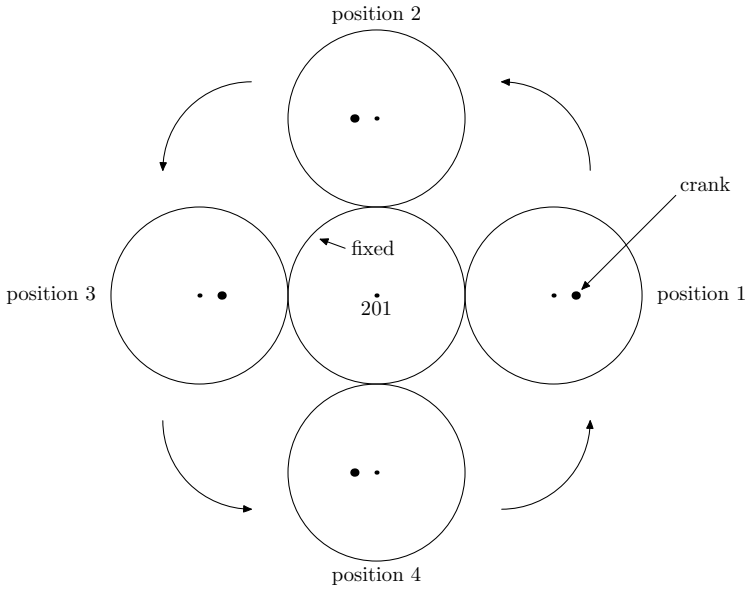
and we find that the period of oscillation is exactly the tropical year.

This is actually what is expected, because the acceleration and slowing down take place at one year intervals. To be fair, the period of oscillation should be the anomalistic year and not the tropical year, but these two years are very close to each other.

But we can also take a different perspective and compute the (rotation) motion of arbor 202 with respect to G. In other words, we can examine the orientation of the crank with respect to the globe. How does it move? Is it in fact moving at all? It is easy to compute:

$$\mathrm{V}_{202}^{G} = \mathrm{V}_{202}^{200} + \mathrm{V}_{200}^{G} = \mathrm{V}_{200}^{G} + \mathrm{V}_{200}^{G} = 2\mathrm{V}_{200}^{G} \tag{1.38}$$

So, in fact, arbor 202 moves twice as fast as wheel 200 in the reference frame of G. We can look at the two 48-teeth wheels from above:



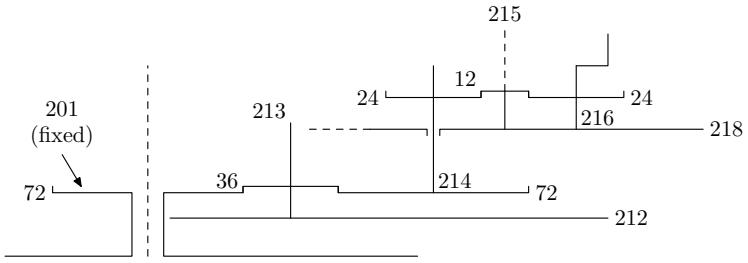
From position 1 to position 3, the motion of 200′ will be ahead of that of 200. And from position 3 to position 1, it will be behind.

Similar constructions are used to produce the oscillatory motions of the Moon, of Mars, Jupiter and Saturn.

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1.3.3.1.2 The motion of Mercury in the Nuremberg *Weltmaschine*

In the case of Mercury, Hahn is attempting to take into account both the elliptic orbit of Mercury and the retrogradations due to the Earth orbiting around the Sun. Hahn's construction is the following one, where I am only focusing on the elliptic orbit:



The fixed frame 201 is that of the globe. The frame 212 represents the corrected motion of the Sun, which is the mean motion of Mercury from a geocentric perspective. This is the same motion as the one of tube 200′ above. The arbor 214 is the mean direction of Mercury on this machine. Arbor 216 represents the motion of Mercury corrected for the geocentric perspective and I will examine it later. The frame 218 is only partly shown and rotates around the arbor 214. It is part of a train not shown. Finally, the crank on arbor 216 is meant to account for the elliptic motion of Mercury.

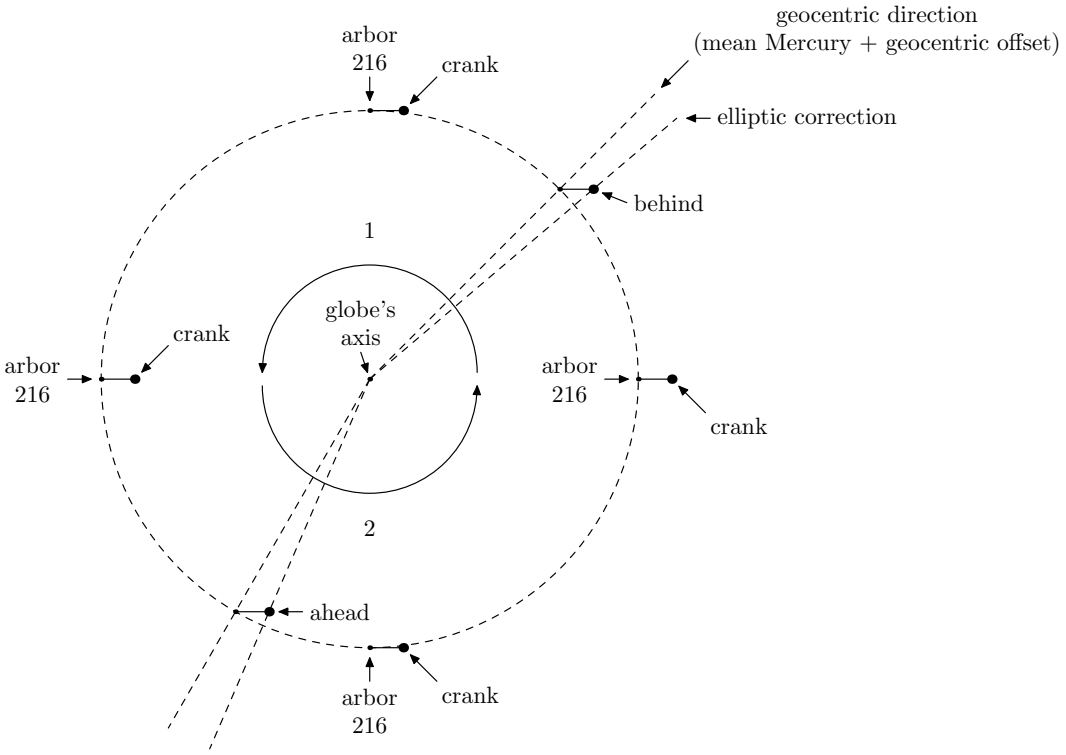
The question we have to ask is how does the crank alter the direction of arbor 216? The crank provides a correction to the direction of arbor 216 and this corrected direction is sometimes ahead and sometimes behind that of arbor 216.

It is easy to see that when the support 218 rotates around arbor 214, it also causes the crank to turn, and therefore that there is an oscillation. But what is the period of this oscillation? The arbor 216 actually replicates the motion of arbor 214, no matter the orientation of support 218. How much the crank moves Mercury on one side or another of arbor 216 therefore does not depend on where arbor 216 is located, or does not depend much on it. There is admittedly a small parallax effect, but we will ignore it here. We therefore do not need to care about how 218 moves, that is on how the retrogradations are taken care of.

Similarly, the arbor 214 replicates the motion of tube 201, no matter the orientation of the frame 212. As a consequence, the crank on arbor 216 has the same orientation as tube 201 which is fixed. The crank 216 therefore has a fixed orientation. It always points in the same direction.

Now, as Mercury revolves around the sky, we have the following configuration:

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Here I have shown the (counterclockwise) revolution of arbor 216 around the globe, that is the revolution of the apparent position of Mercury assuming that there is no correction for the elliptic motion.

On the first half of the revolution, Mercury's motion is slowed down. On the second half, it is accelerated.

In this case, the crank actually points towards Mercury's aphelion and the aphelion is at a fixed position on the celestial sphere.

This construction is different from the one seen above, but in the first one, there is also an anchor to the celestial sphere. Although the crank doesn't have a fixed orientation for the Sun, Mars, Jupiter, Saturn and the Moon, there is a position for the epicyclic wheel (the one carrying the crank) where the crank is farthest from the center, this position corresponds to a fixed direction and this direction is the perihelion.

1.3.3.2 The geocentric cranks in the Nuremberg *Weltmaschine*

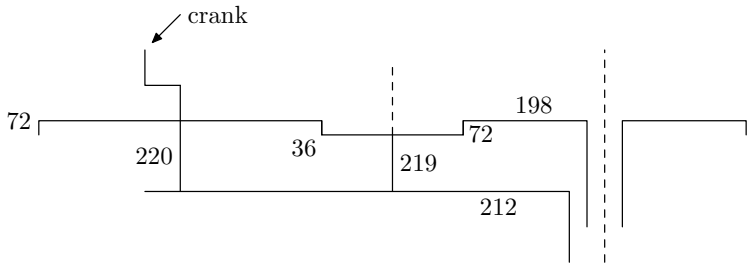
Now, it remains to see how the geocentric cranks for Mercury, Venus, Mars, Jupiter and Saturn work. I am only considering the case of the Nuremberg *Weltmaschine*. These cranks are used to produce the retrogradations of the planets.[12]

1.3.3.2.1 The cases of Mercury and Venus

In the case of Venus, we have the following configuration:

[12]On the parameters for the retrogradations, see in particular [19, p. 185-189].

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Tube 198 moves with the tropical period of Venus. Tube 212 is the corrected motion of the Sun. In the case of Mercury, we have a similar configuration, with tube 196 replacing tube 198.

The crank should represent the synodic motion of Mercury or Venus, that is, it should make one turn whenever the angular difference between tubes 198 and 212 increases by one turn. And indeed, we have

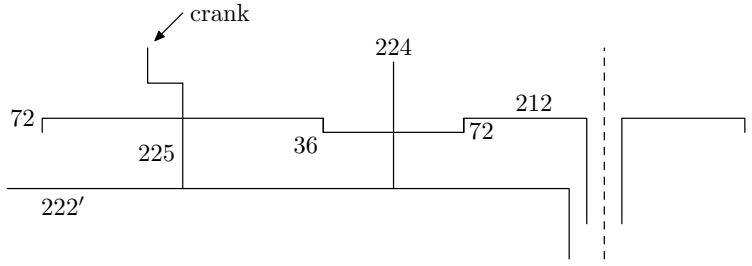
$$\mathrm{V}_{220}^{212} = \mathrm{V}_{198}^{212} \times \left(-\frac{72}{36}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}_{198}^{212} \tag{1.39}$$

Hence, the motion of the crank on plate 212 is that of Venus with respect to the Sun.

The case of Mercury is similar. The geocentric correction is applied first, before the elliptic correction (see above).

1.3.3.2.2 The case of Mars

In the case of Mars, we have the configuration



where 222′ is the corrected motion of Mars (for the eccentricity). In this case, the elliptical correction is applied first, then the geocentric one. (For Mercury, it was the opposite.)

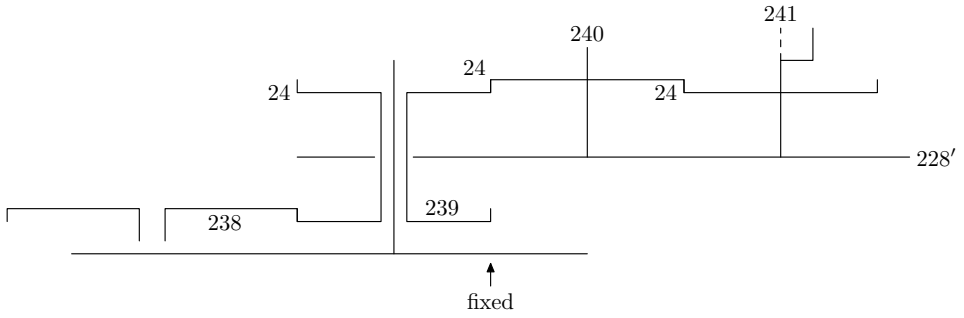
In the case of Mercury, the eccentricity should have been applied first, but it doesn't change much.

Like for Venus, the crank's motion is in par with the difference between 212 and 222′, that is between the corrected motion of Mars and the corrected motion of the Sun.

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1.3.3.2.3 The cases of Jupiter and Saturn

The cases of Jupiter and Saturn are similar. Hahn first corrects the mean motions for the eccentricities (see above). Then he has the following configuration for Jupiter:



The direction of arbor 241, that is the frame 228′, corresponds to the true longitude of Jupiter. This arbor rotates like tube 239. This tube makes one turn clockwise in a tropical year.

In summary, the motion of the crank is the difference between the true motion of Jupiter (228′) and the motion of the Sun (239).

The main difference with Mars is that the motions are flipped, and that they still need to be reversed, which is done when the 90-teeth wheel on arbor 228″ meshes with the 90-teeth wheel on tube 244.

The case of Saturn is similar.

1.3.4 Periods of rotation

Once we have the velocity of a wheel, it is easy to compute its period of rotation. The period of rotation is merely the inverse of the velocity. For instance, if a wheel (or arbor) has the velocity $V = 0.5$ turns/day, the period if $P = 1/V = 2$ days/turn.

Of course, if the velocity is not measured in turns per day, but for instance in degrees per second, then the period must be expressed differently. In the latter case, if $V = 20$ degrees/s, then $1/V = 1$ second for 20 degrees and $P = \frac{360}{20} \times 1$ second $= 18$ seconds.

It is therefore simplest to restrict the velocities to turns/day and to express the periods in days. Given these assumptions, the units can be made implicit. The periods can always be converted to other units later.

1.4 The extent and limits of this work

1.4.1 Methodology

As I wrote before, Oechslin's work is daunting, and even deterring. Probably only very few people have worked through all of its details, and I am not one of

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them. My plan, when working through Oechslin's book, was to do something else. I wanted to redo all the calculations in an axiomatic way and in particular to compute the exact ratios involved in the motions of the planets, something that Oechslin had not done. This is in fact also seldom done by other authors, and it is the cause of many approximations and sometimes serious errors.

All the clocks and mechanisms described by Oechslin have been reanalyzed directly from Oechslin's excellent plates of the gear layouts. In general, studying a clock or mechanism amounts to break it down into parts, and this is also the case with Oechslin's plans. I have often viewed these plans as forests, or labyrinths, and I had to find a beginning, and then slowly walk my way through them. This sometimes took time. When I made calculations, it was not unusual for me to get an obviously incorrect result, and I had to see if the error was mine, if it was Oechslin who got it wrong, or the constructor of the clock. More than once, the error was mine, and Oechslin was my safeguard. But the other possibilities occurred too.

I didn't use Oechslin's calculations, because I found them too difficult to follow, and because they were lacking the explanations I wished to provide. As a result, my computations are longer than Oechslin's, because I have strived to explain them and I wanted to give systematically the exact ratios.[13] In each case, I have compared my final ratios or values with those given by Oechslin, and, in case of discrepancies, I have sought the origin of the discrepancies and tried to correct them. My work thus provides an independent recalculation of the gears. By providing a detailed step-by-step derivation of the periods, I hope to make Oechslin's work more accessible and also more useful.

The exact ratios often involve very large integers but these integers were of course not used by Hahn or others. It is however important to do the exact computations, and not to carry approximate values from one equation to another. It is only in the final stages that I give numerical decimal values, usually to four decimal places, truncated (and not rounded). This is intentional. In some cases, I give the values to more decimal places, if there appears a need, for instance if some ambiguity would have arisen with only four places.

As far as the revolution periods are concerned, I have mainly compared those of the clocks or mechanisms with the then-known values, but I did not go so far as quantifying the errors. And of course, it would be meaningless to compare the values chosen by Hahn or someone else with those of modern almanacs.[14] The only meaningful comparison is between the periods on the clocks and those that were known and available at the time of the construc-

[13]For the exact fractions, I have not given the decompositions in prime factors, as these decompositions are not relevant here. It might be done in the future, but it would ne anecdotical. For tables of primes and factors, I refer to the numerous tables given in my LOCOMAT project (`https://locomat.loria.fr`).

[14]I find it anecdotical and of little interest to evaluate the accuracy of gear trains by the deviations after many years, as many sensationalist writers do (an example is Hahn's article [12]). Astronomical clocks never work uninterrupted, they always end up neglected, and arguing about a few seconds here or there is pointless. These discussion often miss the real points of a clock.

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tion of these clocks. But even these comparisons are objectionable, because different constructors may regard the accuracies in different ways. Some constructors may have contented themselves with crude approximations, and may not have wanted to go the trouble of complex trains, when it suffices to set the mechanism once in a while.

As mentioned above, Oechslin's plates are in general very good, and there is not much that could be improved, except for a clearer naming of the arbors and frames. Perhaps his notation for worms might be changed, but at this point I do not have a replacement, and this would be a minor problem, because there are not that many worms. They mainly appear in Neßtfell's clocks. It is in any case clear that Oechslin gave a lot of thought to his graphical notations. The main improvement that would have been useful, would have been to split many large plates into smaller parts, and this would have made the mechanisms much easier to study. It is also sometimes difficult to relate the plans to the actual clocks, especially when there are many dials (such as in Frater David's 1769 clock, or in Seige's 1791 clock).

I ended up making enlarged copies of many plates, but these enlargements were then often even more difficult to manipulate. In several cases, it took me some time to understand the plates, because I didn't have any guide. For instance, on the plate for Frater David's clock in Vienna, the gears are divided in three groups, and it is not readily obvious that these three groups are actually more or less superimposed. Oechslin split them for clarity, but if you only look at the plates, which is how I approached the clocks (I only read the first volume after I studied all the plates), this may be misleading. Finally, another problem in Oechslin's plates has to do with the dotted lines which are too light, and sometimes escape the attention.

Regarding the use of complex formulas, it is fitting here to recall that a serious horologist should not use rules of thumb. Some horologists compute the frequency of a pendulum by dividing the product of some numbers of teeth by the product of other numbers of teeth, and this may give the expected result in some cases, but it is easy to get it wrong. Horologists should use simple relations that they understand. In fact, one should try as much as possible to stick to the two following basic rules:

1. two wheels on the same arbor have the same speed;
2. the ratio of the speeds of two meshing wheels with parallel and fixed axes is the inverse of the ratio of the numbers of teeth.

In addition, there is sometimes the need to add velocities or to change reference frames, but this has to be done with caution.

Horologists should not use complex formulas, they should not use Willis' formula for epicyclic gears, or anything like that. There is no need for it! A general formula such as Willis' formula is only useful when it will simplify the analysis, in particular when many similar planetary constructions occur. This is seldom the case in the analysis of clocks. And I didn't find it necessary for any of the 29 clocks and mechanisms described here.

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1.4.2 The limits of this work

In my analyses of Oechslin's drawings, I have mainly concentrated on computing the exact ratios and on understanding and explaining how the various machines are obtaining their displays. I have, however, not expanded on how these constructions were obtained, although I have given some general principles in this chapter. For further details, I refer to Oechslin's first volume where he explains the various constructions used by the *Priestermechaniker*. However, in most cases it is easy to see how the ratios have been obtained.

In some cases, one can have the feeling that I am giving huge (or even monstruous) gear ratios. However, such ratios are commonplace in astronomical clocks and they help understand the real motions, even if the authors of the clocks have almost never used them. It is better to manipulate the exact ratios than numerical approximations of these ratios.

1.4.3 How to use this work

After this introduction, the book is organized in 29 chapters, one for each of the mechanism or clock described by Oechslin. The clocks are not given in the same order as Oechslin. Oechslin more or less organized his clocks chronologically, but with notable exceptions. He often described larger or complex machines first, and Hahn's clocks, for instance, are not described chronologically. I have tried to put the clocks and machines of each author in chronological order, so that it is easier to see what has been reused. Hahn, Neßtfell, Seige, and others, have reused ratios from one machine to another, and this can only well be perceived if the clocks are shown in chronological order. I have also decided to depart from how Frater David a Sancto Cajetano, Pater Aurelius a San Daniele and Frater Fridericus a S. Christophoro, are often called in various articles, namely Cajetano, Daniele and Christophoro. I see no reason to call them that way, and I have chosen to call them Frater David, Pater Aurelius and Frater Fridericus. This should not give rise to any ambiguity in this work. Moreover, I am in no way proposing a radical change, as others, such as Czermak [3], have already adopted this convention.

However, in order to facilitate the location of a clock, I have decided to list the authors alphabetically, thus starting with Adams and ending with Stuart. It is only within a given author that the clocks have been shown in chronological order, or in the best chronological order I could think of.

This, however, does not mean that the chapters need to be read in that order. The chapters are in fact all independent and can be read in any order. It would, however, seem more interesting to study the mechanisms of an author in chronological order. It should also be noted that each chapter may have references to other mechanisms, but the notations are independent. A velocity such as V_5^0 may appear in several chapters, with entirely different meanings. I will never reuse the velocity given in one chapter in another chapter. It should also be noted that there is some overlap between the chapters of a same author,

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and I have in particular repeated most of the bibliographic references.

Now, even though the chapters are independent from eachother, the book is not self-contained. It is meant to be a companion to Oechslin's book, and especially to his plates. When reading this book, it is best to have Oechslin's plates at hand (after having them enlarged), and to follow my derivations at the same time. Reading my derivations without the plates makes little sense. Unfortunately, at this point, I cannot include Oechslin's plates, and redrawing them would take months, if not years. They might be included in a future version.

1.4.4 Future versions

This work is a preliminary work. It is a work in progress which is intentionally made available in that state, not entirely polished, with few or no illustrations, and minimal historical notices, although I have tried to give the main bibliographical references in each cases. Some of the photographs are currently hidden, because they are not free to use. The plans of the gear trains are also not shown, for the same reasons.

This work is of course above all a companion to Oechslin's work, and I hope that it will ease the study of these 29 mechanisms by others, and also provide some impetus for new studies. In order to understand how these mechanisms work, and in particular the periods involved, it is in fact not necessary to read the first volume of Oechslin's work, and one can very well limit oneself to Oechslin's plates and my complements. But in order to have more insight on these mechanisms, or on the constructions used by their authors, Oechslin's text remains very useful.

I have also intentionally left aside everything that concerns the calendar, and in particular how the lengths of the months are taken into account. This is a different topic that I might tackle elsewhere some day. I have also not analyzed in detail the irregular motions caused by the eccentric orbits, or by the retrogradation of the planets, but they too might be described in more detail in the future.[15] In the meantime, I refer the reader to Oechslin's work. Oechslin took precise measurements of the eccentricities, he analyzed and compared them, both between the clocks, and with respect to the actual astronomical motions.

I am of course interested by any feedback, comments, corrections, etc., which may help me to improve my work. If some of the computations are not clear enough, I can try to improve them.

[15] I have always given the mean motions or the mean periods, but it is possible to provide more accurate expressions for the irregular motions, for instance the retrogradations in geocentric motions, although these expressions tend to become very complex. These analyses are interesting, but they are mainly exercises in theoretical kinematics, not really anything that brings an insight in the thoughts of the makers. None of these analyses were made by Oechslin. Incidentally, many years ago I worked out such expression for a complex clock by Antide Janvier, and those to whom I showed my work didn't understand what I was doing (which doesn't mean it was useless).

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I am also perfectly aware of many shortcomings. To take only one example, I am far from happy with the descriptions of Neßtfell's machines. There is much more to say, on the construction, on the dials, etc. In front of these machines, I still feel some frustration. In fact, an entire book should be devoted to them, but I cannot do it, because I do not have the necessary data and photographs. Moreover, I believe that this should be a team work, such great is the task.

In the future, I hope to complete my descriptions, in particular with photographs, but this will probably take place rather irregularly.

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Chapter 2

(Oechslin: 11.1)

Adams' orrery in Munich (c1748-1757)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

Although Adams is not one of the *Priestermechaniker*, the analysis of Adams' orrery is included here because it was part of Oechslin's original work. It also sheds some light in the inspiration of Neßtfell's orreries.

2.1 Introduction

This chapter describes a grand orrery (figure 2.1) made by George Adams (1709-1772).

George Adams was an instrument maker based in Fleet Street, London [4]. After his death, his business was taken over by his son George Adams Jr. (1750-1795). Both George Adams father and son made a number of grand orreries, which are horizontal orreries on twelve-sided bases, with a half-hemisphere of colures and parallels. Among these orreries, we can mention those of the British Museum, of the Whipple museum, of the history of science museum in Geneva, and of the Vanderbilt Mansion National Historic Site.

Other grand orreries are the one extended by Thomas Wright in 1733 and exhibited in the Science Museum in London (see figures 2.4 and 2.5), Thomas Wright's orrery at Dunham Massey Hall, Benjamin Cole's grand orrery made c1758 or earlier, and exhibited in Dumfries House, and Joseph Pope's grand orrery (1776-1787) exhibited in Harvard.

The grand orrery kept in Munich was made by George Adams Sr. in London between 1748 and 1757.[1] It was acquired around 1758 by Charles

[1] On Adams's orreries, see Millburn [4]. King also has some pages on Adams [3, p. 204-205]. This orrery was also shown at the 1989 Hahn exhibition [10, p. 61-63].

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Figure 2.1: General view of Adams' orrery (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`).

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Figure 2.2: General view of Adams' orrery. (photography by the author)

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Figure 2.3: The dial showing the time and the name of the constructor in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

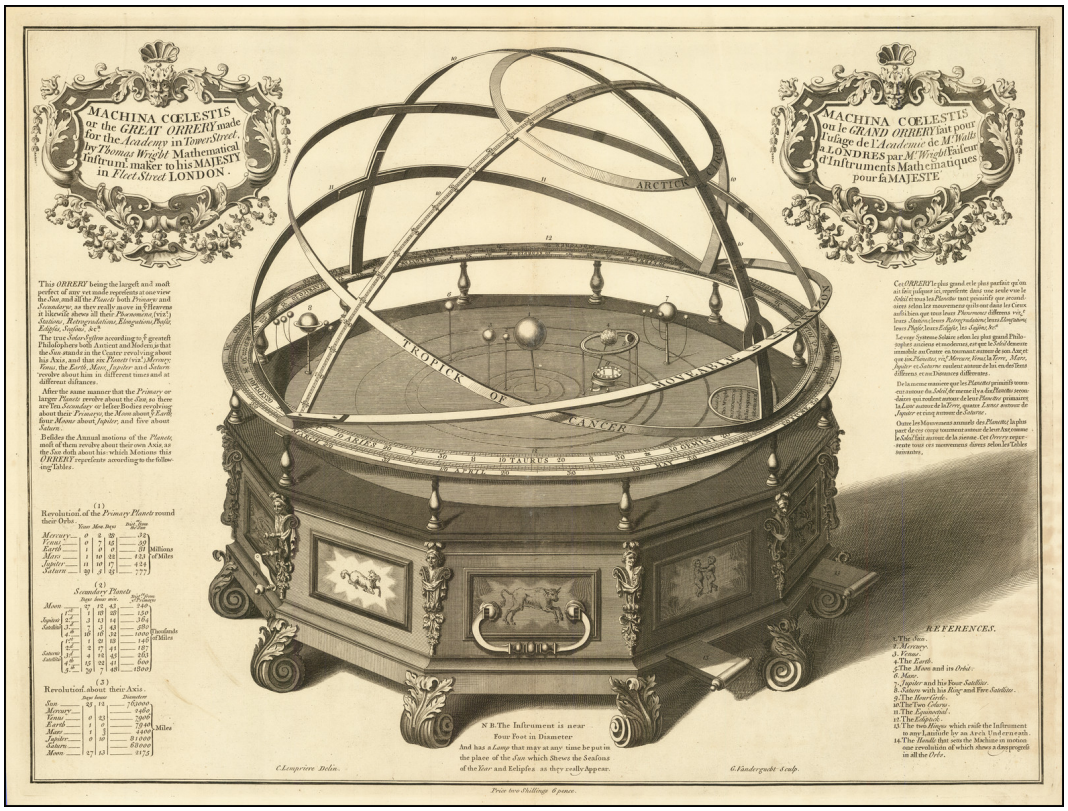


Figure 2.4: A woodcut of Thomas Wright's grand orrery.

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Figure 2.5: Joseph Wright of Derby's painting "A Philosopher Lecturing on the Orrery" (1766). (source: Wikipedia)

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Theodore (1724-1799), elector of Bavaria, for the court library of his palace in Mannheim. The piece came to Munich in about 1802/03 and is today kept in the *Bayerisches Nationalmuseum*. It appeared for the first time in the museum's guide in 1868.[2]

Although this is not an astronomical clock or mechanism made by a Priest-mechanic, Oechslin included it in its survey [5] as a comparison with the clocks and machines of Hahn and others.[3] Of course, Oechslin also happened to have worked on several clocks in the *Bayerisches Nationalmuseum*, and so he had access to Adams's orrery.[4]

[2] Cf. [8, p. 440].

[3] See [5, p. 34] for Oechslin's description.

[4] Adams' orrery was part of the exhibition of Neßtfell's restored Munich machine in 1988-1989 [9, p. 16].

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Figure 2.6: The planets on Adams' orrery. Around the Sun in the center, we can see Mercury (behind the Sun), Venus (to the right), then the Earth (in white) with the Moon, then Mars (towards us), then Jupiter (at the top) and Saturn (at the bottom), with their satellites. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

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Figure 2.7: The large driving wheels of Adams' orrery, when the top cover is removed. Saturn is visible on the left. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

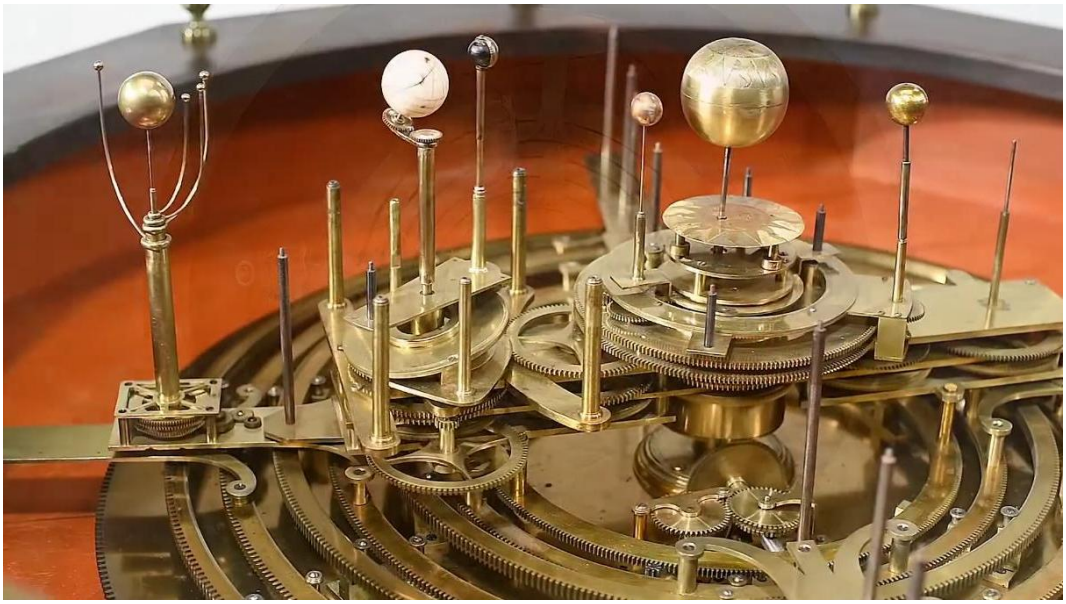


Figure 2.8: From left to right, the gears for Jupiter, the Earth/Moon system, Mercury and Venus in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

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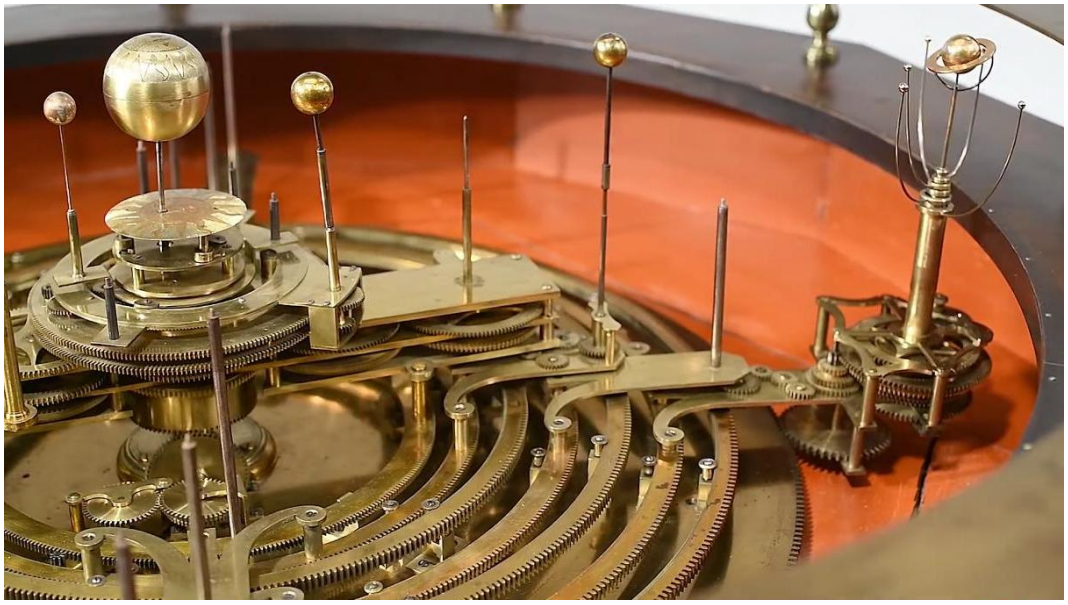


Figure 2.9: From left to right, the gears for Mercury, Venus, Mars and Saturn in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)



Figure 2.10: Detail of the gears of Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

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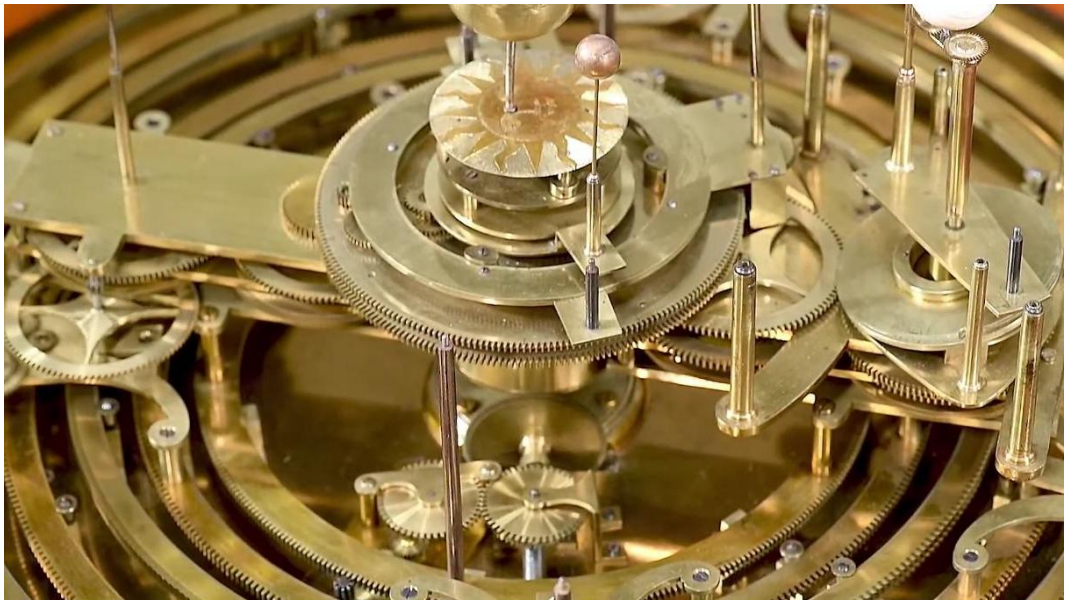


Figure 2.11: Detail of the gears of Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

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2.2 The base motion

All the gears in Adams' orrery are driven by an arbor originating from the clockwork. However, it isn't immediately clear what motion this arbor has or should have. In order to get a better understanding of the gear ratios, we can in fact start with the dial set on the Earth platform, namely that of the hours on arbor 70 which is rotating on the moving frame 15. This arbor is supposed to make one turn in 24 hours on this frame. That is, we can start with

$$\mathrm{V}_{70}^{15} = 1 \tag{2.1}$$

This arbor rotates counterclockwise.

If we start instead with the clockwork, we will have to make assumptions on the clockwork or the transmission to the orrery, and these assumptions will be entirely artificial and will only find their justifications later.

Now, we have

$$\mathrm{V}_{70}^{15} = \mathrm{V}_{70}^{0} + \mathrm{V}_{0}^{15} = \mathrm{V}_{70}^{0} - \mathrm{V}_{15}^{0} \tag{2.2}$$

And then

$$\mathrm{V}_{15}^{0} = \mathrm{V}_{14}^{0} \times \left(-\frac{8}{366}\right) \tag{2.3}$$

$$= \mathrm{V}_{9}^{0} \times \frac{18}{48} \times \left(-\frac{48}{48}\right) \times \left(-\frac{20}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{80}\right) \times \left(-\frac{8}{366}\right) \tag{2.4}$$

$$= -\mathrm{V}_{9}^{0} \times \frac{1}{488} = -\mathrm{V}_{7}^{0} \times \left(-\frac{20}{48}\right) \times \left(-\frac{48}{15}\right) \times \frac{1}{488} = -\mathrm{V}_{7}^{0} \times \frac{1}{366} \tag{2.5}$$

The 20-teeth wheel on arbor 7 meshes with an intermediate 48-teeth wheel which itself meshes with a 15-teeth wheel. On its arbor 9, there is a wheel with 60 teeth meshing with a wheel with 80 teeth on arbor/tube 56. This tube is the central mobile vertical tube of the orrery, rotating around the fixed vertical arbor 76. The resulting motion of tube 56 is obtained as follows:

$$\mathrm{V}_{56}^{0} = \mathrm{V}_{7}^{0} \times \left(-\frac{20}{48}\right) \times \left(-\frac{48}{15}\right) \times \left(-\frac{60}{80}\right) = -\mathrm{V}_{7}^{0} \tag{2.6}$$

Then we have:

$$\mathrm{V}_{70}^{15} = \mathrm{V}_{56}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) = \mathrm{V}_{56}^{15} \tag{2.7}$$

$$\mathrm{V}_{70}^{0} = \mathrm{V}_{70}^{15} + \mathrm{V}_{15}^{0} = \mathrm{V}_{56}^{15} + \mathrm{V}_{15}^{0} = \mathrm{V}_{56}^{0} = -\mathrm{V}_{7}^{0} \tag{2.8}$$

And now

$$\mathrm{V}_{70}^{15} = -\mathrm{V}_{7}^{0} + \mathrm{V}_{7}^{0} \times \frac{1}{366} = -\mathrm{V}_{7}^{0} \times \left(1 - \frac{1}{366}\right) = -\mathrm{V}_{7}^{0} \times \frac{365}{366} \tag{2.9}$$

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We obtain

$$\mathrm{V}_{70}^{15} = -\mathrm{V}_7^0 \times \frac{365}{366} = 1 \tag{2.10}$$

and therefore

$$\mathrm{V}_7^0 = -\frac{366}{365} \tag{2.11}$$

This is the velocity of the sidereal day. We conclude that arbor 7 must make one turn clockwise (from the right) in one sidereal day.

This derivation may seem contrived, but starting with the clockwork will be worse, because the period of the pendulum will have to be given without justification, or one will have to posit that arbor 7 has the velocity of a sidereal day, when in fact this is in no way obvious.

2.3 The period of the pendulum

Now that we know that arbor 7 must make one turn in a sidereal day, we can go backwards until the escapement. We find that the escape wheel E must make one turn in

$$T_E = \frac{365}{366} \times 86400 \times \frac{20}{40} \times \frac{10}{40} \times \frac{12}{72} \times \frac{8}{64} \times \frac{8}{60} \tag{2.12}$$

$$= \frac{1825}{61} = 29.9180\ldots \text{ s} \tag{2.13}$$

The pendulum (if there is a pendulum) makes a half-oscillation in $\frac{T_E}{2\times 30}$, that is a little less than 0.5 seconds, and is about 25 cm long.

2.4 The tropical year

The motion of tube 56 is transferred inside the mobile cage supporting the Earth-Moon system, the inner planets and the Sun. This cage rests on tube 15 which carries at its base a 366-teeth wheel. As we have already seen above, the motion of this tube is also obtained from the arbor 9 mentioned above. We first compute the velocities of arbor 14 and tube 15, which have not been fully explicited above. Arbor 14 will in particular later be used for the motion of Mars.

$$\mathrm{V}_{14}^0 = \mathrm{V}_9^0 \times \frac{18}{48} \times \left(-\frac{48}{48}\right) \times \left(-\frac{20}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{80}\right) \tag{2.14}$$

$$= \mathrm{V}_9^0 \times \frac{3}{32} = \mathrm{V}_7^0 \times \frac{4}{3} \times \frac{3}{32} = \left(-\frac{366}{365}\right) \times \frac{4}{3} \times \frac{3}{32} = -\frac{183}{1460} \tag{2.15}$$

$$\mathrm{V}_{15}^0 = \mathrm{V}_{14}^0 \times \left(-\frac{8}{366}\right) = \left(-\frac{183}{1460}\right) \times \left(-\frac{8}{366}\right) = \frac{1}{365} \tag{2.16}$$

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In other words, the rotation of the cage 15 takes place in exactly 365 days. And since

$$\mathrm{V}_{15}^{0} = -\mathrm{V}_{7}^{0} \times \frac{1}{366} \tag{2.17}$$

it also takes place in 366 sidereal days. Or, put differently, Adams takes the ratio of the tropical year to the sidereal day to be exactly 366 which is an approximation, as the actual ratio is about 366.24. The motion of the cage is counterclockwise seen from above.

2.5 The motions inside cage 15

The motions inside cage 15 are entirely determined by the relative motion of tube 56 with respect to the cage:

$$\mathrm{V}_{56}^{15} = \mathrm{V}_{56}^{0} + \mathrm{V}_{0}^{15} = -\mathrm{V}_{7}^{0} - \mathrm{V}_{15}^{0} = \frac{366}{365} - \frac{1}{365} = 1 \tag{2.18}$$

With respect to frame 15, the tube 56 makes exactly one rotation counterclockwise in one day.

2.5.1 Tubes 70 and 72

And so does the arbor 70:

$$\mathrm{V}_{70}^{15} = \mathrm{V}_{56}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) = 1 \tag{2.19}$$

With respect to frame 15, the arbor 72 makes one rotation counterclockwise in 60 days:

$$\mathrm{V}_{72}^{15} = \mathrm{V}_{70}^{15} \times \left(-\frac{24}{96}\right) \times \left(-\frac{8}{120}\right) = \frac{1}{60} \tag{2.20}$$

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2.5.2 The Earth and the Moon

The orientation of Earth's axis is determined by tube 67. We can compute the velocity of this tube in frame 15 and in the absolute frame 0:

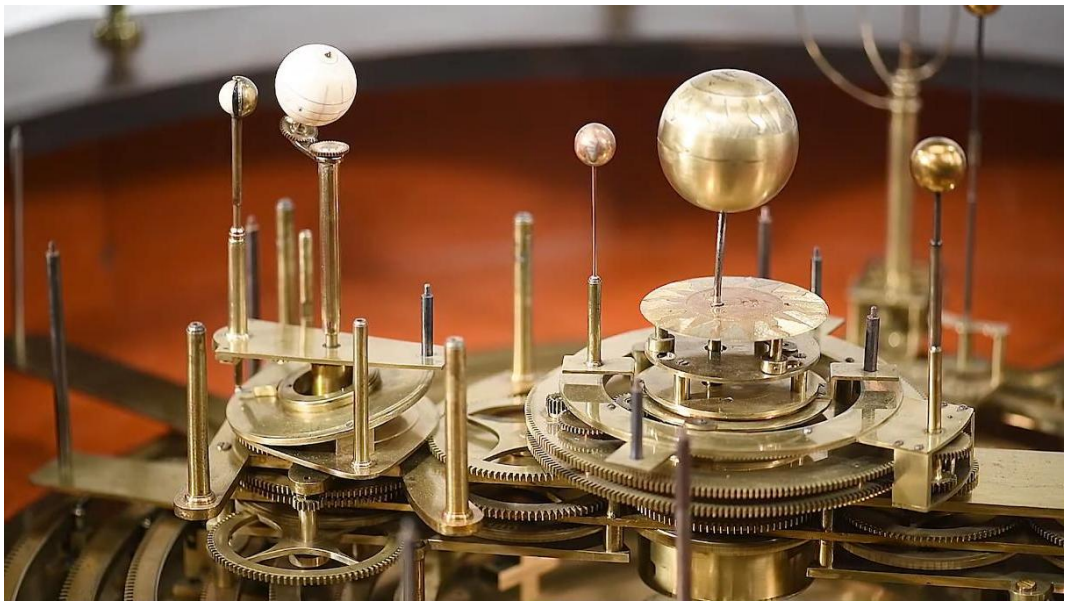


Figure 2.12: The Earth/Moon system and the tilted ramp for the latitude of the Moon in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

$$\mathrm{V}_{67}^{15} = \mathrm{V}_{25}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) \tag{2.21}$$

The arbor 25 is the central arbor of the orrery and is fixed in the absolute frame. Its relative velocity in frame 15 is:

$$\mathrm{V}_{25}^{15} = -\mathrm{V}_{15}^{25} = -\mathrm{V}_{15}^{0} = -\frac{1}{365} \tag{2.22}$$

Hence

$$\mathrm{V}_{67}^{15} = -\frac{1}{365} \tag{2.23}$$

And

$$\mathrm{V}_{67}^{0} = \mathrm{V}_{67}^{15} + \mathrm{V}_{15}^{0} = 0 \tag{2.24}$$

This is what we should have expected, namely that the Earth's axis has a fixed orientation in the absolute frame.

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The rotation of the Earth in frame 15 is also easy to compute. The Earth is on arbor 60. We first compute the velocity of arbor 60 in the frame of tube 67:

$$\mathrm{V}_{60}^{67} = \mathrm{V}_{58}^{67} \times \left(-\frac{40}{10}\right) \times \left(-\frac{10}{40}\right) = \mathrm{V}_{58}^{67} \tag{2.25}$$

Then

$$\mathrm{V}_{58}^{67} = \mathrm{V}_{58}^{15} + \mathrm{V}_{15}^{67} = \mathrm{V}_{58}^{15} + \frac{1}{365} = \mathrm{V}_{56}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{96}{96}\right) + \frac{1}{365} \tag{2.26}$$

$$= 1 + \frac{1}{365} \tag{2.27}$$

$$\mathrm{V}_{60}^{67} = \mathrm{V}_{60}^{0} = 1 + \frac{1}{365} = \frac{366}{365} \tag{2.28}$$

This is the velocity of the sidereal day.

Then, the average velocity of the Earth's axis in frame 15:

$$\mathrm{V}_{60}^{15} = \mathrm{V}_{60}^{67} + \mathrm{V}_{67}^{15} = \frac{366}{365} - \frac{1}{365} = 1 \tag{2.29}$$

The Earth rotates on average in one day in frame 15. This is only an average, because the axis is wobbling in frame 15.

The direction of the Moon is determined by tube 62. Its velocity in frame 15 is

$$\mathrm{V}_{62}^{15} = \mathrm{V}_{58}^{15} \times \left(-\frac{24}{96}\right) \times \left(-\frac{16}{118}\right) = \mathrm{V}_{58}^{15} \times \frac{2}{59} = \frac{2}{59} \tag{2.30}$$

Hence, the synodic revolution of the Moon is $\frac{59}{2} = 29.5$ days. Oechslin obtains the same value.

The tropical revolution of the Moon is obtained from

$$\mathrm{V}_{62}^{0} = \mathrm{V}_{62}^{15} + \mathrm{V}_{15}^{0} = \frac{2}{59} + \frac{1}{365} = \frac{789}{21535} \tag{2.31}$$

$$\mathrm{P}_{62}^{0} = \frac{21535}{789} = 27.2940\ldots \text{ days} \tag{2.32}$$

$$= 27.3688\ldots \text{ sidereal days} \tag{2.33}$$

Oechslin gives the same two values.

The phase of the Moon is determined by tube 65. We expect the motion of tube 65 to be still in frame 15, so that the lit part of the Moon is always in

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the same direction in that frame. So, we compute

$$\mathrm{V}_{65}^{15} = \mathrm{V}_{65}^{62} + \mathrm{V}_{62}^{15} = \mathrm{V}_{65}^{62} + \frac{2}{59} \tag{2.34}$$

$$= \mathrm{V}_{63}^{62} \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) + \frac{2}{59} \tag{2.35}$$

$$= \mathrm{V}_{63}^{62} + \frac{2}{59} = \mathrm{V}_{15}^{62} + \frac{2}{59} = -\frac{2}{59} + \frac{2}{59} = 0 \tag{2.36}$$

The latitude of the Moon is determined by a tilted plane on tube 68. The velocity of this tube is computed as follows:

$$\mathrm{V}_{68}^{0} = \mathrm{V}_{68}^{15} + \mathrm{V}_{15}^{0} = \mathrm{V}_{25}^{15} \times \left(-\frac{96}{96}\right) \times \left(-\frac{100}{95}\right) + \mathrm{V}_{15}^{0} \tag{2.37}$$

$$= \mathrm{V}_{25}^{15} \times \frac{20}{19} + \frac{1}{365} \tag{2.38}$$

$$= \left(-\frac{1}{365}\right) \times \frac{20}{19} + \frac{1}{365} = \left(-\frac{1}{365}\right) \times \frac{1}{19} = -\frac{1}{6935} \tag{2.39}$$

This gives a draconic period of 6935 days, the actual value being about 6798.383 days. This value is negative, because there is a precession of the lunar nodes. Oechslin didn't give this value.

The velocity of tube 68 in frame 15 is

$$\mathrm{V}_{68}^{15} = \left(-\frac{1}{365}\right) \times \frac{20}{19} = -\frac{4}{1387} \tag{2.40}$$

and the associated period is

$$\mathrm{P}_{68}^{15} = -\frac{1387}{4} = -346.75 \text{ days} \tag{2.41}$$

which is the eclipse year. This period was also omitted by Oechslin.

2.6 The Sun, Mercury and Venus

The Sun's axis is offset from the center of the orbit of the Earth. Venus and Mercury's orbits are also offset. The centers of these two orbits, as well as the Sun's axis are located on the fixed plate 25. These offsets approximate the elliptic orbits of the planets. Arbor 25 actually represents the center of Earth's orbit.

All the motions on plate 25 are obtained from wheel/tube 76 located under that plate. This tube 76 gets its motion from wheels located on frame 15. It has following velocity:

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$$\mathrm{V}_{76}^{15} = \mathrm{V}_{72}^{15} \times \left(-\frac{120}{100}\right) \times \left(-\frac{50}{36}\right) \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{73}\right) \tag{2.42}$$

$$= \mathrm{V}_{72}^{15} \times \frac{60}{73} = \frac{1}{60} \times \frac{60}{73} = \frac{1}{73} \tag{2.43}$$

$$\mathrm{V}_{76}^{0} = \mathrm{V}_{76}^{15} + \mathrm{V}_{15}^{0} = \frac{1}{73} + \frac{1}{365} = \frac{6}{365} \tag{2.44}$$

The rotation of the Sun is computed as follows:

$$\mathrm{V}_{85}^{0} = \mathrm{V}_{76}^{0} \times \left(-\frac{73}{30}\right) \times \left(-\frac{58}{59}\right) = \frac{58}{1475} \tag{2.45}$$

$$\mathrm{P}_{85}^{0} = \frac{1475}{58} = 25.4310\ldots \text{ days} \tag{2.46}$$

$$= 25.5007\ldots \text{ sidereal days} \tag{2.47}$$

Oechslin gave the same two periods.

The actual rotation period of the Sun is 25.67 days at the equator, but it increases at higher latitudes.

Mercury is attached to an 88-teeth wheel 84 moving on the fixed plate 25. The velocity of Mercury is

$$\mathrm{V}_{84}^{0} = \mathrm{V}_{76}^{0} \times \left(-\frac{73}{30}\right) \times \left(-\frac{25}{88}\right) = \mathrm{V}_{76}^{0} \times \frac{365}{528} = \frac{6}{365} \times \frac{365}{528} = \frac{1}{88} \tag{2.48}$$

$$\mathrm{P}_{84}^{0} = 88 \text{ days} \tag{2.49}$$

$$= 88.2410\ldots \text{ sidereal days} \tag{2.50}$$

Oechslin gave the same two periods.

Venus has both a motion of revolution around the Sun (or rather around a point near the Sun) and a motion of rotation around its axis. The revolution of Venus is that of wheel 80. It is obtained in a similar way as that of Mercury:

$$\mathrm{V}_{80}^{0} = \mathrm{V}_{76}^{0} \times \left(-\frac{73}{30}\right) \times \left(-\frac{25}{48}\right) \times \left(-\frac{48}{20}\right) \times \left(-\frac{20}{225}\right) \tag{2.51}$$

$$= \mathrm{V}_{76}^{0} \times \frac{73}{270} = \frac{6}{365} \times \frac{73}{270} = \frac{1}{225} \tag{2.52}$$

$$\mathrm{P}_{80}^{0} = 225 \text{ days} \tag{2.53}$$

$$= 225.6164\ldots \text{ sidereal days} \tag{2.54}$$

Oechslin gave the same two periods.

Venus is rotating around its axis 81.

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The rotation about its axis is obtained as follows:

$$\begin{aligned}
\mathrm{V}_{81}^{0} &= \mathrm{V}_{81}^{80} + \mathrm{V}_{80}^{0} && (2.55)\\
&= \mathrm{V}_{25}^{80} \times \left(-\frac{122}{24}\right) + \frac{1}{225} && (2.56)\\
&= \mathrm{V}_{0}^{80} \times \left(-\frac{61}{12}\right) + \frac{1}{225} && (2.57)\\
&= \left(-\frac{1}{225}\right) \times \left(-\frac{61}{12}\right) + \frac{1}{225} && (2.58)\\
&= \frac{73}{2700} && (2.59)\\
\mathrm{P}_{81}^{0} &= \frac{2700}{73} = 36.9863\ldots \text{ days} && (2.60)\\
&= 37.0876\ldots \text{ sidereal days} && (2.61)
\end{aligned}$$

Oechslin gave the same value, but only in sidereal days.

It is not clear how Adams came to this value,[5] because at that time there were two conflicting values for the rotation of Venus: Cassini's 1666 estimation of about 23 hours[6] and Bianchini's estimation of about 24 days.[7]

There is also a tube 83 around the axis 81, that Oechslin claims to be unused. The velocity of this tube is the following:

$$\begin{aligned}
\mathrm{V}_{83}^{0} &= \mathrm{V}_{83}^{80} + \mathrm{V}_{80}^{0} && (2.62)\\
&= \mathrm{V}_{25}^{80} \times \left(-\frac{225}{25}\right) \times \left(-\frac{6}{54}\right) + \mathrm{V}_{80}^{0} && (2.63)\\
&= \mathrm{V}_{0}^{80} + \mathrm{V}_{80}^{0} = 0 && (2.64)
\end{aligned}$$

We have $\mathrm{V}_{25}^{80} = \mathrm{V}_{0}^{80}$ because 25 is still.

The tube 83 is therefore not rotating in the absolute frame.

2.7 The motions of Mars

Mars has a motion of revolution and rotation. The revolution of Mars around the Sun is determined by wheel 18 which has 360 teeth. Its motion is obtained

[5] See Oechslin's brief mention of Venus in this context [5, p. 209].

[6] Cf. *Journal des sçavans*, 12 december 1667.

[7] cf. *Hesperi et Phosphori nova phaenomena sive observationes circa planetam Veneris* (Rome, 1728), p. 58.

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similarly to that of the Earth frame 15. We have

$$\mathrm{V}^0_{17} = \mathrm{V}^0_{14} \times \left(-\frac{8}{18}\right) \times \left(-\frac{18}{16}\right) = \mathrm{V}^0_{14} \times \frac{1}{2} = -\frac{183}{2920} \tag{2.65}$$

$$\mathrm{V}^0_{18} = \mathrm{V}^0_{17} \times \left(-\frac{8}{360}\right) = \frac{61}{43800} \tag{2.66}$$

$$\mathrm{P}^0_{18} = \frac{43800}{61} = 718.0327\ldots \text{ days} \tag{2.67}$$

$$= 720 \text{ sidereal days} \tag{2.68}$$

Oechslin gave the same two periods.[8]

This value seems very inaccurate, the correct value being 687 days. Something must be wrong here. Perhaps the number of teeth of wheel 18 is not 360? The value 345 would for instance have been better.

The rotation of Mars' axis 55 is obtained from a wheel 51 rotating around the Sun, around the same excentric point as wheel 18 for Mars' revolution. The velocity of this wheel 51 is

$$\mathrm{V}^0_{51} = \mathrm{V}^0_{9} \times \left(-\frac{32}{360}\right) = \left(-\frac{488}{365}\right) \times \left(-\frac{32}{360}\right) = \frac{1952}{16425} \tag{2.69}$$

This wheel 51 then meshes with a wheel whose axis is on the Mars rotating frame 18:

$$\mathrm{V}^{18}_{55} = \mathrm{V}^{18}_{51} \times \left(-\frac{360}{16}\right) \times \left(-\frac{18}{27}\right) \times \left(-\frac{27}{48}\right) \times \left(-\frac{35}{36}\right) \tag{2.70}$$

$$= \mathrm{V}^{18}_{51} \times \frac{525}{64} \tag{2.71}$$

and since

$$\mathrm{V}^{18}_{51} = \mathrm{V}^0_{51} + \mathrm{V}^{18}_0 = \frac{1952}{16425} - \frac{61}{43800} = \frac{15433}{131400} \tag{2.72}$$

therefore

$$\mathrm{V}^{18}_{55} = \frac{15433}{131400} \times \frac{525}{64} = \frac{108031}{112128} \tag{2.73}$$

$$\mathrm{V}^0_{55} = \mathrm{V}^{18}_{55} + \mathrm{V}^0_{18} = \frac{108031}{112128} + \frac{61}{43800} = \frac{2704679}{2803200} \tag{2.74}$$

$$\mathrm{P}^0_{55} = \frac{2803200}{2704679} = 1.0364 \text{ days} \tag{2.75}$$

$$= 1.0392\ldots \text{ sidereal days} \tag{2.76}$$

Oechslin gave the same period, but only in sidereal days.

The actual period of the tropical rotation of Mars is 1.0259 days.

[8] See Oechslin's brief mention [5, p. 209].

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2.8 The motions of Jupiter and its satellites

Jupiter has a motion of revolution and rotation, and it also has four satellites. The revolution of Jupiter around the Sun is determined by wheel 21 which has 450 teeth. Its motion is obtained similarly to that of the Earth frame 15.

$$\mathrm{V}^0_{20} = \mathrm{V}^0_{17} \times \left(-\frac{16}{24}\right) \times \left(-\frac{24}{77}\right) = \mathrm{V}^0_{17} \times \frac{16}{77} = \left(-\frac{183}{2920}\right) \times \frac{16}{77} \tag{2.77}$$

$$= -\frac{366}{28105} \tag{2.78}$$

$$\mathrm{V}^0_{21} = \mathrm{V}^0_{20} \times \left(-\frac{8}{450}\right) = \left(-\frac{366}{28105}\right) \times \left(-\frac{8}{450}\right) = \frac{488}{2107875} \tag{2.79}$$

$$\mathrm{P}^0_{21} = \frac{2107875}{488} = 4319.4159\ldots \text{ days} \tag{2.80}$$

$$= 4331.25 \text{ sidereal days} \tag{2.81}$$

Oechslin gave the same two values of the period.

The actual orbital period is about 4332.6 days.

The rotation of Jupiter and of its satellites is obtained from the rotation of a 450-teeth wheel 40 driven from arbor 9:

$$\mathrm{V}^0_{40} = \mathrm{V}^0_{9} \times \left(-\frac{34}{450}\right) = \left(-\frac{488}{365}\right) \times \left(-\frac{34}{450}\right) = \frac{8296}{82125} \tag{2.82}$$

The rotation of Jupiter's axis 46 is obtained as follows:

$$\mathrm{V}^{21}_{43} = \mathrm{V}^{21}_{40} \times \left(-\frac{450}{22}\right) \times \left(-\frac{24}{36}\right) \times \left(-\frac{36}{48}\right) = \mathrm{V}^{21}_{40} \times \left(-\frac{225}{22}\right) \tag{2.83}$$

$$\mathrm{V}^{21}_{46} = \mathrm{V}^{21}_{43} \times \left(-\frac{48}{20}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{20}{20}\right) = \mathrm{V}^{21}_{43} \times \left(-\frac{12}{5}\right) \tag{2.84}$$

$$= \mathrm{V}^{21}_{40} \times \left(-\frac{225}{22}\right) \times \left(-\frac{12}{5}\right) = \mathrm{V}^{21}_{40} \times \frac{270}{11} \tag{2.85}$$

Then

$$\mathrm{V}^{21}_{40} = \mathrm{V}^0_{40} + \mathrm{V}^{21}_{0} = \frac{8296}{82125} - \frac{488}{2107875} = \frac{637328}{6323625} \tag{2.86}$$

Hence

$$\mathrm{V}^{21}_{46} = \frac{637328}{6323625} \times \frac{270}{11} = \frac{3823968}{1545775} \tag{2.87}$$

$$\mathrm{V}^0_{46} = \mathrm{V}^{21}_{46} + \mathrm{V}^0_{21} = \frac{3823968}{1545775} + \frac{488}{2107875} = \frac{8194984}{3312375} \tag{2.88}$$

$$\mathrm{P}^0_{46} = \frac{3312375}{8194984} = 0.4041\ldots \text{ days} = 9.7006\ldots \text{ hours} \tag{2.89}$$

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Oechslin made a mistake and gave an entirely different period of 2.025756 sidereal days.

The actual rotation period of Jupiter is about 9.92 hours.

The motion of the four satellites is obtained from four wheels on arbor 43:

$$\mathrm{V}_{47}^{21} = \mathrm{V}_{43}^{21} \times \left(-\frac{48}{84}\right) \text{ (Io)} \tag{2.90}$$

$$\mathrm{V}_{48}^{21} = \mathrm{V}_{43}^{21} \times \left(-\frac{24}{86}\right) \text{ (Europe)} \tag{2.91}$$

$$\mathrm{V}_{49}^{21} = \mathrm{V}_{43}^{21} \times \left(-\frac{12}{86}\right) \text{ (Ganymede)} \tag{2.92}$$

$$\mathrm{V}_{50}^{21} = \mathrm{V}_{43}^{21} \times \left(-\frac{6}{100}\right) \text{ (Callisto)} \tag{2.93}$$

We have seen above that

$$\mathrm{V}_{43}^{21} = \mathrm{V}_{40}^{21} \times \left(-\frac{225}{22}\right) = \frac{637328}{6323625} \times \left(-\frac{225}{22}\right) = -\frac{318664}{309155} \tag{2.94}$$

We can now compute the tropical revolutions of the satellites:

$$\mathrm{V}_{47}^{0} = \mathrm{V}_{47}^{21} + \mathrm{V}_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{48}{84}\right) + \frac{488}{2107875} \tag{2.95}$$

$$= \frac{95636776}{162306375} \tag{2.96}$$

$$\mathrm{P}_{47}^{0} = \frac{162306375}{95636776} = 1.6971\ldots \text{ days} \tag{2.97}$$

$$= 1.7017\ldots \text{ sidereal days} \tag{2.98}$$

$$\mathrm{V}_{48}^{0} = \mathrm{V}_{48}^{21} + \mathrm{V}_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{24}{86}\right) + \frac{488}{2107875} \tag{2.99}$$

$$= \frac{287028424}{997024875} \tag{2.100}$$

$$\mathrm{P}_{48}^{0} = \frac{997024875}{287028424} = 3.4736\ldots \text{ days} \tag{2.101}$$

$$= 3.4831\ldots \text{ sidereal days} \tag{2.102}$$

$$\mathrm{V}_{49}^{0} = \mathrm{V}_{49}^{21} + \mathrm{V}_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{12}{86}\right) + \frac{488}{2107875} \tag{2.103}$$

$$= \frac{143629624}{997024875} \tag{2.104}$$

$$\mathrm{P}_{49}^{0} = \frac{997024875}{143629624} = 6.9416\ldots \text{ days} \tag{2.105}$$

$$= 6.9606\ldots \text{ sidereal days} \tag{2.106}$$

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$$\mathrm{V}_{50}^{0} = \mathrm{V}_{50}^{21} + \mathrm{V}_{21}^{0} = \left(-\frac{318664}{309155}\right) \times \left(-\frac{6}{100}\right) + \frac{488}{2107875} \tag{2.107}$$

$$= \frac{1439356}{23186625} \tag{2.108}$$

$$\mathrm{P}_{50}^{0} = \frac{23186625}{1439356} = 16.1090\ldots \text{ days} \tag{2.109}$$

$$= 16.1531\ldots \text{ sidereal days} \tag{2.110}$$

Oechslin gave the same values of the periods.

The actual orbital periods of the four first satellites are 1.76 days, 3.53 days, 7.16 days and 16.69 days.

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2.9 The motions of Saturn and its satellites

Saturn has a motion of revolution and rotation, and it also has five satellites. The revolution of Saturn around the Sun is determined by wheel 24 which has 450 teeth. Its motion is obtained similarly to that of the Earth frame 15.

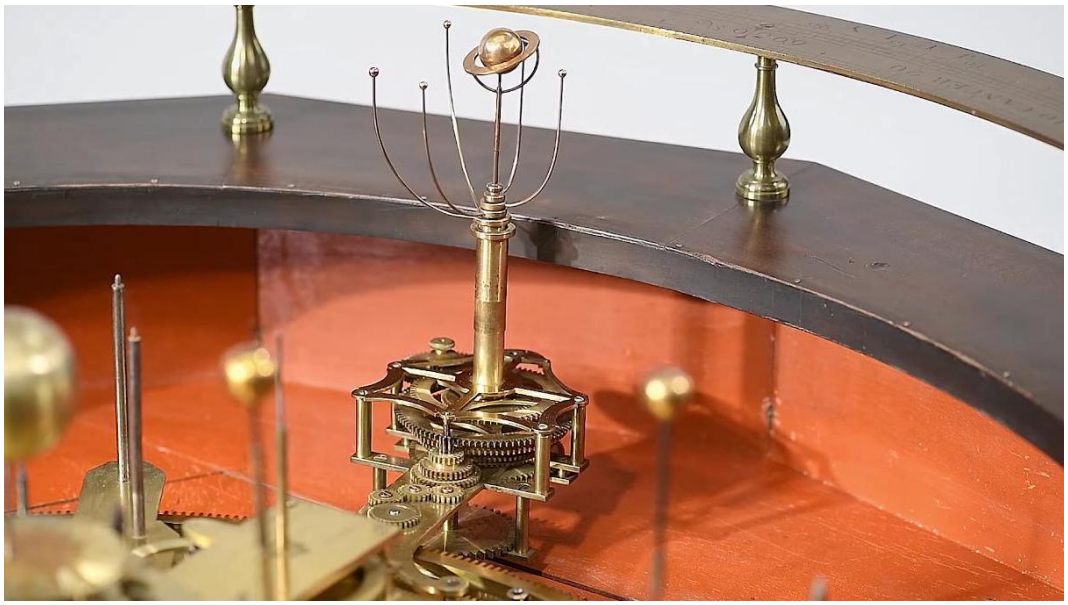


Figure 2.13: Detail of the gears for Saturn in Adams' orrery. The largest wheel at the bottom is the 450-teeth wheel which is used for the rotation of Saturn. This wheel is fixed, and the wheels on arbor 8 and 9 and going through an opening of this wheel. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

$$\mathrm{V}_{24}^0 = \mathrm{V}_{20}^0 \times \left(-\frac{31}{18}\right) \times \left(-\frac{18}{77}\right) \times \left(-\frac{8}{450}\right) \tag{2.111}$$

$$= \mathrm{V}_{20}^0 \times \left(-\frac{124}{17325}\right) = \left(-\frac{366}{28105}\right) \times \left(-\frac{124}{17325}\right) \tag{2.112}$$

$$= \frac{15128}{162306375} \tag{2.113}$$

$$\mathrm{P}_{24}^0 = \frac{162306375}{15128} = 10728.8719\ldots \text{ days} \tag{2.114}$$

$$= 10758.2661\ldots \text{ sidereal days} \tag{2.115}$$

Oechslin gave the same two values of the period.

The actual orbital period is about 10755.7 days.

The rotation of Saturn on its axis 27 is obtained using a fixed 450-teeth wheel and the intermediate arbor 26 located on frame 24. We first compute

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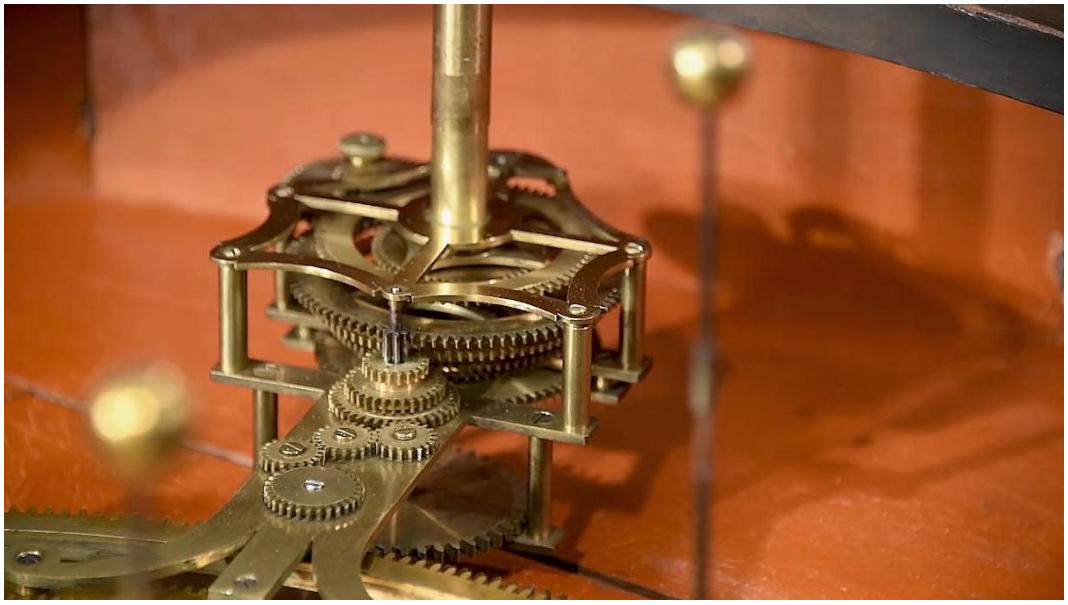


Figure 2.14: Detail of the gears for Saturn in Adams' orrery. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

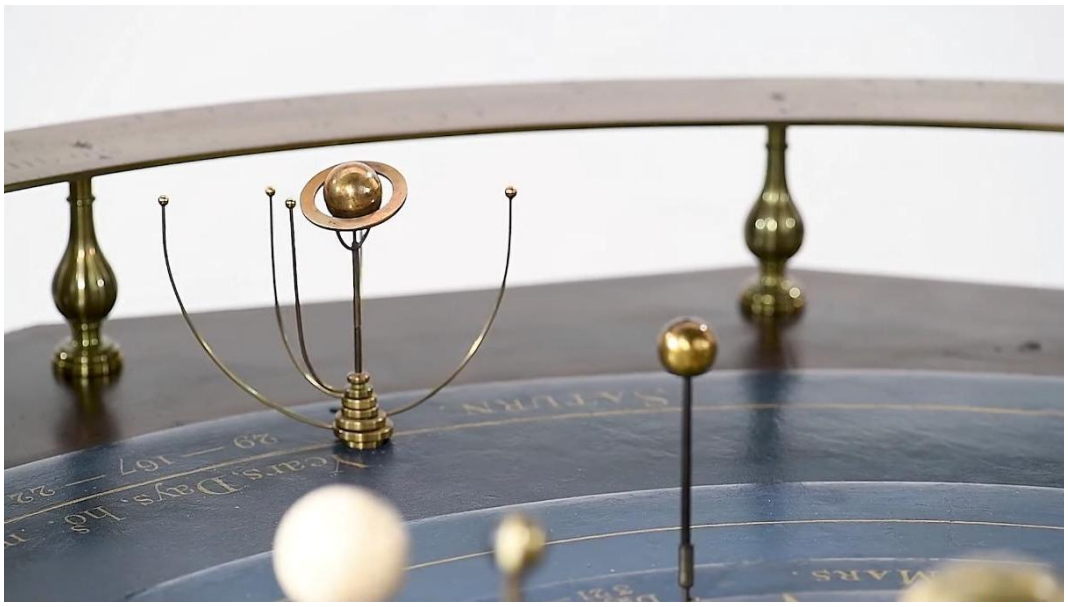


Figure 2.15: Saturn and its satellites when the top cover of Adams' orrery conceals the gears. (excerpt from the video `https://www.youtube.com/watch?v=OL3G-lVG7Mw`)

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the velocity of Saturn's axis with respect to wheel 24:

$$\mathrm{V}_{27}^{24} = \mathrm{V}_{0}^{24} \times \left(-\frac{450}{45}\right) \times \left(-\frac{8}{89}\right) = \mathrm{V}_{0}^{24} \times \frac{80}{89} \tag{2.116}$$

But we have found:

$$\mathrm{V}_{0}^{24} = -\frac{15128}{162306375} \tag{2.117}$$

Hence

$$\mathrm{V}_{27}^{24} = \left(-\frac{15128}{162306375}\right) \times \frac{80}{89} = -\frac{242048}{2889053475} \tag{2.118}$$

$$\mathrm{V}_{27}^{0} = \mathrm{V}_{27}^{24} + \mathrm{V}_{24}^{0} = -\frac{242048}{2889053475} + \frac{15128}{162306375} \tag{2.119}$$

$$= \frac{45384}{4815089125} \tag{2.120}$$

$$\mathrm{P}_{27}^{0} = \frac{4815089125}{45384} = 106096.6227\ldots \text{ days} \tag{2.121}$$

$$= 106387.2983\ldots \text{ sidereal days} \tag{2.122}$$

Oechslin gave the same period, but only in sidereal days.

The value produced by Adams' orrery is entirely incorrect, because Saturn rotates around its axis in about 10 hours, and this was already known by Huygens. It is however not clear where Adams erred and it would be interesting to compare the gears of this grand orrery with other orreries made by Adams. Perhaps some other orreries could help understand Adams' error.

The rotation of Saturn's satellites is obtained from another 450-teeth wheel 28 driven from arbor 9, as in the case of Jupiter:

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{9}^{0} \times \left(-\frac{15}{450}\right) = \left(-\frac{488}{365}\right) \times \left(-\frac{15}{450}\right) = \frac{244}{5475} \tag{2.123}$$

$$\mathrm{V}_{33}^{24} = \mathrm{V}_{28}^{24} \times \left(-\frac{450}{15}\right) \times \left(-\frac{30}{20}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{20}{40}\right) \tag{2.124}$$

$$= \mathrm{V}_{28}^{24} \times \left(-\frac{45}{2}\right) = \left(\mathrm{V}_{28}^{0} + \mathrm{V}_{0}^{24}\right) \times \left(-\frac{45}{2}\right) \tag{2.125}$$

$$= \left(\frac{244}{5475} - \frac{15128}{162306375}\right) \times \left(-\frac{45}{2}\right) \tag{2.126}$$

$$= \frac{2406084}{54102125} \times \left(-\frac{45}{2}\right) = -\frac{10827378}{10820425} \tag{2.127}$$

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The motion of the five satellites is obtained from five wheels on frame 24:

$$\mathrm{V}_{34}^{24} = \mathrm{V}_{33}^{24} \times \left(-\frac{40}{75}\right) = \frac{28873008}{54102125} \text{ (Tethys)} \tag{2.128}$$

$$\mathrm{V}_{35}^{24} = \mathrm{V}_{33}^{24} \times \left(-\frac{32}{88}\right) = \frac{43309512}{119024675} \text{ (Dione)} \tag{2.129}$$

$$\mathrm{V}_{36}^{24} = \mathrm{V}_{33}^{24} \times \left(-\frac{20}{90}\right) = \frac{2406084}{10820425} \text{ (Rhea)} \tag{2.130}$$

$$\mathrm{V}_{37}^{24} = \mathrm{V}_{33}^{24} \times \left(-\frac{6}{96}\right) = \frac{5413689}{86563400} \text{ (Titan)} \tag{2.131}$$

$$\mathrm{V}_{39}^{24} = \mathrm{V}_{33}^{24} \times \left(-\frac{6}{96}\right) \times \left(-\frac{58}{59}\right) \times \left(-\frac{16}{78}\right) = \mathrm{V}_{33}^{24} \times \left(-\frac{29}{2301}\right) \tag{2.132}$$

$$= \frac{104664654}{8299265975} \text{ (Iapetus)} \tag{2.133}$$

And in the absolute frame:

$$\mathrm{V}_{34}^{0} = \mathrm{V}_{34}^{24} + \mathrm{V}_{24}^{0} = \frac{28873008}{54102125} + \frac{15128}{162306375} = \frac{7875832}{14755125} \tag{2.134}$$

$$\mathrm{P}_{34}^{0} = \frac{14755125}{7875832} = 1.8734\ldots \text{ days (Tethys)} \tag{2.135}$$

$$= 1.8786\ldots \text{ sidereal days} \tag{2.136}$$

$$\mathrm{V}_{35}^{0} = \mathrm{V}_{35}^{24} + \mathrm{V}_{24}^{0} = \frac{43309512}{119024675} + \frac{15128}{162306375} = \frac{649809088}{1785370125} \tag{2.137}$$

$$\mathrm{P}_{35}^{0} = \frac{1785370125}{649809088} = 2.7475\ldots \text{ days (Dione)} \tag{2.138}$$

$$= 2.7550\ldots \text{ sidereal days} \tag{2.139}$$

$$\mathrm{V}_{36}^{0} = \mathrm{V}_{36}^{24} + \mathrm{V}_{24}^{0} = \frac{2406084}{10820425} + \frac{15128}{162306375} = \frac{36106388}{162306375} \tag{2.140}$$

$$\mathrm{P}_{36}^{0} = \frac{162306375}{36106388} = 4.4952\ldots \text{ days (Rhea)} \tag{2.141}$$

$$= 4.5075\ldots \text{ sidereal days} \tag{2.142}$$

$$\mathrm{V}_{37}^{0} = \mathrm{V}_{37}^{24} + \mathrm{V}_{24}^{0} = \frac{5413689}{86563400} + \frac{15128}{162306375} = \frac{81326359}{1298451000} \tag{2.143}$$

$$\mathrm{P}_{37}^{0} = \frac{1298451000}{81326359} = 15.9659\ldots \text{ days (Titan)} \tag{2.144}$$

$$= 16.0096\ldots \text{ sidereal days} \tag{2.145}$$

$$\mathrm{V}_{39}^{0} = \mathrm{V}_{39}^{24} + \mathrm{V}_{24}^{0} = \frac{104664654}{8299265975} + \frac{15128}{162306375} = \frac{225938998}{17784141375} \tag{2.146}$$

$$\mathrm{P}_{39}^{0} = \frac{17784141375}{225938998} = 78.7121\ldots \text{ days (Iapetus)} \tag{2.147}$$

$$= 78.9277\ldots \text{ sidereal days} \tag{2.148}$$

Oechslin has the same periods, except for Dione, where his values are slightly different from mine.

The actual periods are 1.9 days (Tethys), 2.7 days (Dione), 4.5 days (Rhea), 16 days (Titan) and 79 days (Iapetus).

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2.10 Conclusion

Using velocities, we have found the same periods as Oechslin, except for the rotation of Jupiter and the orbital period of Dione, where Oechslin made mistakes. Sometimes Oechslin also gives the periods only expressed in sidereal days.

A huge discrepancy was also identified in the rotation of Saturn, and perhaps the comparison with other grand orreries by Adams could help understand its origin.

2.11 References

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Chapter 3

(Oechslin: 5.1)

Pater Aurelius's clock in Munich (1769)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

3.1 Introduction

The clock described in this chapter was constructed in 1769 by Michael Fras, who became a discalced Augustinian known as Pater Aurelius a San Daniele (1728-1782).

Very little is known of Michael Fras. According to Bertele, he was born in Shavers (sp.?) in Thuringia.[1] He entered the Mariabrunn Augustinian monastery near Vienna, the same that Frater David attended. We know that he constructed at least three clocks in the 1760s and 1770s. Pater Aurelius died in Vienna in 1782.

The present clock was an order from the former Cistercian abbey at Salem, near Konstanz, for which abbot Anselm II. Schwab paid 150 ducats to Pater Aurelius in 1771.[2] This clock was located in the Prelature of the abbey.

In 1938, Frischholz mentioned that a clockmaker had remade the escapement of Pater Aurelius' clock about 100 years before his article, so around 1830, and that the clock was restored by the clockmakers Anton and Karl Jagemann after it was donated to the museum.[3]

In any case, the ownership of the clock was forgotten until 1919, when it came in the possession of Ernst von Bassermann-Jordan who then bequeathed

[1] Most of this biographical notice is taken from Bertele [6]. I do unfortunately not know what Shavers refers to and the current spelling may be different. This location was first given by Bertele and repeated by a number of later authors. See also Lloyd [14], Maurice [19, v.1, p. 271, 276, 278], Abeler [2], Brusa [7] and Mattl-Wurm's short notice [18, p. 42-43].

[2] See the 1989 exhibition catalogue [23, p. 54-55] and Oechslin [20, p. 222].

[3] See [12, p. 198].

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it to the *Bayerisches Nationalmuseum* in Munich in 1933.[4]

Another similar clock by Pater Aurelius is located in Innsbruck. It was completed in 1775 and was particularly described by Czermak.[5]

3.2 Description of the clock

Pater Aurelius' clock is a tall case clock, with a very ornate case made by Johann Georg Dirr (1723-1779).[6] There is a large dial surrounded by a number of smaller dials and spheres (figure 3.2). There are a number of calendrical indications, but they will only cursorily be described here. I will first describe the going work, then the main dial, the lunar sphere, the celestial sphere, and then the other dials.

3.2.1 The going work

The clock is weight-driven and regulated by a pendulum. The 30-teeth escapement wheel is on arbor 9 and makes a turn in one minute (all the motions are measured from the front).

$$\mathrm{V}_9^0 = -1440 \tag{3.1}$$

$$\mathrm{P}_9^0 = -\frac{1}{1440}\ \text{days} = -1\ \text{m} \tag{3.2}$$

The pendulum thus makes a half-oscillation in one second.

This motion is then transferred to the central arbor 7 of the main dial:

$$\mathrm{V}_7^0 = \mathrm{V}_9^0 \times \left(-\frac{6}{60}\right) \times \left(-\frac{6}{72}\right) = \mathrm{V}_9^0 \times \frac{1}{120} = -12 \tag{3.3}$$

$$\mathrm{P}_7^0 = -\frac{1}{12} \tag{3.4}$$

[4]See [4], [5, p. 13-16], [21, p. 435] and [20, p. 233].

[5]See [8], [9], Bertele [6, p. 108-110], Trapp [22] and [18, p. 116]. Fischer described the musical tunes of the clock [11]. As noted by Czermak, this clock was mistakenly attributed to Frater David by Luca [17, p. 86]. There are a number of other mentions in the 19th century, which I am not enumerating here. See more recently the account by Vrabec [24]. There is a third equation clock by Pater Aurelius, made in 1771 and kept in Vienna, see [6, p. 110-112], [1, p. 203], [16, p. 241] and [18, p. 115].

[6]When discussing Pater Aurelius' clock, Engelmann does illustrate another clock supposedly by Eberhart Camerhueber (or Camenhueber?) which has a case very similar to that of Pater Aurelius' clock, and dials which are somewhat simpler. This clock has a plate which gives Camerhueber's birth in 1749 [10], This clock was accompanied by a manuscript which seems to have been copied from the description of Pater Aurelius' clock [3]. It isn't clear who made the clockwork, and if Frater David was involved in the cases, as Engelmann believes. Oechslin mentions Camerhueber's clock, but doesn't have any information about it [20, p. 228]. According to a mention in the *Alt-Wien Kunst Chronik* from ca. 1948, p. 224, but which I wasn't able to check, Camerhueber's clock was previously in Dresden and its current (1948) location was not known. Perhaps it was destroyed during WWII.

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Figure 3.1: Pater Aurelius' clock in Munich (photograph by the author).

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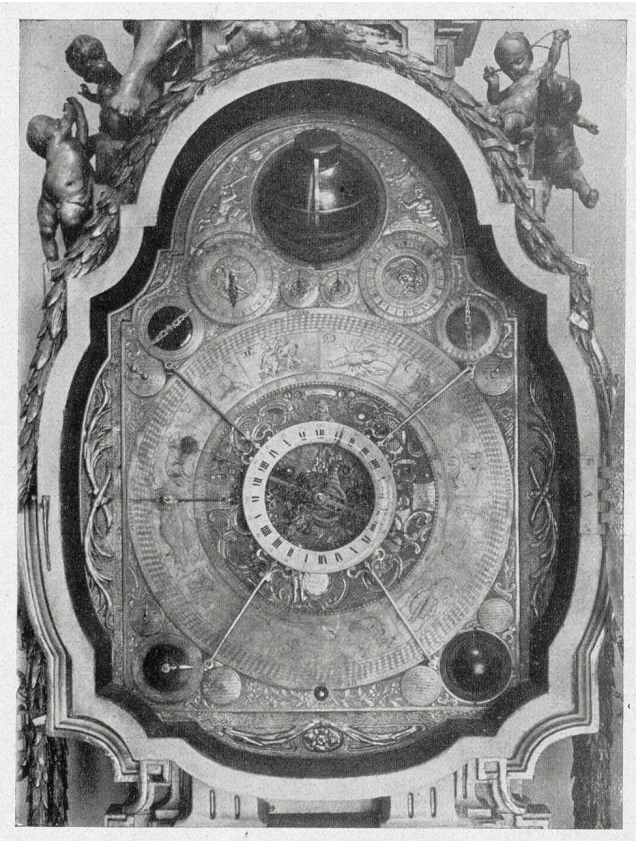


Figure 3.2: The dials of Pater Aurelius' clock in Munich. (source: [10])

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Figure 3.3: The dials of Pater Aurelius' clock in Munich (source: `https://www.bayerisches-nationalmuseum.de/sammlung/00070980`, licence: CC BY-NC-ND 4.0).

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Figure 3.4: The lower left part of the dials of Pater Aurelius' clock in Munich. (photograph by the author)

This arbor makes one turn clockwise in two hours.

The drum is on arbor 5 and its velocity is

$$V_5^0 = V_5^0 \times \left(-\frac{16}{76}\right) \times \left(-\frac{16}{80}\right) = V_5^0 \times \frac{4}{95} = -\frac{48}{95} \tag{3.5}$$

The drum thus makes a turn in a little less than two days. It can be rewound using a crank on arbor 1. The train rotates the winding wheel on arbor 4. The velocity of that arbor is

$$V_4^0 = V_5^0 \text{ (when the clock is not beeing rewound)} \tag{3.6}$$

and

$$V_1^0 = V_4^0 \times \left(-\frac{48}{35}\right) \times \left(-\frac{35}{35}\right) \times \left(-\frac{35}{35}\right) = V_5^0 \times \left(-\frac{48}{35}\right) \tag{3.7}$$

$$= \left(-\frac{48}{95}\right) \times \left(-\frac{48}{35}\right) = \frac{2304}{3325} \tag{3.8}$$

$$P_1^0 = \frac{3325}{2304} \text{ days} = 34.6354\ldots \text{ days} \tag{3.9}$$

One turn of the crank thus gets the clock working for almost 35 hours.

3.2.2 The main dial

The main dial has a wide ring with the signs of the zodiac and calendar rings. In the center, a smaller ring suspended by four arms shows the 24 hours. There are hands for the Sun, the Moon, the lunar apsides and the lunar nodes.

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Inside the large ring, there are four small openings for the year, openings for the day, the month, the epact, the dominical letter, the golden number, the indiction, the day of the week and the regent of the day, as well as an indicator for the leap year.

These motions are all derived from that of the central arbor 7 which makes one turn in two hours.

3.2.2.1 The time

The time is shown by two hands, one for the hours (on twice 12 hours) and one for the minutes (on twice 60 minutes). The use of such a dial for the minutes is rather unusual, but the minute hand is carried on arbor 7 which makes a turn in two hours.



Figure 3.5: The 24-hour dial on Pater Aurelius' clock in Munich (photograph by the author).

The hour hand is carried by tube 11 whose velocity is obtain via the intermediate arbor 10:

$$\mathrm{V}_{10}^0 = \mathrm{V}_7^0 \times \left(-\frac{22}{88}\right) = \mathrm{V}_7^0 \times \left(-\frac{1}{4}\right) = (-12) \times \left(-\frac{1}{4}\right) = 3 \tag{3.10}$$

$$\mathrm{V}_{11}^0 = \mathrm{V}_{10}^0 \times \left(-\frac{24}{72}\right) = 3 \times \frac{1}{3} = -1 \tag{3.11}$$

This hand thus makes a turn clockwise in one day.

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3.2.2.2 The motion of the Sun

The Sun is carried by the tube 20 and its motion is derived from that of arbor 18, which is itself derived from that of arbor 10 seen above. We have

$$\mathrm{V}^0_{18} = \mathrm{V}^0_{10} \times \left(-\frac{16}{134}\right) = 3 \times \left(-\frac{8}{67}\right) = -\frac{24}{67} \tag{3.12}$$

$$\mathrm{V}^0_{20} = \mathrm{V}^0_{18} \times \left(-\frac{12}{80}\right) \times \left(-\frac{8}{157}\right) = \mathrm{V}^0_{18} \times \frac{6}{785} \tag{3.13}$$

$$= -\frac{24}{67} \times \frac{6}{785} = -\frac{144}{52595} \tag{3.14}$$

$$\mathrm{P}^0_{20} = -\frac{52595}{144} = -365.2430\ldots \text{ days} \tag{3.15}$$

This is a (not so good) approximation of the tropical year. The same value is given by Oechslin. The signs of the zodiac and the calendar are given clockwise and the Sun moves clockwise around the dial in one year.

3.2.2.3 The motion of the Moon

The mean motion of the Moon is given by frame 13 whose motion is derived from that of tube 11. We first obtain the velocity of arbor 12 which will be useful later:

$$\mathrm{V}^0_{12} = \mathrm{V}^0_{11} \times \left(-\frac{27}{73}\right) = \frac{27}{73} \tag{3.16}$$

$$\mathrm{V}^0_{13} = \mathrm{V}^0_{12} \times \left(-\frac{19}{192}\right) = \frac{27}{73} \times \left(-\frac{19}{192}\right) = -\frac{171}{4672} \tag{3.17}$$

$$\mathrm{P}^0_{13} = -\frac{4672}{171} = -27.3216\ldots \text{ days} \tag{3.18}$$

This is an approximation of the tropical month, but not the best one. The same value is given by Oechslin.

3.2.2.4 The motion of the lunar apsides

The hands of the lunar apsides are carried by tube 17. The motion of this tube is derived from that of arbor 16, which is itself derived from that of arbor 12 seen above. We have

$$\mathrm{V}^0_{16} = \mathrm{V}^0_{12} \times \left(-\frac{19}{44}\right) \times \left(-\frac{6}{72}\right) = \mathrm{V}^0_{12} \times \frac{19}{528} = \frac{27}{73} \times \frac{19}{528} = \frac{171}{12848} \tag{3.19}$$

$$\mathrm{V}^0_{17} = \mathrm{V}^0_{16} \times \left(-\frac{4}{172}\right) = \frac{171}{12848} \times \left(-\frac{1}{43}\right) = -\frac{171}{552464} \tag{3.20}$$

$$\mathrm{P}^0_{17} = -\frac{552464}{171} = -3230.7836\ldots \text{ days} \tag{3.21}$$

This is an approximation of the precession of the lunar apsides in about 8.85 years. The same value is given by Oechslin. The sign is negative, because

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the motion of the apsides is prograde and the signs of the zodiac are arranged clockwise.

3.2.2.5 The motion of the lunar nodes

The hands (dragon) of the lunar nodes are carried by tube 22. Their motion is derived from that of arbor 16:

$$\mathrm{V}^0_{22} = \mathrm{V}^0_{16} \times \left(-\frac{8}{52}\right) \times \left(-\frac{12}{167}\right) = \mathrm{V}^0_{16} \times \frac{24}{2171} \tag{3.22}$$

$$= \frac{171}{12848} \times \frac{24}{2171} = \frac{513}{3486626} \tag{3.23}$$

$$\mathrm{P}^0_{22} = \frac{3486626}{513} = 6796.5419\ldots \text{ days} \tag{3.24}$$

This is an approximation of the precession of the lunar nodes in about 18.6 years. The same value is given by Oechslin. The sign is positive because the nodes move counterclockwise on the zodiac which is arranged clockwise, but it is in fact a retrogradation.

3.2.2.6 The corrected motion of the Moon

The motion of the mean Moon is corrected in order to take into account its elliptic orbit. The Moon seems to be corrected as a function of the angle between the mean Moon and the line of apsides. The tube 17 carries an eccentric cam under it, and this cam moves the lunar hand back and forth. How exactly this is done is not entirely clear and Oechslin mentions a carriage with a rack.

But on the front of the dial, the tube 17 also carries an eccentric ring (visible on photographs) which moves the lunar hand (of which the figure seems missing) farther or closer to the center. Pater Aurelius therefore both wanted to display the elliptic motion of the Moon, and to have the hand be ahead or behind the mean motion, depending on where the Moon is located with respect to the line of apsides.

This is interesting, but altering the distance of the Moon to the center seems to be a misunderstanding. Perhaps Pater Aurelius wanted to display the eclipses, but in that case, the lengthening of the lunar hand should have been a function of the line of nodes, not of the line of apsides, as it is for instance done in Schwilgué's astronomical clock in Strasbourg.

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3.2.3 The lunar sphere

The lunar sphere is at the lower right of the main dial and it is carried by the arbor 47. Its motion is derived from that of arbor 12 seen above. We have

$$\mathrm{V}^0_{47} = \mathrm{V}^0_{12} \times \left(-\frac{73}{54}\right) \times \left(-\frac{28}{41}\right) \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{44}\right) \times \frac{44}{44} \times \left(-\frac{6}{33}\right) \tag{3.25}$$

$$= \mathrm{V}^0_{12} \times \frac{4088}{44649} = \frac{27}{73} \times \frac{4088}{44649} = \frac{168}{4961} \tag{3.26}$$

$$\mathrm{P}^0_{47} = \frac{4961}{168} = 29.5297\ldots \text{ days} \tag{3.27}$$

This is an approximation of the synodic month, but also a rather bad one. The same value is given by Oechslin.



Figure 3.6: The lower right part of the dials of Pater Aurelius' clock in Munich, with the lunar sphere. (photograph by the author)

This sphere has a vertical axis and moves counterclockwise as seen from above (assuming Oechslin's drawing is correct). This is rather counterintuitive, as it means that the lunar crescent after a New Moon appears at the left, whereas it should appear on the right.

3.2.4 The celestial sphere

The celestial sphere above the main dial shows the apparent clockwise motion (from above) of the sky in one sidereal day. It also shows the motion of the Sun on the sphere. Finally, a fixed cage carries the meridian and the 24-hour dial at the top.

This system obtains its motion from the arbor 55. This arbor was apparently missing, as Oechslin puts the number of the teeth of its two pinions in

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parentheses. The velocity of this arbor (measured from the left on Oechslin's drawing) is derived from that of arbor 10 seen above in the motion of the hour hand:

$$\mathrm{V}^0_{55} = \mathrm{V}^0_{10} \times \frac{88}{11} = \mathrm{V}^0_{10} \times 8 = 3 \times 8 = 24 \tag{3.28}$$

$$\mathrm{P}^0_{55} = \frac{1}{24} = 1 \text{ hour} \tag{3.29}$$

This arbor makes a turn clockwise (seen from above) in one hour.

The arbor 55 drives the central arbor 56 of the sphere, which goes through the dial on the top. Its velocity is

$$\mathrm{V}^0_{56} = \mathrm{V}^0_{55} \times \left(-\frac{5}{120}\right) = \mathrm{V}^0_{55} \times \left(-\frac{1}{24}\right) = -1 \tag{3.30}$$

This arbor makes one turn clockwise (seen from above) in a day.

The globe itself is carried by the tube 60 whose motion is obtained through epicyclic gears located on a 120-teeth wheel on the frame 56 and using a 24-teeth wheel on the fixed tube 57. We first compute the motion of the globe with respect to the moving frame 56. We have

$$\mathrm{V}^{56}_{60} = \mathrm{V}^{56}_{57} \times \left(-\frac{24}{67}\right) \times \left(-\frac{12}{40}\right) \times \left(-\frac{4}{157}\right) = \mathrm{V}^{60}_{57} \times \left(-\frac{144}{52595}\right) \tag{3.31}$$

$$= -\mathrm{V}^{57}_{56} \times \left(-\frac{144}{52595}\right) = -\mathrm{V}^0_{56} \times \left(-\frac{144}{52595}\right) \tag{3.32}$$

$$= -\frac{144}{52595} \tag{3.33}$$

$$\mathrm{P}^{56}_{60} = -\frac{52595}{144} = -365.2430\ldots \text{ days} \tag{3.34}$$

This is the same ratio as the one used for the motion of the Sun in the main dial. The same value is given by Oechslin.

Then, in the absolute frame

$$\mathrm{V}^0_{60} = \mathrm{V}^{56}_{60} + \mathrm{V}^0_{56} = -\frac{144}{52595} - 1 = -\frac{52739}{52595} \tag{3.35}$$

$$\mathrm{P}^0_{60} = -\frac{52595}{52739} = -0.9972\ldots \text{ days} = -23 \text{ h } 56 \text{ m } 4.0910\ldots \text{ s} \tag{3.36}$$

This is the sidereal day and the globe thus makes one turn clockwise (from above) in one sidereal day.

The figure of the Sun moves on the sphere as it is carried by a ring with an internal gearing of 183 teeth on arbor 66. This ring presumably moves in the ecliptic. This motion is also obtained by an epicyclic gearing using another 24-teeth on the fixed tube 61. We can compute the motion of the Sun (ring 66) with respect to the globe (frame 60) (beware of the last fraction which is

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positive):

$$\mathrm{V}_{66}^{60} = \mathrm{V}_{61}^{60} \times \left(-\frac{24}{36}\right) \times \left(-\frac{20}{40}\right) \times \left(-\frac{10}{40}\right) \times \left(-\frac{24}{24}\right) \times \frac{6}{183} \tag{3.37}$$

$$= \mathrm{V}_{61}^{60} \times \frac{1}{366} = -\mathrm{V}_{60}^{61} \times \frac{1}{366} = -\mathrm{V}_{60}^{0} \times \frac{1}{366} = \frac{52739}{52595} \times \frac{1}{366} \tag{3.38}$$

$$= \frac{52739}{19249770} \tag{3.39}$$

$$\mathrm{P}_{66}^{60} = \frac{19249770}{52739} = 365.0006\ldots \text{ days} \tag{3.40}$$

The Sun thus moves counterclockwise with respect to the celestial sphere and makes a turn in slightly more than 365 days. The Sun should indeed move counterclockwise, but the motion should take place in a tropical year, and the value found here (not given by Oechslin) is extremely bad.

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3.2.5 The other dials

All the other dials and indications derive their motion from the motion of arbor 7 which makes one turn clockwise in two hours.

3.2.5.1 The oscillating indications

Six of the dials show indications with oscillate during the year. For instance, at our latitudes, the sunrise varies between about 4 a.m. in Summer and 8 a.m. in Winter. These motions are all based on the motion of an annual cam.

3.2.5.1.1 The Italic and Babylonian hours

The two medium-sized dials at the lower left and lower right of the celestial globe give the Italic and Babylonian hours, that is the number of hours since the last sunset and since the last sunrise. I believe that the Italic hours are shown on the dial left, and the Babylonian hours on the dial right.

There is only one hand on each dial and these hands make a turn in 24 hours. The hand of the dial on the left is carried by arbor 32, whereas the hand of the dial on the right is carried by arbor 30. The motion of these two arbors is derived from arbor 18, via arbor 23:

$$\mathrm{V}^0_{23} = \mathrm{V}^0_{18} \times \left(-\frac{134}{48}\right) = -\frac{24}{67} \times \left(-\frac{134}{48}\right) = 1 \tag{3.41}$$

$$\mathrm{V}^0_{30} = \mathrm{V}^0_{23} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) \times \left(-\frac{45}{45}\right) = -\mathrm{V}^0_{23} = -1 \tag{3.42}$$

$$\mathrm{V}^0_{32} = \mathrm{V}^0_{30} \times \frac{45}{24} \times \frac{24}{45} = \mathrm{V}^0_{30} = -1 \tag{3.43}$$

The dial on the left shows the astronomical hours from 1 to 24, presumably counted from noon. The dial on the right shows the common hours on a twice 12 hours dial. The Italic and Babylonian hours are shown using outer moving rings which have an oscillating motion. This is similar to the construction used by Frater David a Sancto Cajetano in the two clocks described in this volume.

The oscillation of the Italic and Babylonian hours is controlled by a cam located on arbor 33. This arbor makes a turn in one tropical year:

$$\mathrm{V}^0_{33} = \mathrm{V}^0_{20} \times \left(-\frac{157}{157}\right) = -\mathrm{V}^0_{20} = \frac{144}{52595} \tag{3.44}$$

$$\mathrm{P}^0_{33} = \frac{52595}{144} = 365.2430\ldots \text{ days} \tag{3.45}$$

This cam is used to move the tube 49 on the axis of the Babylonian hours, this causing the oscillation of the outer ring.

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3.2.5.1.2 The other oscillating motions

This oscillating motion of the Babylonian hour ring is also transferred to five other dials: the Italic hours (frame 53), the hour of sunrise (arbor 51), the hour of sunset (arbor 52), the lengths of day and night (arbor 50), and the declination of the Sun (arbor 54). During the year, all of these indications oscillate between two values. However, the amplitude of the oscillation varies. The small dials for the sunset (left) and sunrise (right), which are located under the celestial sphere, show variations of ± 2 hours around a median value, and this amplitude of four hours represents about 150 degrees, so that there would be only about ten hours on each dial. More precisely, the ratio between the wheels controlling the Babylonian hours and the sunrise is such as to adapt the amplitude of the Babylonian hours to the amplitude of the sunrise. In his clock, Pater Aurelius took the ratio 68/160. This means that the dials for the sunrise and sunset are virtually on $\frac{68}{160} \times 24$ hours, that is 10 hours and 12 minutes. This is a matter of choice, and Pater Aurelius could have chosen a different ratio. Only the small dials would have had to be changed.

The dial for the lengths of the day and night is located at the right of the dial for the Babylonian hours. It has one hand (arbor 50) of which the upper part gives the length of the night and the lower part the length of the day. The sum of the two lengths is always equal to 24. This dial seems to be divided in about 32 hours. Unfortunately, Oechslin didn't give the exact ratio between these two dials, he only stated that behind the dial for the lengths of the day and night there is a 27-teeth sector. However, whenener the Babylonian hour changes by one hour, the length of the day or night should change by two hours. Since there are about 32 hours in the day/night lengths dial, this corresponds to about 16 hours in the Babylonian hours dial. Hence, there should be a ratio of about 24/16 between the two dials. The full wheel should be about 240 teeth. More precise measurements should be made to obtain the exact value.

The same applies to the dial for the solar declination which is located at the left of the dial for the Italic hours. But here only the upper part of the hand (on arbor 54) is used to read the declination which varies between $-23.5°$ and $+23.5°$.

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3.2.5.2 The calendrical indications

A number of calendrical indications are shown between the wide zodiac ring and the 24-hour suspended dial (figure 3.7). These are the four digits of the year, the day of the month, the day of the week, the month, whether the year is a leap year or not, the day ruler (planet associated to the day), the indiction the dominical letter, the epact, and the golden number.

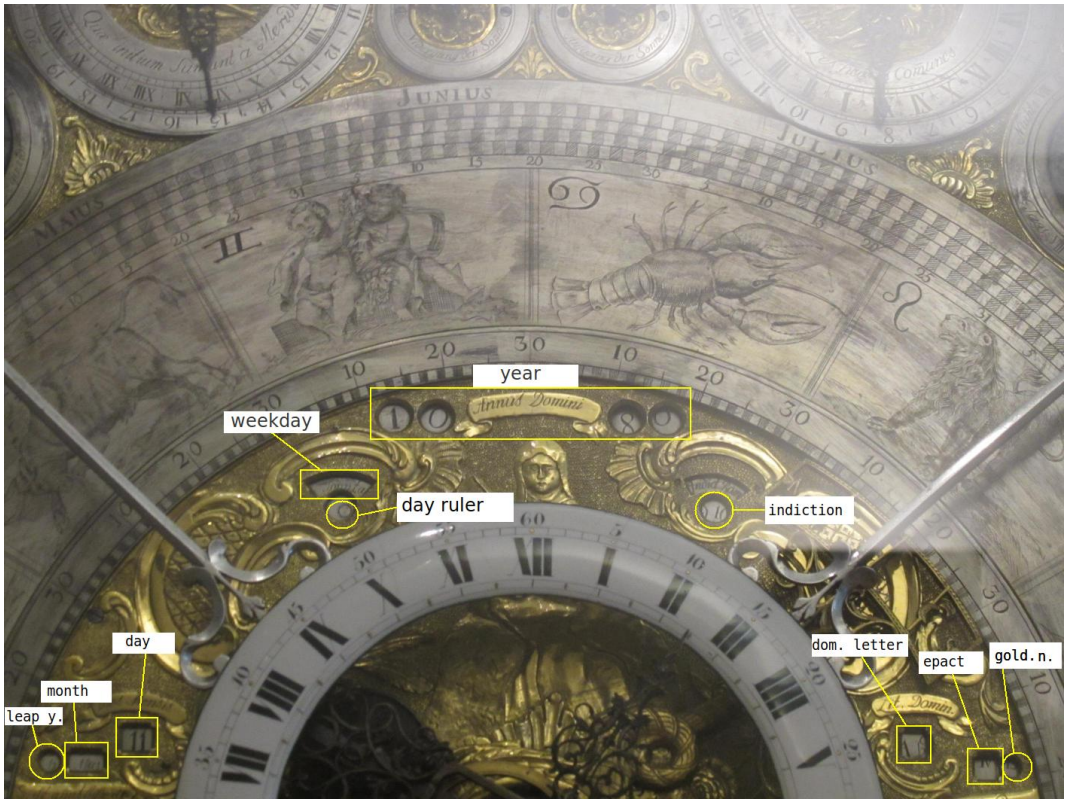


Figure 3.7: Detail of the signs of the zodiac and the openings for the year and other indications (photograph by the author).

3.2.5.2.1 The day and the derived indications

The day is updated through arbor 25 which makes a turn in a day:

$$V^0_{25} = V^0_{23} \times \left(-\frac{48}{48}\right) = -1 \tag{3.46}$$

A lever on this arbor moves the day of the month and there is a 4-year wheel for the lengths of the months. I will not describe this part in more details here.

The day of the week is on arbor 24 and its motion is also derived from

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arbor 23:

$$\mathrm{V}^0_{24} = \mathrm{V}^0_{23} \times \left(-\frac{1}{7}\right) = -\frac{1}{7} \tag{3.47}$$

$$\mathrm{P}^0_{24} = -7 \text{ days} \tag{3.48}$$

This arbor makes a turn clockwise in seven days. The dial with the names of the day probably also carries the planets ruling the days.

3.2.5.2.2 The yearly indications

All the other indications only change when the year changes. They are derived from the motion of arbor 33 which makes a turn in one tropical year.

The arbor 33 carries a finger which moves a 7-teeth wheel on arbor 34 by one tooth every year. This arbor thus makes a turn in seven years:

$$\mathrm{V}^0_{34} = \mathrm{V}^0_{33} \times \left(-\frac{1}{7}\right) \tag{3.49}$$

$$\mathrm{P}^0_{34} = -7 \text{ years} \tag{3.50}$$

Next, we obtain the motions of the arbors for the dominical letter (one turn in 28 years), the epact and golden number (one turn in 19 years) and the indiction (one turn in 15 years):

$$\mathrm{V}^0_{35} = \mathrm{V}^0_{34} \times \left(-\frac{21}{84}\right) = \mathrm{V}^0_{33} \times \frac{1}{28} \tag{3.51}$$

$$\mathrm{P}^0_{35} = 28 \text{ years} \tag{3.52}$$

$$\mathrm{V}^0_{36} = \mathrm{V}^0_{35} \times \left(-\frac{84}{57}\right) = \mathrm{V}^0_{33} \times \left(-\frac{1}{19}\right) \tag{3.53}$$

$$\mathrm{P}^0_{36} = -19 \text{ years} \tag{3.54}$$

$$\mathrm{V}^0_{37} = \mathrm{V}^0_{34} \times \left(-\frac{21}{45}\right) = \mathrm{V}^0_{33} \times \frac{1}{15} \tag{3.55}$$

$$\mathrm{P}^0_{37} = 15 \text{ years} \tag{3.56}$$

The arbor 37 is used to derive the motion of arbor 39:

$$\mathrm{V}^0_{39} = \mathrm{V}^0_{37} \times \left(-\frac{45}{30}\right) \times \left(-\frac{30}{30}\right) = \mathrm{V}^0_{37} \times \frac{3}{2} = \mathrm{V}^0_{33} \times \frac{1}{10} \tag{3.57}$$

$$\mathrm{P}^0_{39} = 10 \text{ years} \tag{3.58}$$

This arbor carries the units of the year and makes a turn in 10 years.

It also carries a finger which moves a 10-teeth wheel on arbor 40 by one tooth every 10 years:

$$\mathrm{V}^0_{40} = \mathrm{V}^0_{39} \times \left(-\frac{1}{10}\right) = \mathrm{V}^0_{33} \times \frac{1}{10} \times \left(-\frac{1}{10}\right) = \mathrm{V}^0_{33} \times \left(-\frac{1}{100}\right) \tag{3.59}$$

$$\mathrm{P}^0_{40} = -100 \text{ years} \tag{3.60}$$

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This arbor carries the tens of the year and makes a turn in 100 years.

This arbor also carries a finger which moves a 10-teeth wheel on arbor 41 by one tooth every 100 years:

$$\mathrm{V}_{41}^{0} = \mathrm{V}_{40}^{0} \times \left(-\frac{1}{10}\right) = \mathrm{V}_{33}^{0} \times \left(-\frac{1}{100}\right) \times \left(-\frac{1}{10}\right) = \mathrm{V}_{33}^{0} \times \frac{1}{1000} \tag{3.61}$$

$$\mathrm{P}_{41}^{0} = 1000 \text{ years} \tag{3.62}$$

This arbor carries the hundreds of the year and makes a turn in 1000 years.

It isn't clear if there is a wheel for the thousands of the year, or if other digits than 1 are possible.

3.3 Conclusion

Compared to other clocks in this volume, Pater Aurelius' clock is not as complex as it seems. The most surprising, though, is that many of the periods used in this clock (for instance the motion of the Sun on the celestial sphere) are not very accurate.

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3.4 References

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Chapter 4

(Oechslin: 6.1)

Frater David's first clock in Vienna (1769)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

4.1 Introduction

The clock described here was constructed in 1769 by David Ruetschmann, who went by the name Frater David a Sancto Cajetano (1726-1796). I am giving here a summary of his life, of which we don't know very much.[1] He was born in Lembach, in the Black Forest (Germany). When he was 12 years old, he became apprentice to a cabinetmaker. When he was 20 years old, he travelled as a journeyman to Vienna. He worked there as a cabinetmaker for 8 years. In 1754 he made the religious vows in the order of the Augustinians in the Mariabrunn monastery near Vienna. In 1760, he was moved to Vienna, so that he could have better opportunities for his scientific development. There he studied mathematics and astronomy. He became close to the mathematics professors Joseph Walcher and Wilhelm Bauer and this started his inventive activity.[2]

[1] This account is mostly based on Kunitsch's account [37, p. 71-75] and lost writings by Joseph Rendler and told by Engelmann [21]. See also Czermak [10, 11], Bassermann-Jordan [7], Kaftan [31], Bertele [8, 9], Lloyd [38], Killian [35, 36], Maurice [47, v.1, p. 275, 278-279], Abeler [1], and the collective "*Himmlisches Räderwerk*" [46]. There are many other mentions and short biographies of Frater David, and I am not making any attempt at exhaustivity. See the references at the end of this chapter. In his *Cornelius Nepos der Uhrmacher* [20], Dietzschold compared Frater David to Antide Janvier in France, but this comparison seems excessive. An account of Frater David's encounter with the Emperor Joseph II is given in [4, 5] and by Killian [35, p. 27-33].

[2] Kunitsch writes that Frater David attended Walcher's Sunday lectures on mechanics for craftsmen and artists and consequently developed new mathematical theories [37, p. 74-75]. Kunitsch, however, does not mention the construction of any clock.

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The present clock is Frater David's first great clock and he built several more, including the one for the Prince of Schwarzenberg (Oechslin 6.2).[3] Around 1770, Frater David also restored the astronomical clock made by Christoph Schenner (or Schöner?) or Carl Graff from Augsburg in 1702. This clock was almost 6 meters tall and was presumably destroyed in 1848.[4] In 1772, he has also repaired the clock of St. Stephen's Cathedral in Vienna [30].

In the 1780s, Frater David restored Neßtfell's planetarium in Vienna.[5] This may have brought him to develop his theory of epicyclic gears.[6]

Frater David also had a nephew Joseph Ruetschmann (1755-1801) who was clockmaker in Vienna.[7] He constructed the clock designed by his uncle for the Palais Schwarzenberg around 1793 (Oechslin 6.2), as well as a clock with a small orrery on top of it.[8]

The clock described in this chapter was located from 1769 to 1835 in the Vienna Augustine monastery.[9] Then, it was sold to someone in Hungary, the city of Vienna having failed to acquire it. It was bought back in 1860 by the Viennese clockmaker Ignaz Lutz who restored it in 1862 and sold it in 1866 to abbot Augustin Steininger in the Zwettl abbey for 1200 gulden. This abbot was also a mechanician and built several clocks. Lutz died around 1880 and Frater David's clock was again restored in 1886 and 1901 by Mathias Römer,

[3]Bertele ([8, p. 24-25], [9, p. 110-115]) lists three clocks, those described in this book, and a clock made in 1793 by Ignaz Berlinger after Frater David's design, which only slightly differs from the Schwarzenberg clock. Kaltenböck mentions four clocks, those mentioned by Bertele, and a clock made in 1777 [33, p. 38-41]. In the 1996 exhibition catalogue, only three clocks are shown, not the one from 1777 [46].

[4]See the accounts of Fidler [22, p. 204-208], Scheiger [56, p. 60-65], Czermak [11] and especially Maurice [47, v.1, p. 188-192] and Killian [35] who show engravings of the clock. An engraving of the clock is also shown by Mattl-Wurm [46, p. 36].

[5]See [22, p. 4] and [52, p. 227]. Czermak also mentions the restoration of Neßtfell's planetarium, but around 1773 [11], and he does not seem to know that this clock is not lost.

[6]See Frater David's books [12, 13, 14] and the series of articles published shortly before his death [15, 16, 17, 18, 19].

[7]Some accounts have listed Joseph Ruetschmann as Frater David's brother [8, p. 24], but this is incorrect.

[8]See [29], [33, p. 43] and [25, p. 127-128]. This orrery shows the planets up to Uranus and was made very compact. It uses a number of interior gearings [29]. An anonymous notice in 1798 mentions another orrery-clock by Joseph Rutschmann, made in 1798, but which seems different from the one kept in Francfort [2]. It is possibly the same orrery-clock as the one mentioned in the Salzburger Zeitung in 1884 [6]. The author of the note moreover assumed that the Salzburg orrery was made by David Ruetschmann. When Hügin described the Francfort orrery clock, he didn't mention Salzburg [29]. Another tall-case clock by Joseph Ruetschmann is illustrated by Lunardi [44, p. 68]. See also [45, p. 434-435].

[9]There are many mentions, reproductions and sometimes detailed descriptions of this clock. It was first described by Joseph Rendler in 1771 [53]. Other descriptions are those of Schulte [57, p. 104-109], Frischholz [24, p. 198-200] and Kames [34, p. 415]. The most comprehensive description is that of the "*Himmlisches Räderwerk*" volume [46]. Detailed descriptions and analysis of the gears were also published by Lunardi [42, 43, 44, 45] and White [60, 61, 62] who focused on Frater David's new techniques for gear trains, especially for the Schwarzenberg clock. See also the 1989 exhibition catalogue [59, p. 55-56] and the c1989 catalogue from the *Uhrenmuseum* [55, p. 57-58].

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Lutz's successor. During that time, the clock was located in the library of the Zwettl abbot. The clock was finally bought back from this abbey in 1928 and is now located in the Vienna clock museum.[10]

The clock is housed in a case 265 cm tall, 77 cm wide and 49 cm deep. It has dials on the front and rear sides. The front side (figure 4.3) has a number of dials and carries the date and signature of Fr. David a. S. Cajetano.

Although the clock looks very complicated, this is mainly an illusion due to the great number of dials. The clock is in fact rather simple, as we will see. However, making this simplicity visible has been difficult, because of Oechslin's drawing which splits the gears in three parts, with many overlaps. The difficulty is the intrication of the gear trains, not the trains themselves.

I will distinguish the front and the rear of the clock.

The front side of the clock is made of two parts. The lower part has a central dial showing the time, and it is surrounded by twelve smaller dials of the same size and showing various astronomical or calendrical indications.

These dials are in turn surrounded by a large ring which is the annual calendar. A number of hands rotate around the center of this part and they will be described later.

The upper part of the front side shows the motion of the Sun, the Moon and the lunar nodes, pictured as a dragon. It also shows the italic hours counted from last sunset.

The rear side of the clock gives the day of the month as well as the true solar time.

4.2 The going work

The clock is driven by a 26 kg lead weight and regulated by a Graham escapement. The pendulum is a seconds pendulum, that is it makes a half oscillation in one second. Once rewound, the clock can work for about 32 days.

The arbor 6 is the arbor driving the dials on the rear side and it makes one turn couterclockwise in one hour as seen from the back:

$$\mathrm{V}_6^0 = 24 \tag{4.1}$$

From this, we derive the velocity of arbor 5, but this time measured from the front side. (This may seem a bit annoying, but I do not want to measure the velocities of an arbor from two different vantage points, and one motion goes to the front, whereas the other one goes to the rear side.) We have

$$\mathrm{V}_5^0 = \mathrm{V}_6^0 \times \frac{12}{96} = 24 \times \frac{1}{8} = 3 \tag{4.2}$$

[10]Details of the history of the clock are given by Rössler [54], Czermak [11], Loeske [39], Dietzschold [20], Kaftan [31], and Oechslin [52, p. 233]. However the older accounts (for instance Czermak's) are partly inaccurate. Loeske also mentions the "*Oesterreichisch-Ungarische Uhrmacher-Zeiting*" which I haven't seen.

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Figure 4.1: Frater David's clock. (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (`https://sammlung.wienmuseum.at/objekt/382414`))

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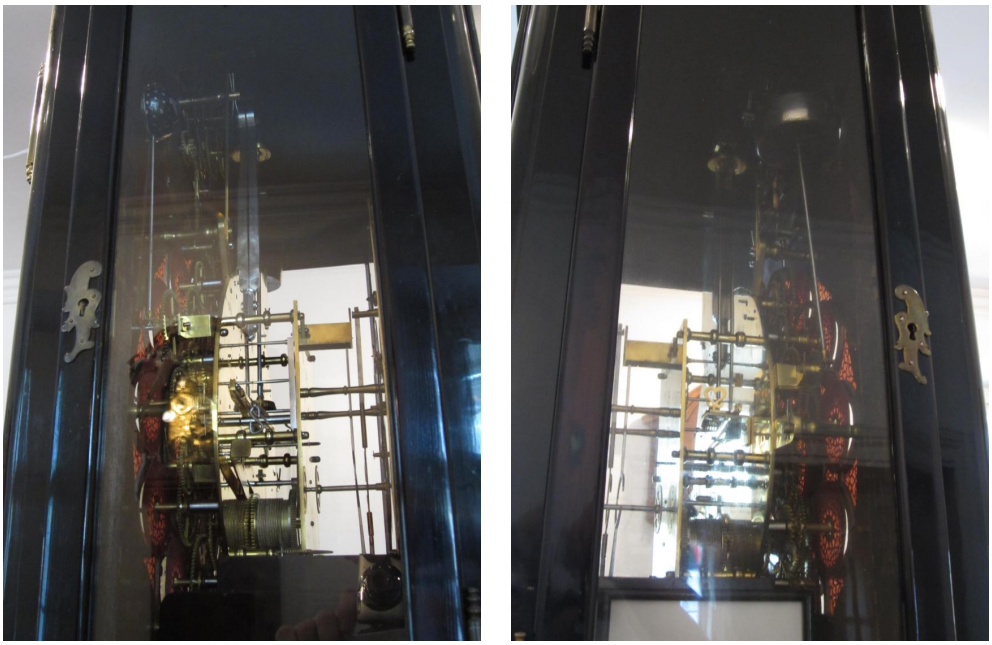


Figure 4.2: The mechanism of Frater David's clock seen from the sides. (photograph by the author)

The drum is on arbor 4 and we also measure it from the front:

$$\mathrm{V}_4^0 = \mathrm{V}_5^0 \times \left(-\frac{12}{96}\right) = 3 \times \left(-\frac{1}{8}\right) = -\frac{3}{8} \tag{4.3}$$

$$\mathrm{P}_4^0 = -\frac{8}{3} \text{ days} = 64 \text{ hours} \tag{4.4}$$

Arbor 1 is the winding and it is located below the bottom hour dial, just outside of the calendar squares. If the arbor 1 were to turn with the drum, we would have

$$\mathrm{V}_1^0 = \mathrm{V}_4^0 \times \left(-\frac{50}{35}\right) \times \left(-\frac{35}{35}\right) \times \left(-\frac{35}{35}\right) = \mathrm{V}_4^0 \times \left(-\frac{10}{7}\right) \tag{4.5}$$

$$= \left(-\frac{3}{8}\right) \times \left(-\frac{10}{7}\right) = \frac{15}{28} \tag{4.6}$$

$$\mathrm{P}_1^0 = \frac{28}{15} \text{ days } = 44 \text{ h } 48 \text{ m} \tag{4.7}$$

So, one turn of the crank makes the clock work for almost 45 hours.

Finally, the escape wheel is on arbor 10 and its velocity (measured from the rear) is:

$$\mathrm{V}_{10}^0 = \mathrm{V}_6^0 \times \left(-\frac{80}{10}\right) \times \left(-\frac{75}{10}\right) = \mathrm{V}_6^0 \times 60 = 24 \times 60 = 1440 \tag{4.8}$$

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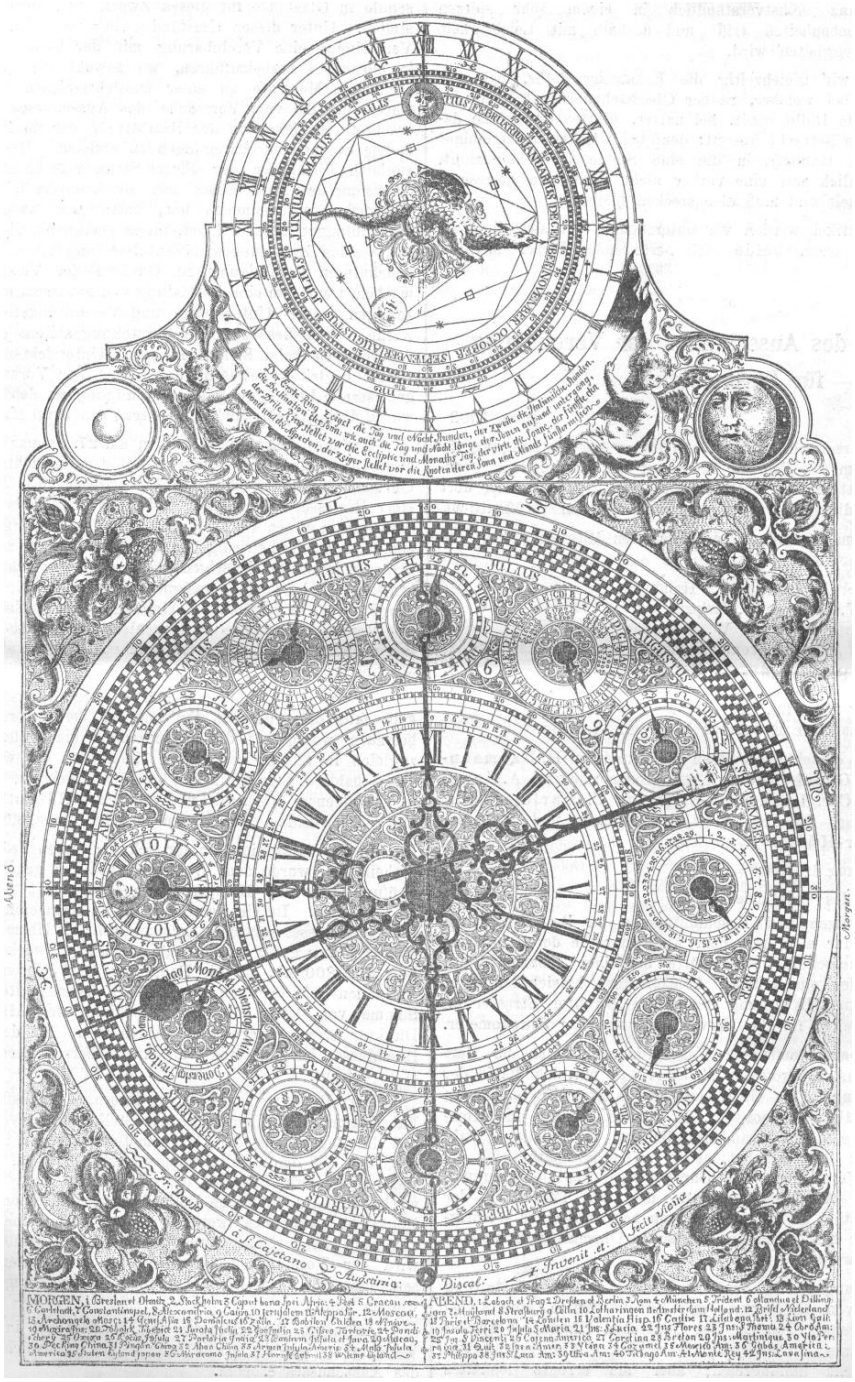


Figure 4.3: The front side of Frater David's clock, in Loeske's reproduction [39] of the engraving published by Rendler in 1771 [53].

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The escape wheel therefore makes a turn counterclockwise (from the rear) in one minute. It carries 30 teeth and the pendulum therefore must make a half-oscillation in one second. Its length is about 1 meter.

Next, we describe the rear side, which is the simplest part.

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4.3 The rear side

The rear side of the clock shows a dial for the time, topped by a semi-circulair dial for the day of the month.

The input for the rear side is arbor 6 which makes one turn counterclockwise (as seen from the rear) in one hour. All the motions on this side are measured from this side and this will be implicit from now on.



Figure 4.4: The rear side of Frater David's clock (from the museum page). (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (`https://sammlung.wienmuseum.at/objekt/382414`))

From arbor 6, we obtain the motion of the central arbor 9 of the time dial:

$$\mathrm{V}_9^0 = \mathrm{V}_6^0 \times \left(-\frac{60}{60}\right) = -24 \tag{4.9}$$

This arbor makes one turn clockwise in an hour and carries the hand of the minutes of mean time.

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4.3.1 The day of the month

The day of the month is shown on a semi-circular and goes from 1 to 31, then springs again back to 1.

The motion of arbor 9 is first used to obtain the motion of arbor 21. We have:

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{9}^{0} \times \left(-\frac{16}{48}\right) \times \left(-\frac{20}{80}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}_{9}^{0} \times \left(-\frac{1}{24}\right) = 1 \qquad (4.10)$$

This arbor thus makes one turn counterclockise in a day. It is used to update the hand for the days, but I will not describe this part in detail. The lengths of the months, as well as the ordinary leap years, are taken into account, so that the hand sometimes jumps from 28 to 1, from 29 to 1, or from 30 to 1.

4.3.2 The time dial

The central arbor 9 actually moves an entire frame containing a number of gears. The rotation of this frame causes the motion of the internal gears, as a consequence of the fixed tube 10. The last arbor 14 of the internal train carries a 10-teeth wheel meshing with a 180-teeth wheel on tube 15 which does itself carry the equation cam. We only compute the motion of tube 15 in the rotating frame 9:

$$\mathrm{V}_{10}^{9} = -\mathrm{V}_{9}^{10} = -\mathrm{V}_{9}^{0} = 24 \qquad (4.11)$$

$$\mathrm{V}_{15}^{9} = \mathrm{V}_{10}^{9} \times \left(-\frac{24}{40}\right) \times \left(-\frac{9}{48}\right) \times \left(-\frac{8}{48}\right) \times \left(-\frac{8}{73}\right) \times \left(-\frac{10}{180}\right) \qquad (4.12)$$

$$= \mathrm{V}_{10}^{9} \times \left(-\frac{1}{8760}\right) = 24 \times \left(-\frac{1}{8760}\right) = -\frac{1}{365} \qquad (4.13)$$

It would not be meaningful to compute the motion of tube 15 in the absolute frame, because Frater David has the feeler and rack of the equation mechanism fixed on the moving frame 9. It is therefore important that the cam makes a turn in a year with respect to the moving frame, which it does. Actually, Frater David could have done better, and it is surprising that he contented himself with such an approximation (365 days) of the tropical year.

The rack linked to the feeler meshes with a 24-teeth wheel on tube 16 which carries the hand for the minutes of true time. In addition the hours of true time are obtained with a train fixed on the fixed frame 10.

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4.4 The front side

The front side of the clock is made of two parts. The lower part has a central dial showing the time, and it is surrounded by twelve smaller dials of the same size and showing various astronomical or calendrical indications. Starting at the bottom and going clockwise, we have 1) the time, with hour and minute hands, 2) the tropical motion of Mercury, 3) the day of the week, 4) the draconic month, 5) the tropical motion of Jupiter, 6) the epacts, the golden number and the indiction, 7) the tropical motion of Saturn as well as the prograde motion of the Earth's perihelion, 8) the dominical letter and the solar cycle, 9) the tropical motion of Mars, 10) the age of the Moon, 11) the anomalistic month (angle from perigee), and 12) the tropical motion of Venus.

These dials have been one source of inspiration for Seige's 1791 clock (Oechslin 10.1), but Seige arranged the dials on two opposite faces, and also added the motion of Uranus.

These dials are in turn surrounded by a large ring which is the annual calendar [52, p. 44]. A number of hands rotate around the center of this part and they will be described later.

The upper part of the front side shows the motion of the Sun, the Moon and the lunar nodes, pictured as a dragon. It also shows the italic hours counted from last sunset.

There are also two openings on the sides of this dial and showing the size of a lunar eclipse and the lunar phase.

The figures of the year are shown through for small apertures.

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Figure 4.5: The front side of Frater David's clock (from the museum page). (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (`https://sammlung.wienmuseum.at/objekt/382414`))

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Figure 4.6: The gears behind the dials of the front side of Frater David's clock (from the museum page). (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (`https://sammlung.wienmuseum.at/objekt/382414`))

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I will first analyze the yearly motions (motion of the Sun and computus), then the dials of the time and the day of the week, then those of the planets, then the motion of the line of the apsides (on the Saturn dial), then those of the Moon, including the display of the lunar phases and the eclipse window. Afterwards, I will consider the motion of the other central hands on the main dial, then the upper part of the clock. All of these motions take their source in the motion of arbor 5 from the going work, whose velocity (measured from the front) was computed above and is

$$\mathrm{V}_5^0 = 3 \tag{4.14}$$

$$\mathrm{P}_5^0 = 8 \text{ hours} \tag{4.15}$$

All of the other velocities on the front side will implicitely be measured from the front.

4.4.1 The yearly motions

4.4.1.1 The annual motion of the Sun

The Sun is pictured by a hand on the central dial. It is carried by tube 34. The motion of this tube is obtained through a train in which we also explicit the motions of tube 26, 30 and 33 which will be useful later. The last wheel has 269 teeth, but these teeth have been unevenly cut [52, p. 137]. We have:

$$\mathrm{V}_{26}^0 = \mathrm{V}_5^0 \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{72}\right) = \mathrm{V}_5^0 \times \frac{1}{3} = 1 \tag{4.16}$$

$$\mathrm{P}_{26}^0 = 1 \text{ day} \tag{4.17}$$

$$\mathrm{V}_{30}^0 = \mathrm{V}_{26}^0 \times \left(-\frac{24}{78}\right) = -\frac{4}{13} \tag{4.18}$$

$$\mathrm{V}_{33}^0 = \mathrm{V}_{30}^0 \times \left(-\frac{24}{94}\right) \times \left(-\frac{15}{15}\right) \times \left(-\frac{15}{64}\right) \tag{4.19}$$

$$= \mathrm{V}_{30}^0 \times \left(-\frac{45}{752}\right) = \left(-\frac{4}{13}\right) \times \left(-\frac{45}{752}\right) = \frac{45}{2444} \tag{4.20}$$

and finally

$$\mathrm{V}_{34}^0 = \mathrm{V}_{33}^0 \times \left(-\frac{40}{269}\right) = \frac{45}{2444} \times \left(-\frac{40}{269}\right) = -\frac{450}{164359} \tag{4.21}$$

$$\mathrm{P}_{34}^0 = -\frac{164359}{450} = -365.2422\ldots \text{ days} - 365 \text{ d } 5 \text{ h } 48 \text{ m } 48 \text{ s} \tag{4.22}$$

The same value is given by Oechslin. This is an approximation of the tropical year.[11] The solar hand moves clockwise along the signs of the zodiac, but also along the calendar rings [52, p. 44].

[11] Incidentally, this value for the tropical year is exactly the one used by Jean-Baptiste Schwilgué (1776-1856) in the Strasbourg astronomical clock [58].

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However, because the 269-teeth wheel has unevenly cut teeth (Oechslin counted 132 teeth on one half and 137 on the other), its motion is also uneven, and the solar hand has an uneven motion. This is meant to account for the equation of center, and it is the only mechanism in this book with such features.

The 269-teeth wheel also carries a finger or pin which is used to update the indication of the year using a carry mechanism.

4.4.1.2 The epacts, the golden number and the indiction

This dial shows the golden number (from 1 to 19), the indiction (from 1 to 15) and two series of epacts. The outer series is that valid for the 18th and 19th centuries, the inner series is valid for the 20th, 21st and 22nd centuries. The same dial is found in Seige's 1791 clock (Oechslin 10.1) who obviously copied it.



Figure 4.7: The dial for the epacts, the golden number and the indiction on Frater David's clock. (photograph by the author)

The motions of the hands for the epacts, the golden number and the indiction are derived from the motion of tube 34 seen above. We first compute the motion of arbor 99, isolating arbor 66 which is useful later:

$$\mathrm{V}^0_{66} = \mathrm{V}^0_{34} \times \left(-\frac{18}{72}\right) \times \left(-\frac{24}{70}\right) = \mathrm{V}^0_{34} \times \frac{3}{35} \tag{4.23}$$

$$\mathrm{V}^0_{99} = \mathrm{V}^0_{66} \times \left(-\frac{70}{90}\right) = \mathrm{V}^0_{34} \times \left(-\frac{1}{15}\right) \tag{4.24}$$

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and then

$$V^0_{100} = V^0_{99} \times \left(-\frac{90}{90}\right) = -\frac{450}{164359} \times \left(-\frac{1}{15}\right) \tag{4.25}$$

$$P^0_{100} = -\frac{164359}{450} \times 15 = -15 \text{ tropical years} \tag{4.26}$$

$$V^0_{101} = V^0_{99} \times \left(-\frac{45}{57}\right) = V^0_{34} \times \left(-\frac{1}{15}\right) \times \left(-\frac{45}{57}\right) = V^0_{34} \times \frac{1}{19} \tag{4.27}$$

$$= -\frac{450}{164359} \times \frac{1}{19} \tag{4.28}$$

$$P^0_{101} = -\frac{164359}{450} \times 19 = -19 \text{ tropical years} \tag{4.29}$$

So, the central arbor 100 of this dial makes a turn clockwise in 15 tropical years and it carries the indiction hand. The tube 101 makes a turn clockwise in 19 years and carries the hand indicating the epacts and the golden number. (Note that Oechslin's drawing incorrectly swaps the indiction and the epacts.)

It is of course surprising that Frater David doesn't switch the epacts and the indiction at the New Year, but has them gradually change throughout the year. This is far from adequate.

Although this motion is derived from that of tube 34 which is uneven, this is of no consequences, because the uneveness of tube 34 has a period of one year. So, after one year, the tube 34 will always have made one turn.

4.4.1.3 The dominical letter and the solar cycle



Figure 4.8: The dial for the dominical letter and the solar cycle on Frater David's clock. (photograph by the author)

The hand of the dominical letter and solar cycle is on arbor 68 and its motion is derived from that of arbor 66 (the motion of arbor 67 is used below

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for the motion of the apsides on the dial of Saturn):

$$V^0_{67} = V^0_{66} \times \left(-\frac{30}{72}\right) = V^0_{66} \times \left(-\frac{5}{12}\right) = V^0_{34} \times \frac{3}{35} \times \left(-\frac{5}{12}\right) \tag{4.30}$$

$$= V^0_{34} \times \left(-\frac{1}{28}\right) \tag{4.31}$$

$$V^0_{68} = V^0_{67} \times \left(-\frac{72}{72}\right) = V^0_{34} \times \frac{1}{28} = -\frac{450}{164359} \times \frac{1}{28} \tag{4.32}$$

$$P^0_{68} = -\frac{164359}{450} \times 28 = -28 \text{ tropical years} \tag{4.33}$$

This hand makes a turn clockwise in 28 years. It is also moving gradually throughout the year.

The correspondence between the dominical letter and the solar cycle is only valid in the 18th century.

4.4.2 The time of the day and the day of the week

4.4.2.1 The time dial



Figure 4.9: The dial for the time on Frater David's clock. (photograph by the author)

This dial gives the minutes and hours of mean time. The minute hand is carried by arbor 114 and the hour hand by tube 116. Both are derived from the motion of arbor 5, but the motion of the hours is not derived as usual from the motion of the minutes. We first derive the motion of arbor 112:

$$V^0_{112} = V^0_5 \times \left(-\frac{96}{24}\right) = 3 \times (-4) = -12 \tag{4.34}$$

$$P^0_{112} = -2 \text{ hours} \tag{4.35}$$

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Then we have

$$V^0_{114} = V^0_{112} \times \left(-\frac{60}{30}\right) \times \left(-\frac{60}{60}\right) = V^0_{112} \times 2 = -24 \tag{4.36}$$

$$P^0_{114} = -1 \text{ hour} \tag{4.37}$$

$$V^0_{116} = V^0_{112} \times \left(-\frac{60}{60}\right) \times \left(-\frac{12}{72}\right) = V^0_{112} \times \frac{1}{6} = -2 \tag{4.38}$$

$$P^0_{116} = -12 \text{ hours} \tag{4.39}$$

Hence, arbor 114 makes a turn clockwise in one hour, and tube 116 makes a turn clockwise in twelve hours.

4.4.2.2 The day of the week



Figure 4.10: The dial for the day of the week on Frater David's clock. (photograph by the author)

The day of the week is given by the hand carried by arbor 92. The motion of this arbor is derived from that of arbor 30 seen above. We have

$$V^0_{92} = V^0_{30} \times \left(-\frac{78}{56}\right) \times \left(-\frac{24}{72}\right) = V^0_{30} \times \frac{13}{28} = -\frac{4}{13} \times \frac{13}{28} = -\frac{1}{7} \tag{4.40}$$

$$P^0_{92} = -7 \text{ days} \tag{4.41}$$

Arbor 92 makes a turn clockwise in seven days.

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Figure 4.11: The dial for the tropical motion of Mercury on Frater David's clock. (photograph by the author)

4.4.3 The motion of the planets

4.4.3.1 The tropical motion of Mercury

The motion of Mercury is given by the arbor 94 and is derived from arbor 92. We have

$$\mathrm{V}^0_{94} = \mathrm{V}^0_{92} \times \left(-\frac{16}{58}\right) \times \left(-\frac{15}{52}\right) = \mathrm{V}^0_{92} \times \frac{30}{377} = -\frac{1}{7} \times \frac{30}{377} \tag{4.42}$$

$$= -\frac{30}{2639} \tag{4.43}$$

$$\mathrm{P}^0_{94} = -\frac{2639}{30} = -87.9666\ldots \text{ days} = -87 \text{ d } 23 \text{ h } 12 \text{ m} \tag{4.44}$$

The same value is given by Oechslin. This is an approximation of Mercury's tropical orbital period. The velocity is negative, because the signs of the zodiac are marked clockwise.

Incidentally, the 2010 museum guide states that Mercury rotates with its sidereal period, and that the period is 87 days 23 hours 14 minutes and 4 seconds [23]. It isn't clear where this value was taken, but Oechslin finds the same value as I do. I see no reason to assume that the motions of the planets on the clock are not the tropical ones.

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Figure 4.12: The dial for the tropical motion of Venus on Frater David's clock. (photograph by the author)

4.4.3.2 The tropical motion of Venus

The motion of Venus is given by the arbor 86 and is derived from tube 26. We have:

$$\mathrm{V}^0_{74} = \mathrm{V}^0_{26} \times \left(-\frac{24}{75}\right) = -\frac{8}{25} \tag{4.45}$$

$$\mathrm{V}^0_{86} = \left(-\frac{8}{25}\right) \times \left(-\frac{10}{10}\right) \times \left(-\frac{10}{53}\right) \times \left(-\frac{18}{66}\right) \times \left(-\frac{20}{74}\right) \tag{4.46}$$

$$= -\frac{96}{21571} \tag{4.47}$$

$$\mathrm{P}^0_{86} = -\frac{21571}{96} = -224.6979\ldots \text{ days} = -224 \text{ d } 16 \text{ h } 45 \text{ m} \tag{4.48}$$

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Venus. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin, but, again, the 2010 museum guide gives a different period of 224 days 16 hours 48 minutes, without giving any source [23].

4.4.3.3 The tropical motion of Mars

The hand of Mars is located on arbor 89. This motion is derived from that of arbor 80. We first obtain the motion of arbor 75, which is derived from that of arbor 74 seen above. We have

$$\mathrm{V}^0_{75} = \mathrm{V}^0_{74} \times \left(-\frac{75}{48}\right) = \left(-\frac{8}{25}\right) \times \left(-\frac{75}{48}\right) = \frac{1}{2} \tag{4.49}$$

$$\mathrm{V}^0_{80} = \mathrm{V}^0_{75} \times \left(-\frac{12}{31}\right) \times \left(-\frac{16}{31}\right) = \mathrm{V}^0_{75} \times \frac{192}{961} = \frac{1}{2} \times \frac{192}{961} = \frac{96}{961} \tag{4.50}$$

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Figure 4.13: The dial for the tropical motion of Mars on Frater David's clock. (photograph by the author)

and finally

$$V^0_{89} = V^0_{80} \times \left(-\frac{20}{61}\right) \times \left(-\frac{9}{45}\right) \times \left(-\frac{12}{54}\right) = V^0_{80} \times \left(-\frac{8}{549}\right) \tag{4.51}$$

$$= \frac{96}{961} \times \left(-\frac{8}{549}\right) = -\frac{256}{175863} \tag{4.52}$$

$$P^0_{89} = -\frac{175863}{256} = -686.9648\ldots \text{ days} = -686 \text{ d } 23 \text{ h } 9 \text{ m } 22.5 \text{ s} \tag{4.53}$$

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Mars. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin, but, again, the 2010 museum guide gives a different period of 1y 321d 23h 31m 56s, without giving any source [23]. Moreover, if we subtract the value found above for the tropical year to Mars' orbital period, we find 321 d 17 h 20 m 34.5 s, and not the value stated in the guide.

4.4.3.4 The tropical motion of Jupiter

The hand of Jupiter is located on arbor 60. Its motion is derived from that of arbor 33 seen earlier. We have

$$V^0_{60} = V^0_{33} \times \left(-\frac{18}{48}\right) \times \left(-\frac{16}{82}\right) \times \left(-\frac{12}{70}\right) = V^0_{33} \times \left(-\frac{18}{1435}\right) \tag{4.54}$$

$$= \frac{45}{2444} \times \left(-\frac{18}{1435}\right) = -\frac{81}{350714} \tag{4.55}$$

$$P^0_{60} = -\frac{350714}{81} = -4329.8024\ldots \text{ days} \tag{4.56}$$

$$= -4329 \text{ d } 19 \text{ h } 15 \text{ m } 33.3333\ldots \text{ s} \tag{4.57}$$

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Figure 4.14: The dial for the tropical motion of Jupiter on Frater David's clock. (photograph by the author)

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Jupiter. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin, although he did not correctly convert it to hours, minutes and seconds. If we assume the tropical year of 365d 5h 48m 48s, this is then 11 years and 312 days 3 hours 18 minutes and 45.3333... s, but the 2010 museum guide gives a different period of 11 years 314d 22h, without giving any source [23].

4.4.3.5 The tropical motion of Saturn



Figure 4.15: The dial for the tropical motion of Saturn as well as the prograde motion of the Earth's perihelion on Frater David's clock. (photograph by the author)

The hand of Saturn is carried by the arbor 64. Its motion is derived from

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the motion of Jupiter. We have

$$\mathrm{V}^0_{64} = \mathrm{V}^0_{60} \times \left(-\frac{54}{54}\right) \times \left(-\frac{54}{54}\right) \times \left(-\frac{54}{54}\right) \times \left(-\frac{29}{72}\right) = \mathrm{V}^0_{60} \times \frac{29}{72} \tag{4.58}$$

$$= -\frac{81}{350714} \times \frac{29}{72} = -\frac{261}{2805712} \tag{4.59}$$

$$\mathrm{P}^0_{64} = -\frac{2805712}{261} = -10749.8544\ldots \text{ days} \tag{4.60}$$

$$= -10749 \text{ d } 20 \text{ h } 30 \text{ m } 20.6896\ldots \text{ s} \tag{4.61}$$

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Saturn. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin. If we assume the tropical year of 365d 5h 48m 48s, this is then 29 years and 157 days 19 hours 55 minutes and 8.6896… s, but the 2010 museum guide gives a different period of 29 years 167d 22h, without giving any source [23].

4.4.4 The line of apsides

On the dial for Saturn, there is another hand carried by tube 73. This hand shows the motion of the Earth's perihelion which moves around the zodiac in about 21000 years. The motion is derived form arbor 67 which is also used for the solar cycle.

$$\mathrm{V}^0_{73} = \mathrm{V}^0_{67} \times \left(-\frac{14}{14}\right) \times \left(-\frac{14}{52}\right) \times \left(-\frac{12}{67}\right) \times \left(-\frac{12}{72}\right) \times \left(-\frac{13}{78}\right) \tag{4.62}$$

$$= \mathrm{V}^0_{67} \times \left(-\frac{7}{5226}\right) \tag{4.63}$$

$$= \mathrm{V}^0_{34} \times \left(-\frac{1}{28}\right) \times \left(-\frac{7}{5226}\right) = \mathrm{V}^0_{34} \times \frac{1}{20904} \tag{4.64}$$

$$= -\frac{450}{164359} \times \frac{1}{20904} \tag{4.65}$$

$$\mathrm{P}^0_{73} = -\frac{164359}{450} \times 20904 = -20904 \text{ years} \tag{4.66}$$

This is an approximation of the tropical period of precession of the Earth's apsides. The value is negative, because the signs are given clockwise. However, Oechslin's calculations use the ratio 13/76 instead of 13/78 and arrive at a period of 20368 tropical years. This is certainly a typo.

The same value of 20904 years is given in the 2010 museum guide, albeit without any source [23], and claiming incorrectly that this dial gives the period of precession of the equinoxes.

The period of about 21000 years is in fact due to the difference between the anomalistic and the tropical year. The anomalistic year is about 365 days 6 hours and 14 minutes. This was already known at Frater David's time, and it does for instance appear in Ferguson's *Astronomy explained* published in

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1764. The difference between the anomalistic and the tropical years is about 25 minutes, and it will take about 21000 years to accumulate to a full revolution.

4.4.5 The motion of the Moon

4.4.5.1 The draconic month



Figure 4.16: The dial for the draconic month on Frater David's clock. (photograph by the author)

The hand on arbor 96 shows the number of days elapsed since the mean Moon was at its ascending node, which may be called the draconic age, or dial of the draconic month. This motion is derived from that of arbor 92. We have

$$\mathrm{V}^0_{96} = \mathrm{V}^0_{92} \times \left(-\frac{25}{41}\right) \times \left(-\frac{27}{64}\right) = \mathrm{V}^0_{92} \times \frac{675}{2624} \tag{4.67}$$

$$= -\frac{1}{7} \times \frac{675}{2624} = -\frac{675}{18368} \tag{4.68}$$

$$\mathrm{P}^0_{96} = -\frac{18368}{675} = -27.2118\ldots \text{ days} \tag{4.69}$$

$$= -27 \text{ d } 5 \text{ h } 5 \text{ m } 4 \text{ s} \tag{4.70}$$

This is an approximation of the draconic month. The same value was found by Oechslin. The 2010 museum guide doesn't give a value for this period [23].

4.4.5.2 The age of the Moon

The hand of the age of the Moon is located on arbor 90. This motion is derived from that of arbor 80, like the motion of Mars. We have:

$$\mathrm{V}^0_{90} = \mathrm{V}^0_{80} \times \left(-\frac{20}{59}\right) = \frac{96}{961} \times \left(-\frac{20}{59}\right) = -\frac{1920}{56699} \tag{4.71}$$

$$\mathrm{P}^0_{90} = -\frac{56699}{1920} = -29.5307\ldots \text{ days} - 29 \text{ d } 12 \text{ h } 44 \text{ m } 15 \text{ s} \tag{4.72}$$

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Figure 4.17: The dial for the age of the Moon on Frater David's clock. (photograph by the author)

This is an approximation of the synodic month. The same value was found by Oechslin, but, again, the 2010 museum guide gives the truncated period of 29d 12h 44m without giving any source [23].

4.4.5.3 The mean anomaly of the Moon



Figure 4.18: The dial for the anomalistic month on Frater David's clock. (photograph by the author)

The hand on arbor 78 shows the motion of the mean Moon with respect to

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its perigee. This motion is derived from that of arbor 75:

$$\mathrm{V}^0_{78} = \mathrm{V}^0_{75} \times \left(-\frac{12}{37}\right) \times \left(-\frac{47}{51}\right) \times \left(-\frac{17}{70}\right) = \mathrm{V}^0_{75} \times \left(-\frac{94}{1295}\right) \tag{4.73}$$

$$= \frac{1}{2} \times \left(-\frac{94}{1295}\right) = -\frac{47}{1295} \tag{4.74}$$

$$\mathrm{P}^0_{78} = -\frac{1295}{47} = -27.5531\ldots \text{ days} \tag{4.75}$$

$$= -27 \text{ d } 13 \text{ h } 16 \text{ m } 35.7446 \text{ s} \tag{4.76}$$

This is an approximation of the anomalistic month. The same value was given by Oechslin. However, the 2010 museum guide gives a slightly different period of 27d 13h 16m 35s without giving any source [23].

4.4.5.4 The Moon phases

The phase of the Moon is shown on a lunar sphere on arbor 82. The velocity of this arbor is

$$\mathrm{V}^0_{82} = \mathrm{V}^0_{75} \times \left(-\frac{12}{31}\right) \times \left(-\frac{16}{31}\right) \times \left(-\frac{20}{59}\right) \times \frac{12}{12} \tag{4.77}$$

$$= \mathrm{V}^0_{75} \times \left(-\frac{3840}{56699}\right) = \frac{1}{2} \times \left(-\frac{3840}{56699}\right) = -\frac{1920}{56699} \tag{4.78}$$

$$\mathrm{P}^0_{82} = -\frac{56699}{1920} = -29.5307\ldots \text{ days} \tag{4.79}$$

The same value is given by Oechslin. This is an approximation of the synodic month, using the same ratio as the one used for the age of the Moon.

4.4.5.5 The eclipse "window"

The arbor 98 is used to show the eclipse "window," that is, when an eclipse can occur [52, p. 31]. The velocity of this arbor is computed as follows:

$$\mathrm{V}^0_{98} = \mathrm{V}^0_{96} \times \left(-\frac{64}{32}\right) \times \frac{16}{16} = \frac{675}{9184} \tag{4.80}$$

$$\mathrm{P}^0_{98} = \frac{9184}{675} = 13.6059\ldots \text{ days} \tag{4.81}$$

The same value is given by Oechslin. This is the period on the display of the eclipses, and it is actually a "window" for possible eclipses. Every 13.6 days, the Moon goes through a node, and if the longitude of the Sun is close to that of the Moon, an eclipse may take place.

4.4.6 The main dial

The main dial shows the motion of the Sun (with a hand carrying a golden Sun), the Moon (with a hand carrying the Moon on one side, and a black disk

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Figure 4.19: The center of the front dials of Frater David's clock. (photograph by the author)

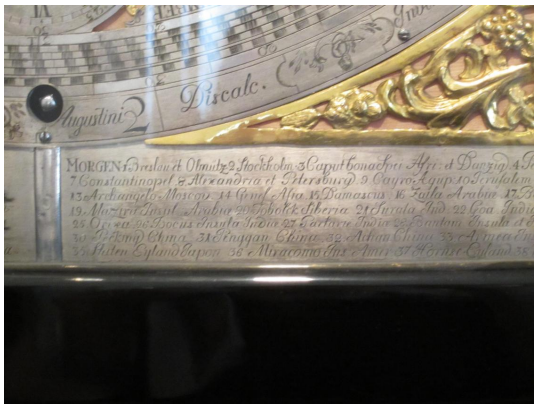


Figure 4.20: Excerpt of the list of places on Frater David's clock. This corresponds to the numbers written outside the 24-hours dial. When the Sun is over a given value, the table below tells where the Sun is culminating at this moment. (photograph by the author)

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on the other), the lunar nodes (with a hand carrying the signs for ascending and descending nodes) and the lunar apsides (a smaller double-hand slightly extending beyond the hours). The position of the apsides was used to alter the position of the mean Moon and an additional hand is given for this purpose.

First, I will consider the display of time, which is actually absent from the main dial.

4.4.6.1 The 24-hour wheel

The main dial contains a 24-hour dial, and one would expect to be able to read the time, at least the hours. Indeed, the central tube 43 makes one turn in 24 hours. The motion of this tube is derived from arbor 5. We have

$$\mathrm{V}_{43}^0 = \mathrm{V}_5^0 \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) \times \left(-\frac{12}{36}\right) \times \left(-\frac{90}{90}\right) \tag{4.82}$$

$$= \mathrm{V}_5^0 \times \frac{1}{3} = 3 \times \frac{1}{3} = 1 \tag{4.83}$$

$$\mathrm{P}_{43}^0 = 1 \text{ day} \tag{4.84}$$

However, the motion of this tube is counterclockwise, and the dial with the hours runs clockwise. No hand is carried by this tube. This is likely some unfinished part of the clock.

4.4.6.2 The motion of the Sun

The motion of the Sun is given by the hand on tube 34 and was computed earlier. We found that

$$\mathrm{P}_{34}^0 = -\frac{164359}{450} = -365.2422\ldots \text{ days} \tag{4.85}$$

This is actually the mean motion of the Sun and Frater David did not take into account any correction for the elliptical orbit of the Earth.

4.4.6.3 The mean motion of the Moon

The mean motion of the Moon is given by the hand on tube 29 and this motion is derived from that of tube 26. We have

$$\mathrm{V}_{29}^0 = \mathrm{V}_{26}^0 \times \left(-\frac{20}{61}\right) \times \left(-\frac{19}{69}\right) \times \left(-\frac{30}{74}\right) = \mathrm{V}_{26}^0 \times \left(-\frac{1900}{51911}\right) \tag{4.86}$$

$$= -\frac{1900}{51911} \tag{4.87}$$

$$\mathrm{P}_{29}^0 = -\frac{51911}{1900} = -27.3215\ldots \text{ days} \tag{4.88}$$

The same value is given by Oechslin. This is an approximation of the tropical month.

This motion is later altered by the motion of the lunar apsides.

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4.4.6.4 The lunar nodes

The hand of the lunar nodes, the dragon, is carried by tube 39. Its motion is derived from that of tube 34. We have

$$\mathrm{V}^0_{39} = \mathrm{V}^0_{34} \times \left(-\frac{18}{67}\right) \times \left(-\frac{16}{16}\right) \times \left(-\frac{16}{80}\right) = \mathrm{V}^0_{34} \times \left(-\frac{18}{335}\right) \tag{4.89}$$

$$= \left(-\frac{450}{164359}\right) \times \left(-\frac{18}{335}\right) = \frac{1620}{11012053} \tag{4.90}$$

$$\mathrm{P}^0_{39} = \frac{11012053}{1620} = 6797.5635\ldots \text{ days} \tag{4.91}$$

The same value is given by Oechslin, also in years. This is an approximation of the period of precession of the lunar nodes. The value is positive, because the lunar nodes are retrograding (which corresponds to a direct motion here, because the signs of the zodiac are given clockwise).

4.4.6.5 The lunar apsides

The hand of the lunar apsides is carried by tube 36. Its motion is also derived from that of tube 34. We have

$$\mathrm{V}^0_{36} = \mathrm{V}^0_{34} \times \left(-\frac{13}{45}\right) \times \left(-\frac{20}{51}\right) = \mathrm{V}^0_{34} \times \frac{52}{459} = \left(-\frac{450}{164359}\right) \times \frac{52}{459} \tag{4.92}$$

$$= -\frac{200}{644793} \tag{4.93}$$

$$\mathrm{P}^0_{36} = -\frac{644793}{200} = -3223.965 \text{ days} \approx -8.83 \text{ years} \tag{4.94}$$

The same value is given by Oechslin, also in years. This is an approximation of the period of precession of the lunar apsides.

It should be noted that the motion of the lunar apsides is thus slightly irregular, because the motion of tube 34 is uneven, as we have seen above.

4.4.6.6 The correction of the Moon

The actual hand of the Moon pivots first on the hand of the mean Moon, and then on a pin rotating around the center of the dial on arbor 118. Both are visible on photographs of the dial. It isn't clear if arbor 118 is still rotating, as Oechslin's drawing seems to imply that a fixed 31-teeth on the central arbor is missing. And this wheel would have meshed with a 25-teeth wheel on arbor 118. This arbor moves on the apsides hand and therefore it moves around the center in the period of precession of the lunar apsides. But the pin is actually slightly offset from arbor 118, so that an additional correction is introduced, and this correction depends on the ratio between the central wheel (31 teeth) and the satellite wheel (25 teeth).

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With respect to tube 36 (the apsides hand), arbor 118 has the velocity:

$$\mathrm{V}_{118}^{36} = \mathrm{V}_{117}^{36} \times \left(-\frac{31}{25}\right) \tag{4.95}$$

where 117 is the central fixed arbor.

$$= -\mathrm{V}_{36}^{0} \times \left(-\frac{31}{25}\right) = \frac{200}{644793} \times \left(-\frac{31}{25}\right) \tag{4.96}$$

$$= -\frac{248}{644793} \tag{4.97}$$

$$\mathrm{P}_{118}^{36} = -\frac{644793}{248} = -2599.9717\ldots \text{ days} \tag{4.98}$$

$$\mathrm{V}_{118}^{0} = \mathrm{V}_{118}^{36} + \mathrm{V}_{36}^{0} = -\frac{248}{644793} - \frac{200}{644793} = -\frac{448}{644793} \tag{4.99}$$

$$\mathrm{P}_{118}^{0} = -\frac{644793}{448} = -1439.2700\ldots \text{ days} \approx= -3.94 \text{ years} \tag{4.100}$$

It isn't clear what Frater David had in mind here, but Oechslin obtains the same value.[12] Frater David may have tried to obtain the motion of the lunar evection, and a simple way to obtain such a motion would have been to put a 28-teeth wheel on tube 29, and having it mesh with a 32-teeth wheel on arbor 118 *under* the apsides hand. The relative motion of the wheel on tube 29 (the mean Moon) to the apsides hand is the velocity of the anomalistic month and the period of arbor 118 with respect to tube 36 would have been about 31.5 days. Then, adding this motion to the motion of the apsides would have resulted in a motion whose period is about 31.8 days, the period of the evection.

4.4.7 The upper dial

The upper dial is a relatively conventional astrolabe dial, showing the motions of the zodiac, the Sun, the Moon, and the dragon (line of nodes). Moreover, this dial also shows the italic hours, that is the number of hours since the last sunset, and which are derived from the annual rotation of a cam.

4.4.7.1 The italic hours

The italic hours are shown on a mobile 24-hour ring just inside the fixed 24-hour ring. Since the difference between the hours of mean time and the italic hours oscillates and is about 6 hours on average, this ring has a back-and-forth motion controlled by a cam. This cam makes a turn in one year and is located

[12] See [52, p. 208-209] on this problem.

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Figure 4.21: The upper dial of the front side of Frater David's clock. (photograph by the author)

on arbor 104. The motion of this cam is derived from that of tube 34 which also makes a turn in a year:

$$V^0_{104} = V^0_{34} \times \left(-\frac{18}{72}\right) \times \left(-\frac{72}{18}\right) \times \frac{16}{16} \times \left(-\frac{18}{18}\right) = -V^0_{34} \tag{4.101}$$

The cam on arbor 104 moves a lever which moves another toothed segment. This segment meshes with a 14-teeth wheel on arbor 106, which carries a 30-teeth wheel meshing with a 32-teeth circular segment on the italic hours ring. Since I do not have the dimensions of the cam and the lever, I will not go further into this description, but the shape of the cam could be recomputed and the motion of the ring could be obtained from the dimensions of the various parts. However, this also raises the question of whether the actual cam takes the equation of time into account.

The italic hours are also shown on the 1793 clock made by Frater David for the Prince of Schwarzenberg (Oechslin 6.2). They are also shown on Pater Aurelius' clock in Munich (Oechslin 5.1).

4.4.7.2 The motion of the Sun

The main input to the upper dial is provided by arbor 44, whose motion is derived from tube 26. We have

$$V^0_{44} = V^0_{26} \times \left(-\frac{72}{36}\right) = V^0_{26} \times (-2) = -2 \tag{4.102}$$

This motion is then transferred to tube 45:

$$V^0_{45} = V^0_{44} \times \frac{40}{80} = -1 \tag{4.103}$$

$$P^0_{45} = -1 \text{ day} \tag{4.104}$$

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This tube, or frame, which carries a number of gears, makes one turn clockwise in one day. This is also the motion of the Sun. The Sun is pictured by a golden disk and thin hand, and this hand passes through a narrow groove between the disk of the Moon and the ring of the ecliptic or zodiac.

The motions of the various internal gears of frame 45 are obtained from two fixed wheels on the central arbor 46 of this dial.

4.4.7.3 The motion of the Moon

The Moon is located on tube 50 but corresponds to an entire disk on the front of the dial. This disk shows the position of the Moon, and also the various aspects that can occur between the Sun and the Moon (sextile, quartile, trine, opposition, etc.). The motion of this tube is derived from the fixed 16-teeth wheel on the central arbor 46. We have

$$\mathrm{V}_{50}^{45} = \mathrm{V}_{46}^{45} \times \left(-\frac{16}{59}\right) \times \left(-\frac{15}{15}\right) \times \left(-\frac{15}{62}\right) \times \left(-\frac{32}{62}\right) \tag{4.105}$$

$$= \mathrm{V}_{46}^{45} \times \frac{1920}{56699} = -\mathrm{V}_{45}^{0} \times \frac{1920}{56699} = \frac{1920}{56699} \tag{4.106}$$

$$\mathrm{P}_{50}^{45} = \frac{56699}{1920} = 29.5307\ldots \text{ days} \tag{4.107}$$

The same value is given by Oechslin. This is an approximation of the synodic month, using the same ratio as the one used earlier.

We can also compute the motion of the Moon with respect to the fixed frame:

$$\mathrm{V}_{50}^{0} = \mathrm{V}_{50}^{45} + \mathrm{V}_{45}^{0} = \frac{1920}{56699} - 1 = -\frac{54779}{56699} \tag{4.108}$$

$$\mathrm{P}_{50}^{0} = -\frac{56699}{54779} = -1.0350\ldots \text{ days} \tag{4.109}$$

$$= -24 \text{ h } 50 \text{ m } 28.3137 \text{ s} \tag{4.110}$$

The same value is given by Oechslin. The disk of the Moon rotates clockwise.

4.4.7.4 The motion of the zodiac

The zodiac is located on tube 55. It is pictured by a mobile ring on the dial which carries the names of the months. Its motion is derived from the fixed 15-teeth wheel on the central arbor 46. We have

$$\mathrm{V}_{55}^{45} = \mathrm{V}_{46}^{45} \times \left(-\frac{15}{15}\right) \times \left(-\frac{15}{39}\right) \times \left(-\frac{12}{47}\right) \times \left(-\frac{12}{64}\right) \times \left(-\frac{40}{269}\right) \tag{4.111}$$

$$= \mathrm{V}_{46}^{45} \times \left(-\frac{450}{164359}\right) = -\mathrm{V}_{45}^{0} \times \left(-\frac{450}{164359}\right) \tag{4.112}$$

$$= -\frac{450}{164359} \tag{4.113}$$

$$\mathrm{P}_{55}^{45} = -\frac{164359}{450} = -365.2422\ldots \text{ days} \tag{4.114}$$

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The same value is given by Oechslin.

We can also compute the motion of the zodiac with respect to the fixed frame:

$$\mathrm{V}_{55}^{0} = \mathrm{V}_{55}^{45} + \mathrm{V}_{45}^{0} = -\frac{450}{164359} - 1 = -\frac{164809}{164359} \tag{4.115}$$

$$\mathrm{P}_{55}^{0} = -\frac{164359}{164809} = -0.9972\ldots \text{ days} \tag{4.116}$$

$$= -23 \text{ h } 56 \text{ m } 4.0905\ldots \text{ s} \tag{4.117}$$

The same value is given by Oechslin. The zodiac rotates clockwise.

4.4.7.5 The motion of the dragon

The dragon is located on tube 57. This motion is derived from that of the ecliptic tube 55. We have

$$\mathrm{V}_{57}^{45} = \mathrm{V}_{55}^{45} \times \left(-\frac{30}{73}\right) \times \left(-\frac{72}{28}\right) = \mathrm{V}_{55}^{45} \times \frac{540}{511} \tag{4.118}$$

$$= -\frac{450}{164359} \times \frac{540}{511} = -\frac{243000}{83987449} \tag{4.119}$$

$$\mathrm{P}_{57}^{45} = -\frac{83987449}{243000} = -345.6273\ldots \text{ days} \tag{4.120}$$

The same value is given by Oechslin. This is an approximation of the eclipse year.

We can also compute the motion of the dragon with respect to the fixed frame:

$$\mathrm{V}_{57}^{0} = \mathrm{V}_{57}^{45} + \mathrm{V}_{45}^{0} = -\frac{243000}{83987449} - 1 = -\frac{84230449}{83987449} \tag{4.121}$$

$$\mathrm{P}_{57}^{0} = -\frac{83987449}{84230449} = 0.9971\ldots \text{ days} \tag{4.122}$$

$$= -23 \text{ h } 55 \text{ m } 50.7409 \text{ s} \tag{4.123}$$

The same value is given by Oechslin. The dragon moves clockwise.

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Chapter 5

(Oechslin: 6.2)

Frater David's Schwarzenberg clock (1793)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

5.1 Introduction

The clock described here was commissioned by Joseph I Adam Prince of Schwarzenberg and is sometimes called the "Schwarzenberg clock." It was designed by David Ruetschmann (1726-1796), who went by the name Frater David a Sancto Cajetano.[1] The clock is probably located in the Schwarzenberg Palais in Vienna.

The clock was probably completed in 1793 by Frater David's nephew Joseph Ruetschmann (1755-1801) in Vienna.[2] For the mechanism of this clock, Joseph Ruetschmann was paid 2000 gulden in 1793 and the sculptor Adam Vogl 700 gulden.[3] In the past, there has been some confusion about the identity of this clock, when the 1769 clock was no longer in Vienna, or when it was at the Zwettl abbey.[4]

[1] For biographical information on Frater David, see the description of the 1769 clock in the previous chapter.

[2] The clock is illustrated in a number of articles and books, for instance [10] and more recently in the collective "*Himmlisches Räderwerk*" [31]. See also the 1989 exhibition catalogue [41, p. 67, pl. 24] where it is dated from 1786. It was also described by Oechslin in 1991, but I haven't seen his article [34]. About the authorship of the clock, see Oechslin [35, p. 218]. The same Joseph Ruetschmann has also built a clock with a small orrery on top of it [14, p. 127-128]. This orrery shows the planets up to Uranus and was made very compact. It uses a number of interior gearings [18]. Some accounts have listed Joseph Ruetschmann as Frater David's brother [5, p. 24].

[3] See [33], [6], [13], [5] and [35, p. 223].

[4] For instance, in 1896, Mörath [33] corrected Ilg [19] who thought that the Schwarzenberg clock was the same one as the 1769 clock.

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The Schwarzenberg clock is the first clock by Frater David where he puts in practice his theory of epicyclic gears and achieves the precise ratios he aims at. This theory was then published in 1791 and 1793 [7, 8, 9]. Frater David wanted in particular to avoid using wheels with more than 100 teeth.

As observed by Oechslin, Frater David may have been influenced by Neßtfell's planetarium in Vienna, which he restored around 1785, and this may have brought him to develop his theory of epicyclic gears.[5] This connection was not put forward by Bertele when he wrote about this clock, and Bertele still viewed the origin of epicyclic gears in Frater David's clock as a mystery we would never be able to elucidate.[6]

The clock is a tall-case clock with ten dials. A description of the indications of the clock was given by Bertele.[7] The largest dial is the one for the time, but it also shows the motion of the Sun, the Moon, and the zodiac. This dial is set in a square whose corners contain four small dials showing the seconds, the day of the week, the synodic month, and the power reserve.

Above these dials, there is a sphere showing the phase of the Moon, and two dials, one left for the italic hours, and one right for the draconic month. On top of these dials, another dial shows the months on a four-year cycle. The day of the month is also shown on a circular arc.

5.2 The going work

The clock is spring-driven and regulated by a pendulum. However, Oechslin did not give the details of the going work train, and we only know that the arbor 5 is the central arbor of the main dial and that it makes a turn clockwise in one hour. We therefore have

$$\mathrm{V}_5^0 = -24 \tag{5.1}$$

$$\mathrm{P}_5^0 = -\frac{1}{24} = -1 \text{ h} \tag{5.2}$$

5.3 The main dial

The main dial shows the time, but also the motion of the Sun, the Moon, the zodiac, the lunar nodes and the lunar apsides. The central arbor is the one making a turn in one hour, but in fact this arbor is connected by friction to a tube which is also named 5 by Oechslin. This tube therefore also makes a turn in one hour. It is the one carrying the minutes hand.

[5] [35, p. 227]
[6] [5, p. 24]
[7] [5, p. 25]

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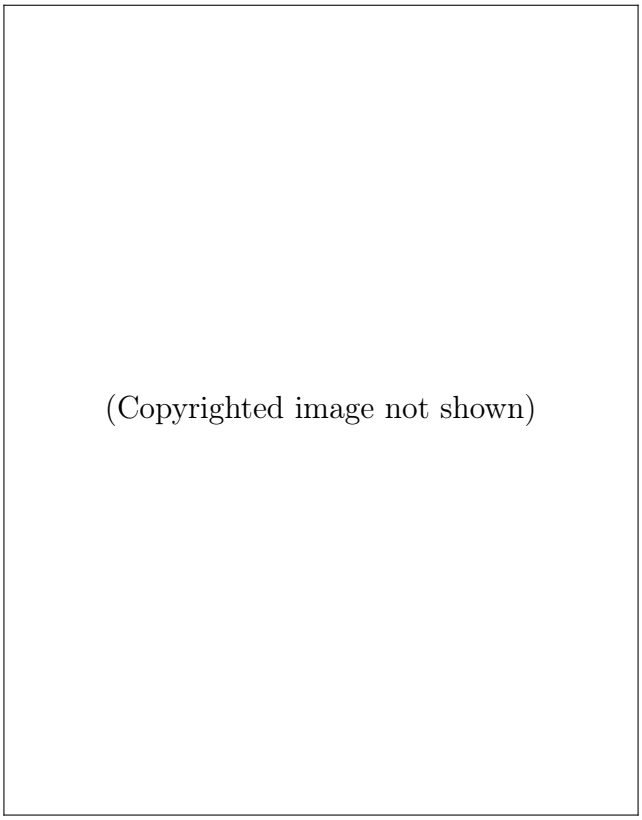


Figure 5.1: The dials of the Schwarzenberg clock. (source: [10])

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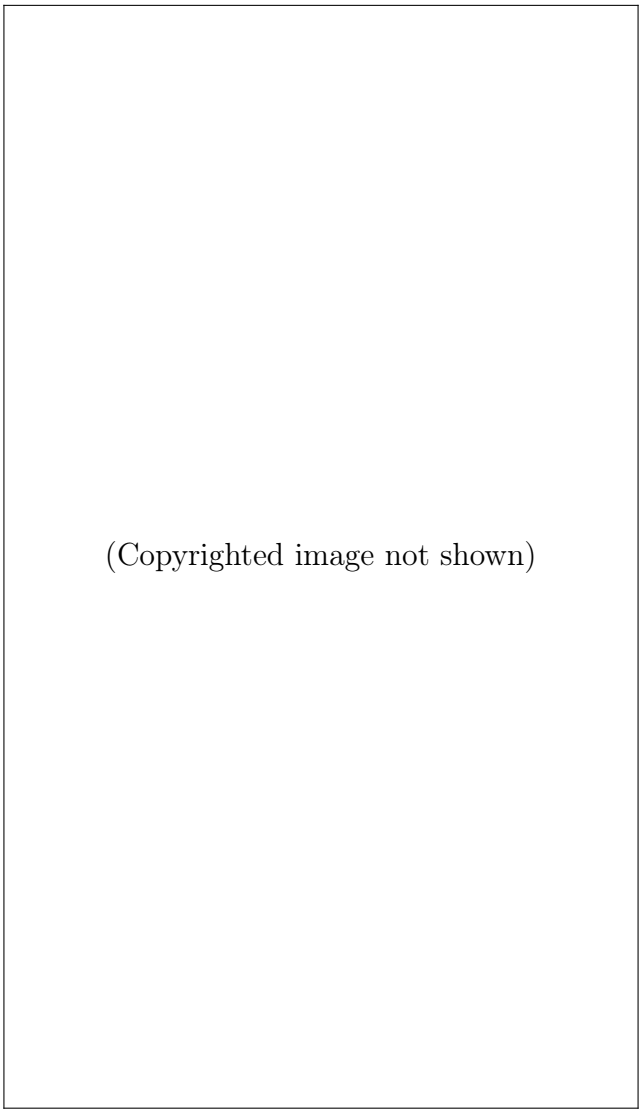


Figure 5.2: A clock by Joseph Ruetschmann with almost the same dials as the Schwarzenberg clock, auctioned by Leonard Joel, 14 May 2017.

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The second tube is tube 9 and it carries the hours hand. We have

$$\mathrm{V}_9^0 = \mathrm{V}_5^0 \times \left(-\frac{20}{40}\right) \times \left(-\frac{8}{48}\right) = \mathrm{V}_5^0 \times \frac{1}{12} = -2 \tag{5.3}$$

$$\mathrm{P}_9^0 = -\frac{1}{2} = -12 \text{ h} \tag{5.4}$$

The other motions (Sun, Moon, lines of nodes and apsides) are obtained using epicyclic (satellite) wheels, such that an input motion on a fixed arbor causes the motion of a satellite wheel, whose arbor moves around the initial fixed arbor, as a result of a constraint introduced by a fixed wheel. These constructions are much more elaborate than those used in Frater David's 1769 clock, and obviously the result of his research on gear ratios.

The reason for using these constructions is that the ratios that Frater David wanted to obtain cannot be obtained exactly with fixed arbors and teeth-counts smaller than 100. The sought ratios were

- for the Sun: $\frac{164359}{450}$ and 164359 contains the prime factor 269;
- for the Moon: $\frac{295073}{10800}$ and 295073 is prime;
- for the lunar apsides: $\frac{145951}{45}$ and 145951 contains the factors 103 and 109
- for the lunar nodes: $\frac{9789403}{1440}$ and 9789403 contains the prime factor 753031.

The use of epicyclic constructions makes it possible to use gears with smaller teeth-counts, smaller primes, and still obtain the same final ratios. Frater David also needed tables of primes, and he apparently used the unpublished work of Florian Ulbrich [37]. Of course, in most cases, constructors will use slightly different ratios, so that the gears can be cut. There is in fact little point trying to get some exact target ratio, first because these ratios are always approximations and will be improved over time, and second because astronomical clocks never work for ever without being stopped or restored, and therefore also reset.

Interestingly, the ratio $\frac{164359}{450}$ was also used by Jean-Baptiste Schwilgué in the Strasbourg astronomical clock [40], but Schwilgué resorted to 269-teeth and 270-teeth wheels, something he could have avoided, had he known about Frater David's work.

Frater David was however not the first one to use such constructions. In the 1750s, Neßtfell used the same constructions in his two planetary machines (Oechslin 3.1 and 3.2). Frater David may have taken his idea from Neßtfell's machine in Vienna, which he restored around 1785.[8] However, there is an important difference. Neßtfell's constructions were used because in his machines the planets are set on moving carriages. He didn't use them for the purpose of obtaining exact ratios, although one can be tempted to think so at first

[8] [35, p. 227-228]

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sight. Frater David was probably the first one to use these constructions for the purpose of obtaining specific ratios with trains involving only small primes.

I will not go into the details of the theory of the epicyclic constructions here, and I refer the reader to the introduction of this book and to Frater David's writings [7, 8, 9] and those of Graham White [42, 43, 44]. See also Lunardi on this clock [30, p. 431-434].

5.3.1 The motion of the Sun

As a first application of the epicyclic layout of the gears, we can consider the motion of the Sun. The satellite wheels are located on frame 48, so that we first compute the motion with respect to this frame. There are two motions of special interest here, namely the input motion of arbor 42 (which is derived from tube 5), and the output motion of tube 47. The latter is in fact a fixed tube, and frame 48 rotates around it, but with respect to frame 48, tube 47 also rotates. And what we are interested in is how much frame 48 rotates with respect to tube 47 as a function of how much arbor 42 rotates with respect to the fixed frame. We therefore first seek the ratio between V_{47}^{48} and V_{42}^{48}:

$$\mathrm{V}_{47}^{48} = \mathrm{V}_{42}^{48} \times \left(-\frac{21}{21}\right) \times \left(-\frac{21}{71}\right) \times \left(-\frac{14}{74}\right) \times \left(-\frac{14}{75}\right) \times \left(-\frac{8}{80}\right) \tag{5.5}$$

$$= \mathrm{V}_{42}^{48} \times \left(-\frac{343}{328375}\right) = \left(\mathrm{V}_{42}^{47} + \mathrm{V}_{47}^{48}\right) \times \left(-\frac{343}{328375}\right) \tag{5.6}$$

Then, we have:

$$\mathrm{V}_{47}^{48} \times \left(1 + \frac{343}{328375}\right) = \mathrm{V}_{42}^{47} \times \left(-\frac{343}{328375}\right) \tag{5.7}$$

$$\mathrm{V}_{47}^{48} \times \frac{328718}{328375} = \mathrm{V}_{42}^{47} \times \left(-\frac{343}{328375}\right) \tag{5.8}$$

$$\mathrm{V}_{47}^{48} = \mathrm{V}_{42}^{47} \times \left(-\frac{343}{328718}\right) \tag{5.9}$$

$$\mathrm{V}_{48}^{47} = \mathrm{V}_{48}^{0} = \mathrm{V}_{42}^{0} \times \frac{343}{328718} \tag{5.10}$$

Now, before going on, we need to compute the motion of arbor 42:

$$\mathrm{V}_{42}^{0} = \mathrm{V}_{5}^{0} \times \left(-\frac{20}{49}\right) \times \left(-\frac{15}{56}\right) \times \left(-\frac{42}{42}\right) = \mathrm{V}_{5}^{0} \times \left(-\frac{75}{686}\right) \tag{5.11}$$

$$= (-24) \times \left(-\frac{75}{686}\right) = \frac{900}{343} \tag{5.12}$$

Combining the two, we obtain

$$\mathrm{V}_{48}^{0} = \frac{900}{343} \times \frac{343}{328718} = \frac{450}{164359} \tag{5.13}$$

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Now, the frame 48 carries a 94-teeth wheel meshing with another 94-teeth wheel on tube 49 carrying the solar hand. We have

$$\mathrm{V}^0_{49} = \mathrm{V}^0_{48} \times \left(-\frac{94}{94}\right) = -\mathrm{V}^0_{48} = -\frac{450}{164359} \tag{5.14}$$

$$\mathrm{P}^0_{49} = -\frac{164359}{450} = -365.2422\ldots \text{ days} \tag{5.15}$$

The same value is given by Oechslin. This is an excellent approximation of the tropical year, the same which was used in the Vienna clock constructed in 1769.

Since the above construction is used several times, we can also generalize it somewhat. There is an input motion x on arbor 42, a ratio y in the rotating frame, and a final ratio z to obtain. In the above case, we have

$$x = \frac{900}{343} \tag{5.16}$$

$$y = -\frac{343}{328375} \tag{5.17}$$

$$z = \frac{450}{164359} \tag{5.18}$$

The relationship between these three ratios is

$$z = x \times \frac{y}{y-1} \tag{5.19}$$

Now, in the special case where $x = a/b$ and $y = -b/c$, this reduces to

$$z = \frac{a}{b+c} = \frac{900}{343+328375} \tag{5.20}$$

How Frater David obtained these trains is explained in the introduction of this book. He took the tropical year to be 31556928 seconds, and introduced an input motion of 32928 seconds (P^0_{42} above), and thus came up with smaller primes.

5.3.2 The motion of the line of apsides

The motion of the line of apsides is obtained like that of the Sun. There is a rotating frame 38 with a number of gears and whose input is the motion of arbor 34. So, we first compute the motion of arbor 34. We have

$$\mathrm{V}^0_{34} = \mathrm{V}^0_9 \times \left(-\frac{24}{51}\right) \times \left(-\frac{12}{52}\right) \times \left(-\frac{10}{64}\right) \times \left(-\frac{64}{24}\right) \times \left(-\frac{24}{64}\right) \tag{5.21}$$

$$= \mathrm{V}^0_9 \times \left(-\frac{15}{884}\right) = (-2) \times \left(-\frac{15}{884}\right) = \frac{15}{442} \tag{5.22}$$

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This will be our x above.

Then,

$$\mathrm{V}_{37}^{38} = \mathrm{V}_{34}^{38} \times \left(-\frac{21}{50}\right) \times \left(-\frac{17}{70}\right) \times \left(-\frac{8}{89}\right) = \mathrm{V}_{34}^{38} \times \left(-\frac{102}{11125}\right) \tag{5.23}$$

The ratio $-\frac{102}{11125}$ is y.

We can now immediately compute the value of z:

$$\mathrm{V}_{38}^{0} = x \times \frac{y}{y-1} \tag{5.24}$$

$$= \frac{15}{442} \times \frac{-\frac{102}{11125}}{-\frac{102}{11125}-1} = \frac{15}{442} \times \frac{102}{11227} = \frac{45}{145951} \tag{5.25}$$

A 90-teeth wheel on frame 38 meshes with an identical wheel on tube 39 carrying the hand of the lunar apsides. We therefore have

$$\mathrm{V}_{39}^{0} = \mathrm{V}_{38}^{0} \times \left(-\frac{90}{90}\right) = -\mathrm{V}_{38}^{0} = -\frac{45}{145951} \tag{5.26}$$

$$\mathrm{P}_{39}^{0} = -\frac{145951}{45} = -3243.3555\ldots \text{ days} \tag{5.27}$$

The same value is given by Oechslin.

5.3.3 The mean motion of the Moon

The mean motion of the Moon is obtained like those of the Sun and the line of apsides. There is a rotating frame 27 with a number of gears and whose input is the motion of arbor 21. We first compute the motions of arbor 20 (which is used later) and of arbor 21. We have

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{5}^{0} \times \left(-\frac{20}{42}\right) \times \left(-\frac{30}{42}\right) = \mathrm{V}_{5}^{0} \times \frac{50}{147} \tag{5.28}$$

$$= (-24) \times \frac{50}{147} = -\frac{400}{49} \tag{5.29}$$

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{20}^{0} \times \left(-\frac{42}{42}\right) = \frac{400}{49} \tag{5.30}$$

Likewise, within the frame 27, we have

$$\mathrm{V}_{26}^{27} = \mathrm{V}_{21}^{27} \times \left(-\frac{21}{21}\right) \times \left(-\frac{21}{40}\right) \times \left(-\frac{9}{47}\right) \times \left(-\frac{14}{50}\right) \times \left(-\frac{8}{50}\right) \tag{5.31}$$

$$= \mathrm{V}_{21}^{27} \times \left(-\frac{1323}{293750}\right) \tag{5.32}$$

Hence

$$\mathrm{V}_{27}^{0} = \frac{400}{49} \times \frac{-\frac{1323}{293750}}{-\frac{1323}{293750}-1} = \frac{400}{49} \times \frac{1323}{295073} = \frac{10800}{295073} \tag{5.33}$$

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A 88-teeth wheel on frame 27 meshes with an identical wheel on tube 28. We therefore have

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{27}^{0} \times \left(-\frac{88}{88}\right) = -\mathrm{V}_{27}^{0} = -\frac{10800}{295073} \tag{5.34}$$

$$\mathrm{P}_{28}^{0} = -\frac{295073}{10800} = -27.3215\ldots \text{ days} \tag{5.35}$$

The same value is given by Oechslin. This is an approximation of the tropical month. This motion is then modified (see below) to obtain the corrected Moon.

We can also compute the synodic month:

$$\mathrm{V}_{28}^{49} = \mathrm{V}_{28}^{0} - \mathrm{V}_{49}^{0} = -\frac{10800}{295073} + \frac{450}{164359} = -\frac{1642294350}{48497903207} \tag{5.36}$$

$$\mathrm{P}_{28}^{49} = -\frac{48497903207}{1642294350} = -29.5305\ldots \text{ days} \tag{5.37}$$

5.3.4 The line of the nodes (dragon)

The motion of the nodes is obtained like those of the Sun, the line of apsides and the Moon. There is a rotating frame 18 with a number of gears and whose input is the motion of tube 12. So, we first compute the motion of arbor 12. We have

$$\mathrm{V}_{12}^{0} = \mathrm{V}_{9}^{0} \times \left(-\frac{24}{54}\right) \times \left(-\frac{10}{33}\right) \times \left(-\frac{12}{66}\right) = \mathrm{V}_{9}^{0} \times \left(-\frac{80}{3267}\right) \tag{5.38}$$

$$= (-2) \times \left(-\frac{80}{3267}\right) = \frac{160}{3267} \tag{5.39}$$

Likewise, within the frame 18, we have

$$\mathrm{V}_{17}^{18} = \mathrm{V}_{12}^{18} \times \left(-\frac{27}{50}\right) \times \left(-\frac{11}{11}\right) \times \left(-\frac{11}{50}\right) \times \left(-\frac{11}{50}\right) \times \left(-\frac{11}{64}\right) \tag{5.40}$$

$$= \mathrm{V}_{12}^{18} \times \left(-\frac{35937}{8000000}\right) \tag{5.41}$$

Hence

$$\mathrm{V}_{18}^{0} = \frac{160}{3267} \times \frac{-\frac{35937}{8000000}}{-\frac{35937}{8000000} - 1} = \frac{160}{3267} \times \frac{35937}{8035937} = \frac{1760}{8035937} \tag{5.42}$$

$$\mathrm{P}_{18}^{0} = \frac{8035937}{1760} = 4565.8732\ldots \text{ days} \tag{5.43}$$

This value is wrong, but Oechslin found the same one. However, Oechslin suggested that one of the ratios 11/50 should be replaced by 9/61.[9] In that

[9] See [35, p. 205] on this error.

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case, we obtain

$$\mathrm{V(h)}_{17}^{18} = \mathrm{V}_{12}^{18} \times \left(-\frac{27}{50}\right) \times \left(-\frac{11}{11}\right) \times \left(-\frac{11}{50}\right) \times \left(-\frac{9}{61}\right) \times \left(-\frac{11}{64}\right) \quad (5.44)$$

$$= \mathrm{V}_{12}^{18} \times \left(-\frac{29403}{9760000}\right) \quad (5.45)$$

Hence

$$\mathrm{V(h)}_{18}^{0} = \frac{160}{3267} \times \frac{-\frac{29403}{9760000}}{-\frac{29403}{9760000} - 1} = \frac{160}{3267} \times \frac{29403}{9789403} = \frac{1440}{9789403} \quad (5.46)$$

$$\mathrm{P(h)}_{18}^{0} = \frac{9789403}{1440} = 6798.1965\ldots \text{ days} \quad (5.47)$$

The same value is given by Oechslin.

The sign of the velocity is positive, because there is a retrogradation of the nodes (which is here a counterclockwise motion, since the signs of the zodiac are given clockwise).

5.3.5 The corrected motion of the Moon

The frame 39, which has the motion of the line of apsides, also carries an eccentric cam which is used to correct the motion of the Moon. The mean Moon corresponds to frame 28, and the corrected Moon to frame 29. The period of this correction is that of the motion of frame 28 with respect to frame 39, that is the draconic month:

$$\mathrm{V}_{28}^{39} = \mathrm{V}_{28}^{0} - \mathrm{V}_{39}^{0} = -\frac{10800}{295073} + \frac{45}{145951} = -\frac{1562992515}{43066199423} \quad (5.48)$$

$$\mathrm{P}_{28}^{39} = -\frac{43066199423}{1562992515} = -27.5536\ldots \text{ days} \quad (5.49)$$

5.4 The other dials

5.4.1 The italic hours

The medium-sized dial at the left of the Moon sphere gives the mean time as well as the italic hours. These hours are counted from the last sunset, and the difference between the two is about six hours on average. The dial has an outside fixed ring of twice 12 hours (in Arabic numerals), and an inner moving ring of also twice 12 hours (in Roman numerals). The inner ring oscillates and its motion is obtained like on Frater David's 1769 clock (Oechslin 6.1). The ring has a back-and-forth motion controlled by a cam. This cam makes a turn in one year and is located on arbor 51. The motion of this cam is derived from that of frame 48 which also makes a turn in a year:

$$\mathrm{V}_{51}^{0} = \mathrm{V}_{48}^{0} \times \left(-\frac{94}{26}\right) \times \left(-\frac{26}{94}\right) = \mathrm{V}_{48}^{0} \quad (5.50)$$

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The cam on arbor 51 moves a rack meshing with a toothed circular segment on the italic hours ring 52. Since I do not have the dimensions of the cam and the lever, I will not go further into this description, but the shape of the cam could be recomputed and the motion of the ring could be obtained from the dimensions of the various parts. However, like in 1769 clock, this also raises the question of whether the actual cam takes the equation of time into account.

The italic hours are also shown on Pater Aurelius' clock in Munich (Oechslin 5.1).

5.4.2 The phase of the Moon

The phase of the Moon, the synodic month and the draconic months are derived from the motion of arbor 54, which is itself derived from the motion of arbor 20. We therefore first compute the motion of arbor 54:

$$\mathrm{V}^0_{54} = \mathrm{V}^0_{20} \times \left(-\frac{14}{32}\right) \times \left(-\frac{14}{50}\right) = \mathrm{V}^0_{20} \times \frac{49}{400} = -\frac{400}{49} \times \frac{49}{400} = -1 \quad (5.51)$$

This arbor makes one turn clockwise per day.

Now, this motion is used to derive the motion of arbor 58:

$$\mathrm{V}^0_{58} = \mathrm{V}^0_{54} \times \left(-\frac{12}{31}\right) \times \left(-\frac{16}{31}\right) \times \left(-\frac{10}{59}\right) \times \left(-\frac{59}{59}\right) \quad (5.52)$$

$$= \mathrm{V}^0_{54} \times \frac{1920}{56699} = -\frac{1920}{56699} \quad (5.53)$$

$$\mathrm{P}^0_{58} = -\frac{56699}{1920} = -29.5307\ldots \text{ days} \quad (5.54)$$

The same value is given by Oechslin. This is also an approximation of the synodic month, but not the same that was used above.

The motion of arbor 58 is transferred to the vertical arbor 59 through two 25-teeth wheels and this arbor carries the lunar sphere which is located in the center and above the main dial. The sphere rotates clockwise as seen from above (from the right on the drawing).

5.4.3 The synodic month

This dial is located in the upper left corner of the main dial and shows the number of days since the last new moon. Its period is therefore the synodic month. The hand is on arbor 63. Its motion is derived from that of arbor 58. We have

$$\mathrm{V}^0_{63} = \mathrm{V}^0_{58} \times \left(-\frac{59}{59}\right) \times \left(-\frac{59}{30}\right) \times \left(-\frac{30}{59}\right) \times \left(-\frac{59}{59}\right) = \mathrm{V}^0_{58} \quad (5.55)$$

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5.4.4 The draconic month

This dial is located in the upper right corner of the main dial. The hand is on arbor 68 and makes a turn in a draconic month. Its motion is also derived from the motion of arbor 54. We have

$$\mathrm{V}^0_{68} = \mathrm{V}^0_{54} \times \left(-\frac{12}{41}\right) \times \left(-\frac{25}{32}\right) \times \left(-\frac{9}{28}\right) \times \frac{28}{22} \times \left(-\frac{22}{56}\right) \tag{5.56}$$

$$= \mathrm{V}^0_{54} \times \frac{675}{18368} = -\frac{675}{18368} \tag{5.57}$$

$$\mathrm{P}^0_{68} = -\frac{18368}{675} = -27.2118\ldots \text{ days} \tag{5.58}$$

The same value is given by Oechslin.

5.4.5 The day of the week

This dial is located in the lower left corner of the main dial. The hand for the day of the week is on arbor 72 and its motion is derived from arbor 54 seen above. We first obtain the motions of arbor 70 and 71:

$$\mathrm{V}^0_{70} = \mathrm{V}^0_{54} \times \left(-\frac{50}{50}\right) \times \frac{50}{25} = \mathrm{V}^0_{54} \times (-2) = 2 \tag{5.59}$$

$$\mathrm{V}^0_{71} = \mathrm{V}^0_{70} \times \left(-\frac{20}{40}\right) = 2 \times \left(-\frac{1}{2}\right) = -1 \tag{5.60}$$

The arbor 71 makes on turn in a day and carries a finger which moves a wheel on arbor 72 by a 7th at each turn. The hand on arbor 72 therefore makes a turn clockwise in seven days.

5.4.6 The calendar dial

The calendar dial derives its motion from arbor 73 which makes a turn in 24 hours:

$$\mathrm{V}^0_{73} = \mathrm{V}^0_{70} \times \left(-\frac{25}{50}\right) = 2 \times \left(-\frac{1}{2}\right) = -1 \tag{5.61}$$

This arbor is used to move the arbor 75 also in one day. The latter carries a finger which moves the day of the month on arbor 76 by one unit every day. The length of the month is taken into account on the basis of a 4-year cycle. I am not detailing this here, as I do not have sufficient information to do so. But when the day returns to 1, a 48-teeth wheel on arbor 77 advances by one tooth, and the motion of this arbor is transferred using two 60-teeth wheels to the tube 78 which carries the hand of the month. This hand therefore makes a turn in 48 months, that is four years.

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Chapter 6

(Oechslin: 4.1)

Frater Fridericus's mechanism in Bamberg (1772)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

6.1 Introduction

The machine described in this chapter was designed in 1772 by Frater Fridericus a S. Christophoro,[1] a discalced Carmelite from Würzburg, and constructed by the mechanician Johann Georg Fellwöck (sometimes spelled Felbäch or Felbeck) (1728-1810).[2] It was acquired for 300 to 400 gulden by the Banz Abbey, near Bamberg.[3]

By 1813, the machine was located in Bamberg, but in 1900 it was in Munich. Since at least 1937, it is again in Bamberg, now in its historical museum.[4]

The Fridericus/Fellwöck machine shows the motion of the Earth around the Sun (not pictured), the motion of Mercury, and the Earth-Moon system positioned opposite the main Earth. The Earth therefore appears twice on this machine. The representation of the motion of Mercury is clearly inspired by Neßtfell's machines.

[1] For other descriptions of this machine, see Hess [4, p. 92-98], Bassermann-Jordan [1, p. 101], Stoehr [10, p. 80], King [7, p. 232], Henck [3, p. 106-109], the 1989 exhibition catalogue [11, p. 68-69, pl. 23], and Oechslin [8, p. 35, 37, 218].

[2] See [5, p. 26] and [6, p. 112].

[3] [8, p. 222]

[4] For details on the history of this machine, see [8, p. 234] and [2]. Fowler has observed that there are similarities between the front dials of Neßtfell's machines, and the dials of some "Gutwein-clocks," clocks with geographical dials made in or around Würzburg around 1760. One of these clocks may have been made by Johann Georg Fellwöck [2, p. 26-29]. This suggests an influence of the making of these clocks on the Neßtfell and Fridericus/Fellwöck machines, or conversely.

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Figure 6.1: General view of Frater Fridericus's clock in Bamberg. (photograph by the author)



Figure 6.2: The upper part of Frater Fridericus's clock in Bamberg. The Earth globe is at the front. (photograph by the author)

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Neßtfell was in fact an important influence in Frater Fridericus's work, as he was his teacher, and because Fellwöck also worked with Neßtfell for the construction of his second machine.[5] This explains why Frater Fridericus' machine has an appearance reminiscent of Neßtfell's planetariums and, at first sight, Frater Fridericus' machine looks like a simplified version of Neßtfell's machine. Not only is the general appearance similar, but the front Earth-Mercury system of Neßtfell's machine is here laid horizontally. Instead, the machine of Frater Fridericus and Johann Georg Fellwöck does not display the orrery which fills most of Neßtfell's machines.

6.2 The going work

The going work is driven by a weight and regulated by a pendulum, but Oechslin doesn't give the details of the gears. All we know is that the rope arbor 4 makes a turn in 12 hours, clockwise, as seen from the left. We therefore have

$$\mathrm{V}_4^0 = -2 \tag{6.1}$$

Then

$$\mathrm{V}_5^0 = \mathrm{V}_4^0 \times \left(-\frac{13}{26}\right) = 1 \tag{6.2}$$

$$\mathrm{V}_6^0 = \mathrm{V}_5^0 \times \left(-\frac{26}{26}\right) = -1 \tag{6.3}$$

6.3 The main driving motion

The 26-teeth wheel on arbor 6 meshes with a 340-teeth contrate wheel with a vertical axis 7 (see figure 6.3). A crank can be put on arbor 6 and used to move the machine in the future or in the past. The velocity of the 340-teeth wheel is

$$\mathrm{V}_7^0 = \mathrm{V}_6^0 \times \left(-\frac{26}{340}\right) = \frac{13}{170} \tag{6.4}$$

$$\mathrm{P}_7^0 = \frac{170}{13} = 13.0769\ldots \text{ days} \tag{6.5}$$

6.4 The motion of the Earth around the Sun

The axis of the Earth is fixed on frame 26 and this frame rotates around the central axis which may be thought of the Sun. The motion of this frame is obtained using epicyclic gears located on the frame. This is somewhat similar

[5] [8, p. 226-227]

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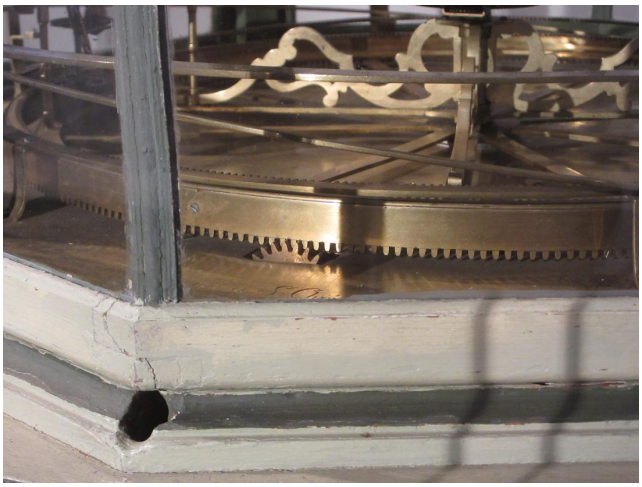


Figure 6.3: Detail of Frater Fridericus's clock in Bamberg. At the bottom, we can see the 26-teeth wheel which meshes with the 340-teeth wheel with a vertical arbor. A crank can be put in the hole to rotate that wheel. Frater Fridericus's signature is also partly visible. (photograph by the author)

to the constructions used by Neßtfell in his planetary machines. There is a train which is constrained by two relative motions. First, there is the motion of the frame 7 seen above. Second, there is the relative motion of the fixed frame 8 which carries a 498-teeth internal gear. We can therefore compute the motion of frame 8 as a function of the motion of frame 7, but with respect to the moving frame 26. We have

$$\mathrm{V}_8^{26} = \mathrm{V}_7^{26} \times \frac{434}{32} \times \left(-\frac{14}{50}\right) \times \left(-\frac{7}{20}\right) \times \left(-\frac{20}{69}\right) \times \frac{48}{498} \tag{6.6}$$

$$= \mathrm{V}_7^{26} \times \left(-\frac{10633}{286350}\right) \tag{6.7}$$

that is

$$\mathrm{V}_0^{26} = \left(\mathrm{V}_7^0 + \mathrm{V}_0^{26}\right) \times \left(-\frac{10633}{286350}\right) \tag{6.8}$$

and

$$\mathrm{V}_0^{26} \times \left(1 + \frac{10633}{286350}\right) = \mathrm{V}_7^0 \times \left(-\frac{10633}{286350}\right) \tag{6.9}$$

$$\mathrm{V}_0^{26} \times \frac{296983}{286350} = \mathrm{V}_7^0 \times \left(-\frac{10633}{286350}\right) \tag{6.10}$$

therefore

$$\mathrm{V}_0^{26} = \mathrm{V}_7^0 \times \left(-\frac{10633}{296983}\right) \tag{6.11}$$

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and

$$\mathrm{V}_{26}^{0} = \mathrm{V}_{7}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{6.12}$$

$$\mathrm{P}_{26}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{6.13}$$

This is an approximation of the tropical year. The same value is given by Oechslin. The Earth moves counterclockwise around the center of the machine in one year.

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6.5 The rotation of the Earth



Figure 6.4: The Earth globe on Frater Fridericus's clock in Bamberg. (photograph by the author)

The Earth is carried by the arbor 12 which is located on frame 26. We can compute the motion of this arbor with respect to its supporting frame:

$$\mathrm{V}_{12}^{26} = \mathrm{V}_{7}^{26} \times \frac{434}{32} = \mathrm{V}_{7}^{26} \times \frac{217}{16} \tag{6.14}$$

but we saw earlier that

$$\mathrm{V}_{8}^{26} = \mathrm{V}_{7}^{26} \times \left(-\frac{10633}{286350}\right) \tag{6.15}$$

therefore

$$\mathrm{V}_{7}^{26} = \mathrm{V}_{8}^{26} \times \left(-\frac{286350}{10633}\right) = \mathrm{V}_{26}^{0} \times \frac{286350}{10633} \tag{6.16}$$

$$= \frac{138229}{50487110} \times \frac{286350}{10633} = \frac{372255}{5048711} \tag{6.17}$$

and so

$$\mathrm{V}_{12}^{26} = \frac{372255}{5048711} \times \frac{217}{16} = \frac{80779335}{80779376} \tag{6.18}$$

$$\mathrm{P}_{12}^{26} = \frac{80779376}{80779335} = 1.0000005\ldots \text{ days} = 24 \text{ h } 0.0438\ldots \text{ s} \tag{6.19}$$

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The Earth thus turns counterclockwise in slightly more than 24 hours with respect to the supporting frame. In such a model, the Earth should actually make a turn in exactly 24 hours and this is not entirely true here.[6]

6.6 The shadowline on Earth

The boundary between the day and the night is shown by a circle surrounding the Earth and this ring is made to oscillate with the help of a feeler in contact with a large tilted ring. The inclination of this ring is not clearly visible in Oechslin's photographs. This large ring is currently (2025) not correctly positioned, and the feeler is not in contact with it (see figure 6.5).

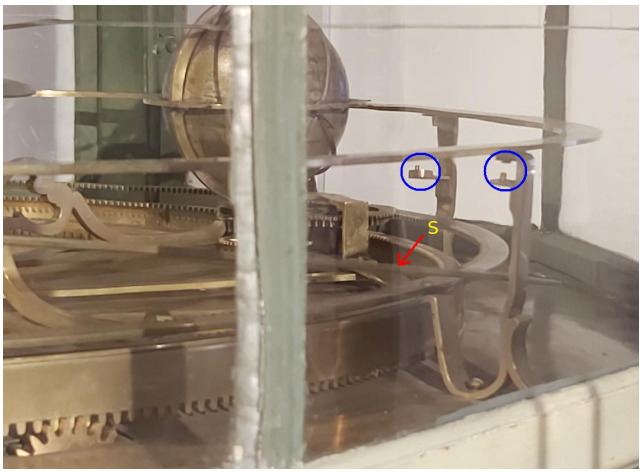


Figure 6.5: The tilted ring S for the shadowline of the Earth on Frater Fridericus's clock in Bamberg. This ring should go through the two fixations circled in blue, but doesn't. The feeler for the shadowline therefore is not in touch with the tilted ring, as it should normally be. (photograph by the author)

6.7 The Mercury system

The Mercury system is located on a fixed central frame which is not given any particular name by Oechslin. Mercury rotates on an circular orbit which is offset from the center of the machine, thus producing an eccentric motion which is that of frame 28. The motion of this frame within the fixed frame is

[6] See also Oechslin's observations on Frater Fridericus' machine [8, p. 198].

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obtained by the rotation of the frame 26. We have

$$\mathrm{V}^0_{28} = \mathrm{V}^0_{26} \times \left(-\frac{108}{26}\right) \times \left(-\frac{40}{40}\right) = \mathrm{V}^0_{26} \times \frac{54}{13} \tag{6.20}$$

$$= \frac{138229}{50487110} \times \frac{54}{13} = \frac{287091}{25243555} \tag{6.21}$$

$$\mathrm{P}^0_{28} = \frac{25243555}{287091} = 87.9287\ldots \text{ days} \tag{6.22}$$

This is an approximation of the orbital period of Mercury. The same value is given by Oechslin.

Around the path of Mercury, a tilted slope of about 5.6° (figure 6.6) is used to move Mercury up and down, thus simulating the angle of its orbital plane, whose actual value is about 7°.



Figure 6.6: The central part of Frater Fridericus's clock in Bamberg, with the motion of Mercury. The tilted ring causes Mercury to move up and down. (photograph by the author)

A hand goes from the axis of the Earth to Mercury and extends to the outside dial. This supposedly serves to show the geocentric position of Mercury, but it can only work if 1) the ratios of the orbits of Mercury and the Earth are correct, and 2) if the outside dial is at an infinite distance, which of course it isn't. We can however see if the orbits are represented to scale. Mercury's semimajor axis is about 57.9 million kilometers, and Earth's is about 149.6 million kilometers, thus a ratio of about 2.58. On Frater Fridericus' machine, the ratio is about $221/61 = 3.62$, and this is a very bad approximation, and also a surprising one, given that the relative distances of the planets were already well known at the beginning of the 17th century. Consequently, it appears that the representation of the geocentrical position of Mercury is quite bad. It is not very accurate and only gives a vague idea of where Mercury is located in the sky. A better result would have been obtained if the orbits had been to

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scale, and if the hand for Mercury's position had been obtained as a parallel to the Earth-Mercury line and going through the center of the machine.

Finally, the central Mercury system also carries the year. This is obtained as follows. The arbor 29 is part of the fixed frame of Mercury. This arbor carries a 4-teeth or 4-pointed star wheel which advances by one tooth when the frame 26 makes one turn, that is, every year. The arbor 29 thus makes a turn in four years and carries a hand showing the cycle of four years.

This arbor also carries a 4-teeth wheel meshing with a 10-teeth (or stars) wheel on tube 30, which carries the units of the year and makes a turn in 10 years. This motion is transferred to a wheel on arbor 31, and this arbor carries a finger which advances a 10-pointed star wheel on arbor 32. The latter carries the tens of the year and makes a turn in 100 years. The machine also seems to show the hundreds and thousands, but Oechslin did not describe them in his drawing. They may be constructed similarly, or perhaps only set manually.

6.8 The lunar system

The lunar system is located on the frame 26 and also makes a turn in one tropical year. It is always opposite the main Earth sphere.

The motion within this system is derived from arbor 13, which parallels the Earth arbor 12 seen earlier. Both of these arbors carry 32-teeth wheels which mesh with the internal 434-teeth gear on frame 7. We therefore have

$$\mathrm{V}_{13}^{26} = \mathrm{V}_{12}^{26} = \frac{80779335}{80779376} \approx 1 \tag{6.23}$$

6.8.1 The mean motion of the Moon

The Moon is on frame 19 which carries an 127-teeth internal gear. The motion of this frame is derived from that of arbor 13:

$$\mathrm{V}_{19}^{26} = \mathrm{V}_{13}^{26} \times \left(-\frac{15}{101}\right) \times \left(-\frac{50}{37}\right) \times \left(-\frac{50}{28}\right) \times \left(-\frac{12}{12}\right) \times \frac{12}{127} \tag{6.24}$$

$$= \mathrm{V}_{13}^{26} \times \frac{112500}{3322193} = \frac{80779335}{80779376} \times \frac{112500}{3322193} = \frac{324559828125}{9584452767556} \tag{6.25}$$

$$\mathrm{P}_{19}^{26} = \frac{9584452767556}{324559828125} = 29.5306\ldots \text{ days} \tag{6.26}$$

This is an approximation of the synodic month. The same value is given by Oechslin.

The phases of the Moon are shown on a fixed ring fixed on frame 26.

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6.8.2 The lunar nodes

The lunar nodes are on frame 25. The motion of this frame is also derived from that of arbor 13, using in particular two worms. We have

$$\mathrm{V}_{25}^{26} = \mathrm{V}_{13}^{26} \times \left(-\frac{1}{66}\right) \times \left(-\frac{20}{20}\right) \times \left(-\frac{1}{103}\right) \tag{6.27}$$

$$= \mathrm{V}_{13}^{26} \times \left(-\frac{1}{6798}\right) \approx -\frac{1}{6798} \tag{6.28}$$

$$\mathrm{P}_{25}^{26} = -\mathrm{P}_{13}^{26} \times 6798 \approx -6798 \text{ days} \tag{6.29}$$

This is an approximation of the period of precession of the lunar nodes and the value is negative because the nodes are retrograding. The same value is given by Oechslin.

The frame 25 carries a tilted slope (figure 6.7) which is used to move the Moon up and down, so as to simulate its motion in an inclined orbit. The inclination in this machine is a little bit more than 5 degrees.

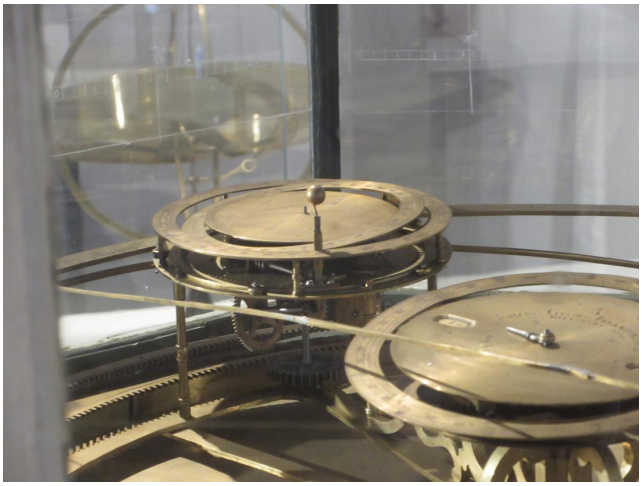


Figure 6.7: Detail of Frater Fridericus's clock in Bamberg. In the back, there is the part for the motion of the Moon. The internal tilted ring is used to move the Moon up and down. (photograph by the author)

6.8.3 The line of apsides

The lunar apsides are on frame 22. The motion of this frame is also derived from that of arbor 13, using in particular two worms. We have

$$\mathrm{V}_{22}^{26} = \mathrm{V}_{13}^{26} \times \frac{1}{32} \times \left(-\frac{32}{32}\right) \times \left(-\frac{1}{101}\right) = \mathrm{V}_{13}^{26} \times \frac{1}{3232} \approx \frac{1}{3232} \tag{6.30}$$

$$\mathrm{P}_{22}^{26} = \mathrm{P}_{13}^{26} \times 3232 \approx 3232 \text{ days} \tag{6.31}$$

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This is an approximation of the period of precession of the lunar apsides and the value is positive because the motion is prograde. The same value is given by Oechslin.

The frame 22 is not used to change the motion of the Moon, but rather to change the position of the Earth with respect to the Moon. The shortest distance between the Moon and the Earth, that is the perigee, moves like frame 22.

6.8.4 The rotation of the Earth

Within the lunar system, the Earth would have been on arbor 14, but it is now missing. The motion of arbor 14 is derived from that of arbor 13, but the arbor 14 is located on frame 22. So, we first compute the motion of arbor 14 with respect to frame 22:

$$\mathrm{V}_{14}^{22} = \mathrm{V}_{13}^{22} \times \left(-\frac{6}{6}\right) = -\mathrm{V}_{13}^{22} \tag{6.32}$$

then

$$\mathrm{V}_{14}^{26} = \mathrm{V}_{14}^{22} + \mathrm{V}_{22}^{26} = -\mathrm{V}_{13}^{22} + \mathrm{V}_{22}^{26} = -\left(\mathrm{V}_{13}^{26} + \mathrm{V}_{26}^{22}\right) + \mathrm{V}_{22}^{26} \tag{6.33}$$

$$= -\mathrm{V}_{13}^{26} + 2\mathrm{V}_{22}^{26} = -\mathrm{V}_{13}^{26} + 2\mathrm{V}_{13}^{26} \times \frac{1}{3232} \tag{6.34}$$

$$= -\mathrm{V}_{13}^{26} \times \left(1 - 2 \times \frac{1}{3232}\right) = -\mathrm{V}_{13}^{26} \times \frac{1615}{1616} \tag{6.35}$$

$$= -\frac{80779335}{80779376} \times \frac{1615}{1616} = -\frac{7674036825}{7678792448} \approx -1 \tag{6.36}$$

So, the Earth makes a turn clockwise in about one day. This motion is however wrong, as the Earth should move counterclockwise with respect to frame 26, as it does on arbor 12.

6.9 References

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[5] Joachim Heinrich Jäck. *Bamberg und dessen Umgebungen. Ein Taschenbuch*. Erlangen: J. J. Palm, 1813.

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[7] Henry Charles King and John Richard Millburn. *Geared to the stars — The evolution of planetariums, orreries, and astronomical clocks*. Toronto: University of Toronto Press, 1978.

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[10] August Stoehr. Das Planetarium von Johann Zick und andere Planetenmaschinen des 18. Jahrhunderts in den ehemaligen hochstiften Würzburg und Bamberg. *Frankenland. Zeitschrift für alle Franken und Frankrenfreunde zur Kenntnis und Pflege des fränkischen Volkstums*, 6-7(2):77–92, 1919/1920. [p. 78-80 on Neßtfell and Father Fridericus].

[11] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

[12] Ernst Zinner. *Der Sternenmantel Kaiser Heinrichs — Himmelskunde und Rechenkunst im alten Bamberg*. Bamberg: Bamberger Verlagshaus Meisenbach, 1939. [p. 42 on Neßtfell and Frater Fridericus].

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Chapter 7

(Oechslin: 8.1)

Hahn's *Weltmaschine* in Stuttgart (1769)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

7.1 Introduction

The "world machine", or *Weltmaschine*, described in this chapter was constructed under the supervision of Philipp Matthäus Hahn (1739-1790) in 1768/69 when he was pastor in Onstmettingen. Of all the *Priestermechaniker* mentioned in this book, Hahn is the one who made the most clocks, the one on which the most has been written, and also the one who left the most written traces. I will therefore not attempt to give a complete biography of Hahn and I will only sketch the main events in his life and direct the interested reader to more comprehensive sources.[1]

Hahn was born in Scharnhausen, near Stuttgart. His father Georg Gottfried Hahn (1705-1766) was a pastor who taught him Greek, Latin and Hebrew when he was only four years old. He was also early on interested in astronomy

[1] A number of very good accounts of Hahn's life have been published, in particular by Engelmann [8], Roeßle [34], Munz [27, 29], Bertsch [7] and King [21, p. 232-242]. Hahn himself has left journals which have been published [11, 12], workshop journals [15, 16, 17, 18] and a description of some of his works [10, 13, 14]. And in 1989, two great volumes have been published for the Hahn exhibition that was then organized [38, 39]. Some references to that work are given later, but there are many other interesting parts not cited here, for instance on the cases of Hahn's clocks. Zinner also gave an interesting summary of Hahn's life [44, p. 351-355], largely based on Engelmann's monography. Maurice also gave a short summary of Hahn's clocks with a number of illustrations [26, v.1, p. 269-273]. The *Allgemeine deutsche Biographie* has a very short notice by Hartmann [19]. The *Neue deutsche Biographie* has a more comprehensive notice by Freytag-Löringhoff [9]. See also Bassermann-Jordan [2] and Abeler's notice [1]. Besides Oechslin's work [33], there are many other short accounts of Hahn's life, such as the one by Roloff [36].

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Figure 7.1: Portait of Philipp Matthäus Hahn by Johann Philipp Weisbrod. (source: Wikimedia, Historisches Museum Basel, Inv. 1913.94.1.)

and constructed sundials when he was eight years old. He went to the nearby school in Esslingen then to the school in Nürtingen, not far from Scharnhausen. There he became under the influence of pietism.

In 1756 his father was transferred to Onstmettingen (now part of Albstadt) and he became acquainted with the teacher Philipp Gottfried Schaudt (1739-1809) who had been to an horology school.[2] He made a sundial for the church of Balingen and later other sundials in order to earn some money. At that time, he also read Wolff's *Anfangs-Gründe aller mathematischen Wissenschaften* and made excerpts of it.[3] From 1757 to 1759, Hahn studied theology at the University of Tübingen. After the end of his studies, Hahn occupied several positions, and was vicar in several parishes.

In 1763 he built his first "*Öhrsonnenuhr*," a portable sundial that could be used to set a clock (figure 7.3).[4] This is a sundial where a pinhole (*Öhr*) projects the image of the sun, instead of the Sun projecting the shadow of a gnomon. In other words, the time is read from the inverted shadow. However, the image of the sun is not read on a scale. Instead, the user must set the time on the dial until the image of the sun is located at a certain position.[5]

[2]On Schaudt, see especially Alfred Munz's chapter for the 1989 Hahn exhibition [39, p. 380-390].

[3]Among the interesting parts, see the one on gears [41]. Engelmann illustrates a page copied from Wolf [8, fig. 8]. See also the 1989 Hahn exhibition catalogue [38, pl .4].

[4]See Engelmann [8, p. 128-132, fig. 28-29], Zinner [44, p. 352], Syndram [37, p. 188-190] and especially the 1989 Hahn exhibition catalogue [38, p. 367-373].

[5]Hahn was not the first to use such devices. Sundials with alidades and hour/minute dials did exist before him, see for instance Johann Michael Bergauer's sundial made in Innsbruck

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In 1764, he became pastor in Onstmettingen, as successor to his father. In addition to his work as a pastor, he maintained from then on a precision mechanical workshop in the presbytery, which involved a number of workers and later some of his sons.[6] There, with the help of Schaudt, he invented a pendulum scale (figure 7.2)[7] and had constructed other machines and instruments, without constructing them himself. This workshop is considered as the germ cell of the contemporary precision industry in Württemberg.[8]

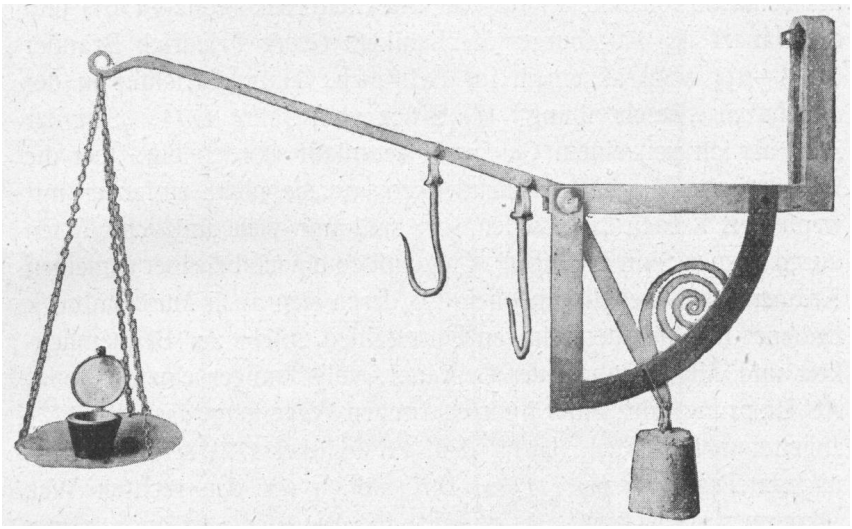


Figure 7.2: Hahn's pendulum scale. (source: [8])

After 1764, he commissioned a weaver to build a complex wooden clock, and then an orrery. Then, he let Schaudt make an astronomical clock in wood for Charles Eugene, Duke of Württemberg (1728-1793), for which he received 300 gulden in 1767.[9]

Hahn then received an order from the duke for an astronomical clock in metal, which is the machine described in this chapter, and which was completed in 1769.[10] The machine was sold to the duke for 8000 gulden and Hahn obtained the wooden clock back.

As a result of this new machine, Hahn obtained in 1770 the parish of Kornwestheim. He was also offered a position of professor of mathematics in Tübingen, but didn't accept it. In Kornwestheim, Hahn also had a workshop

before 1717 and kept at the *Germanisches Nationalmuseum* in Nuremberg (Inv. Nr. WI 1216).

[6]On Hahn's workshop, see especially Ulrike Zubal's chapter for the 1989 Hahn exhibition [39, p. 391-401]. See also the part on Hahn's collaborators [38, p. 44-52].

[7]See [20].

[8]On the work of Hahn on scales and the development of the scales industry, see especially Hans Jenemann's chapters for the 1989 Hahn exhibition [38, p. 150-187, 357-366], [39, p. 479-499].

[9][33, p. 221]

[10]See Engelmann [8, p. 142-148] and Zinner [44, p. 352].

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Figure 7.3: Hahn's "Öhrsonnenuhr" at the *Landesmuseum für Technik und Arbeit*, Mannheim. (source: Wikimedia, photograph by H. Zell)

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which involved a number of people who constructed instruments according to his plans.

For his astronomical clocks, Hahn constructed five or six rotary calculating machines, of which two survive. These machines use Leibniz' principle of the stepped drum in Leupold circular layout. Hahn's calculating machine was the first really usable such machine. There are also several reconstructions of these machines.[11]

Hahn received more and more orders and besides Schaudt, Hahn employed Johann Gottfried Ewald Sechting (1749-1818), his brothers Georg David and Gottfried, and later also his children.

Hahn also made tall-case clocks, so-called "Dielenuhren."[12]

In 1781, he became pastor of Echterdingen, not far from his birthplace. There the workshop mostly made pocket watches with improved cylinder escapements.[13]

Hahn was married twice and had a number of children. He published several books on theology and died in 1790 in Echterdingen.

After his death, Hahn's sons continued the manufacture of watches and clocks. His portable sundial was also copied by Georg Matthias Burger (1750-1825) in Nurnberg,[14] Hollrich in Mainz[15] and J. J. Sauter in Stockholm and St Petersburg.[16]

Among Hahn's sons, there was Christoph Matthäus (1767-1833) who helped his father, was court mechanician in Stuttgart, and made watches and clocks after his father's death. Christian Gottfried was also court mechanician in Stuttgart, Berlin and then went to America. Johann Georg also made watches and was court mechanician in Stuttgart from 1807 to 1813.

[11] On Hahn's calculating machines and their context, see especially Erhard Anthes's chapter for the 1989 Hahn exhibition [38, p. 463-466], [39, p. 456-478].

[12] Cf. [8, fig. 32].

[13] See [8, fig. 24-27] and [23, 22]. See especially Günther Oestmann's chapters on Hahn's watches for the 1989 catalogue [38, p. 467-514], [39, p. 500-510].

[14] See Zinner [44, p. 276-277].

[15] See Zinner [44, p. 387].

[16] See Zinner [44, p. 498].

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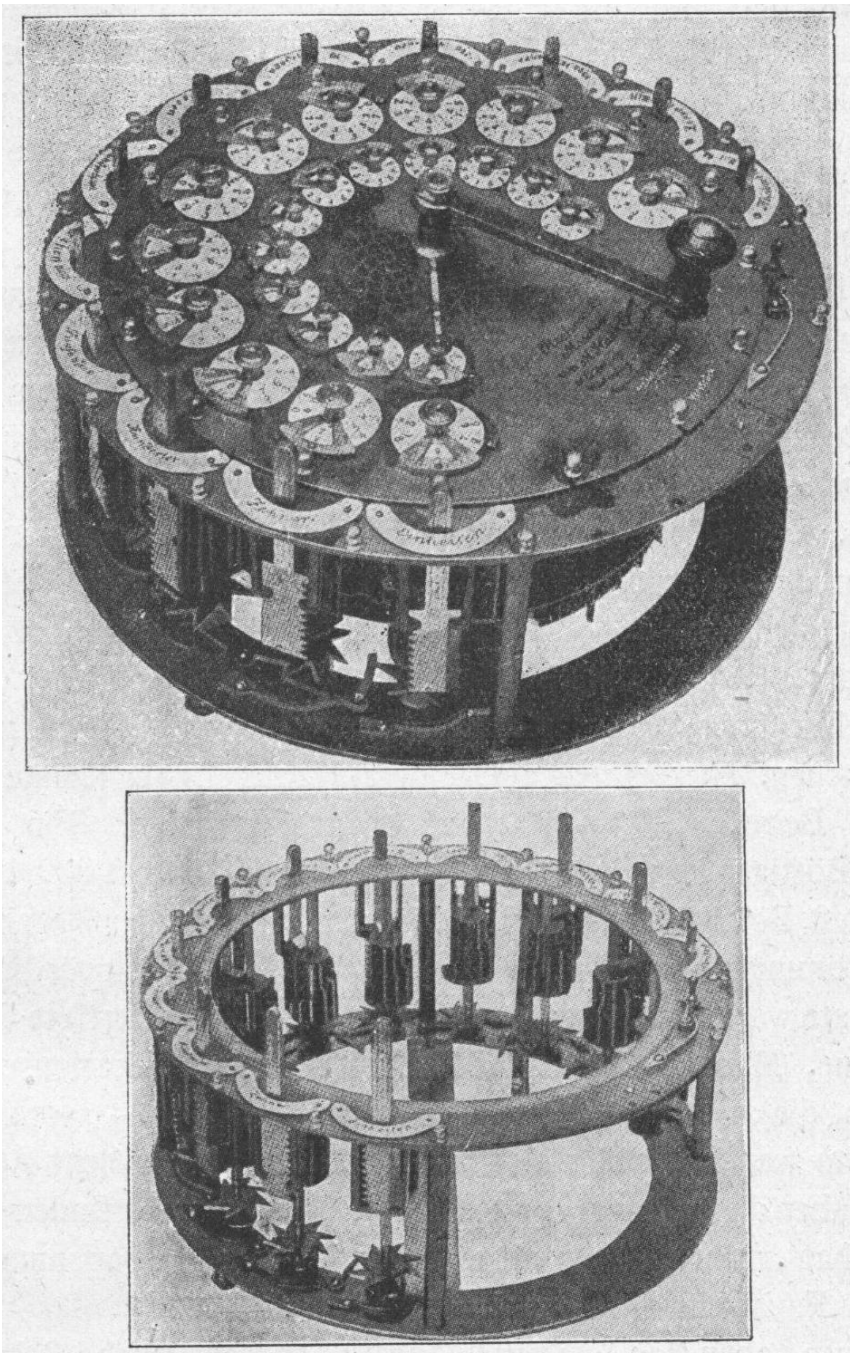


Figure 7.4: One of Hahn's calculating machines. (source: [8])

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7.2 History and general description

For the machine described in this chapter, Hahn was helped by the teacher Gottfried Schaudt (1739-1809).[17] This machine is Hahn's first large *Weltmaschine* and, as mentioned above, it was an order from Charles Eugene, duke of Württemberg (1728-1793), who installed it in the library of his castle in Ludwigsburg. This machine is therefore sometimes called the *Ludwigsburger Weltmaschine*.[18]

It is now exhibited in the *Landesmuseum Württemberg* (Württemberg State Museum) in Stuttgart.[19]

Hahn's machine is made of three parts (figure 7.5). The central part shows the time and the calendar, the left part shows the solar system in a heliocentric representation, and the right part shows a geocentric representation as a celestial sphere. These parts are connected through the base of the furniture. The current appearance of the furniture is not the original one (figure 7.8).[20]

[17][33, p. 221]

[18]Descriptions of the machine were given by Vischer [40], Hahn [10] and Oechslin [30]. See especially the 1989 Hahn exhibition catalogue [38, p. 375-382]. See also Oechslin [33, p. 55-57].

[19]For details on the history of the machine, see Oechslin [33, p. 234-235]. In 2008, a movie accompanying the machine was made by Christoph Prenosil and Steffen Schönbrunn as part of their final project in the *Hochschule für Medien* Stuttgart, under the direction of Prof. Dr. Bernd Eberhardt. This five minute movie is shown next to the machine, but it does not describe the clock in detail. The heliocentric system has been modelled in 3D, at least for a global rendering, but without any animation. The gears of the celestial sphere are not shown in the movie, and have probably not been modelled in 3D. The sphere itself is shown, probably using a 3D textured sphere, together with the various hands. (movie seen on April 11, 2025)

[20]The Hahn museum in Onstmettingen shows a 1/4 model of the original Ludwigsburg machine. This model is presumably non functional. The original engraving of the Ludwigsburg machine was presumably the one published in 1774 [10].

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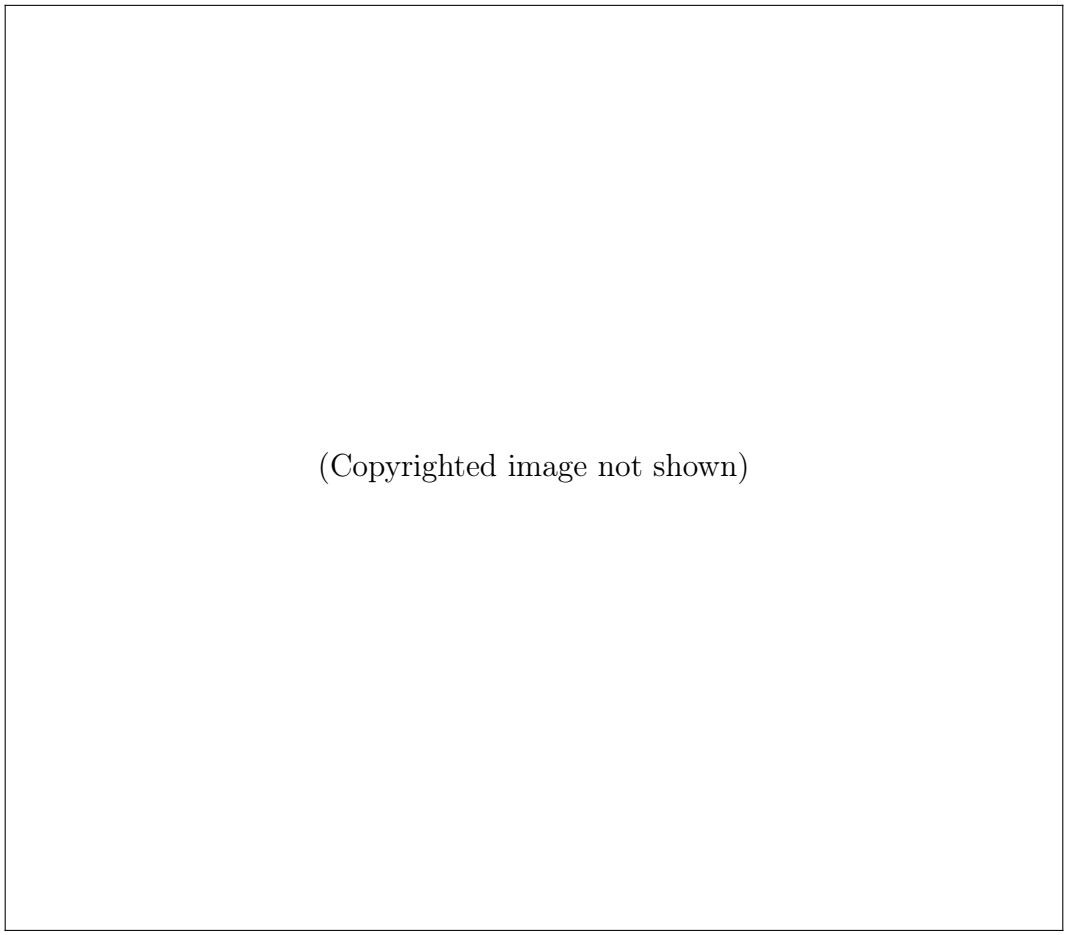


Figure 7.5: Hahn's "Weltmaschine" in Stuttgart out of its glass window.
(source: `https://bawue.museum-digital.de/object/81`)

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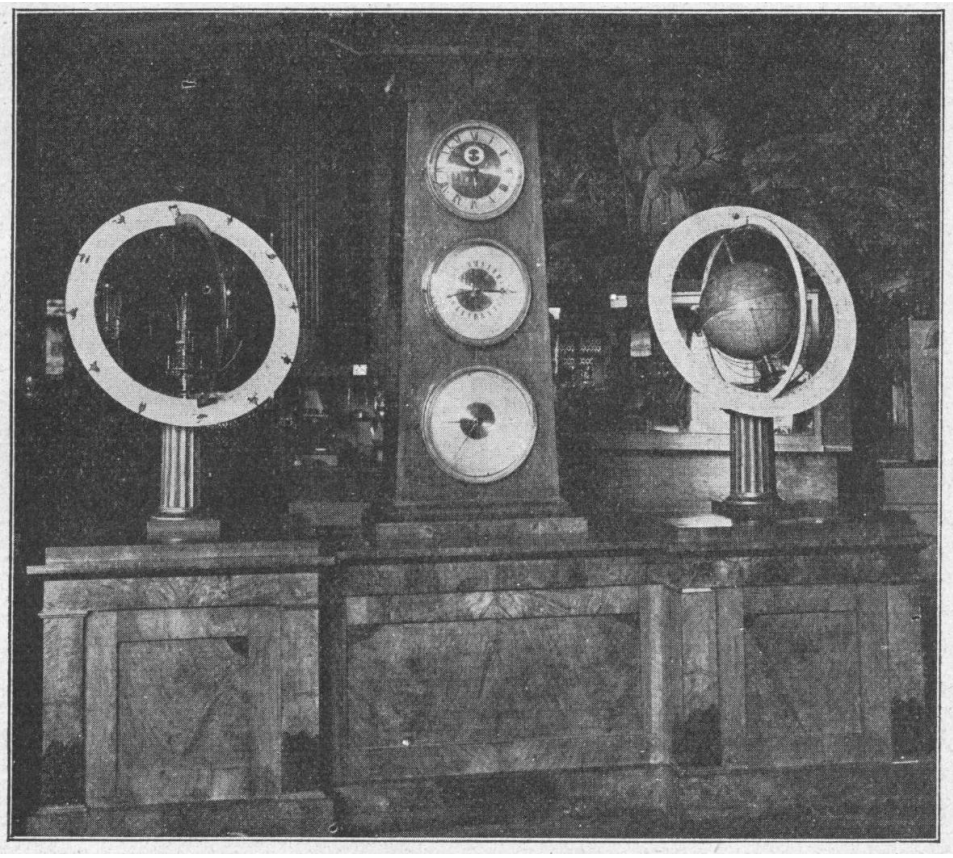


Figure 7.6: Hahn's *Weltmaschine* as it appeared in 1923. (source: [8])

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Figure 7.7: General view of Hahn's *Weltmaschine*. It is very difficult to photograph the machine in its entirety, because of the windows and lighting. (photograph by the author, 2025)

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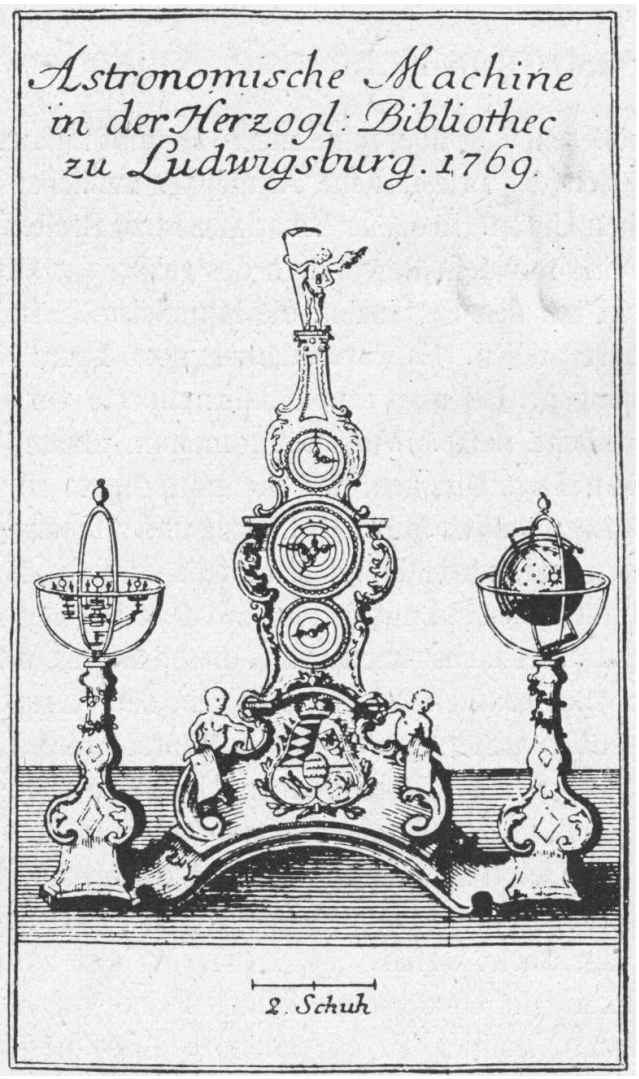


Figure 7.8: The original appearance of Hahn's *Weltmaschine*. (source: [8], from [10]) A reduced-scale non functional model of this case is exhibited in the Onstmettingen museum.

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The central part contains a clockwork with a 30-teeth escape wheel making one turn in one minute and leading to an arbor making one turn in an hour. Using four wheels, the hour hand cannon is made to make one turn in twelve hours. This cannon carries a 24-teeth wheel which meshes with a 48-teeth wheel which is making one turn in a day. This is the input motion for the other parts of the machine.

The first vertical arbor of that part is arbor 8 and it moves counterclockwise as seen from above (I assume that the motion of vertical arbors is measured from above):

$$\mathrm{V}_8^0 = 1 \tag{7.1}$$

This motion is transfered to the vertical arbor 31, but the direction of the motion is reversed (details of this reversal are given in the section on the calendar part):

$$\mathrm{V}_{31}^0 = -1 \tag{7.2}$$

This motion is again transfered to the left to the orrery part through the horizontal arbor 33 and to the right to the globe through the horizontal arbor 141. Both arbors make one turn in a day counterclockwise when looked from the right:

$$\mathrm{V}_{33}^0 = \mathrm{V}_{141}^0 = 1 \tag{7.3}$$

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7.3 The celestial globe

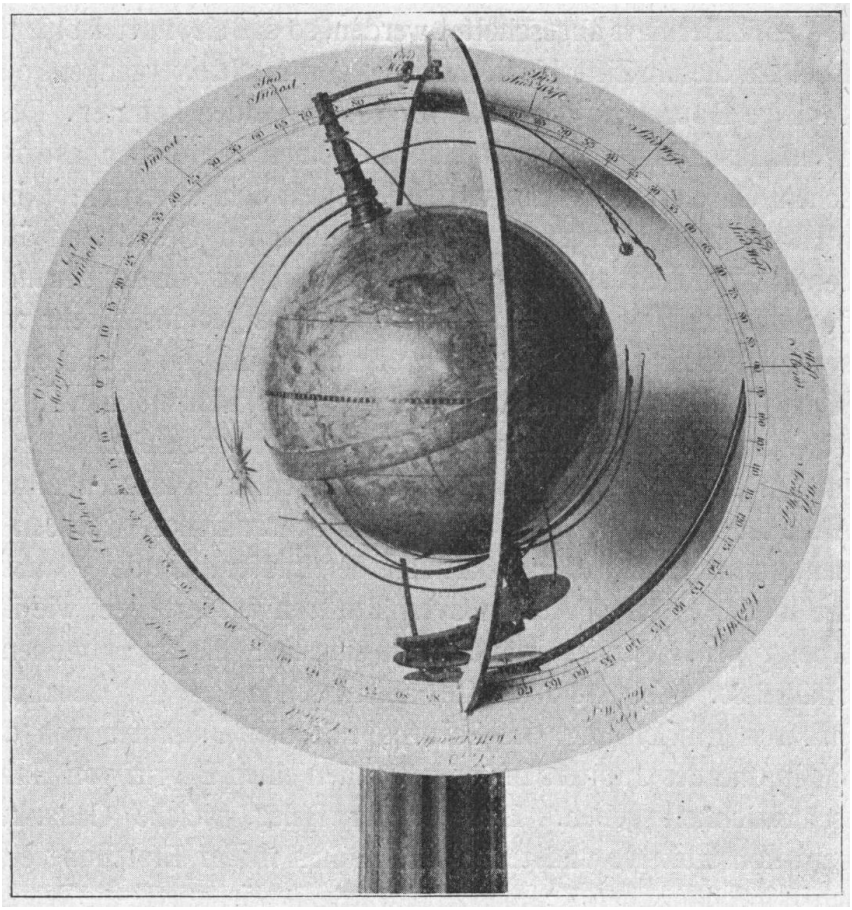


Figure 7.9: The celestial globe at the right of Hahn's *Weltmaschine*. (source: [8])

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Figure 7.10: Hahn's *Weltmaschine*: the celestial globe. (photograph by the author, 2025)

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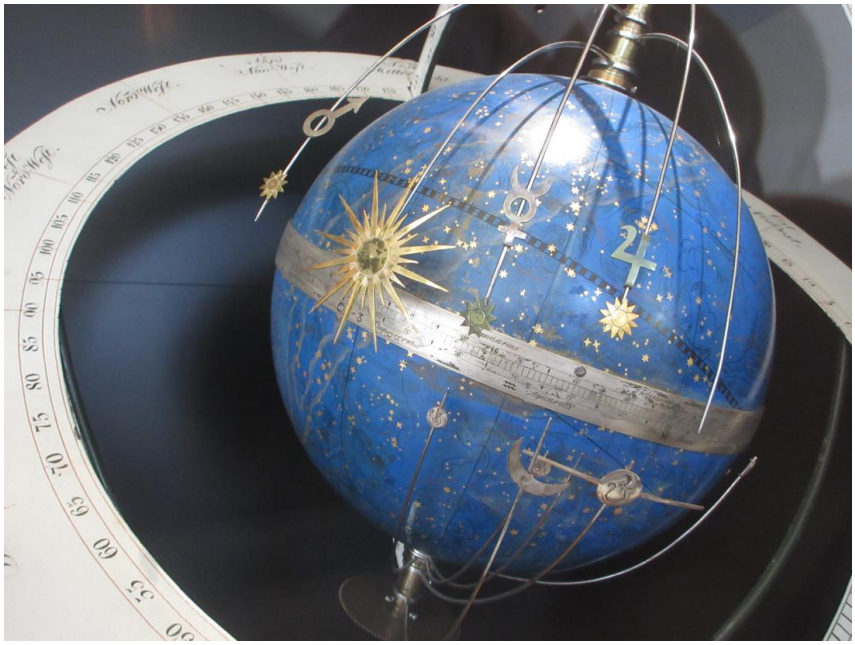


Figure 7.11: Hahn's *Weltmaschine*: the celestial globe. (photograph by the author, 2025)



Figure 7.12: Hahn's *Weltmaschine*: detail of the celestial globe. (photograph by the author, 2025)

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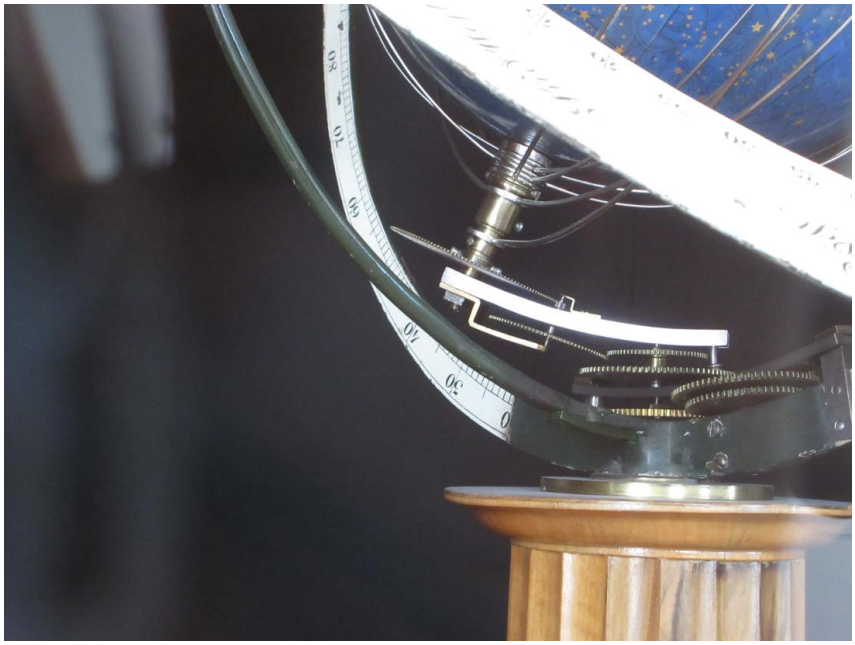


Figure 7.13: Hahn's *Weltmaschine*: detail of the transmission to the celestial globe. (photograph by the author, 2025)

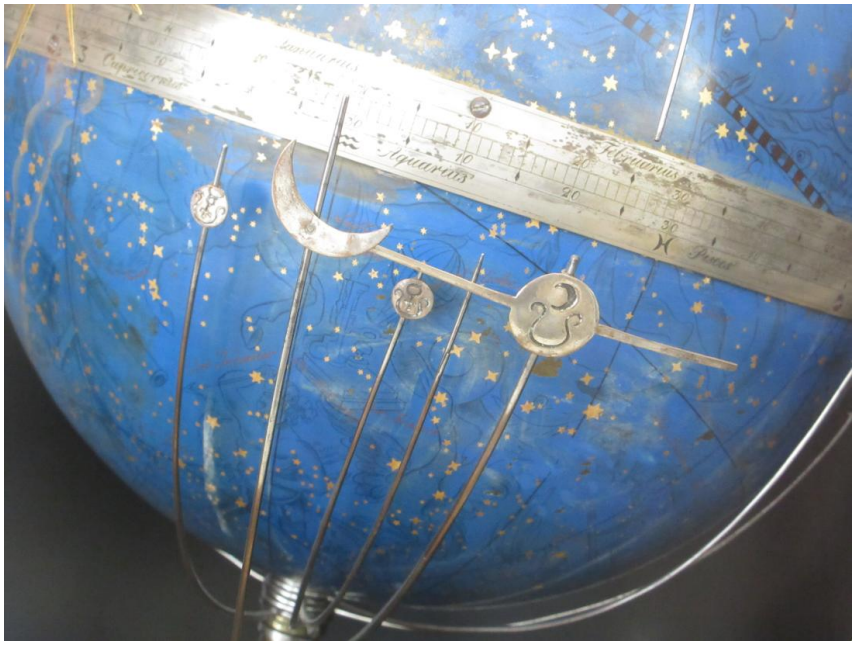


Figure 7.14: Hahn's *Weltmaschine*: detail of the celestial globe. From left to right, the figures of Mercury, the Moon, Venus, and the descending node of the Moon. It is possible that some of the figures are attached to the wrong hands. (photograph by the author, 2025)

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The globe in the right part of the machine can be thought of as a geocentric representation of the universe, with the Earth at its center.[21] This representation complements the heliocentric view of the left part.

The globe not only shows the apparent motion of the stars, but also represents the geocentric motion of the Moon, the Sun and the planets. The vertical axis represents the axis of the Earth, and the globe rotates around this axis. However, the planets, the Sun and the Moon rotate nearly in the plane of the ecliptic which is inclined by 23.5 degrees with the axis of the Earth. The main axis of the globe is therefore that of the planets, but this axis does therefore also rotate around the vertical axis.

A comparison with the gears of this globe and the globes of the *Weltmaschine* of Gotha (Oechslin 8.3) and the Aschaffenburg globe clock (Oechslin 8.4) shows that the structures of these three globes are very similar, and there are mainly small differences in teeth counts, but without altering the ratios.

On this globe, there are according to Oechslin's drawing two hands driven at the bottom of the globe (the corrected Moon and the lunar nodes), and six hands driven at the top of the globe (the Sun and the five planets). On the photographs, we can indeed see six hands fixed at the top of the globe, but also seven (?) at the bottom of the globe. I believe therefore that some of the hands are fixed at both ends of the globe, but only driven either at the top or the bottom, something that Oechslin did not indicate.

There are probably two problems in the mounting of the globe. First, some of the upper parts do not seem to bet connected to their corresponding lower parts. Second, some of the figures of the planets do not seem to be on the tube with which they should move. For instance, the figure of the Sun seems to be on the tube corresponding to Jupiter, the figure of Mercury seems to be on the tube of Venus, the figure of Jupiter seems to be on the tube for Mercury, and there may be other anomalies of that sort, which should be sorted out. In the sequel, I will only use Oechslin's drawing as a basis, but future work should look into this matter and clarify it.

Consequently, we can divide the motions in two parts. First, there is the apparent motion of the sky, then there is the geocentric motion of the other bodies with respect to the globe. The stars, moreover, must be thought of having fixed coordinates, in other words the precession of equinoxes is not taken into account by Hahn.

The motion of the globe originates from the horizontal arbor 141 mentioned earlier. This arbor makes one turn in a day and carries a 24-teeth wheel meshing with another 24-teeth wheel on a vertical arbor 142. This arbor therefore makes in one day one turn counterclockwise as seen from above:

$$\mathrm{V}_{142}^{0} = 1 \tag{7.4}$$

[21] Besides the previous photographs, the globe is also pictured in the 1989 Hahn exhibition catalogue [38, pl .12].

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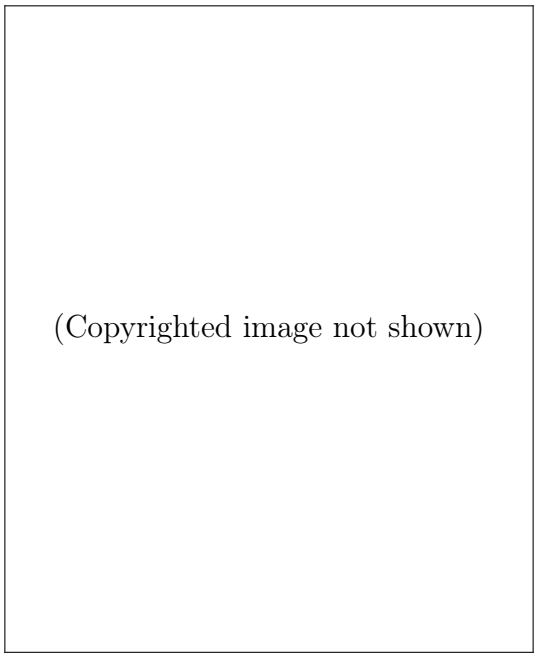


Figure 7.15: Detail of the globe. It is possible that some figures are attached to the wrong hands. (source: `https://www.leo-bw.de`)

The globe is contained in a fixed meridian ring. The arbor 142 carries a 57-teeth wheel meshing with a 58-teeth wheel on arbor 143. This motion is transfered to the arbor 144 and eventually to the tube 145. This tube has the following velocity:

$$\mathrm{V}^0_{145} = \mathrm{V}^0_{142} \times \left(-\frac{57}{58}\right) \times \left(-\frac{70}{66}\right) \times \left(-\frac{76}{79}\right) \tag{7.5}$$

$$= -\frac{25270}{25201} \tag{7.6}$$

which corresponds to the period

$$\mathrm{P}^0_{145} = -\frac{25201}{25270} \text{ days } = -\text{ 23h 56mn 4.0838... s} \tag{7.7}$$

The rotation of tube 145 is therefore clockwise. The motion of tube 145 is transfered to an inclined arbor 156 which does therefore rotate in one sidereal day around the vertical axis. This arbor 156 carries the globe.

Within the globe, all the motions are obtained from tube 148 whose motion is obtained from a 92-teeth wheel rotating on a fixed 71-teeth wheel bound to the meridian ring. As a consequence, in the reference frame of the globe (156), this 71-teeth wheel on arbor 146 is making one turn counterclockwise in one sidereal day:

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$$\mathrm{V}_{146}^{156} = -\mathrm{V}_{145}^{0} = \frac{25270}{25201} \tag{7.8}$$

Therefore

$$\mathrm{V}_{148}^{156} = \mathrm{V}_{146}^{156} \times \left(-\frac{71}{92}\right) \times \left(-\frac{7}{148}\right) \tag{7.9}$$

$$= \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \tag{7.10}$$

and

$$\mathrm{P}_{148}^{156} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{7.11}$$

This is an excellent approximation of the tropical month. It is the revolution of the Moon with respect to the zodiac. This motion is counterclockwise. Oechslin gives the same value, also in sidereal days.

The inside of the globe is organized in five compartments. The first compartment at the bottom takes care of the motions of the Moon. The second compartment takes care of the motions of Mercury and Venus. The third compartment takes care of the motions of Mars and the Sun. The fourth compartment takes care of the motion of Jupiter and the last compartment takes care of the motion of Saturn.

7.3.1 The motions of the Moon

The tube 148 carries a 112-teeth wheel which meshes with a 53-teeth wheel on arbor 149. This arbor carries a 4-leaves pinion meshing with a tube 150 carrying a 113-teeth wheel in the second compartment. This tube therefore has the velocity

$$\mathrm{V}_{150}^{156} = \mathrm{V}_{148}^{156} \times \left(-\frac{112}{53}\right) \times \left(-\frac{4}{113}\right) \tag{7.12}$$

$$= \frac{6279595}{171568408} \times \frac{448}{5989} = \frac{351657320}{128440399439} \tag{7.13}$$

$$\mathrm{P}_{150}^{156} = \frac{128440399439}{351657320} = 365.2430\ldots \text{ days} \tag{7.14}$$

The same value is given by Oechslin, also in sidereal days. This is a (not so good) approximation of the tropical year, which should be about 365.2422 days. The tube 150 therefore represents the mean longitude of the Sun.

This motion is then used to produce the motion of the lunar nodes on tube 155 in the first compartment, via the arbor 151:

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$$\mathrm{V}_{151}^{156} = \mathrm{V}_{150}^{156} \times \left(-\frac{28}{78}\right) = -\frac{4923202480}{5009175578121} \tag{7.15}$$

$$\mathrm{V}_{155}^{156} = \mathrm{V}_{151}^{156} \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{86}\right) \tag{7.16}$$

$$= \left(-\frac{4923202480}{5009175578121}\right) \times \frac{425}{2838} = -\frac{1046180527000}{7108020145353699} \tag{7.17}$$

$$\mathrm{P}_{155}^{156} = -\frac{7108020145353699}{1046180527000} = -6794.2577\ldots \text{ days} \tag{7.18}$$

This is an excellent approximation of the period of precession of the nodes which is about 6793 days. This precession is clockwise, hence the negative sign. Oechslin gives the same value, also in sidereal days.

The motion of tube 151 is also used to produce the motion of tube 152:

$$\mathrm{V}_{152}^{156} = \mathrm{V}_{151}^{156} \times \left(-\frac{23}{73}\right) = \left(-\frac{4923202480}{5009175578121}\right) \times \left(-\frac{23}{73}\right) \tag{7.19}$$

$$= \frac{4923202480}{15898687704471} \tag{7.20}$$

$$\mathrm{P}_{152}^{156} = \frac{15898687704471}{4923202480} = 3229.3385\ldots \text{ days} \tag{7.21}$$

Oechslin gives the same value, also in sidereal days.

Now, the 112-teeth wheel on tube 148 carries an eccentric arbor 153 with a 60-teeth wheel meshing with another 60-teeth wheel on tube 152. We can compute the period of arbor 153 with respect to tube 148 as follows:

$$\mathrm{V}_{153}^{148} = -\mathrm{V}_{152}^{148} = -\left(\mathrm{V}_{152}^{156} - \mathrm{V}_{148}^{156}\right) = \mathrm{V}_{148}^{156} - \mathrm{V}_{152}^{156} \tag{7.22}$$

$$= \frac{6279595}{171568408} - \frac{4923202480}{15898687704471} \tag{7.23}$$

$$= \frac{106165514457065}{2925358537622664} \tag{7.24}$$

$$\mathrm{P}_{153}^{148} = \frac{2925358537622664}{106165514457065} = 27.5546\ldots \text{ days} \tag{7.25}$$

This is an excellent approximation of the anomalistic month which is about 27.55455 days. It is not given by Oechslin. This motion is used to move an eccentric pin and create the correction known as the anomaly to the mean motion of the moon.

The motion of the nodes and the corrected motion of the moon are shown below the globe.

7.3.2 The motions of the Sun, Mercury and Venus

The second compartment the globe is mainly focused on the motions of Mercury and Venus. Hahn's purpose was to show the geocentric motion of the

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planets and in that perspective the inner planets Mercury and Venus oscillate around the Sun. Their mean geocentric motion is the mean motion of the Sun, which has a period of a tropical year with respect to the zodiac.

The periods of oscillations are those of the synodic revolutions of the planets, and so should be 115.88 days for Mercury and 583.92 days for Venus.

Let's see how Hahn tried to achieve this. We have already mentioned the tube 150 which corresponds to the tropical year. As a first step, the true motion of the Sun is obtained. In order to do so, the mean position of the Sun is corrected by the equation of center. This is done as follows. The 113-teeth wheel on tube 150 carries an arbor 158 whose motion is derived from a fixed 36-teeth wheel on the globe arbor 156. Since the train only contains 36-teeth wheels, the period of the wheel on arbor 158 is the same as that of tube 150 with respect to the globe, except that the motion of 158 is clockwise with respect to tube 150. Consequently the period of 158 with respect to tube 150 is the tropical year, but the wheel on arbor 158 is in fact still with respect to the globe.

Now, the wheel on arbor 158 carries an eccentric pin which makes the entire structure for Mercury and Venus oscillate with the period of the tropical year. However, the period of the equation of center is not the tropical year (365.2422 days), but the anomalistic year (about 365.2596 days), and so tube 150 should actually have been designed with that motion in mind. Moreover, as we have seen above, the mean motion of the Sun is not as accurate as it could have been. In any case, this still provides a good approximation of the longitude of the Sun. All that remains to be seen is how the actual motions of Mercury and Venus are obtained.

Now, Oechslin numbers the frame with the true motion of the Sun also 150, but I will name it $150'$. This frame carries a number of wheels which derive their motion from two fixed wheels on the globe's central arbor. The velocity of these two wheels (on arbor 156) with respect to the frame $150'$ is

$$\mathrm{V}_{156}^{150'} \approx \mathrm{V}_{156}^{150} = -\mathrm{V}_{150}^{156} = -\frac{351657320}{128440399439} \tag{7.26}$$

The oscillation of the Venus hand therefore occurs with velocity

$$\mathrm{V}_{168}^{150'} = \mathrm{V}_{156}^{150'} \times \left(-\frac{62}{40}\right) \times \left(-\frac{52}{44}\right) \times \left(-\frac{28}{82}\right) \tag{7.27}$$

$$= \mathrm{V}_{156}^{150'} \times \left(-\frac{2821}{4510}\right) \tag{7.28}$$

$$\approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{2821}{4510}\right) \tag{7.29}$$

$$\approx \frac{99202529972}{57926620146989} \tag{7.30}$$

$$\mathrm{P}_{168}^{150'} \approx \frac{57926620146989}{99202529972} = 583.92\ldots \text{ days} \tag{7.31}$$

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Venus having a near circular orbit, Hahn did not take its eccentricity into account. But he did so for Mercury. Its motion is obtained likewise from the true longitude of the Sun by adding an oscillation of period the synodic period of Mercury, but in addition Hahn adds a small term to account for Mercury's eccentric orbit.

We can also compute the motion of Venus with respect to the globe:

$$\mathrm{V}_{168}^{156} = \mathrm{V}_{168}^{150'} + \mathrm{V}_{150'}^{156} \approx \mathrm{V}_{168}^{150'} + \mathrm{V}_{150}^{156} \tag{7.32}$$

$$\approx \frac{99202529972}{57926620146989} + \frac{351657320}{128440399439} = \frac{257799981292}{57926620146989} \tag{7.33}$$

$$\mathrm{P}_{168}^{156} \approx \frac{57926620146989}{257799981292} = 224.6959\ldots \text{ days} \tag{7.34}$$

The same value is given by Oechslin.

The oscillation of the Mercury hand due to motion of the Earth therefore occurs with velocity

$$\mathrm{V}_{163}^{150'} = \mathrm{V}_{156}^{150'} \times \left(-\frac{58}{39}\right) \times \left(-\frac{58}{34}\right) \times \left(-\frac{41}{33}\right) \tag{7.35}$$

$$= \mathrm{V}_{156}^{150'} \times \left(-\frac{68962}{21879}\right) \tag{7.36}$$

$$\approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{68962}{21879}\right) \tag{7.37}$$

$$\approx \frac{836241106960}{96901637907789} \tag{7.38}$$

$$\mathrm{P}_{163}^{150'} \approx \frac{96901637907789}{836241106960} = 115.8776\ldots \text{ days} \tag{7.39}$$

See also Oechslin's additional observations [33, p. 135-136].

We can also compute the motion of Mercury with respect to the globe:

$$\mathrm{V}_{163}^{156} = \mathrm{V}_{163}^{150'} + \mathrm{V}_{150'}^{156} \approx \mathrm{V}_{163}^{150'} + \mathrm{V}_{150}^{156} \tag{7.40}$$

$$\approx \frac{836241106960}{96901637907789} + \frac{351657320}{128440399439} = \frac{31944902606120}{2810147499325881} \tag{7.41}$$

$$\mathrm{P}_{168}^{156} \approx \frac{2810147499325881}{31944902606120} = 87.9685\ldots \text{ days} \tag{7.42}$$

The same value is given by Oechslin.

We can observe that Hahn obtained very good approximations of the periods of the main oscillations of Mercury and Venus.

The eccentricity of Mercury's orbit is taken into account as follows. Hahn used a similar device as for the eccentricity of the Earth. He put the eccentric pin moving the Mercury hand on a wheel (arbor 165) which is still with respect to the globe frame. This means, like for the Sun, that a specific direction is defined in the sky and the motion of Mercury will be accelerated or decelerated according to the closeness to this direction. This will account for Mercury's equation of center.

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The motion of the frame 150′ is also transfered to the third compartment, using two parallel trains of 116 teeth (arbors 150′ and 177), 22 teeth (arbors 169 and 176) and again 22 teeth (arbor 170). On Oechslin's plan, it seems that arbor 170 goes through the frames of Mars and Jupiter, but perhaps Oechslin's plan is not entirely faithful to the actual layout and the wheels of Mars and Jupiter are smaller than shown. In any case, it is through this transfer that the motion of the solar hand is obtained on top of the globe and this hand moves with the true longitude of the Sun.

7.3.3 The motions of Mars, Jupiter and Saturn

The motions of Mars, Jupiter and Saturn are obtained in very similar ways, all from the true motion of the Sun. First Hahn obtains the tropical orbit periods.

Then he adds a correction for the equation of center. This is done using merely two identical wheels and an eccentric pin. The deviation due to the pin is positive during half of the tropical orbit period, and negative during the other half.

Finally, Hahn adds another correction to take into account the retrogradation due to the motion of the Earth and which has the period of the synodic revolution of the planet. This is also done in a very simple way, in that in all three cases, Hahn puts a wheel having the motion of the true sun (the mean sun with the equation of center) on the frame which has taken the equation of center of the planet into account. This wheel has a pin which puts the hand of the planet in its final position. Since the position of the pin depends of the relative angle of the motion of the Sun and the mean position of the planet, it does naturally have the period of the synodic revolution of the planet.

7.3.3.1 The motion of Mars

First, the mean motion of Mars is obtained from the true motion of the Sun. This motion corresponds to tube 172 whose velocity is

$$\mathrm{V}_{172}^{156} = \mathrm{V}_{150'}^{156} \times \left(-\frac{116}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{12}{30}\right) \times \left(-\frac{30}{119}\right) \tag{7.43}$$

$$= \mathrm{V}_{150'}^{156} \times \frac{696}{1309} \tag{7.44}$$

$$\approx \mathrm{V}_{150}^{156} \times \frac{696}{1309} \tag{7.45}$$

$$\approx \frac{351657320}{128440399439} \times \frac{696}{1309} \tag{7.46}$$

$$\approx \frac{1205682240}{828219127417} \tag{7.47}$$

$$\mathrm{P}_{172}^{156} \approx \frac{828219127417}{1205682240} = 686.9298\ldots \text{ days} \tag{7.48}$$

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This is a good approximation of the tropical orbit period of Mars which is about 686.972 days. The same value is given by Oechslin, also in sidereal days.

The synodic revolution of Mars can be computed as follows. We compute the difference of velocities between the tube for the mean motion of the Sun and the tropical period of Mars:

$$\mathrm{V}_{150}^{172} = \mathrm{V}_{150}^{156} - \mathrm{V}_{172}^{156} \approx \frac{351657320}{128440399439} - \frac{1205682240}{828219127417} \tag{7.49}$$

$$\approx \frac{30795133880}{24018354695093} \tag{7.50}$$

$$\mathrm{P}_{150}^{172} \approx \frac{24018354695093}{30795133880} = 779.9399\ldots \text{ days} \tag{7.51}$$

This is very close to the actual value of 779.94 days. This value is not given by Oechslin.

7.3.3.2 The motion of Jupiter

Like for Mars, the mean motion of Jupiter is obtained from the true motion of the Sun. This motion corresponds to tube 181 whose velocity is

$$\mathrm{V}_{181}^{156} = \mathrm{V}_{172}^{156} \times \left(-\frac{119}{30}\right) \times \left(-\frac{30}{31}\right) \times \left(-\frac{5}{34}\right) \times \left(-\frac{34}{121}\right) \tag{7.52}$$

$$= \mathrm{V}_{172}^{156} \times \frac{595}{3751} \tag{7.53}$$

$$\approx \frac{1205682240}{828219127417} \times \frac{595}{3751} \tag{7.54}$$

$$\approx \frac{42198878400}{182744114525951} \tag{7.55}$$

$$\mathrm{P}_{181}^{156} \approx \frac{182744114525951}{42198878400} = 4330.5443\ldots \text{ days} \tag{7.56}$$

This is a good approximation of the tropical orbit period of Jupiter which is about 4330.595 days. The same value is given by Oechslin, also in sidereal days.

We could similarly also compute the synodic revolution of Jupiter.

7.3.3.3 The motion of Saturn

Like for Mars and Jupiter, the mean motion of Jupiter is obtained from the true motion of the Sun. This motion corresponds to tube 187 whose velocity

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is

$$\mathrm{V}_{187}^{156} = \mathrm{V}_{181}^{156} \times \left(-\frac{121}{34}\right) \times \left(-\frac{24}{212}\right) \tag{7.57}$$

$$= \mathrm{V}_{181}^{156} \times \frac{363}{901} \tag{7.58}$$

$$\approx \frac{42198878400}{182744114525951} \times \frac{363}{901} \tag{7.59}$$

$$\approx \frac{126596635200}{1360764026346131} \tag{7.60}$$

$$\mathrm{P}_{187}^{156} \approx \frac{1360764026346131}{126596635200} = 10748.8166\ldots \text{ days} \tag{7.61}$$

This is a good approximation of the tropical orbit period of Saturn which is about 10746.94 days. The same value is given by Oechslin, also in sidereal days.

We could similarly also compute the synodic revolution of Saturn.

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7.4 The planetary system or orrery

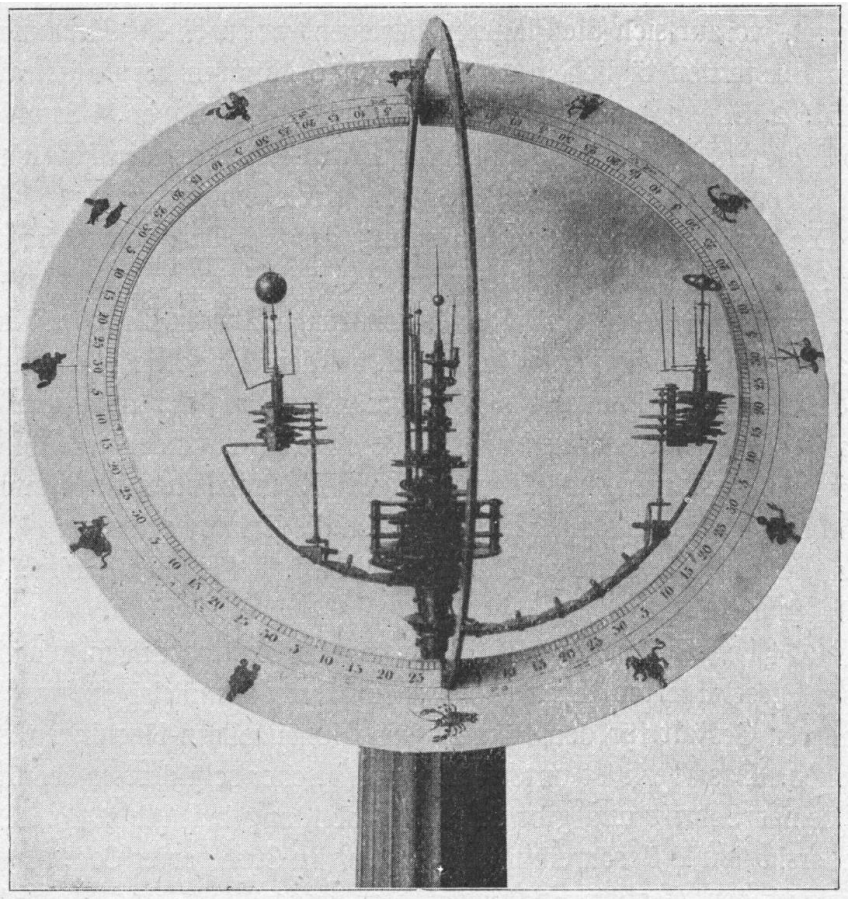


Figure 7.16: The orrery at the left of Hahn's *Weltmaschine*. (source: [8])

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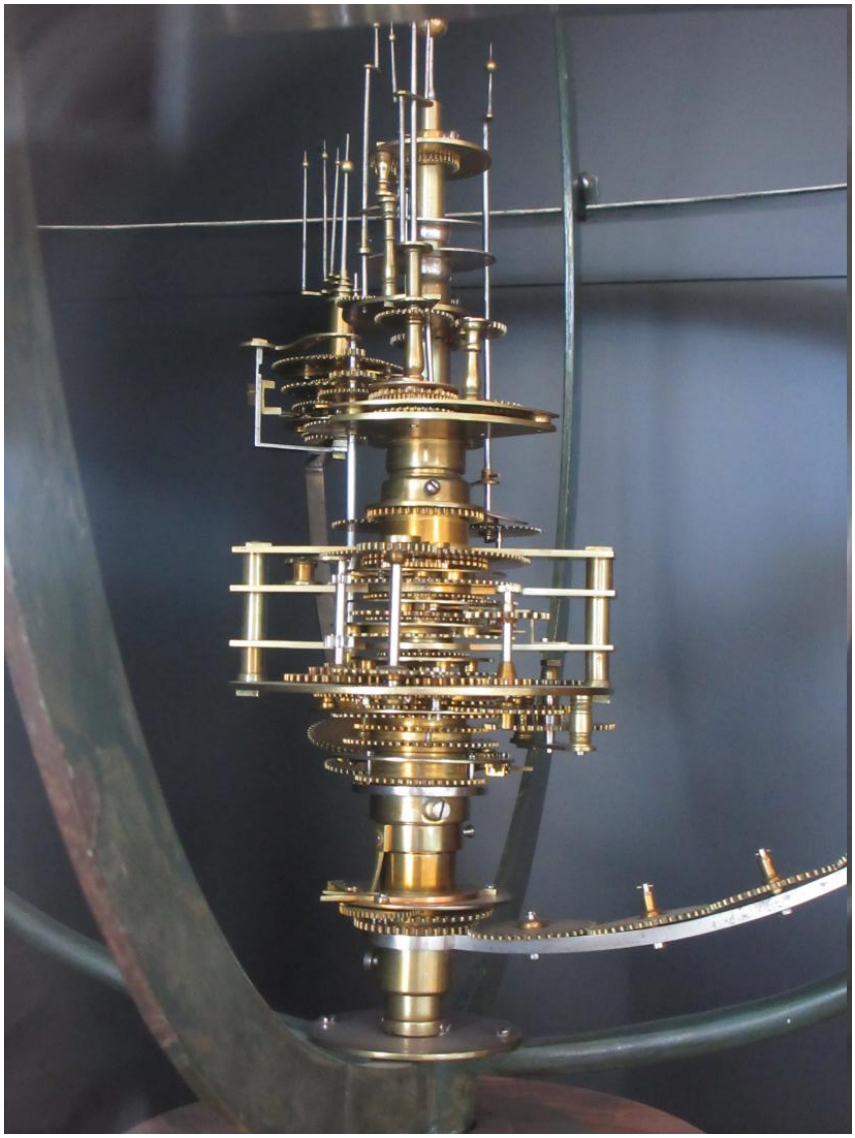


Figure 7.17: The central part of the orrery of Hahn's *Weltmaschine*. (photograph by the author, 2025)

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The orrery on the left of the machine shows the heliocentric motions of the planets Mercury to Saturn, as well as the Moon, the four then known satellites of Jupiter and the five then known satellites of Saturn. Mercury, Venus, Mars and the Moon are moreover given a motion in tilted planes.

As mentioned above, the orrery is driven by the horizontal arbor 33 coming from the central part. This arbor makes one turn counterclockwise in one day when when looked from the right:

$$\mathrm{V}_{33}^{0} = 1 \tag{7.62}$$

When reaching the orrery part, this arbor carries a 24-teeth wheel which meshes with a similar wheel on the vertical arbor 34. This arbor makes one turn clockwise in a day:

$$\mathrm{V}_{34}^{0} = -1 \tag{7.63}$$

7.4.1 The Sun

The central vertical arbor 34 actually goes through the entire planetary system and carries the Sun which then makes one turn in a day. As the Sun has presumably no features, this is in fact of no consequences, merely a side-effect of the construction.

7.4.2 The fixed cage

The central vertical arbor 34 goes through a fixed cage 54 which contains a number of gears that are used to produce the mean motions of Mercury, Venus, the Moon, the Earth and Mars. All these motions are derived from a single 24-teeth wheel located on that arbor 34.

Moreover, this motion is also transfered below the cage to the arms of Jupiter and Saturn, and eventually to the satellites of Jupiter and Saturn.

7.4.3 The mean motions of Mercury, Venus, the Moon, the Earth and Mars

The input motion of arbor 34 is first used to obtain the motion of a number of intermediary arbors 37, 39, 42, 43 and 45. The velocities of these arbors are

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the following:

$$V_{37}^{54} = V_{34}^{54} \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) \times \left(-\frac{9}{45}\right) = \frac{1}{5} \tag{7.64}$$

$$V_{39}^{54} = V_{37}^{54} \times \left(-\frac{15}{42}\right) = \frac{1}{5} \times \left(-\frac{15}{42}\right) = -\frac{1}{14} \tag{7.65}$$

$$V_{42}^{54} = V_{39}^{54} \times \left(-\frac{14}{28}\right) = \left(-\frac{1}{14}\right) \times \left(-\frac{14}{28}\right) = \frac{1}{28} \tag{7.66}$$

$$V_{43}^{54} = V_{42}^{54} \times \left(-\frac{4}{32}\right) = \frac{1}{28} \times \left(-\frac{4}{32}\right) = -\frac{1}{224} \tag{7.67}$$

$$V_{45}^{54} = V_{42}^{54} \times \left(-\frac{28}{30}\right) = \frac{1}{28} \times \left(-\frac{28}{30}\right) = -\frac{1}{30} \tag{7.68}$$

Then, these arbors are used to obtain a number of "base motions." The base motion of Mercury is given by tube 46 surrounding the central vertical arbor 34. This tube has the velocity

$$V_{46}^{54} = V_{45}^{54} \times \left(-\frac{15}{44}\right) = \left(-\frac{1}{30}\right) \times \left(-\frac{15}{44}\right) = \frac{1}{88} \tag{7.69}$$

$$P_{46}^{54} = 88 \text{ days} \tag{7.70}$$

The base motion of Venus is given by tube 47. This tube has the velocity

$$V_{47}^{54} = V_{45}^{54} \times \left(-\frac{6}{45}\right) = \left(-\frac{1}{30}\right) \times \left(-\frac{6}{45}\right) = \frac{1}{225} \tag{7.71}$$

$$P_{47}^{54} = 225 \text{ days} \tag{7.72}$$

The base motion of the Moon is given by tube 40. This tube has the velocity

$$V_{40}^{54} = V_{39}^{54} \times \left(-\frac{20}{39}\right) = \left(-\frac{1}{14}\right) \times \left(-\frac{20}{39}\right) = \frac{10}{273} \tag{7.73}$$

$$P_{40}^{54} = 27.3 \text{ days} \tag{7.74}$$

The base motion of the Earth is given by tube 48. This tube has the velocity

$$V_{48}^{54} = V_{45}^{54} \times \left(-\frac{6}{73}\right) = \left(-\frac{1}{30}\right) \times \left(-\frac{6}{73}\right) = \frac{1}{365} \tag{7.75}$$

$$P_{48}^{54} = 365 \text{ days} \tag{7.76}$$

The motion of Mars is given by tube 44. It is actually not the base motion, but the mean motion of Mars. Tube 44 has the velocity

$$V_{44}^{54} = V_{43}^{54} \times \left(-\frac{30}{92}\right) = \left(-\frac{1}{224}\right) \times \left(-\frac{30}{92}\right) = \frac{15}{10304} \tag{7.77}$$

$$P_{44}^{54} = \frac{10304}{15} = 686.9333\ldots \text{ days} \tag{7.78}$$

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The same value is given by Oechslin.

Each of the base wheels, except the one for Mars, carries a small pinion which meshes with the actual wheel corresponding to the mean motion of the planet or the Moon. These pinions are made to rotate slowly and thus slightly change the velocity of the base wheels. Similar constructions are also used for the satellites of Jupiter and Venus.

More precisely, this is done as follows. Mercury's base wheel 46 carries a train of wheels whose last element is a 6-pointed star wheel. This star wheel is advanced by one tooth for every turn of the base wheel. The tube 58 whose mean motion is the mean motion of Mercury makes x_1 turns with respect to the base wheel when the base wheel makes one turn. We have

$$x_1 = \left(-\frac{1}{6}\right) \times \left(-\frac{6}{18}\right) \times \left(-\frac{3}{27}\right) \times \left(-\frac{3}{51}\right) = \frac{1}{2754} \tag{7.79}$$

and

$$\mathrm{V}_{58}^{46} = x_1 \times \mathrm{V}_{46}^{54} \tag{7.80}$$

Consequently

$$\mathrm{V}_{58}^{54} = \mathrm{V}_{58}^{46} + \mathrm{V}_{46}^{54} = \mathrm{V}_{46}^{54} \times (1 + x_1) \tag{7.81}$$

$$= \frac{1}{88} \times \frac{2755}{2754} = \frac{2755}{242352} \tag{7.82}$$

and

$$\mathrm{P}_{58}^{54} = \frac{242352}{2755} = 87.9680\ldots \text{ days} \tag{7.83}$$

This is the (mean) period of tube 58 and it is very close to the actual orbital period of Mercury which is about 87.9691 days. Oechslin gives a slightly incorrect value of the period, because he mistakenly used the ratio $\frac{1}{9}$ instead of $\frac{1}{6}$ in the computation of x_1.

It is such a construction that enabled Hahn to obtain accurate periods without resorting to many-numbered teeth counts. The greatest number of teeth for a wheel in the orrery is 92.

In the case of Venus, we have

$$x_2 = \left(-\frac{1}{6}\right) \times \left(-\frac{3}{24}\right) \times \left(-\frac{24}{27}\right) \times \left(-\frac{3}{43}\right) = \frac{1}{774} \tag{7.84}$$

and

$$\mathrm{V}_{64}^{47} = x_2 \times \mathrm{V}_{47}^{54} \tag{7.85}$$

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Consequently

$$\mathrm{V}_{64}^{54} = \mathrm{V}_{64}^{47} + \mathrm{V}_{47}^{54} = \mathrm{V}_{47}^{54} \times (1 + x_2) \tag{7.86}$$

$$= \frac{1}{225} \times \frac{775}{774} = \frac{31}{6966} \tag{7.87}$$

and

$$\mathrm{P}_{64}^{54} = \frac{6966}{31} = 224.7096\ldots \text{ days} \tag{7.88}$$

This is the (mean) period of tube 64 and it is again an excellent approximation of the actual orbital period of Venus. Oechslin gives the same value.

In the case of the Moon, we have

$$x_3 = \left(-\frac{1}{9}\right) \times \left(-\frac{3}{23}\right) \times \left(-\frac{3}{55}\right) = -\frac{1}{1265} \tag{7.89}$$

and

$$\mathrm{V}_{69}^{40} = x_3 \times \mathrm{V}_{40}^{54} \tag{7.90}$$

Consequently

$$\mathrm{V}_{69}^{54} = \mathrm{V}_{69}^{40} + \mathrm{V}_{40}^{54} = \mathrm{V}_{40}^{54} \times (1 + x_3) \tag{7.91}$$

$$= \frac{10}{273} \times \frac{1264}{1265} = \frac{2528}{69069} \tag{7.92}$$

and

$$\mathrm{P}_{69}^{54} = \frac{69069}{2528} = 27.3215\ldots \text{ days} \tag{7.93}$$

This is the (mean) period of tube 69 and it is again an excellent approximation of the actual tropical month. Oechslin gives the same value.

In the case of the Earth, we have

$$x_4 = \left(-\frac{1}{9}\right) \times \left(-\frac{3}{36}\right) \times \left(-\frac{3}{43}\right) = -\frac{1}{1548} \tag{7.94}$$

and

$$\mathrm{V}_{78}^{48} = x_4 \times \mathrm{V}_{48}^{54} \tag{7.95}$$

Consequently

$$\mathrm{V}_{78}^{54} = \mathrm{V}_{78}^{48} + \mathrm{V}_{48}^{54} = \mathrm{V}_{48}^{54} \times (1 + x_4) \tag{7.96}$$

$$= \frac{1}{365} \times \frac{1547}{1548} = \frac{1547}{565020} \tag{7.97}$$

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and

$$\mathrm{P}_{78}^{54} = \frac{565020}{1547} = 365.2359\ldots \text{ days} \tag{7.98}$$

This is the (mean) period of tube 78 and it should be equal to the tropical year which is about 365.2422 days. Oechslin gives the same value, but there is an important discrepancy here, and it is to be noted that in Oechslin's figure the arbor with the 36-teeth wheel and the 3-leaves pinion has its teeth counts parenthesized, perhaps meaning that this arbor was missing. If this was the case, it appears that replacing 36 by 35 would give a more accurate value for the tropical year, namely $\frac{549325}{1504} = 365.24268\ldots$ days.

7.4.4 The upper part of the orrery

All the motions that I have described are transfered to the upper part of the orrery. From the central arbor to the outer tube fixed on the cage, we have in that order 1) the central arbor carrying the Sun, 2) the mean motion of Mercury, 3) a tube fixed the cage, 4) the mean motion of Venus, 5) the mean motion of the Moon, 6) another fixed tube, 7) the mean motion of the Earth, 8) the mean motion of Mars, and 9) another fixed tube.

I will now go again from the inside to the outside, except for the Earth and the Moon which I will consider last. After the central arbor moving the Sun we have the tube 58 for the mean motion of Mercury. However, Mercury is given two additional motions. First, the axis of Mercury is actually sliding on a tilted plane in order to have Mercury move on a tilted orbit. This tilted plane is fixed on the fixed tube surrounding the Mercury tube.

The second additional motion of Mercury is to account for Mercury's eccentric orbit. Hahn put Mercury slightly off its axis and Mercury's axis rotates. The elliptic motion of Mercury is therefore approximated by an epicycle. In order to keep the perihelion in a fixed direction, Hahn rotates Mercury's axis in such a way that it is only moving in translation with respect to frame 54. This can be observed by computing the velocity of Mercury's axis 60 with respect to the fixed frame 54:

$$\mathrm{V}_{60}^{54} = \mathrm{V}_{60}^{58} + \mathrm{V}_{58}^{54} \tag{7.99}$$

$$= \mathrm{V}_{54}^{58} \times \left(-\frac{36}{18}\right) \times \left(-\frac{12}{24}\right) + \mathrm{V}_{58}^{54} \tag{7.100}$$

$$= \mathrm{V}_{54}^{58} + \mathrm{V}_{58}^{54} = 0 \tag{7.101}$$

The same is done with Venus, but with only three 36-teeth wheels, and also with Mars, but with three 42-teeth wheels. In all three cases, the trains of three wheels starts with a fixed wheel. In the case of Mercury and Venus, the tilted planes are fixed to a fixed tube, but in the case of Mars, the tilted plane is fixed to the wheel which replicates the motion of a fixed central tube.

It now remains to see how the motions of the Earth and the Moon are obtained. We have seen above that the motion of tube 78 is that of the tropical

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year. This tube carries the axis of the Earth. Like for Mercury, Venus and Mars, the Earth is offset and moves on an epicycle, with the perihelion always in the same direction, again with three 36-teeth wheels, as for Venus. The motion of the Earth occurs in the plane of the ecliptic, and there is therefore no tilt.

Hahn next has the Moon move around the Earth and he achieves this by replicating the motion of a central tube like for the epicycles for the elliptic motions of Mercury, Venus, the Earth and Mars, but this time the central tube is not fixed. It is instead the tube whose motion is the tropical month. Consequently, the Moon revolves around the Earth, with a motion with respect to the fixed frame equal to that of the tropical month.

Finally, Hahn takes the tilt of the lunar orbit into account. But this tilt is not fixed in time. For the other planets Hahn considered the orientation of the orbital planes to be fixed, but in the case of the Moon, there is a greater precession of the nodes which needs to be taken into account if one wishes to display the solar and lunar eclipses, for instance. In order to better appreciate the motion of this plane, Hahn also shows the plane of the ecliptic around the lunar plane. The ecliptic is moved by three wheels again mimicking the motion of a fixed central tube.

Now, we can compute the motion of the lunar plane as follows. The lunar plane is fixed on tube 74. Let us compute the velocity of this tube with respect to the fixed frame 54:

$$\begin{aligned}
\mathrm{V}_{74}^{54} &= \mathrm{V}_{74}^{78} + \mathrm{V}_{78}^{54} && (7.102)\\
&= \mathrm{V}_{54}^{78} \times \left(-\frac{36}{36}\right) \times \left(-\frac{59}{56}\right) + \mathrm{V}_{78}^{54} && (7.103)\\
&= \mathrm{V}_{54}^{78} \times \frac{59}{56} + \mathrm{V}_{78}^{54} && (7.104)\\
&= \mathrm{V}_{54}^{78} \times \left(\frac{59}{56} - 1\right) && (7.105)\\
&= \mathrm{V}_{54}^{78} \times \frac{3}{56} = \left(-\frac{1547}{565020}\right) \times \frac{3}{56} && (7.106)\\
&= -\frac{221}{1506720} && (7.107)
\end{aligned}$$

The sign is negative, because this is a precession.

$$\mathrm{P}_{74}^{54} = -\frac{1506720}{221} = -6817.737\ldots \text{ days} \qquad (7.108)$$

Oechslin gives the same value.

This is a somewhat less accurate value than the one used in the globe, which was 6794.257 days, the correct period being about 6793 days. However, given that Hahn has only used the pair 59/56, this is in fact quite remarkable.

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7.4.5 The lower part of the orrery

The lower part of the orrery is concerned with the motions of Jupiter, Saturn and their satellites. Things are a bit simpler here, in that there are no eccentric motions, and no tilted planes. Moreover, the motions are obtained like for Mercury and other bodies, namely by first creating a base motion, the correcting it at regular intervals.

But first, from the fixed cage, Hahn obtains the motions of four tubes around the central vertical arbor of the orrery, tubes 51 (Jupiter), 38 (Jupiter's satellites), 53 (Saturn) and 41 (Saturn's satellites). These motions are themselves first obtained from those of arbors 50, 37, 52 and 39. We have already found above that

$$\mathrm{V}_{37}^{54} = \frac{1}{5} \tag{7.109}$$

$$\mathrm{V}_{39}^{54} = -\frac{1}{14} \tag{7.110}$$

and we have

$$\mathrm{V}_{49}^{54} = \mathrm{V}_{43}^{54} \times \left(-\frac{16}{16}\right) = \frac{1}{224} \tag{7.111}$$

$$\mathrm{V}_{52}^{54} = \mathrm{V}_{49}^{54} \times \left(-\frac{16}{49}\right) = \frac{1}{224} \times \left(-\frac{16}{49}\right) = -\frac{1}{686} \tag{7.112}$$

$$\mathrm{V}_{50}^{54} = \mathrm{V}_{49}^{54} \times \left(-\frac{16}{16}\right) = -\frac{1}{224} \tag{7.113}$$

Now

$$\mathrm{V}_{51}^{54} = \mathrm{V}_{50}^{54} \times \left(-\frac{3}{58}\right) = \left(-\frac{1}{224}\right) \times \left(-\frac{3}{58}\right) = \frac{3}{12992} \tag{7.114}$$

$$\mathrm{P}_{51}^{54} = \frac{12992}{3} = 4330.6666\ldots \text{ days} \tag{7.115}$$

$$\mathrm{V}_{38}^{54} = \mathrm{V}_{37}^{54} \times \left(-\frac{45}{45}\right) = -\frac{1}{5} \tag{7.116}$$

$$\mathrm{V}_{53}^{54} = \mathrm{V}_{52}^{54} \times \left(-\frac{3}{47}\right) = \left(-\frac{1}{686}\right) \times \left(-\frac{3}{47}\right) = \frac{3}{32242} \tag{7.117}$$

$$\mathrm{P}_{53}^{54} = \frac{32242}{3} = 10747.3333\ldots \text{ days} \tag{7.118}$$

$$\mathrm{V}_{41}^{54} = \mathrm{V}_{39}^{54} \times \left(-\frac{42}{42}\right) = \frac{1}{14} \tag{7.119}$$

We can see that the motions of tubes 51 and 53 are those of the orbital periods of Jupiter and Saturn. Oechslin gives the same values.

The motions of tubes 38 and 41 are used to obtain the motions of tubes 83 and 109, which are the base motions for the satellites of Jupiter and Saturn.

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The motion of tube 83 is obtained like the corrective motions of Mercury, Venus, etc. When tube 38 makes one turn, tube 83 makes an additional rotation of x_5 turns

$$x_5 = \left(-\frac{1}{7}\right) \times \left(-\frac{5}{32}\right) \times \left(-\frac{3}{58}\right) = -\frac{15}{12992} \tag{7.120}$$

and

$$\mathrm{V}_{83}^{38} = x_5 \times \mathrm{V}_{38}^{54} \tag{7.121}$$

Consequently

$$\mathrm{V}_{83}^{54} = \mathrm{V}_{83}^{38} + \mathrm{V}_{38}^{54} = \mathrm{V}_{38}^{54} \times (1 + x_5) \tag{7.122}$$

$$= \left(-\frac{1}{5}\right) \times \frac{12977}{12992} = -\frac{12977}{64960} \tag{7.123}$$

$$\mathrm{P}_{83}^{54} = -\frac{64960}{12977} = -5.00577\ldots \text{ days} \tag{7.124}$$

It should be noted that Oechslin shows the direction of rotation of tube 83 in two ways: above, it shows the motion as counterclockwise, that is the opposite direction to tube 38 (hence the negative sign of x_5), but below it shows it clockwise which corresponds to the negative sign of V_{83}^{54}.

The motion of tube 109 is obtained similarly:

$$x_6 = \left(-\frac{1}{6}\right) \times \left(-\frac{3}{49}\right) \times \left(-\frac{3}{47}\right) = -\frac{3}{4606} \tag{7.125}$$

and

$$\mathrm{V}_{109}^{41} = x_6 \times \mathrm{V}_{41}^{54} \tag{7.126}$$

Consequently

$$\mathrm{V}_{109}^{54} = \mathrm{V}_{109}^{41} + \mathrm{V}_{41}^{54} = \mathrm{V}_{41}^{54} \times (1 + x_6) \tag{7.127}$$

$$= \frac{1}{14} \times \frac{4603}{4606} = \frac{4603}{64484} \tag{7.128}$$

$$\mathrm{P}_{109}^{54} = \frac{64484}{4603} = 14.00912\ldots \text{ days} \tag{7.129}$$

Oechslin also shows the direction of rotation of tube 109 in two ways: above, it shows the motion as clockwise, that is the opposite direction to tube 41 (hence the negative sign of x_6), but below it shows it counterclockwise which corresponds to the positive sign of V_{109}^{54}.

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7.4.5.1 Jupiter and its satellites

The Jupiter system is located on an arm fixed on tube 51 whose motion is the mean motion of Jupiter. Jupiter is not given any eccentric motion, although Hahn took it into account in the Furtwangen orrery (Oechslin 8.12) in 1774, in the Gotha *Weltmaschine* (Oechslin 8.3) in 1780, and in the Nuremberg *Weltmaschine* (Oechslin 8.2) in the 1780s.

I am first going to compute the velocities and periods in the Jupiter arm reference frame, then compute the absolute velocities and periods.



Figure 7.18: Hahn's *Weltmaschine*: the central part of the orrery and the system of Jupiter. (photograph by the author, 2025)

The input motion on the arm is the relative motion of tube 83 with respect to tube 51.

$$\mathrm{V}_{83}^{51} = \mathrm{V}_{83}^{54} - \mathrm{V}_{51}^{54} = -\frac{12977}{64960} - \frac{3}{12992} = -\frac{1}{5} \tag{7.130}$$

On the Jupiter arm, there is a first train leading to arbor 87. This arbor carries wheels which are driving the four base wheels of the satellites. Arbor

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Figure 7.19: Hahn's *Weltmaschine*: the system of Jupiter. (photograph by the author, 2025)

87 has the following velocity:

$$\mathrm{V}_{87}^{51} = \mathrm{V}_{83}^{51} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{24}\right) \times \left(-\frac{24}{24}\right) \tag{7.131}$$

$$= \mathrm{V}_{83}^{51} \times 2 = -\frac{2}{5} \tag{7.132}$$

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and the base wheels have these motions:

$$\mathrm{V}_{92}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{48}{34}\right) = \frac{48}{85} \tag{7.133}$$

$$\mathrm{P}_{92}^{51} = \frac{85}{48} = 1.7708\ldots \text{ days (Io)} \tag{7.134}$$

$$\mathrm{V}_{93}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{45}{64}\right) = \frac{9}{32} \tag{7.135}$$

$$\mathrm{P}_{93}^{51} = \frac{32}{9} = 3.5555\ldots \text{ days (Europa)} \tag{7.136}$$

$$\mathrm{V}_{98}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{15}{43}\right) = \frac{6}{43} \tag{7.137}$$

$$\mathrm{P}_{98}^{51} = \frac{43}{6} = 7.1666\ldots \text{ days (Ganymede)} \tag{7.138}$$

$$\mathrm{V}_{103}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{10}{67}\right) = \frac{4}{67} \tag{7.139}$$

$$\mathrm{P}_{103}^{51} = \frac{67}{4} = 16.75 \text{ days (Callisto)} \tag{7.140}$$

The base wheels have heliocentric periods which are very close to the actual orbital periods. The absolute velocity of Io's base wheel, for instance, is

$$\mathrm{V}_{92}^{54} = \mathrm{V}_{92}^{51} + \mathrm{V}_{51}^{54} = \frac{48}{85} + \frac{3}{12992} = \frac{623871}{1104320} \tag{7.141}$$

$$\mathrm{P}_{92}^{54} = \frac{1104320}{623871} = 1.7701\ldots \text{ days} \tag{7.142}$$

and this differs little from the heliocentric period, because the satellites move much faster around Jupiter than Jupiter around the Sun.

The figure given by Oechslin for the motion of Io is not clear and is probably erroneous. I will assume below that Io's base wheel of 34 teeth carries an eccentric arbor on which there is a 10-teeth wheel and a 4-leaves pinion. I assume that this pinion meshes with the actual wheel of Io which has 52 teeth. On Oechslin's drawing, the 52-teeth wheel is bound to the base wheel and the actual Io wheel is not toothed.

Assuming that this is correct, the 4-leaves pinion then makes a small angular motion for each rotation of the base wheel, using a similar construction than that described earlier for Mercury and other planets. Eventually, the tube 88 carrying Io makes an additional fraction of a turn x_7 when the base wheel makes one turn. We have:

$$x_7 = \left(-\frac{1}{42}\right) \times \left(-\frac{3}{10}\right) \times \left(-\frac{4}{52}\right) = -\frac{1}{1820} \tag{7.143}$$

Then

$$\mathrm{V}_{88}^{92} = x_7 \times \mathrm{V}_{92}^{51} \tag{7.144}$$

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Consequently

$$\mathrm{V}_{88}^{51} = \mathrm{V}_{88}^{92} + \mathrm{V}_{92}^{51} = \mathrm{V}_{92}^{51} \times (1 + x_7) \tag{7.145}$$

$$= \frac{48}{85} \times \frac{1819}{1820} = \frac{1284}{2275} \tag{7.146}$$

$$\mathrm{P}_{88}^{51} = \frac{2275}{1284} = 1.7718\ldots \text{ days} \tag{7.147}$$

Europa's base wheel of 64 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Europa's actual satellite wheel (30 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 97 carrying Europa makes an additional fraction of a turn x_8 when the base wheel makes one turn. We have:

$$x_8 = \left(-\frac{1}{8}\right) \times \left(-\frac{1}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{3}{30}\right) = \frac{1}{2400} \tag{7.148}$$

Then

$$\mathrm{V}_{97}^{93} = x_8 \times \mathrm{V}_{93}^{51} \tag{7.149}$$

Consequently

$$\mathrm{V}_{97}^{51} = \mathrm{V}_{97}^{93} + \mathrm{V}_{93}^{51} = \mathrm{V}_{93}^{51} \times (1 + x_8) \tag{7.150}$$

$$= \frac{9}{32} \times \frac{2401}{2400} = \frac{7203}{25600} \tag{7.151}$$

$$\mathrm{P}_{97}^{51} = \frac{25600}{7203} = 3.5540\ldots \text{ days} \tag{7.152}$$

The 3-leaves pinion and the 30-teeth wheel on its arbor are parenthesized by Oechslin, and I assume that they were missing.

Ganymede's base wheel of 43 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Ganymede's actual satellite wheel (43 teeth). Eventually, the tube 102 carrying Ganymede makes an additional fraction of a turn x_9 when the base wheel makes one turn. We have:

$$x_9 = \left(-\frac{1}{8}\right) \times \left(-\frac{1}{27}\right) \times \left(-\frac{3}{30}\right) \times \left(-\frac{3}{43}\right) = \frac{1}{30960} \tag{7.153}$$

Then

$$\mathrm{V}_{102}^{98} = x_9 \times \mathrm{V}_{98}^{51} \tag{7.154}$$

Consequently

$$\mathrm{V}_{102}^{51} = \mathrm{V}_{102}^{98} + \mathrm{V}_{98}^{51} = \mathrm{V}_{98}^{51} \times (1 + x_9) \tag{7.155}$$

$$= \frac{6}{43} \times \frac{30961}{30960} = \frac{30961}{221880} \tag{7.156}$$

$$\mathrm{P}_{102}^{51} = \frac{221880}{30961} = 7.1664\ldots \text{ days} \tag{7.157}$$

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Callisto's base wheel of 67 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Callisto's actual satellite wheel (55 teeth). Eventually, the tube 106 carrying Callisto makes an additional fraction of a turn x_{10} when the base wheel makes one turn. We have:

$$x_{10} = \left(-\frac{1}{6}\right) \times \left(-\frac{1}{43}\right) \times \left(-\frac{3}{55}\right) = -\frac{1}{4730} \tag{7.158}$$

Then

$$\mathrm{V}_{106}^{103} = x_{10} \times \mathrm{V}_{103}^{51} \tag{7.159}$$

Consequently

$$\mathrm{V}_{106}^{51} = \mathrm{V}_{106}^{103} + \mathrm{V}_{103}^{51} = \mathrm{V}_{103}^{51} \times (1 + x_{10}) \tag{7.160}$$

$$= \frac{4}{67} \times \frac{4729}{4730} = \frac{9458}{158455} \tag{7.161}$$

$$\mathrm{P}_{106}^{51} = \frac{158455}{9458} = 16.7535\ldots \text{ days} \tag{7.162}$$

We can also compute the absolute motions:

$$\mathrm{V}_{88}^{54} = \mathrm{V}_{88}^{51} + \mathrm{V}_{51}^{54} = \frac{1284}{2275} + \frac{3}{12992} = \frac{2384079}{4222400} \tag{7.163}$$

$$\mathrm{P}_{88}^{54} = \frac{4222400}{2384079} = 1.7710\ldots \text{ days (Io)} \tag{7.164}$$

$$\mathrm{V}_{97}^{54} = \mathrm{V}_{97}^{51} + \mathrm{V}_{51}^{54} = \frac{7203}{25600} + \frac{3}{12992} = \frac{1463409}{5196800} \tag{7.165}$$

$$\mathrm{P}_{97}^{54} = \frac{5196800}{1463409} = 3.5511\ldots \text{ days (Europa)} \tag{7.166}$$

$$\mathrm{V}_{102}^{54} = \mathrm{V}_{102}^{51} + \mathrm{V}_{51}^{54} = \frac{30961}{221880} + \frac{3}{12992} = \frac{50363869}{360333120} \tag{7.167}$$

$$\mathrm{P}_{102}^{54} = \frac{360333120}{50363869} = 7.1545\ldots \text{ days (Ganymede)} \tag{7.168}$$

$$\mathrm{V}_{106}^{54} = \mathrm{V}_{106}^{51} + \mathrm{V}_{51}^{54} = \frac{9458}{158455} + \frac{3}{12992} = \frac{123353701}{2058647360} \tag{7.169}$$

$$\mathrm{P}_{106}^{54} = \frac{2058647360}{123353701} = 16.6889\ldots \text{ days (Callisto)} \tag{7.170}$$

Oechslin gives the same values for the periods of Ganymede and Callisto, but slightly different values for Io and Europa. I believe that he made small errors in his computations.

The actual values are 1.7627 days, 3.5255 days, 7.1556 days and 16.690 days.

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7.4.5.2 Saturn and its satellites

The Saturn system is located on an arm fixed on tube 53 whose motion is the mean motion of Saturn. It is very similar to the Jupiter system, but there is a fifth satellite. And like in the case of Jupiter, Saturn is not given any eccentric motion, although Hahn took it into account in the Furtwangen orrery (Oechslin 8.12) in 1774, in the Gotha *Weltmaschine* (Oechslin 8.3) in 1780, and in the Nuremberg *Weltmaschine* (Oechslin 8.2) in the 1780s.

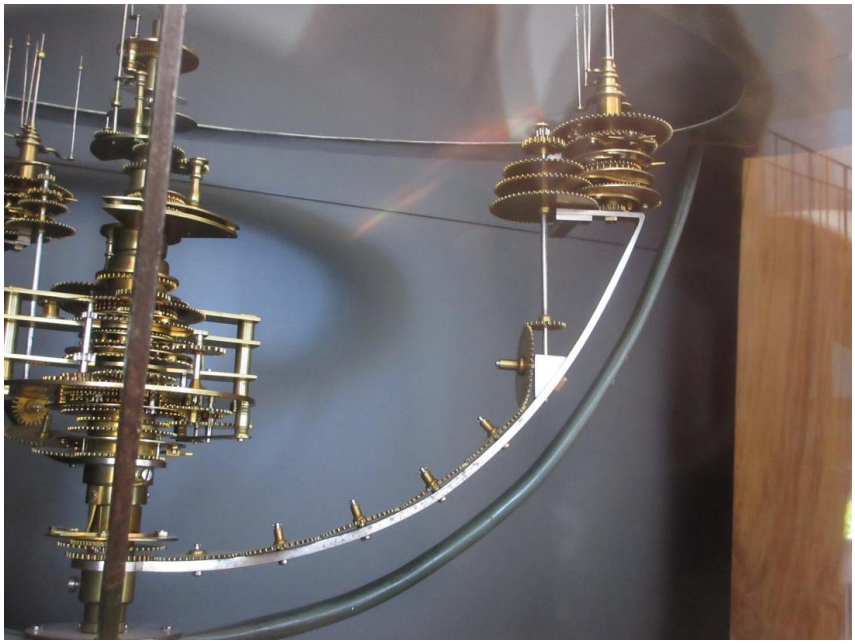


Figure 7.20: Hahn's *Weltmaschine*: the transmission to the system of Saturn in the orrery. (photograph by the author, 2025)

The input motion on the arm is the relative motion of tube 109 with respect to tube 53.

$$\mathrm{V}_{109}^{53} = \mathrm{V}_{109}^{54} - \mathrm{V}_{53}^{54} = \frac{4603}{64484} - \frac{3}{32242} = \frac{4597}{64484} \tag{7.171}$$

On the Saturn arm, there is a first train leading to arbor 116. This arbor carries wheels which are driving the five base wheels of the satellites. Arbor 116 has the following velocity:

$$\mathrm{V}_{116}^{53} = \mathrm{V}_{109}^{53} \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \tag{7.172}$$

$$\times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{21}\right) \tag{7.173}$$

$$= \mathrm{V}_{109}^{53} \times (-2) = -\frac{4597}{32242} \tag{7.174}$$

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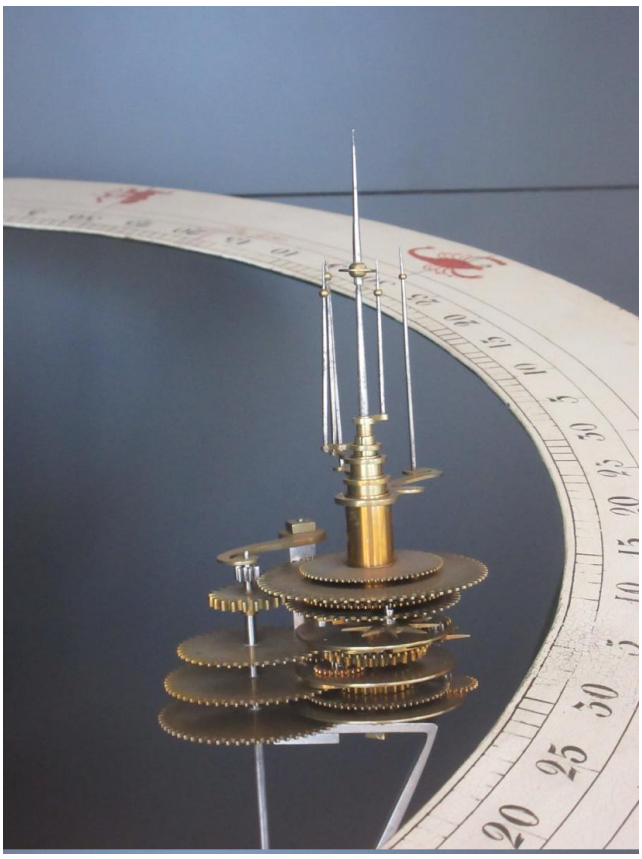


Figure 7.21: Hahn's *Weltmaschine*: detail of the system of Saturn. (photograph by the author, 2025)

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and the base wheels have these motions:

$$\mathrm{V}_{117}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{63}{17}\right) = \left(-\frac{4597}{32242}\right) \times \left(-\frac{63}{17}\right) = \frac{41373}{78302} \tag{7.175}$$

$$\mathrm{P}_{117}^{53} = \frac{78302}{41373} = 1.8925\ldots \text{ days (Tethys)} \tag{7.176}$$

$$\mathrm{V}_{123}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{56}{22}\right) = \frac{9194}{25333} \tag{7.177}$$

$$\mathrm{P}_{123}^{53} = \frac{25333}{9194} = 2.7553\ldots \text{ days (Dione)} \tag{7.178}$$

$$\mathrm{V}_{132}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{42}{27}\right) = \frac{4597}{20727} \tag{7.179}$$

$$\mathrm{P}_{132}^{53} = \frac{20727}{4597} = 4.5088\ldots \text{ days (Rhea)} \tag{7.180}$$

$$\mathrm{V}_{133}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{21}{48}\right) = \frac{4597}{73696} \tag{7.181}$$

$$\mathrm{P}_{133}^{53} = \frac{73696}{4597} = 16.0313\ldots \text{ days (Titan)} \tag{7.182}$$

$$\mathrm{V}_{136}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{6}{68}\right) = \frac{13791}{1096228} \tag{7.183}$$

$$\mathrm{P}_{136}^{53} = \frac{1096228}{13791} = 79.4886\ldots \text{ days (Iapetus)} \tag{7.184}$$

Tethys' base wheel of 17 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Tethys' actual satellite wheel (65 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 122 carrying Tethys makes an additional fraction of a turn x_{11} when the base wheel makes one turn. We have:

$$x_{11} = \left(-\frac{1}{9}\right) \times \left(-\frac{3}{27}\right) \times \left(-\frac{27}{27}\right) \times \left(-\frac{3}{65}\right) = \frac{1}{1755} \tag{7.185}$$

Then

$$\mathrm{V}_{122}^{117} = x_{11} \times \mathrm{V}_{117}^{53} \tag{7.186}$$

Consequently

$$\mathrm{V}_{122}^{53} = \mathrm{V}_{122}^{117} + \mathrm{V}_{117}^{53} = \mathrm{V}_{117}^{53} \times (1 + x_{11}) \tag{7.187}$$

$$= \frac{41373}{78302} \times \frac{1756}{1755} = \frac{4036166}{7634445} \tag{7.188}$$

$$\mathrm{P}_{122}^{53} = \frac{7634445}{4036166} = 1.8915\ldots \text{ days} \tag{7.189}$$

The 3-leaves pinion and the 27-teeth wheel on its arbor are parenthesized by Oechslin, and I assume that they were missing.

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Dione's base wheel of 22 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Dione's actual satellite wheel (23 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 127 carrying Dione makes an additional fraction of a turn x_{12} when the base wheel makes one turn. We have:

$$x_{12} = \left(-\frac{1}{6}\right) \times \left(-\frac{3}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{3}{23}\right) = \frac{3}{1012} \tag{7.190}$$

This ratio was erroneously given as $\frac{3}{572}$ by Oechslin who mistakenly replaced $\frac{3}{23}$ by $\frac{3}{13}$ in his computations.

Then

$$\mathrm{V}_{127}^{123} = x_{12} \times \mathrm{V}_{123}^{53} \tag{7.191}$$

Consequently

$$\mathrm{V}_{127}^{53} = \mathrm{V}_{127}^{123} + \mathrm{V}_{123}^{53} = \mathrm{V}_{123}^{53} \times (1 + x_{12}) \tag{7.192}$$

$$= \frac{9194}{25333} \times \frac{1015}{1012} = \frac{666565}{1831214} \tag{7.193}$$

$$\mathrm{P}_{127}^{53} = \frac{1831214}{666565} = 2.7472\ldots \text{ days} \tag{7.194}$$

The 3-leaves pinion and the 22-teeth wheel on its arbor are parenthesized by Oechslin, and I assume that they were missing.

The structure of the gears driving the satellite Rhea is also confuse in Oechslin's figure. The problem is similar to the one observed for Io, the first satellite of Jupiter. Here, I am assuming that Rhea's base wheel has 27 teeth and that it carries an arbor with a 3-leaves pinion meshing with Rhea's actual satellite wheel (43 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 128 carrying Rhea makes an additional fraction of a turn x_{13} when the base wheel makes one turn. We have:

$$x_{13} = \left(-\frac{1}{5}\right) \times \left(-\frac{5}{18}\right) \times \left(-\frac{18}{18}\right) \times \left(-\frac{3}{43}\right) = \frac{1}{258} \tag{7.195}$$

The same ratio is given by Oechslin.

Then

$$\mathrm{V}_{128}^{132} = x_{13} \times \mathrm{V}_{132}^{53} \tag{7.196}$$

Consequently

$$\mathrm{V}_{128}^{53} = \mathrm{V}_{128}^{132} + \mathrm{V}_{132}^{53} = \mathrm{V}_{132}^{53} \times (1 + x_{13}) \tag{7.197}$$

$$= \frac{4597}{20727} \times \frac{259}{258} = \frac{170089}{763938} \tag{7.198}$$

$$\mathrm{P}_{128}^{53} = \frac{763938}{170089} = 4.4914\ldots \text{ days} \tag{7.199}$$

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Titan's base wheel of 48 teeth carries an eccentric arbor with a 2-leaves pinion meshing with Titan's actual satellite wheel (65 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 135 carrying Titan makes an additional fraction of a turn x_{14} when the base wheel makes one turn. We have:

$$x_{14} = \left(-\frac{1}{9}\right) \times \left(-\frac{2}{65}\right) = \frac{2}{585} \tag{7.200}$$

The same ratio is given by Oechslin.
Then

$$\mathrm{V}_{135}^{133} = x_{14} \times \mathrm{V}_{133}^{53} \tag{7.201}$$

Consequently

$$\mathrm{V}_{135}^{53} = \mathrm{V}_{135}^{133} + \mathrm{V}_{133}^{53} = \mathrm{V}_{133}^{53} \times (1 + x_{14}) \tag{7.202}$$

$$= \frac{4597}{73696} \times \frac{587}{585} = \frac{2698439}{43112160} \tag{7.203}$$

$$\mathrm{P}_{135}^{53} = \frac{43112160}{2698439} = 15.9767\ldots \text{ days} \tag{7.204}$$

Iapetus' base wheel of 68 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Iapetus' actual satellite wheel (70 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 140 carrying Iapetus makes an additional fraction of a turn x_{15} when the base wheel makes one turn. We have:

$$x_{15} = \left(-\frac{1}{8}\right) \times \left(-\frac{3}{18}\right) \times \left(-\frac{4}{34}\right) \times \left(-\frac{3}{70}\right) = \frac{1}{9520} \tag{7.205}$$

The same ratio is given by Oechslin.
Then

$$\mathrm{V}_{140}^{136} = x_{15} \times \mathrm{V}_{136}^{53} \tag{7.206}$$

Consequently

$$\mathrm{V}_{140}^{53} = \mathrm{V}_{140}^{136} + \mathrm{V}_{136}^{53} = \mathrm{V}_{136}^{53} \times (1 + x_{15}) \tag{7.207}$$

$$= \frac{13791}{1096228} \times \frac{9521}{9520} = \frac{131304111}{10436090560} \tag{7.208}$$

$$\mathrm{P}_{140}^{53} = \frac{10436090560}{131304111} = 79.4803\ldots \text{ days} \tag{7.209}$$

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We can also compute the absolute motions:

$$\mathrm{V}_{122}^{54} = \mathrm{V}_{122}^{53} + \mathrm{V}_{53}^{54} = \frac{4036166}{7634445} + \frac{3}{32242} = \frac{56516269}{106882230} \tag{7.210}$$

$$\mathrm{P}_{122}^{54} = \frac{106882230}{56516269} = 1.8911\ldots \text{ days} \tag{7.211}$$

$$\mathrm{V}_{127}^{54} = \mathrm{V}_{127}^{53} + \mathrm{V}_{53}^{54} = \frac{666565}{1831214} + \frac{3}{32242} = \frac{16335017}{44864743} \tag{7.212}$$

$$\mathrm{P}_{127}^{54} = \frac{44864743}{16335017} = 2.7465\ldots \text{ days} \tag{7.213}$$

$$\mathrm{V}_{128}^{54} = \mathrm{V}_{128}^{53} + \mathrm{V}_{53}^{54} = \frac{170089}{763938} + \frac{3}{32242} = \frac{4168922}{18716481} \tag{7.214}$$

$$\mathrm{P}_{128}^{54} = \frac{18716481}{4168922} = 4.4895\ldots \text{ days} \tag{7.215}$$

$$\mathrm{V}_{135}^{54} = \mathrm{V}_{135}^{53} + \mathrm{V}_{53}^{54} = \frac{2698439}{43112160} + \frac{3}{32242} = \frac{18917153}{301785120} \tag{7.216}$$

$$\mathrm{P}_{135}^{54} = \frac{301785120}{18917153} = 15.9529\ldots \text{ days} \tag{7.217}$$

$$\mathrm{V}_{140}^{54} = \mathrm{V}_{140}^{53} + \mathrm{V}_{53}^{54} = \frac{131304111}{10436090560} + \frac{3}{32242} = \frac{132275151}{10436090560} \tag{7.218}$$

$$\mathrm{P}_{140}^{54} = \frac{10436090560}{132275151} = 78.8968\ldots \text{ days} \tag{7.219}$$

Oechslin obtains slightly different values, apart from the above error on x_{12}, and I am inclined to think that some errors have crept in his computations. Moreover, his values (1.83, 2.65, 4.38, 15.45 and 76.42) differ more from the actual ones (1.9 days, 2.7 days, 4.5 days, 16 days and 79 days) than the ones I have obtained. The discrepancies are greater than for Jupiter's satellites.

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7.5 The calendar part (summary)

As mentioned earlier, the calendar part is driven by arbor 8 which makes one rotation counterclockwise (seen from above) in a day. This arbor carries a 50-teeth wheel which meshes with another 50-teeth wheel on tube 9 which drives a hand making one turn clockwise in a day:

$$\mathrm{T}_9^0 = 1 \tag{7.220}$$

A coupling pairs the tube 9 with the central arbor 10 which then has the same motion:

$$\mathrm{T}_{10}^0 = 1 \tag{7.221}$$

A 24-teeth wheel on that arbor meshes with a similar wheel on arbor 11, which in turn meshes with another similar wheel on arbor 31, which goes down vertically towards the orrery and globe.

I will not describe the calendar gears in more detail, partly because I want to focus on the astronomical gears, and partly because I do not have sufficient details in the intermittent motions and cams involved in this part of the machine.

I am merely giving some information on the dials. The upper dial of the middle part of the machine shows the hours, minutes and seconds, as mentioned earlier. The middle dial shows the day of the month, the day of the week and the month. This part of the clock does take into account the lengths of the months, probably also the leap years, but perhaps not the common years such as 1800, 1900, 2100, etc. One of the arbors of that part of the clock makes a turn in 10 years and leads to the lower dial. This dial is a counter for the years and has two hands, one making a turn in 100 years and another in 8000 years. This is used to indicate the chronology of the world as described by Johann Albrecht Bengel.[22]

[22]See in particular Bengel's works [3, 4, 5, 6], Wilfried Veeser's chapter in 1989 [39, p. 327-339], and Marini [25]. This dial is also illustrated in the 1989 Hahn exhibition catalogue [38, p. 374].

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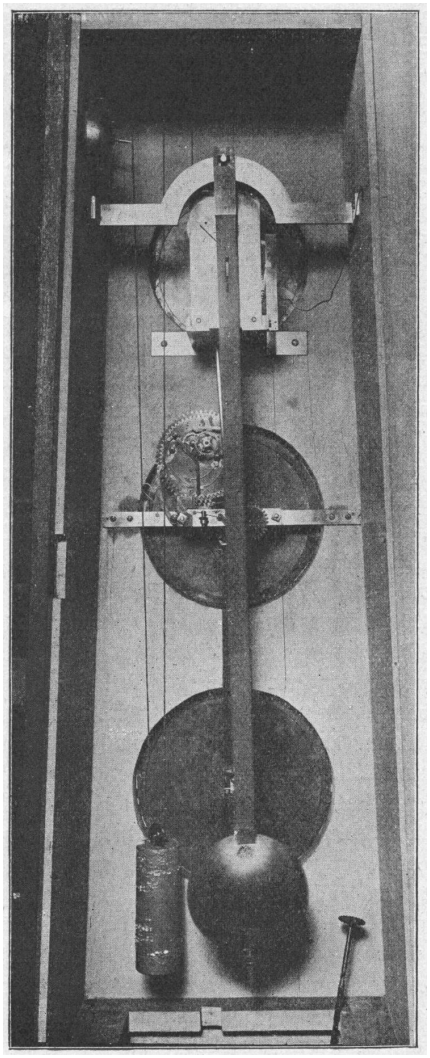


Figure 7.22: Hahn's *Weltmaschine*: the back of the central dials with the pendulum and the weight. (source: [8])

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Figure 7.23: Hahn's *Weltmaschine*: the upper central dial giving the time. (photograph by the author, 2025)



Figure 7.24: Hahn's *Weltmaschine*: the middle central dial giving the day of the week and the month. (photograph by the author, 2025)

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Figure 7.25: Hahn's *Weltmaschine*: detail of the middle central dial giving the day of the week and the month. (photograph by the author, 2025)



Figure 7.26: Hahn's *Weltmaschine*: the lower central dial giving the years, following Bengel's chronology. (photograph by the author, 2025)

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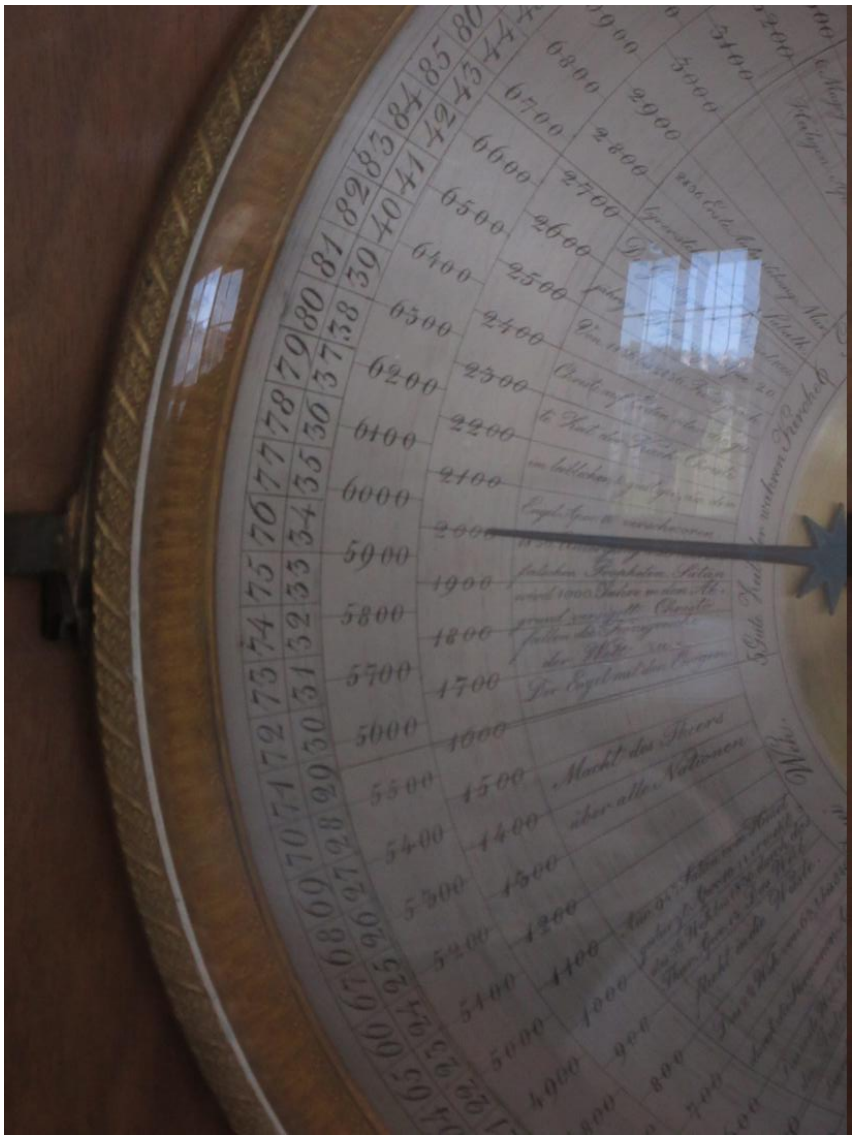


Figure 7.27: Hahn's *Weltmaschine*: detail of the lower central dial giving the years. (photograph by the author, 2025)

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Figure 7.28: Hahn's *Weltmaschine*: detail of the lower central dial giving the years. We can for instance see the birth of Christ (*Jesus Christus geboren*). (photograph by the author, 2025)

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Chapter 8

(Oechslin: 8.5)

Hahn's globe clock in Stuttgart (1770)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

8.1 Introduction

The clock described here was constructed by Philipp Matthäus Hahn (1739-1790)[1] and is located in the Landesmuseum Württemberg in Stuttgart.[2] Another very similar clock is kept in this museum,[3] but it doesn't have a lunar globe on top of the celestial sphere.[4]

The clock has a square base of which three faces have dials, for the indications of time, for a tellurium and for an orrery. On top of this base, a celestial globe shows the geocentric motion of the Sun, the Moon and Venus, as well as the lunar nodes. On top of this sphere, a small sphere shows the phase of the Moon.

Although apparently simpler than other clocks and machines by Hahn, this clock display several idiosyncrasies which are found in only a few of Hahn's clocks. It is for instance unusual to have only Venus as a planet rotating around the celestial globe, although this is also found in the globe clock in Darmstadt (Oechslin 8.6). There is also a rather bad approximation of the motion of the

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [6, p. 408-409]. A description of this clock seems to be given by Hahn [3], but I haven't seen it.

[3] This clock is currently exhibited in the basement of the Landesmuseum.

[4] There are several similar clocks by Hahn, one of which was described by Hahn in 1770 [2]. Another similar clock was described by Hahn in 1774 [1]. The globe clock of Darmstadt (Oechslin 8.6) is also similar. Yet another similar clock was also reconstructed by Alfred Leiter [4].

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Moon in the tellurium.

8.2 The clockwork

The clock is driven by a spring-driven pendulum clockwork, with the barrel wheel on arbor 1 making one turn in 36 hours. We have, from the front

$$\mathrm{T}_1^0 = \frac{24}{36} = \frac{2}{3} \tag{8.1}$$

Arbors 2 and 3 have the following velocities:

$$\mathrm{T}_2^0 = \mathrm{T}_1^0 \times \left(-\frac{54}{9}\right) = -4 \tag{8.2}$$

$$\mathrm{T}_3^0 = \mathrm{T}_2^0 \times \left(-\frac{60}{10}\right) = 24 \tag{8.3}$$

Hence, arbor 3 makes one turn in one hour and it carries the minute hand.

The escape wheel on arbor 5 has the velocity

$$\mathrm{T}_5^0 = \mathrm{T}_3^0 \times \left(-\frac{72}{6}\right) \times \left(-\frac{60}{6}\right) = 24 \times 120 = 2880 \tag{8.4}$$

It makes one turn clockwise in $\frac{86400}{2880} = 30$ seconds. It carries 36 teeth or pins and the pendulum makes half an oscillation in $\frac{30}{2\times 36} = \frac{5}{12}$ seconds. Its length is about 17 cm.

The time dial gives the hours on an large 24-hour dial, the minutes on an internal dial, and the day of the week and of the month on another internal dial. The calendar indications will not be described here. I will only explain the connection between the clockwork and the other parts of the clock.

The arbor 3 is used to make a motion of one turn in one day on tube 7, and then also on arbor 8:

$$\mathrm{T}_7^0 = \mathrm{T}_3^0 \times \left(-\frac{14}{56}\right) \times \left(-\frac{12}{72}\right) = 1 \tag{8.5}$$

$$\mathrm{T}_8^0 = \mathrm{T}_7^0 \tag{8.6}$$

The motion of arbor 8 is also used to obtain the days of the week on tube 10:

$$\mathrm{T}_{10}^0 = \mathrm{T}_8^0 \times \left(-\frac{24}{48}\right) \times \left(-\frac{16}{56}\right) = \frac{1}{7} \tag{8.7}$$

But arbor 8 is used in the back of the clockwork to produce the motion of arbor 18:

$$\mathrm{T}_{18}^0 = \mathrm{T}_8^0 \times \left(-\frac{35}{35}\right) \times \left(-\frac{35}{29}\right) \times \left(-\frac{38}{44}\right) \tag{8.8}$$

$$= -\frac{665}{638} \tag{8.9}$$

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This motion is then used to obtain the input motion of the globe on the vertical arbor 19:

$$\mathrm{V}_{19}^0 = \mathrm{T}_{18}^0 \times \frac{76}{79} = \left(-\frac{665}{638}\right) \times \frac{76}{79} \tag{8.10}$$

$$= -\frac{25270}{25201} \tag{8.11}$$

This velocity is counted positively counterclockwise as seen from above. It is the usual ratio used by Hahn for the sidereal day.

This motion is also used to obtain one of the input motions of the tellurium on arbor 44 and the input motion of the orrery on arbor 20. I will therefore first describe the gears of the orrery.

8.3 The orrery

As mentioned above, the input motion of the orrery is arbor 20 and it makes one turn clockwise (seen from the front) in a sidereal day:

$$\mathrm{V}_{20}^0 = -\frac{25270}{25201} \tag{8.12}$$

8.3.1 The mean motions of the planets

This motion is first used to obtain the mean motion of Mercury on tube 23:

$$\mathrm{V}_{23}^0 = \mathrm{V}_{20}^0 \times \left(-\frac{6}{53}\right) \times \left(-\frac{32}{77}\right) \times \left(-\frac{20}{83}\right) = \mathrm{V}_{20}^0 \times \left(-\frac{3840}{338723}\right) \tag{8.13}$$

$$= \left(-\frac{25270}{25201}\right) \times \left(-\frac{3840}{338723}\right) = \frac{13862400}{1219451189} \tag{8.14}$$

$$\mathrm{P}_{23}^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{8.15}$$

The same value is given by Oechslin. This motion is counterclockwise as seen from the front. This is exactly the same ratio as the one used in the Furtwangen orrery, in the Aschaffenburg globe clock, and in the Gotha and Nuremberg *Weltmaschinen*.

The remaining part of the orrery is also identical to the one used in the Aschaffenburg globe clock, so that my descriptions only differ by the arbor and tube numbers.

Mercury is actually given an irregular motion, in order to account for its elliptic orbit. I will get back to it below, but I will first deal with the mean motions.

Venus (tube 27) only has a mean motion which is obtained from the mean

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motion of Mercury:

$$\mathrm{V}_{27}^0 = \mathrm{V}_{23}^0 \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) \tag{8.16}$$

$$= \mathrm{V}_{23}^0 \times \frac{783}{2000} = \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{8.17}$$

$$\mathrm{P}_{27}^0 = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{8.18}$$

The same value is given by Oechslin.

The Earth (tube 29) also only has a mean motion which is obtained from the (mean) motion of Venus:

$$\mathrm{V}_{29}^0 = \mathrm{V}_{27}^0 \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = \mathrm{V}_{27}^0 \times \frac{3239}{5265} \tag{8.19}$$

$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{8.20}$$

$$\mathrm{P}_{29}^0 = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{8.21}$$

The same value is given by Oechslin. This is an approximation of the tropical year. This motion is also transferred to arbor 42 which is the input to the tellurium.

The motion of the Earth is then used to obtain the mean motion of Mars. The motion of Mars is used to obtain the motion of Jupiter, and the motion of Jupiter is used to obtain the motions of Saturn and of the Moon.

So, first we obtain the mean motion of Mars on tube 31:

$$\mathrm{V}_{31}^0 = \mathrm{V}_{29}^0 \times \left(-\frac{116}{33}\right) \times \left(-\frac{18}{119}\right) = \mathrm{V}_{29}^0 \times \frac{696}{1309} \tag{8.22}$$

$$= \frac{473632}{172990675} \times \frac{696}{1309} = \frac{329647872}{226444793575} \tag{8.23}$$

$$\mathrm{P}_{31}^0 = \frac{226444793575}{329647872} = 686.9293\ldots \text{ days} \tag{8.24}$$

The same value is given by Oechslin.

Note that in the Aschaffenburg clock, there may be an error on the 33-teeth wheel, or perhaps it was Oechslin who made a typo.

From Mars, we obtain the mean motion of Jupiter on tube 34:

$$\mathrm{V}_{34}^0 = \mathrm{V}_{31}^0 \times \left(-\frac{119}{31}\right) \times \left(-\frac{5}{121}\right) = \mathrm{V}_{31}^0 \times \frac{595}{3751} \tag{8.25}$$

$$= \frac{329647872}{226444793575} \times \frac{595}{3751} = \frac{329647872}{1427553648235} \tag{8.26}$$

$$\mathrm{P}_{34}^0 = \frac{1427553648235}{329647872} = 4330.5410\ldots \text{ days} \tag{8.27}$$

The same value is given by Oechslin.

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The mean motion of Jupiter is then used to obtain the mean motion of Saturn on tube 37:

$$\mathrm{V}^0_{37} = \mathrm{V}^0_{34} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) = \mathrm{V}^0_{34} \times \frac{363}{901} \tag{8.28}$$

$$= \frac{329647872}{1427553648235} \times \frac{363}{901} = \frac{988943616}{10629965595535} \tag{8.29}$$

$$\mathrm{P}^0_{37} = \frac{10629965595535}{988943616} = 10748.8085\ldots \text{ days} \tag{8.30}$$

The same value is given by Oechslin.

8.3.2 The motion of the Moon

The mean motion of Jupiter is also used to obtain the motion of tube 40:

$$\mathrm{V}^0_{40} = \mathrm{V}^0_{34} \times \left(-\frac{121}{34}\right) \times \left(-\frac{3}{21}\right) \times \left(-\frac{73}{102}\right) = \mathrm{V}^0_{34} \times \left(-\frac{8833}{24276}\right) \tag{8.31}$$

$$= \frac{329647872}{1427553648235} \times \left(-\frac{8833}{24276}\right) = -\frac{2005357888}{23867281242805} \tag{8.32}$$

$$\mathrm{P}^0_{40} = -\frac{23867281242805}{2005357888} = 11901.7564\ldots \text{ days} \tag{8.33}$$

The same value is given by Oechslin.

The motion of tube 40 is used to rotate the Moon around the Earth. We can compute this motion precisely. We first compute the motion of the Moon with respect to the Earth. It is the motion of arbor 41. We have

$$\mathrm{V}^{29}_{41} = \mathrm{V}^{29}_{40} \times \left(-\frac{72}{6}\right) = -\left(\mathrm{V}^0_{40} - \mathrm{V}^0_{29}\right) \times 12 = \left(\mathrm{V}^0_{29} - \mathrm{V}^0_{40}\right) \times 12 \tag{8.34}$$

$$= \left(\frac{473632}{172990675} + \frac{2005357888}{23867281242805}\right) \times 12 \tag{8.35}$$

$$= \frac{336758509216}{119336406214025} \times 12 \tag{8.36}$$

$$= \frac{4041102110592}{119336406214025} \tag{8.37}$$

$$\mathrm{P}^{29}_{41} = \frac{119336406214025}{4041102110592} = 29.5306\ldots \text{ days} \tag{8.38}$$

As expected, we find an approximation of the synodic month. This value is not given by Oechslin who only gives the tropical month:

$$\mathrm{V}^0_{41} = \mathrm{V}^{29}_{41} + \mathrm{V}^0_{29} = \frac{4041102110592}{119336406214025} + \frac{473632}{172990675} \tag{8.39}$$

$$= \frac{4367833830368}{119336406214025} \tag{8.40}$$

$$\mathrm{P}^0_{41} = \frac{119336406214025}{4367833830368} = 27.3216\ldots \text{ days} \tag{8.41}$$

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8.3.3 The anomalies of the planets

Finally, we examine how the anomalies of the planets Mercury, Mars, Jupiter and Saturn have been obtained. In each case, there is a central wheel fixed to the frame. Their teeth numbers are actually irrelevant, as they all mesh with identical wheels pivoting on the mean motion wheels. It therefore suffices to consider the case of Mercury.

The Mercury *satellite* wheel on arbor 25 rotates as the tube 23 rotates with the mean motion of Mercury. In the frame of tube 23, we have

$$\mathrm{V}_{25}^{23} = \mathrm{V}_{24}^{23} \times \left(-\frac{24}{24}\right) = -\mathrm{V}_{24}^{23} = \mathrm{V}_{23}^{24} = \mathrm{V}_{23}^{0} \tag{8.42}$$

$$\mathrm{P}_{25}^{23} = \mathrm{P}_{23}^{0} \tag{8.43}$$

Therefore, on the Mercury tube 23, the eccentric arbor of Mercury oscillates with the same period as the mean tropical motion of Mercury. This causes the acceleration and slowing down of the motion to occur at the same places in time, and therefore will account for the equation of center. Of course, in this simple case, the tropical and anomalistic motions of Mercury have been considered identical, which they are not exactly.

The same constructions are used for Mars, Jupiter and Saturn.

8.4 The tellurium

The tellurium occupies one side of the base of the clock. It shows the Earth rotating around the Sun and the Moon rotating around the Earth.

As mentioned above, the tellurium has two input motions. The first is that of arbor 44 and it makes one turn counterclockwise (seen from the front) in a sidereal day:

$$\mathrm{V}_{44}^{0} = \frac{25270}{25201} \tag{8.44}$$

The second one is arbor 42 which originates from the orrery. It provides the revolution of one tropical year and turns clockwise. The connection between the orrery and the tellurium is also found in the globe clock of Aschaffenburg (Oechslin 8.4) constructed in 1776/1777, and the structure of the tellurium is similar, except that there is no Moon in the Aschaffenburg tellurium.

$$\mathrm{V}_{42}^{0} = -\frac{473632}{172990675} \tag{8.45}$$

I am only considering the motion of arbor 42 from the tellurium side, hence the negative sign.

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8.4.1 The motion of the Earth

The motion of arbor 42 is used to produce the counterclockwise motion of tube 43:

$$\mathrm{V}^0_{43} = \mathrm{V}^0_{42} \times \left(-\frac{56}{56}\right) = -\mathrm{V}^0_{42} = \frac{473632}{172990675} \tag{8.46}$$

Hence, tube 43 makes one turn counterclockwise in a tropical year. This tube supports the Earth frame.

The meridian of the Earth, which supports its axis, is fixed to a wheel on tube 48. The motion of this wheel replicates the motion of the central arbor 46 which is fixed. Therefore, the meridian, and also the axis of the Earth, always keep the same orientation with respect to the fixed frame, no matter where the Earth is located.

Finally, the arbor 50 replicates the motion of tube 45:

$$\mathrm{V}^0_{50} = \mathrm{V}^0_{45} = \mathrm{V}^0_{44} \times \left(-\frac{40}{40}\right) = -\mathrm{V}^0_{44} = -\frac{25270}{25201} \tag{8.47}$$

And arbor 51, which is that of the Earth's axis, has the velocity:

$$\mathrm{V}^0_{51} = \mathrm{V}^0_{50} \times \left(-\frac{20}{20}\right) = -\mathrm{V}^0_{50} = \frac{25270}{25201} \tag{8.48}$$

Consequently, the Earth rotates counterclockwise with the velocity of the sidereal day, as it should.

8.4.2 The motion of the Moon

The axis of the Moon is located on frame 53 which rotates on the Earth support 43. We can first compute the revolution of frame 53 with respect to the rotating support 43:

$$\mathrm{V}^{43}_{53} = \mathrm{V}^{43}_{45} \times \left(-\frac{60}{60}\right) \times \left(-\frac{3}{41}\right) \times \left(-\frac{24}{52}\right) = \mathrm{V}^{43}_{45} \times \left(-\frac{18}{533}\right) \tag{8.49}$$

$$= \left(\mathrm{V}^0_{45} - \mathrm{V}^0_{43}\right) \times \left(-\frac{18}{533}\right) = \left(\mathrm{V}^0_{44} + \mathrm{V}^0_{43}\right) \times \frac{18}{533} \tag{8.50}$$

$$= \left(\frac{25270}{25201} + \frac{473632}{172990675}\right) \times \frac{18}{533} = \frac{398491850662}{396321636425} \times \frac{18}{533} \tag{8.51}$$

$$= \frac{7172853311916}{211239432214525} \tag{8.52}$$

$$\mathrm{P}^{43}_{53} = \frac{211239432214525}{7172853311916} = 29.4498\ldots \text{ days} \tag{8.53}$$

Oechslin does not give the same value.

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Then we have

$$\mathrm{V}_{53}^{0} = \mathrm{V}_{53}^{43} + \mathrm{V}_{43}^{0} = \frac{7172853311916}{211239432214525} + \frac{473632}{172990675} \tag{8.54}$$

$$= \frac{7751206768012}{211239432214525} \tag{8.55}$$

$$\mathrm{P}_{53}^{0} = \frac{211239432214525}{7751206768012} = 27.2524\ldots \text{ days} \tag{8.56}$$

This appears to be very bad approximations of the synodic and tropical months which are about 29.53 and 27.32 days. This is due to the ratio $\frac{18}{533}$ being a bad approximation of

$$x = \frac{s \cdot a}{m_t(s+a)} \approx 0.03367877 \tag{8.57}$$

where s is the sidereal day, a the tropical year and m_t the tropical month. A much better approximation would have been obtained with $x = \frac{13}{386}$, but this fraction can't be expressed as a product of simple ratios. It is possible that Hahn sticked with the ratio $\frac{18}{533}$ and knew that it wasn't perfect.

Incidentally, the ratio $\frac{18}{533}$ is also used for the small lunar globe at the top of the celestial globe, but in that case the ratio serves to compute a good approximation of the synodic month.

8.5 The celestial globe and the lunar globe

The celestial globe is located above the base of the clock. It is contained in a frame representing the meridian and it rotates around the vertical axis representing the axis of the Earth. The globe shows the motions of the Sun, the Moon and its nodes, as well as Venus, as seen from a geocentric perspective. A small lunar globe is mounted on top of the celestial globe.

As mentioned above, the input motion of the globe is the vertical arbor 19 and it makes one turn clockwise (seen from above) in a sidereal day:

$$\mathrm{V}_{19}^{0} = -\frac{25270}{25201} \tag{8.58}$$

8.5.1 The celestial globe

The Sun, the Moon, the lunar nodes and Venus rotate around the celestial globe, but around the axis of the ecliptic, which is tilted by 23.5° on the axis of the Earth.

The input motion to the globe actually moves directly the globe, but through the frame supporting the axis 62 of the ecliptic. The globe is fixed to this central axis, to which are also attached three wheels that will be described later. It is the rotation of this tilted axis around the vertical axis that causes a motion within the sphere. This is done as follows.

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The tube 57 is attached at the lower part of the fixed meridian of the globe. This tube carries a 71-teeth wheel. It meshes with a 34-teeth wheel on the arbor 58 which is pivoting on the frame supporting the ecliptic. When the tilted frame rotates, so does arbor 58.

Now, we have our first motion of the celestial globe, namely the clockwise (from above) rotation of the entire globe in one sidereal day, because of the motion of the vertical arbor 19. This motion should be clockwise, because it is the apparent motion of the sky and the Earth rotates counterclockwise as seen from above.

8.5.1.1 The motion of the Moon

The rotation of arbor 58 causes the rotation of tube 61 which enters the globe in the inferior part. We can compute the velocity of tube 61 with respect to the frame/axis 62:

$$\mathrm{V}_{61}^{62} = \mathrm{V}_{57}^{62} \times \left(-\frac{71}{34}\right) \times \left(-\frac{34}{48}\right) \times \left(-\frac{28}{46}\right) \times \left(-\frac{3}{74}\right) = \mathrm{V}_{57}^{62} \times \frac{497}{13616} \quad (8.59)$$

$$= \mathrm{V}_{57}^{19} \times \frac{497}{13616} = -\mathrm{V}_{19}^{0} \times \frac{497}{13616} = \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \quad (8.60)$$

$$\mathrm{P}_{61}^{62} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \quad (8.61)$$

This is an approximation of the tropical month and tube 61 is actually the mean motion of the Moon. This value is positive, because the Moon revolves counterclockwise with respect to the stars. The same value is given by Oechslin, also in sidereal days.

The actual motion of the Moon shown around the celestial globe takes the equation of center into account. The "corrected Moon" is accelerated or slowed down with respect to the mean Moon. In principle, the period of the equation of center should be that of the anomalistic month, which is a bit longer than the tropical month. The motion of the corrected Moon is obtained using an eccentric pin on arbor 64. We can compute the period of arbor 64 with respect to frame 61:

$$\mathrm{V}_{64}^{61} = \mathrm{V}_{62}^{61} \times \left(-\frac{67}{43}\right) \times \left(-\frac{21}{33}\right) = -\mathrm{V}_{61}^{62} \times \frac{469}{473} \quad (8.62)$$

$$= -\frac{6279595}{171568408} \times \frac{469}{473} = -\frac{2945130055}{81151856984} \quad (8.63)$$

$$\mathrm{P}_{64}^{61} = -\frac{81151856984}{2945130055} = -27.5545\ldots \text{ days} \quad (8.64)$$

which is an excellent approximation of the anomalistic month. (The fact that the value is negative is irrelevant, as the oscillation of the corrected Moon would be equivalent whether the pins turns clockwise or counterclockwise.)

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Oechslin doesn't give this value, but gives the period of precession of the apsides:

$$\mathrm{V}_{64}^{62} = \mathrm{V}_{64}^{61} + \mathrm{V}_{61}^{62} = -\frac{2945130055}{81151856984} + \frac{6279595}{171568408} \tag{8.65}$$

$$= \frac{6279595}{20287964246} \tag{8.66}$$

$$\mathrm{P}_{64}^{62} = \frac{20287964246}{6279595} = 3230.7759\ldots \text{ days} \tag{8.67}$$

8.5.1.2 The motion of the Sun

The mean motion of the Sun is given by tube 68 and derived from the mean motion of the Moon:

$$\mathrm{V}_{68}^{62} = \mathrm{V}_{61}^{62} \times \left(-\frac{122}{27}\right) \times \left(-\frac{28}{28}\right) \times \left(-\frac{28}{43}\right) \times \left(-\frac{3}{118}\right) \tag{8.68}$$

$$= \mathrm{V}_{61}^{62} \times \frac{1708}{22833} = \frac{6279595}{171568408} \times \frac{1708}{22833} = \frac{2681387065}{979355364966} \tag{8.69}$$

$$\mathrm{P}_{68}^{62} = \frac{979355364966}{2681387065} = 365.2420\ldots \text{ days} \tag{8.70}$$

This is a good approximation of the tropical year and the same value is given by Oechslin. The value is positive, because the Sun moves counterclockwise with respect to the stars.

Hahn has also constructed the "corrected Sun," in that he added the equation of center. He did however assume that the tropical and anomalistic years are identical, that is he assumed the line of apsides to be fixed in the zodiac. The motion of the corrected Sun is obtained with an eccentric pin on an arbor rotating on frame 68. But the rotation of this arbor is such that it always replicates the motion of the central axis 62 of the sphere, therefore the pin is actually always oriented on the same direction. The direction of the pin is that at right angles of the line of apsides. This is a very simple construction that causes the mean Sun to be accelerated or slowed down depending how far it is from the line of apsides.

8.5.1.3 The line of nodes

The corrected Sun (on a tube I will name 68′) is also used to obtain the motion of the line of nodes on tube 76. We have

$$\mathrm{V}_{76}^{62} = \mathrm{V}_{68'}^{62} \times \left(-\frac{70}{43}\right) \times \left(-\frac{20}{24}\right) \times \left(-\frac{4}{101}\right) = \mathrm{V}_{68'}^{62} \times \left(-\frac{700}{13029}\right) \tag{8.71}$$

$$\approx \mathrm{V}_{68}^{62} \times \left(-\frac{700}{13029}\right) \approx \frac{2681387065}{979355364966} \times \left(-\frac{700}{13029}\right) \tag{8.72}$$

$$\approx -\frac{938485472750}{6380010525071007} \tag{8.73}$$

$$\mathrm{P}_{76}^{62} \approx -\frac{6380010525071007}{938485472750} = -6798.1984\ldots \text{ days} \tag{8.74}$$

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This is an excellent approximation of the period of precession of the lunar nodes. The value is negative, because the line of nodes is retrograding. The same value is given by Oechslin.

8.5.1.4 The motion of Venus

The corrected Sun 68′ is also used to obtain the motion of Venus. The mean geocentric motion of Venus is actually that of the Sun. But because of the revolution of the Earth around the Sun, Venus appears to oscillate around the position of the Sun, and sometimes to retrograde. Ths oscillation results from the motion of an eccentric pin on arbor 73. The period of this oscillation should be that of the synodic revolution of Venus. We can compute this motion in the reference frame 68′, that is, that of the corrected Sun:

$$\mathrm{V}_{73}^{68'} = \mathrm{V}_{62}^{68'} \times \left(-\frac{31}{20}\right) \times \left(-\frac{26}{22}\right) \times \left(-\frac{14}{41}\right) = \mathrm{V}_{62}^{68'} \times \left(-\frac{2821}{4510}\right) \tag{8.75}$$

$$\approx \mathrm{V}_{62}^{68} \times \left(-\frac{2821}{4510}\right) = -\mathrm{V}_{68}^{62} \times \left(-\frac{2821}{4510}\right) \tag{8.76}$$

$$\approx \frac{2681387065}{979355364966} \times \frac{2821}{4510} = \frac{1512838582073}{883378539199332} \tag{8.77}$$

$$\mathrm{P}_{73}^{68'} = \frac{883378539199332}{1512838582073} = 583.9212\ldots \text{ days} \tag{8.78}$$

and this is an excellent approximation of the synodic period of Venus. (Like for other similar motions, the sign is irrelevant here, as the same motion would be obtained if arbor 73 were rotating clockwise.)

The ratio $\frac{2821}{4510}$ is also used in the globe clock of Aschaffenburg and in the *Weltmaschinen* of Stuttgart and Gotha.

Oechslin doesn't give the value of the synodic period of Venus, only the tropical one:

$$\mathrm{V}_{73}^{62} = \mathrm{V}_{73}^{68'} + \mathrm{V}_{68'}^{62} \approx \mathrm{V}_{73}^{68'} + \mathrm{V}_{68}^{62} \tag{8.79}$$

$$\approx \frac{1512838582073}{883378539199332} + \frac{2681387065}{979355364966} = \frac{3931449714703}{883378539199332} \tag{8.80}$$

$$\mathrm{P}_{73}^{62} = \frac{883378539199332}{3931449714703} = 224.6953\ldots \text{ days} \tag{8.81}$$

The Darmstadt globe clock (Oechslin 8.6) also displays the motion of Venus, but its irregular motion is obtained using the elongation between Venus and the Sun. That clock contains a wheel making one turn in the tropical period of Venus, which is not the case here.

8.5.2 The lunar globe

The frame causing the rotation of the celestial globe carries at the top a 3-leaves/3-pin pinion which is used to rotate the small lunar globe. This pinion

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makes one turn clockwise in one sidereal day:

$$\mathrm{V}_{19}^{0} = -\frac{25270}{25201} \tag{8.82}$$

The tube carrying the Moon is tube 79 and its velocity is

$$\mathrm{V}_{79}^{0} = \mathrm{V}_{19}^{0} \times \left(-\frac{3}{26}\right) \times \left(-\frac{18}{18}\right) \times \left(-\frac{12}{41}\right) \tag{8.83}$$

$$= \mathrm{V}_{19}^{0} \times \left(-\frac{18}{533}\right) = \frac{454860}{13432133} \tag{8.84}$$

$$\mathrm{P}_{79}^{0} = \frac{13432133}{454860} = 29.5302\ldots \text{ days} \tag{8.85}$$

The same value is given by Oechslin.

The globe makes one turn clock counterclockwise (as seen from above) in one synodic month. It is half lit and is moving within a hemisphere that hides half of the sphere (as in the current Strasbourg astronomical clock). This hemisphere is fixed to the central axis of the lunar globe, which is itself fixed to the meridian frame. The lunar globe therefore shows phases of the Moon. However, the phases seem to appear in the wrong order, and the sphere should in fact rotate clockwise, as it does in the Darmstadt globe clock (Oechslin 8.6) from 1785.

It is interesting to observe that we have again the ratio $\frac{18}{533}$ which was used in the tellurium. We now have a relatively good value of the synodic month, because the ratio $\frac{18}{533}$ is a good approximation of the ratio s/m_s where s is the sidereal day and m_s is the synodic month, but it was not a good approximation for the ratio used in the tellurium. There may have been a mixup between the two ratios. This ratio $\frac{18}{533}$ was reused at least in the Darmstadt globe clock (Oechslin 8.6) and in the Zurich clock (Oechslin 8.7), both from 1785.

8.6 References

[1] Philipp Matthäus Hahn. *Beschreibung mechanischer Kunstwerke.* Stuttgart: Johann Benedict Metzler, 1774.

[2] Philipp Matthäus Hahn. *Kurze Beschreibung einer kleinen beweglichen Welt-Maschine.* Tübingen: Noûs-Verlag Thomas Heck, 1988. [facsimile of the 1770 edition published in Constance, edited by Reinhard Breymayer and with an introduction by Alfred Munz].

[3] Philipp Matthäus Hahn. Beschreibung der astronomischen Uhr von M. Phil. Math. Hahn, ehemals Pfarrer zu Kornwestheim, after 1780. [describes the Stuttgart globe clock (Oechslin 8.5), not seen].

[4] Alfred Leiter. *Die Pforzheimer Uhrenmanufaktur von 1767-1790 — Neue Erkenntnisse durch Publikationen über Philipp Matthäus Hahn, 1739-1790.* Kornwestheim: Minner-Verlag, 1993.

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[5] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert*. Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[6] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

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Chapter 9

(Oechslin: 8.12)

Hahn's orrery in Furtwangen (1774)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

9.1 Introduction

The orrery described here is located in Furtwangen and was constructed in 1774 by Philipp Matthäus Hahn (1739-1790).[1]

It was acquired by Charles Frederick (1728-1811), margrave of Baden, and it was used for teaching in the *Gymnasium illustre*, a school initially founded in Durlach, and that moved to Karlsruhe.[2] This is now part of the University of Karlsruhe. The orrery came to Furtwangen in 1865.

It is the only "pure" orrery by Hahn, all his other clocks and machines contain other parts, such as globes. Nevertheless, there are three other clocks or machines by Hahn that contain similar orreries. The first is that of the Ludwigsburg *Weltmaschine* (Oechslin 8.1) from 1769, now in Stuttgart. After the Furtwangen orrery, we have the Gotha *Weltmaschine* (Oechslin 8.3) from 1780, and finally the Nuremberg *Weltmaschine* (Oechslin 8.2) constructed along the 1770s and 1780s. These orreries are all different, although the Furtwangen one is probably closest to the one in Gotha. I will make more detailed comparisons during my analysis of the gears.

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1). The present machine was described by Kistner in the catalogue of the Furtwangen museum published in 1925 [1, p. 55-66]. It is also mentioned by Zinner [4, p. 353] and especially in the 1989 Hahn exhibition catalogue [3, p. 455-457].

[2] Some accounts claim that the orrery was used in the *Collegium illustre* which was in Tübingen, but this seems incorrect.

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9.2 The going work

The going work provides a rotation of one turn in 24 hours on arbor 1:

$$\mathrm{V}_1^0 = -1 \tag{9.1}$$

This motion is clockwise as seen from the outside.

It is used to produce the motion of the central arbor 5 of the orrery:

$$\mathrm{V}_2^0 = \mathrm{V}_1^0 \times \left(-\frac{35}{35}\right) \tag{9.2}$$

$$\mathrm{V}_3^0 = \mathrm{V}_2^0 \times \frac{76}{79} \text{ (because of how the wheels mesh)} \tag{9.3}$$

$$\mathrm{V}_4^0 = \mathrm{V}_3^0 \times \frac{57}{58} \text{ (same)} \tag{9.4}$$

$$\mathrm{V}_5^0 = \mathrm{V}_4^0 \times \frac{35}{33} \tag{9.5}$$

and therefore

$$\mathrm{V}_5^0 = \frac{25270}{25201} \tag{9.6}$$

This is the velocity of a sidereal day.[3] The motion of the central arbor 5 is counterclockwise as seen from above. This ratio is the usual ratio used by Hahn for the sidereal day.

9.3 The orrery

The main part of the orrery is a fixed frame (0) containing the gears for the Moon and for the planets up to Mars. The motion Mars, as well as the base sidereal motion, are then used to derive the motions of Jupiter and Saturn.

9.3.1 The first planets and the Moon

The motions of the planets are obtained in a rather simple way, and the orrery is in that matter simpler than that of the Ludwigsburg machine (Oechslin 1.1) from 1769. In the latter, the motions of the planets were obtained by intermittent corrections, and the planets were moving in (simulated) tilted planes. In the Furtwangen orrery, all the planets move in the same horizontal plane and the motions are regular. Venus and the Earth have a simple circular motion. Mercury and Mars, on the other hand, move on eccentric circles that approximate ellipses. Finally, the Moon is revolving around the Earth.

[3] According to Oechslin, the wheel which should have had 79 teeth above actually only has 78 teeth [2, p. 204].

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9.3.1.1 The mean motions

We can easily compute these motions. The only input to the fixed frame is the vertical arbor 5 which makes one turn counterclockwise (seen from above) in a sidereal day.

This motion is first used to derive the mean motion of Mercury on tube 9:

$$\mathrm{V}_9^0 = \mathrm{V}_5^0 \times \left(-\frac{52}{52}\right) \times \left(-\frac{6}{53}\right) \times \left(-\frac{32}{77}\right) \times \left(-\frac{20}{83}\right) \tag{9.7}$$

$$= \mathrm{V}_5^0 \times \frac{3840}{338723} = \frac{25270}{25201} \times \frac{3840}{338723} \tag{9.8}$$

$$= \frac{13862400}{1219451189} \tag{9.9}$$

$$\mathrm{P}_9^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{9.10}$$

This motion is counterclockwise. The same value is given by Oechslin.

The same ratio is used in the Stuttgart globe clock (Oechslin 8.5), and in the orreries of Gotha (Oechslin 8.3) and Nuremberg (Oechslin 8.2).

The motion of Mercury is used to obtain the motion of Venus on tube 13:

$$\mathrm{V}_{13}^0 = \mathrm{V}_9^0 \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) = \mathrm{V}_9^0 \times \frac{783}{2000} \tag{9.11}$$

$$= \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{9.12}$$

$$\mathrm{P}_{13}^0 = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{9.13}$$

The same value is given by Oechslin.

The same ratio is also used in the Stuttgart globe clock (Oechslin 8.5), and in the orreries of Gotha (Oechslin 8.3) and Nuremberg (Oechslin 8.2).

The motion of Venus is used to obtain the motion of the Earth on tube 15:

$$\mathrm{V}_{15}^0 = \mathrm{V}_{13}^0 \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = \mathrm{V}_{13}^0 \times \frac{3239}{5265} \tag{9.14}$$

$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{9.15}$$

$$\mathrm{P}_{15}^0 = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{9.16}$$

The same value is given by Oechslin.

The same ratio is also used in the Stuttgart globe clock (Oechslin 8.5), and in the orreries of Gotha (Oechslin 8.3) and Nuremberg (Oechslin 8.2).

The motion of the Earth is used to obtain the motion of Mars on tube 17:

$$\mathrm{V}_{17}^0 = \mathrm{V}_{15}^0 \times \left(-\frac{116}{33}\right) \times \left(-\frac{18}{119}\right) = \mathrm{V}_{15}^0 \times \frac{696}{1309} \tag{9.17}$$

$$= \frac{473632}{172990675} \times \frac{696}{1309} = \frac{329647872}{226444793575} \tag{9.18}$$

$$\mathrm{P}_{17}^0 = \frac{226444793575}{329647872} = 686.9293\ldots \text{ days} \tag{9.19}$$

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The same value is given by Oechslin.

The same ratio is used in the Stuttgart globe clock (Oechslin 8.5), the Aschaffenburg globe clock (Oechslin 8.4), and in the Gotha orrery (Oechslin 8.3), but not in the Nuremberg orrery (Oechslin 8.2).

9.3.1.2 The motion of the Moon

The motion of Mercury is also used to obtain the mean motion of the Moon on tube 11:

$$\mathrm{V}_{11}^0 = \mathrm{V}_9^0 \times \left(-\frac{84}{85}\right) \times \left(-\frac{101}{31}\right) = \mathrm{V}_9^0 \times \frac{8484}{2635} \tag{9.20}$$

$$= \frac{13862400}{1219451189} \times \frac{8484}{2635} = \frac{23521720320}{642650776603} \tag{9.21}$$

$$\mathrm{P}_{11}^0 = \frac{642650776603}{23521720320} = 27.3215\ldots \text{ days} \tag{9.22}$$

This is an approximation of the tropical month. The same value is given by Oechslin.

The same ratio is used in the orreries of Gotha (Oechslin 8.3) and Nuremberg (Oechslin 8.2), but not in the Stuttgart globe clock (Oechslin 8.5).

The Moon is carried by tube 34 which replicates the motion of tube 11.

9.3.1.3 The anomalies of Mercury and Mars

Mercury and Mars are each excentered on wheels that replicate the motion of a central wheel or tube which is fixed. In the case of Mercury, the fixed wheel is a 20-teeth wheel on the central arbor carrying the Sun, and this arbor is fixed to the cage (frame) containing the various gears that have been described. In the case of Mars, the fixed wheel is a 40-teeth wheel fixed on top of the above-mentioned frame.

Consequently, Mercury and Mars merely oscillate around their mean position with the orbital period of the planet. The lines of apsides of both planets are considered fixed.

9.3.2 Jupiter and its satellites

Jupiter and its satellites is carried on an arm which revolves around the Sun. This arm moves with the period of Jupiter's orbital period. In addition, two horizontal arbors are mounted on this arm, to care for the eccentric motion of Jupiter and for the motion of its satellites. We must therefore consider *three* inputs for the motion of the Jupiter system.

The support arm of Jupiter is carried by tube 19. The motion of this tube

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is derived from the mean motion of Mars. This tube has the velocity

$$\mathrm{V}_{19}^0 = \mathrm{V}_{17}^0 \times \left(-\frac{119}{31}\right) \times \left(-\frac{5}{121}\right) = \mathrm{V}_{17}^0 \times \frac{595}{3751} \tag{9.23}$$

$$= \frac{329647872}{226444793575} \times \frac{595}{3751} = \frac{329647872}{1427553648235} \tag{9.24}$$

$$\mathrm{P}_{19}^0 = \frac{1427553648235}{329647872} = 4330.5410\ldots \text{ days} \tag{9.25}$$

The same value is given by Oechslin.

The same ratio is used in the Gotha orrery (Oechslin 8.3) and in the Stuttgart globe clock (Oechslin 8.5).

As mentioned above, two arbors are mounted on the arm supporting Jupiter and its satellites, one (28) for the eccentric motion of Jupiter, and the other (38) for the motion of the satellites of Jupiter. A similar construction was used in the Gotha *Weltmaschine* in 1780, except that the horizontal arbors are replaced by gear trains.

9.3.2.1 The eccentric motion of Jupiter

The eccentric motion of Jupiter is obtained very simply. The system of Jupiter is actually fixed on the frame 29 which rotates excentrically on the support arm 19. The orientation of frame 29 is actually replicating that of a fixed wheel on the fixed central tube of the orrery. Consequently, Jupiter moves on an eccentric circle, an approximation of its elliptic orbit.

9.3.2.2 Jupiter's satellites

The motion of the satellites of Jupiter is obtained from the motion of the arbor 38 mounted on the arm supporting the Jupiter system. The motion of this arbor is derived from the motion of the tube 36, which is itself derived from the fundamental sidereal day motion. We can easily compute the motion of tube 36:

$$\mathrm{V}_{36}^0 = \mathrm{V}_5^0 \times \left(-\frac{52}{52}\right) \times \left(-\frac{24}{24}\right) \times \left(-\frac{52}{52}\right) = -\mathrm{V}_5^0 = -\frac{25270}{25201} \tag{9.26}$$

In other words, tube 36 makes one turn *clockwise* in one sidereal day.

The motion of tube 36 is then replicated in the motion of arbor 39, which goes through tube 29. And since the orientation of tube 29 is fixed in space, we can merely consider the motion of the four satellites of Jupiter with an

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input which is that of the sidereal day motion. We have

$$\mathrm{V}^0_{42} = \mathrm{V}^0_{39} \times \left(-\frac{26}{26}\right) \times \left(-\frac{35}{57}\right) \times \left(-\frac{56}{61}\right) \tag{9.27}$$

$$= \mathrm{V}^0_{39} \times \left(-\frac{1960}{3477}\right) = \mathrm{V}^0_{36} \times \left(-\frac{1960}{3477}\right) \tag{9.28}$$

$$= \left(-\frac{25270}{25201}\right) \times \left(-\frac{1960}{3477}\right) = \frac{2606800}{4611783} \tag{9.29}$$

$$\mathrm{P}^0_{42} = \frac{4611783}{2606800} = 1.7691\ldots \text{ days (Io)} \tag{9.30}$$

The same ratio is used in the *Weltmaschinen* of Gotha (Oechslin 8.3) and Nuremberg (Oechslin 8.2).

It should also be noted that Oechslin mistakenly used the ratio $\frac{36}{57}$ instead of $\frac{35}{57}$ in his computations, and consequently obtained wrong periods for all four satellites of Jupiter. The same mistake was made in the analysis of the Gotha machine, but not in the Nuremberg machine.

Then

$$\mathrm{V}^0_{44} = \mathrm{V}^0_{42} \times \left(-\frac{67}{68}\right) \times \left(-\frac{45}{89}\right) = \mathrm{V}^0_{42} \times \frac{3015}{6052} \tag{9.31}$$

$$= \frac{2606800}{4611783} \times \frac{3015}{6052} = \frac{654958500}{2325875893} \tag{9.32}$$

$$\mathrm{P}^0_{44} = \frac{2325875893}{654958500} = 3.5511\ldots \text{ days (Europa)} \tag{9.33}$$

The same ratio is used in the *Weltmaschine* of Gotha (Oechslin 8.3) but not in that of Nuremberg (Oechslin 8.2).

$$\mathrm{V}^0_{46} = \mathrm{V}^0_{44} \times \left(-\frac{94}{41}\right) \times \left(-\frac{21}{97}\right) = \mathrm{V}^0_{44} \times \frac{1974}{3977} \tag{9.34}$$

$$= \frac{654958500}{2325875893} \times \frac{1974}{3977} = \frac{1292888079000}{9250008426461} \tag{9.35}$$

$$\mathrm{P}^0_{46} = \frac{9250008426461}{1292888079000} = 7.1545\ldots \text{ days (Ganymede)} \tag{9.36}$$

The same ratio is used in the *Weltmaschine* of Gotha (Oechslin 8.3) but not in that of Nuremberg (Oechslin 8.2).

$$\mathrm{V}^0_{48} = \mathrm{V}^0_{46} \times \left(-\frac{66}{53}\right) \times \left(-\frac{21}{61}\right) = \mathrm{V}^0_{46} \times \frac{1386}{3233} \tag{9.37}$$

$$= \frac{1292888079000}{9250008426461} \times \frac{1386}{3233} = \frac{162903897954000}{2718661567522583} \tag{9.38}$$

$$\mathrm{P}^0_{48} = \frac{2718661567522583}{162903897954000} = 16.6887\ldots \text{ days (Calliso)} \tag{9.39}$$

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The same ratio is used in the *Weltmaschine* of Gotha (Oechslin 8.3) but not in that of Nuremberg (Oechslin 8.2).

The periods of Europa, Ganymede and Callisto in the Nuremberg *Weltmaschine* are somewhat less accurate than those of the Furtwangen orrery, even when the Nuremberg machine is corrected of its flaws.

9.3.3 Saturn and its satellites

The structure of the system of Saturn is similar to that of Jupiter. There is a similar arm, and also two similar horizontal arbors used to provide the eccentric motion of Saturn and the motion of its satellites.

The support arm of Saturn is carried by tube 22. Its motion is derived from tube 19, the support arm of Jupiter. When tube 19 rotates, it carries a wheel on arbor 21 which meshes with a wheel on a fixed central tube. Arbor 21 then rotates and this rotation is transmitted to tube 22. We therefore first consider the motion of tube 22 in the reference frame of tube 19:

$$\mathrm{V}_{22}^{19} = \mathrm{V}_{20}^{19} \times \left(-\frac{103}{68}\right) \times \left(-\frac{41}{104}\right) = \mathrm{V}_{20}^{19} \times \frac{4223}{7072} \tag{9.40}$$

where tube 20 is the fixed central tube.

$$= -\mathrm{V}_{19}^{0} \times \frac{4223}{7072} = -\frac{329647872}{1427553648235} \times \frac{4223}{7072} \tag{9.41}$$

$$= -\frac{43503217608}{315489356259935} \tag{9.42}$$

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{22}^{19} + \mathrm{V}_{19}^{0} = \mathrm{V}_{19}^{0} \times \left(1 - \frac{4223}{7072}\right) = \mathrm{V}_{19}^{0} \times \frac{2849}{7072} \tag{9.43}$$

$$= \frac{329647872}{1427553648235} \times \frac{2849}{7072} = \frac{2668087464}{28680850569085} \tag{9.44}$$

$$\mathrm{P}_{22}^{0} = \frac{28680850569085}{2668087464} = 10749.5915\ldots \text{ days} \tag{9.45}$$

The same value is obtained by Oechslin.

The same ratio is used in the Gotha orrery (Oechslin 8.3), but not in other machines described by Oechslin.

As mentioned above, two arbors are mounted on the arm supporting Saturn and its satellites, one (31) for the eccentric motion of Saturn, and the other (51) for the motion of the satellites of Saturn. A similar construction was used in the Gotha *Weltmaschine* in 1780, except that the horizontal arbors are replaced by gear trains.

9.3.3.1 The eccentric motion of Saturn

Like for Jupiter, the system of Saturn is fixed on the frame 32 which rotates excentrically on the support arm 22. The orientation of frame 32 is also

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replicating that of a fixed wheel on the fixed central tube of the orrery. Consequently, Saturn moves on an eccentric circle, an approximation of its elliptic orbit.

9.3.3.2 Saturn's satellites

The motion of the satellites of Saturn is obtained from the motion of the arbor 51 mounted on the arm supporting the Saturn system. The motion of this arbor is derived from the motion of the tube 49, which has the same motion as the tube 36 used for the satellites of Jupiter.

$$\mathrm{V}^0_{49} = \mathrm{V}^0_{36} = -\frac{25270}{25201} \tag{9.46}$$

In other words, tube 49 also makes one turn *clockwise* in one sidereal day. The motion of tube 49 is then replicated in the motion of arbor 52, which goes through tube 29. And since the orientation of tube 32 is fixed in space, we can merely consider the motion of the five satellites of Saturn with an input which is that of the sidereal day motion. If we take the actual teeth numbers as given on Oechslin's drawing, we find that the motions of the five satellites are the following ones:

For Tethys:

$$\mathrm{V}^0_{55} = \mathrm{V}^0_{52} \times \left(-\frac{20}{78}\right) \times \left(-\frac{86}{33}\right) = \mathrm{V}^0_{52} \times \frac{860}{1287} \tag{9.47}$$

$$= \frac{25270}{25201} \times \frac{860}{1287} = \frac{21732200}{32433687} \tag{9.48}$$

$$\mathrm{P}^0_{55} = \frac{32433687}{21732200} = 1.4924\ldots \text{ days} \tag{9.49}$$

For Dione:

$$\mathrm{V}^0_{57} = \mathrm{V}^0_{55} \times \left(-\frac{22}{46}\right) \times \left(-\frac{49}{34}\right) = \mathrm{V}^0_{55} \times \frac{539}{782} \tag{9.50}$$

$$= \frac{21732200}{32433687} \times \frac{539}{782} = \frac{532438900}{1152870147} \tag{9.51}$$

$$\mathrm{P}^0_{57} = \frac{1152870147}{532438900} = 2.1652\ldots \text{ days} \tag{9.52}$$

For Rhea:

$$\mathrm{V}^0_{59} = \mathrm{V}^0_{57} \times \left(-\frac{27}{32}\right) \times \left(-\frac{23}{32}\right) = \mathrm{V}^0_{57} \times \frac{621}{1024} \tag{9.53}$$

$$= \frac{532438900}{1152870147} \times \frac{621}{1024} = \frac{399329175}{1425771776} \tag{9.54}$$

$$\mathrm{P}^0_{59} = \frac{1425771776}{399329175} = 3.5704\ldots \text{ days} \tag{9.55}$$

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For Titan:

$$V^0_{61} = V^0_{59} \times \left(-\frac{40}{36}\right) \times \left(-\frac{12}{47}\right) = V^0_{59} \times \frac{40}{141} \tag{9.56}$$

$$= \frac{399329175}{1425771776} \times \frac{40}{141} = \frac{665548625}{8376409184} \tag{9.57}$$

$$P^0_{61} = \frac{8376409184}{665548625} = 12.5857\ldots \text{ days} \tag{9.58}$$

For Iapetus:

$$V^0_{63} = V^0_{61} \times \left(-\frac{68}{39}\right) \times \left(-\frac{8}{69}\right) = V^0_{61} \times \frac{544}{2691} \tag{9.59}$$

$$= \frac{665548625}{8376409184} \times \frac{544}{2691} = \frac{665548625}{41435509401} \tag{9.60}$$

$$P^0_{63} = \frac{41435509401}{665548625} = 62.2576\ldots \text{ days} \tag{9.61}$$

These values are all wrong by a factor of about 1.27. The actual periods are 1.9 days (Tethys), 2.7 days (Dione), 4.5 days (Rhea), 16 days (Titan) and 79 days (Iapetus).

It does however appear that there are two possible causes to these discrepancies. First, Oechslin's plan gives a ratio of 86/33 for Tethys, but in his computations he uses the ratio 68/33 (values marked "(a)" below), and therefore it is possible that the 86-teeth wheel should have had 68 teeth.[4] This problem is the same in the Gotha *Weltmaschine* from 1780 (Oechslin 8.3) where the system of the satellites of Saturn uses exactly the same ratios. The wrong ratio used in the motion of Tethys propagates to the other satellites, because the motion of each satellite is obtained from the previous one.

It seems that this problem in the Furtwangen planetarium was not noticed by Hahn and made its way to the Gotha *Weltmaschine*.

There is however another possibility. In the Furtwangen orrery, one of the ratios used in the motion of Tethys is $\frac{20}{78}$ and if we replace it by $\frac{20}{98}$, we also obtain much more accurate periods (values marked "(b)" below). This possibility also applies to the Gotha machine, the only difference being that the ratio $\frac{20}{78}$ was replaced by $\frac{10}{39}$.

So, the periods given by Oechslin are those obtained when Tethys uses the ratio 68/33. The motions of the five satellites are then the following, where I have added "(a)" to distinguish this choice from the second one "(b)" below.

[4] See [2, p. 204] on this error.

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For Tethys:

$$\mathrm{V}^0_{55}(a) = \mathrm{V}^0_{52} \times \left(-\frac{20}{78}\right) \times \left(-\frac{68}{33}\right) = \mathrm{V}^0_{52} \times \frac{680}{1287} \tag{9.62}$$

$$= \frac{25270}{25201} \times \frac{680}{1287} = \frac{17183600}{32433687} \tag{9.63}$$

$$\mathrm{P}^0_{55}(a) = \frac{32433687}{17183600} = 1.8874\ldots \text{ days} \tag{9.64}$$

For Dione:

$$\mathrm{V}^0_{57}(a) = \mathrm{V}^0_{55}(a) \times \left(-\frac{22}{46}\right) \times \left(-\frac{49}{34}\right) = \mathrm{V}^0_{55}(a) \times \frac{539}{782} \tag{9.65}$$

$$= \frac{17183600}{32433687} \times \frac{539}{782} = \frac{24764600}{67815891} \tag{9.66}$$

$$\mathrm{P}^0_{57}(a) = \frac{67815891}{24764600} = 2.7384\ldots \text{ days} \tag{9.67}$$

For Rhea:

$$\mathrm{V}^0_{59}(a) = \mathrm{V}^0_{57}(a) \times \left(-\frac{27}{32}\right) \times \left(-\frac{23}{32}\right) = \mathrm{V}^0_{57}(a) \times \frac{621}{1024} \tag{9.68}$$

$$= \frac{24764600}{67815891} \times \frac{621}{1024} = \frac{9286725}{41934464} \tag{9.69}$$

$$\mathrm{P}^0_{59}(a) = \frac{41934464}{9286725} = 4.5155\ldots \text{ days} \tag{9.70}$$

For Titan:

$$\mathrm{V}^0_{61}(a) = \mathrm{V}^0_{59}(a) \times \left(-\frac{40}{36}\right) \times \left(-\frac{12}{47}\right) = \mathrm{V}^0_{59}(a) \times \frac{40}{141} \tag{9.71}$$

$$= \frac{9286725}{41934464} \times \frac{40}{141} = \frac{15477875}{246364976} \tag{9.72}$$

$$\mathrm{P}^0_{61}(a) = \frac{246364976}{15477875} = 15.9172\ldots \text{ days} \tag{9.73}$$

For Iapetus:

$$\mathrm{V}^0_{63}(a) = \mathrm{V}^0_{61}(a) \times \left(-\frac{68}{39}\right) \times \left(-\frac{8}{69}\right) = \mathrm{V}^0_{61}(a) \times \frac{544}{2691} \tag{9.74}$$

$$= \frac{15477875}{246364976} \times \frac{544}{2691} = \frac{526247750}{41435509401} \tag{9.75}$$

$$\mathrm{P}^0_{63}(a) = \frac{41435509401}{526247750} = 78.7376\ldots \text{ days} \tag{9.76}$$

If instead we replace the ratio 20/78 by 20/98, we obtain the following values, marked "(b)" in order to distinguish them from the previous ones:

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For Tethys:

$$\mathrm{V}^0_{55}(b) = \mathrm{V}^0_{52} \times \left(-\frac{20}{98}\right) \times \left(-\frac{86}{33}\right) = \mathrm{V}^0_{52} \times \frac{860}{1617} \tag{9.77}$$

$$= \frac{25270}{25201} \times \frac{860}{1617} = \frac{3104600}{5821431} \tag{9.78}$$

$$\mathrm{P}^0_{55}(b) = \frac{5821431}{3104600} = 1.8750\ldots \text{ days} \tag{9.79}$$

For Dione:

$$\mathrm{V}^0_{57}(b) = \mathrm{V}^0_{55}(b) \times \left(-\frac{22}{46}\right) \times \left(-\frac{49}{34}\right) = \mathrm{V}^0_{55}(b) \times \frac{539}{782} \tag{9.80}$$

$$= \frac{3104600}{5821431} \times \frac{539}{782} = \frac{10866100}{29560773} \tag{9.81}$$

$$\mathrm{P}^0_{57}(b) = \frac{29560773}{10866100} = 2.7204\ldots \text{ days} \tag{9.82}$$

For Rhea:

$$\mathrm{V}^0_{59}(b) = \mathrm{V}^0_{57}(b) \times \left(-\frac{27}{32}\right) \times \left(-\frac{23}{32}\right) = \mathrm{V}^0_{57}(b) \times \frac{621}{1024} \tag{9.83}$$

$$= \frac{10866100}{29560773} \times \frac{621}{1024} = \frac{24448725}{109674752} \tag{9.84}$$

$$\mathrm{P}^0_{59}(b) = \frac{109674752}{24448725} = 4.4859\ldots \text{ days} \tag{9.85}$$

For Titan:

$$\mathrm{V}^0_{61}(b) = \mathrm{V}^0_{59}(b) \times \left(-\frac{40}{36}\right) \times \left(-\frac{12}{47}\right) = \mathrm{V}^0_{59}(b) \times \frac{40}{141} \tag{9.86}$$

$$= \frac{24448725}{109674752} \times \frac{40}{141} = \frac{40747875}{644339168} \tag{9.87}$$

$$\mathrm{P}^0_{61}(b) = \frac{644339168}{40747875} = 15.8128\ldots \text{ days} \tag{9.88}$$

For Iapetus:

$$\mathrm{V}^0_{63}(b) = \mathrm{V}^0_{61}(b) \times \left(-\frac{68}{39}\right) \times \left(-\frac{8}{69}\right) = \mathrm{V}^0_{61}(b) \times \frac{544}{2691} \tag{9.89}$$

$$= \frac{40747875}{644339168} \times \frac{544}{2691} = \frac{13582625}{1062448959} \tag{9.90}$$

$$\mathrm{P}^0_{63}(b) = \frac{1062448959}{13582625} = 78.2211\ldots \text{ days} \tag{9.91}$$

It isn't clear which of these two solutions is the right one.

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9.4 References

[1] Adolf Kistner. *Die historische Uhrensammlung Furtwangen.* Furtwangen: A. Uttenweiler, 1925. [for Rinderle's clock and Hahn's orrery].

[2] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[3] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.

[4] Ernst Zinner. *Deutsche und niederländische astronomische Instrumente des 11.-18. Jahrhunderts.* München: C. H. Beck, 1967. [2nd edition].

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Chapter 10

(Oechslin: 8.10)

Hahn's astronomical clock in Basel (1775)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

10.1 Introduction

The clock described here was constructed in 1775 by Philipp Matthäus Hahn (1739-1790)[1] and has been acquired by the historical museum of Basel in 1913. This clock was commissioned in 1775 by the Basel tradesman Wilhelm Brenner and remained in his family until 1913.[2] The cost of this clock was 200 gulden.[3]

It is a longcase pendulum-clock with six dials. The central dial shows the minutes, the seconds, and the minutes corrected by the equation of time.

Four dials symmetrically surround the central dial and a sixth dial is positionned at the top. The lower left dial shows the hours, the lower right dial shows the day of the week and of the month, the upper right dial shows the motion of the Sun, the date in the year, and the length of the day. The upper left dial shows the motion and phase of the Moon, the sign of the zodiac, as well as the length of moonlight. Finally, the dial at the top is a planispheric representation of the sky.

An almost identical clock was made in 1774/1775 for the physician Diethelm Lavater (1743-1826) and is kept in the Museum of Music Automatons in Seewen (Switzerland), as a loan from the Swiss National Museum (*Landesmuseum Zürich*).[4]

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [5, p. 442-443]. See also Engelmann [1, fig. 31, 33], Reinhardt [4, p. 27] and Gutmann [2, p. 6].

[3] [3, p. 224]

[4] Cf. [6] and [2, p. 19] and especially the 1989 exhibition catalogue [5, p. 440-442]. This

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Figure 10.1: Upper part of the dial of the Basel clock. (source: Wikimedia, Historisches Museum Basel)

10.2 The going work and the indication of time

The clock is driven by a weight of 23.171 kg[5] and the winding drum on arbor 1 makes one turn in 560 hours, that is in 23 days and 8 hours. Its motion is counterclockwise as seen from the front. The clock could work for one year without being wound.

We have

$$\mathrm{T}_1^0 = -\frac{24}{560} = -\frac{3}{70} \tag{10.1}$$

We then derive the motions of arbors 3 and 4:

$$\mathrm{T}_3^0 = \mathrm{T}_1^0 \times \left(-\frac{84}{12}\right) \times \left(-\frac{96}{12}\right) = -\frac{12}{5} \tag{10.2}$$

$$\mathrm{T}_4^0 = \mathrm{T}_3^0 \times \left(-\frac{120}{12}\right) = 24 \tag{10.3}$$

Arbor 4 makes 24 turns in a day, hence one turn clockwise in one hour.

This arbor drives the minute hand on the central dial of the clock.

The escape wheel is on arbor 6 and its velocity is

$$\mathrm{T}_6^0 = \mathrm{T}_4^0 \times \left(-\frac{64}{8}\right) \times \left(-\frac{60}{8}\right) = \mathrm{T}_4^0 \times 60 = 1440 \tag{10.4}$$

clock was exhibited in the Lavater house in Zurich in September 2025 as part of the exhibition "*Das Ticken der Uhren im Zeitalter der Aufklärung — Johann Caspar Lavater und Philipp Matthäus Hahn.*"

[5]Cf. [2, p. 32].

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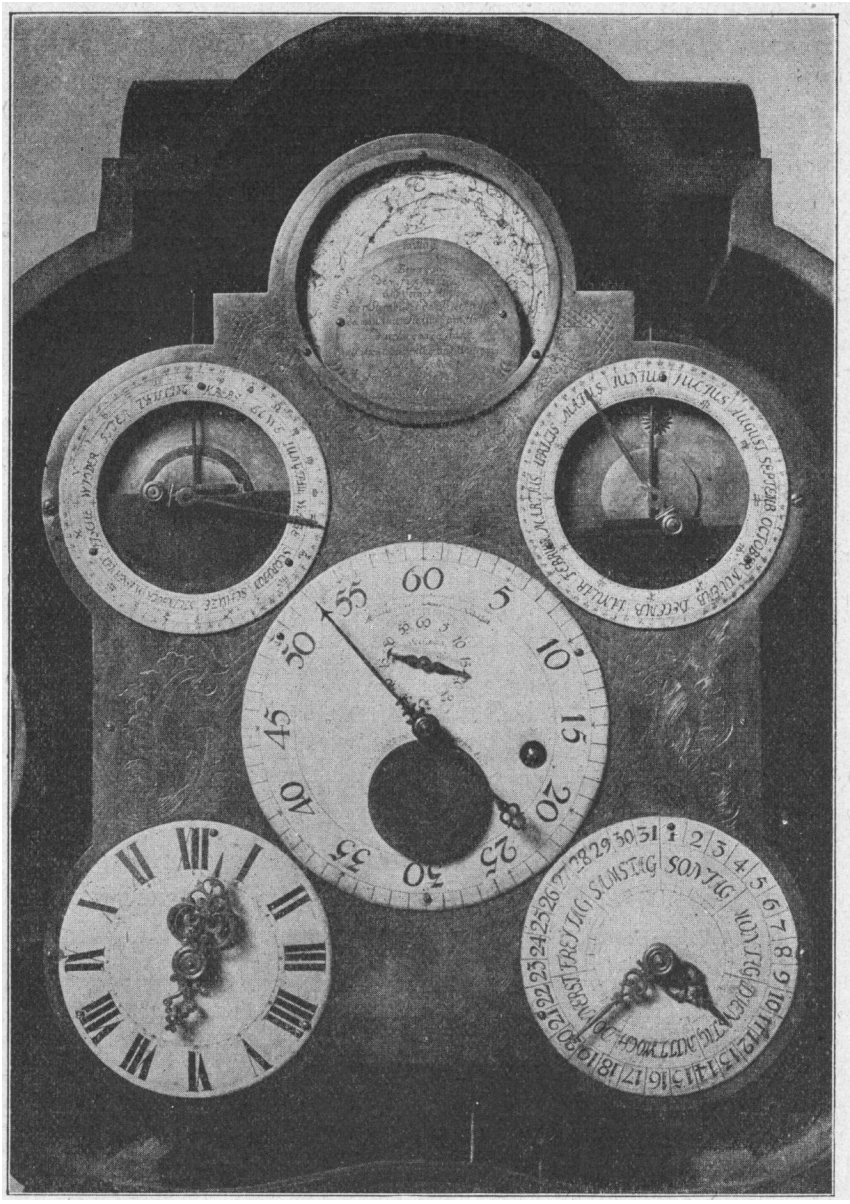


Figure 10.2: Hahn's clock in Basel. (source: [1])

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It does therefore make one turn in one minute. It has 30 pins and the pendulum makes a half-swing in one second. Its length is about one meter.

The arbor 6 carries the second hand which is located on the small upper dial within the large central dial.

The large minute hand is also replicated on a smaller dial within the large central dial:

$$\mathrm{T}_8^0 = \mathrm{T}_4^0 \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{60}\right) = \mathrm{T}_4^0 \tag{10.5}$$

These two hands therefore should always be parallel to each other.

The smaller hand on arbor 8 actually moves on an oscillating dial so that this smaller dial shows the minutes corrected for the equation of time, that is the minutes of true (solar) time, and not mean time. This correction is obtained by a toothed segment meshing with a wheel on tube 38. This segment is controlled by the equation of time cam which is located on the central arbor of the solar dial, which makes one turn in a year.

The motion of arbor 3 is also used to obtain the indication of the hour on tube 11. This tube controls the hour hand on the lower left dial. We have

$$\mathrm{T}_{11}^0 = \mathrm{T}_3^0 \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{54}\right) \times \left(-\frac{54}{72}\right) \tag{10.6}$$

$$= \mathrm{T}_3^0 \times \left(-\frac{5}{6}\right) = \left(-\frac{12}{5}\right) \times \left(-\frac{5}{6}\right) = 2 \tag{10.7}$$

In other words, the tube 11 makes two turns clockwise in one day, hence one turn in twelve hours.

10.3 The lunar dial

The hand on tube 11 actually moves another hand which is fixed on the central arbor 12 of the hours dial. The motion of this arbor is transferred to tube 14 on the lunar dial:

$$\mathrm{V}_{14}^0 = \mathrm{V}_{12}^0 \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}_{12}^0 \times \frac{1}{2} \tag{10.8}$$

and since

$$\mathrm{V}_{12}^0 = -\mathrm{T}_{12}^0 = -\mathrm{T}_{11}^0 \tag{10.9}$$

we have

$$\mathrm{V}_{14}^0 = -\mathrm{T}_{11}^0 \times \frac{1}{2} = -1 \tag{10.10}$$

Tube 14 thus makes one turn clockwise in one day. This tube moves the ring with the age of the Moon. We will see below that its motion should indeed be that of one turn per day.

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The motion of tube 14 is used to move the central arbor 20:

$$\mathrm{V}^0_{20} = \mathrm{V}^0_{14} \times \left(-\frac{28}{54}\right) \times \left(-\frac{6}{85}\right) = -\frac{28}{765} \tag{10.11}$$

$$\mathrm{P}^0_{20} = -\frac{765}{28} = -27.3214\ldots \text{ days} \tag{10.12}$$

This is the period of the tropical month. The same value is given by Oechslin. It is the period of the large hand which is also responsible for the oscillation of the horizon, but it isn't clear why the horizon should move with this period.

Tube 14 also carries an arm on which pivots arbor 16. This arbor carries a 42-teeth wheel which meshes with a fixed 16-teeth wheel on tube 15. The rotation of tube 14 thus produces a rotation of arbor 16. We can compute this motion in the reference frame of tube 14:

$$\mathrm{V}^{14}_{16} = \mathrm{V}^{14}_{15} \times \left(-\frac{16}{42}\right) = -\mathrm{V}^{15}_{14} \times \left(-\frac{8}{21}\right) = \mathrm{V}^0_{14} \times \frac{8}{21} = -\frac{8}{21} \tag{10.13}$$

A 8-leaves pinion on arbor 16 meshes with two wheels: a 90-teeth wheel on tube 18 and a 80-teeth wheel on tube 17. Tube 18carries the disk with the hole, which represents the Moon. We have:

$$\mathrm{V}^{14}_{17} = \mathrm{V}^{14}_{16} \times \left(-\frac{8}{80}\right) = \left(-\frac{8}{21}\right) \times \left(-\frac{1}{10}\right) = \frac{4}{105} \tag{10.14}$$

$$\mathrm{V}^{14}_{18} = \mathrm{V}^{14}_{16} \times \left(-\frac{8}{90}\right) = \left(-\frac{8}{21}\right) \times \left(-\frac{4}{45}\right) = \frac{32}{945} \tag{10.15}$$

$$\mathrm{P}^{14}_{18} = \frac{945}{32} = 29.53125 \text{ days} \tag{10.16}$$

The same value is given by Oechslin. After one synodic month, the Moon has made one turn with respect to the interior ring (on tube 14) showing the age of the Moon. And this motion is counterclockwise, which is why the ring is numbered counterclockwise.

Finally, we can compute the absolute motion of tube 18:

$$\mathrm{V}^0_{18} = \mathrm{V}^{14}_{18} + \mathrm{V}^0_{14} = \frac{32}{945} - 1 = -\frac{913}{945} \tag{10.17}$$

$$\mathrm{P}^0_{18} = -\frac{945}{913} \approx -24 \text{ h } 50 \text{ mn} \tag{10.18}$$

The Moon apparently revolves around the Earth in 24 hours and 50 minutes.

We can also compute the motion of tube 17 with respect to tube 18:

$$\mathrm{V}^0_{17} = \mathrm{V}^{14}_{17} + \mathrm{V}^0_{14} = \frac{4}{105} - 1 = -\frac{101}{105} \tag{10.19}$$

$$\mathrm{V}^{18}_{17} = \mathrm{V}^0_{17} - \mathrm{V}^0_{18} = -\frac{101}{105} + \frac{913}{945} = \frac{4}{945} \tag{10.20}$$

$$\mathrm{P}^{18}_{17} = \frac{945}{4} = 8\mathrm{P}^{14}_{18} \tag{10.21}$$

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We can see that in one synodic month (P_{18}^{14}), tube 17 rotates of an eighth turn with respect to tube 18. This is why tube 17 is tied with a disk contained eight figures of the Moon. This disk revolves counterclockwise behind the disk moved by tube 18.

This dial also has an outer ring with the signs of the zodiac, but its use is not clear, given that no hand in this dial makes a turn in one year.

10.4 The solar dial

The motion of tube 14 in the lunar component is used to derive the motion of tube 22 in the solar dial. We have

$$V_{22}^0 = V_{14}^0 \times \frac{72}{36} \times \frac{36}{72} = V_{14}^0 = -1 \tag{10.22}$$

This corresponds to a clockwise rotation in one day and is used to move the disk carrying the Sun.

The motion of tube 22 is used to provide the motion to the calendar part (see below). It causes in particular the rotation of arbor 28 in one day:

$$V_{28}^0 = V_{22}^0 \times \frac{72}{36} \times \left(-\frac{36}{72}\right) = -V_{22}^0 = 1 \tag{10.23}$$

This motion is used to move the central arbor 30 of the solar dial with the help of a wormgear:[6]

$$V_{30}^0 = V_{28}^0 \times \frac{21}{65} \times \left(-\frac{1}{118}\right) = -\frac{21}{7670} \tag{10.24}$$

$$P_{30}^0 = -\frac{7670}{21} = -365.2380\ldots \text{ days} \tag{10.25}$$

which is an approximation of the tropical year. The same value is given by Oechslin.

This arbor moves the hand showing the months in one year. It also carries the moving horizon.

Finally, this arbor carries on its rear the cam of the equation of the time which is used to display the corrected minutes on the central dial.

10.5 The calendar

I will not describe the calendar dial in detail.[7] It has two hands, one for the day of the week, and another for the day of the month. The motion of the hand for the day of the week is merely obtained from the daily motion of arbor 28 mentioned above.

The day of the month takes the lengths of the months into account, including that of leap years (but not the secular exceptions).

[6] Wormgears have apparently seldom been used by Hahn, cf. [2, p. 32].

[7] For more information on the calendar gears, see [4, p. 30-31] and [2, p. 33, 39].

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10.6 The celestial map

The motion of tube 22 in the solar component is also used to derive the motion of the sky which is that of arbor 26. We have

$$\mathrm{V}_{26}^0 = \mathrm{V}_{22}^0 \times \left(-\frac{72}{36}\right) \times \left(-\frac{35}{29}\right) \times \left(-\frac{38}{44}\right) \times \left(-\frac{38}{79}\right) = \mathrm{V}_{22}^0 \times \frac{25270}{25201} \tag{10.26}$$

$$= -\frac{25270}{25201} \tag{10.27}$$

We find here the familiar value for the sidereal day. This value is negative, because the stars apparently move clockwise, from East to West.

10.7 References

[1] Max Engelmann. *Leben und Wirken des württembergischen Pfarrers und Feintechnikers Philipp Matthäus Hahn.* Berlin: Richard Carl Schmidt & Co., 1923.

[2] Veronika Gutmann. *Die astronomische Uhr von Philipp Matthäus Hahn (1775).* Number 16 in Basler Kostbarkeiten. Basel: Baumann & Cie, Banquiers, 1995.

[3] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[4] Hans Franz Reinhardt. Die Basler astronomische Uhr von Philipp Matthäus Hahn. In *Jahresbericht des Historischen Museums Basel 1964*, pages 25–31. 1964.

[5] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.

[6] Rudolf Wolf. Astronomische Uhr von Hahn. — Geschenkt von Herrn Blass-Lavater in Zürich. In *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, volume 28, pages 146–152. 1883.

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Chapter 11

(Oechslin: 8.4)

Hahn's globe clock in Aschaffenburg (1776/1777)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

11.1 Introduction

The clock described here was constructed in 1776/1777 by Philipp Matthäus Hahn (1739-1790)[1] and is located in Schloss Johannisburg in Aschaffenburg.[2]

It has a base with four faces, three of which containing dials. One of the three dials shows the hour on a 24-hour scale, the minutes on an internal dial, the day of the month and of the week on another internal dial, and the year on four small 0-9 dials.[3] The second dial of the base shows a tellurium and the third shows an orrery with the planets from Mercury to Saturn.

The upper part is a celestial globe showing the geocentric motion of the planets, the Sun, the Moon and the lunar nodes.

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]See especially the 1989 exhibition catalogue [2, p. 405-407]. This clock is also cited by Zinner [4, p. 353]. My photographs of the clock were taken on 24 July 2019.

[3]Oechslin's plate seems to imply that the units are given left, then the tens, then the hundreds, then the thousands, but this should be checked. A similar display of the year was used by Alexius Johann in 1796 (Oechslin 9.1).

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Figure 11.1: General views of Hahn's clock in Aschaffenburg. (photographs by the author)

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Figure 11.2: The time dial of Hahn's clock in Aschaffenburg. (photograph by the author)



Figure 11.3: The orrery of Hahn's clock in Aschaffenburg. (photograph by the author)

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Figure 11.4: The tellurium of Hahn's clock in Aschaffenburg. (photograph by the author)



Figure 11.5: The side opposite the time dial on Hahn's clock in Aschaffenburg. (photograph by the author)

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Figure 11.6: The celestial globe of Hahn's clock in Aschaffenburg. (photograph by the author)

11.2 The base motion

The clockwork has a wheel making one turn in 24 hours on arbor 1. We have (from the front)

$$\mathrm{T}_1^0 = 1 \tag{11.1}$$
$$\mathrm{V}_1^0 = -1 \tag{11.2}$$

because the motion is clockwise.

The motion of arbor 3 is then derived:

$$\mathrm{V}_3^0 = \mathrm{V}_1^0 \left(-\frac{76}{79}\right) \times \left(-\frac{57}{58}\right) = -\frac{2166}{2291} \tag{11.3}$$

Although Oechslin's plan lacks that connexion, a 35-teeth wheel on arbor 3 must mesh with a 33-teeth wheel on arbor 4, which is the vertical arbor driving the celestial globe. We then have

$$\mathrm{V}_4^0 = -\mathrm{V}_3^0 \times \left(-\frac{35}{33}\right) = -\left(-\frac{2166}{2291}\right) \times \left(-\frac{35}{33}\right) = -\frac{25270}{25201} \tag{11.4}$$

This motion is clockwise as seen from above. It corresponds to a period of

$$\mathrm{P}_4^0 = -\frac{25201}{25270} \text{ days} = 86164.0838\ldots \text{ s} \tag{11.5}$$

Hence, the motion of arbor 4 is that of a sidereal day.

This ratio was also used by Hahn in the Gotha, Stuttgart and Nuremberg machines, as well as in the Darmstadt globe clock.

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This velocity is then also used to drive the tellurium and the orrery.

I will first describe the gears of the orrery, then of the tellurium, and finally of the celestial globe.

11.3 The orrery

The orrery shows the motions of the planets from Mercury to Saturn around the Sun, as well as the Moon. Mercury, Mars, Jupiter and Saturn are given an irregular motion in order to account for their elliptical orbits.

The input to the orrery is the motion of arbor 5 which is clockwise as seen from the front of the orrery:

$$\mathrm{V}_5^0 = \mathrm{V}_4^0 = -\frac{25270}{25201} \tag{11.6}$$

11.3.1 The mean motions of the planets

The input motion is first used to produce the motion of tube 8, which is the mean motion of Mercury:

$$\mathrm{V}_8^0 = \mathrm{V}_5^0 \times \left(-\frac{32}{53}\right) \times \left(-\frac{6}{77}\right) \times \left(-\frac{20}{83}\right) \tag{11.7}$$

$$= \mathrm{V}_5^0 \times \left(-\frac{3840}{338723}\right) = \left(-\frac{25270}{25201}\right) \times \left(-\frac{3840}{338723}\right) = \frac{13862400}{1219451189} \tag{11.8}$$

$$\mathrm{P}_8^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{11.9}$$

This motion is counterclockwise as seen from the front. The same value is given by Oechslin.

Mercury is actually given an irregular motion, in order to account for its elliptic orbit. I will get back to it below, but will first deal with the mean motions.

Venus (tube 12) only has a mean motion which is obtained from the mean motion of Mercury:

$$\mathrm{V}_{12}^0 = \mathrm{V}_8^0 \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) \tag{11.10}$$

$$= \mathrm{V}_8^0 \times \frac{783}{2000} = \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{11.11}$$

$$\mathrm{P}_{12}^0 = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{11.12}$$

The same value is given by Oechslin.

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The Earth (tube 14) also only has a mean motion which is obtained from the (mean) motion of Venus:

$$\mathrm{V}_{14}^0 = \mathrm{V}_{12}^0 \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = \mathrm{V}_{12}^0 \times \frac{3239}{5265} \tag{11.13}$$

$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{11.14}$$

$$\mathrm{P}_{14}^0 = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{11.15}$$

This is an approximation of the tropical year. The same value is given by Oechslin. This motion is also transferred to arbor 27 which is the input to the tellurium.

The motion of the Earth is then used to obtain the mean motion of Mars. The motion of Mars is used to obtain the motion of Jupiter, and the motion of Jupiter is used to obtain the motions of Saturn and of the Moon.

So, first we compute the mean motion of Mars on tube 16. The train as pictured on Oechslin's drawing contains a 34-teeth wheel and we obtain:

$$\mathrm{V}_{16}^0? = \mathrm{V}_{14}^0 \times \left(-\frac{116}{34}\right) \times \left(-\frac{18}{119}\right) = \mathrm{V}_{14}^0 \times \frac{1044}{2023} \tag{11.16}$$

$$= \frac{473632}{172990675} \times \frac{1044}{2023} = \frac{494471808}{349960135525} \tag{11.17}$$

$$\mathrm{P}_{16}^0? = \frac{349960135525}{494471808} = 707.7453\ldots \text{ days} \tag{11.18}$$

However, this value is not very accurate. We do in fact obtain a much better value for the orbital period of Mars if we replace the 34-teeth wheel by a 33-teeth one. And in fact, this is what Oechslin does in his calculations. We now have:

$$\mathrm{V}_{16}^0 = \mathrm{V}_{14}^0 \times \left(-\frac{116}{33}\right) \times \left(-\frac{18}{119}\right) = \mathrm{V}_{14}^0 \times \frac{696}{1309} \tag{11.19}$$

$$= \frac{473632}{172990675} \times \frac{696}{1309} = \frac{329647872}{226444793575} \tag{11.20}$$

$$\mathrm{P}_{16}^0 = \frac{226444793575}{329647872} = 686.9293\ldots \text{ days} \tag{11.21}$$

and Oechslin has the same value.

This assumption is also supported by the use of the same ratio $\frac{696}{1309}$ in the *Weltmaschinen* of Gotha and Stuttgart, as well as in the Stuttgart globe clock (Oechslin 8.5).

Continuing with this assumption, from Mars, we obtain the mean motion

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of Jupiter on tube 19:

$$\mathrm{V}^0_{19} = \mathrm{V}^0_{16} \times \left(-\frac{119}{31}\right) \times \left(-\frac{5}{121}\right) = \mathrm{V}^0_{16} \times \frac{595}{3751} \tag{11.22}$$

$$= \frac{329647872}{226444793575} \times \frac{595}{3751} = \frac{329647872}{1427553648235} \tag{11.23}$$

$$\mathrm{P}^0_{19} = \frac{1427553648235}{329647872} = 4330.5410\ldots \text{ days} \tag{11.24}$$

The same value is given by Oechslin.

The mean motion of Jupiter is then used to obtain the mean motion of Saturn on tube 22:

$$\mathrm{V}^0_{22} = \mathrm{V}^0_{19} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) = \mathrm{V}^0_{19} \times \frac{363}{901} \tag{11.25}$$

$$= \frac{329647872}{1427553648235} \times \frac{363}{901} = \frac{988943616}{10629965595535} \tag{11.26}$$

$$\mathrm{P}^0_{22} = \frac{10629965595535}{988943616} = 10748.8085\ldots \text{ days} \tag{11.27}$$

The same value is also given by Oechslin.

11.3.2 The motion of the Moon

The mean motion of Jupiter is also used to obtain the motion of tube 25:

$$\mathrm{V}^0_{25} = \mathrm{V}^0_{19} \times \left(-\frac{121}{34}\right) \times \left(-\frac{3}{21}\right) \times \left(-\frac{73}{102}\right) = \mathrm{V}^0_{19} \times \left(-\frac{8833}{24276}\right) \tag{11.28}$$

$$= \frac{329647872}{1427553648235} \times \left(-\frac{8833}{24276}\right) = -\frac{2005357888}{23867281242805} \tag{11.29}$$

$$\mathrm{P}^0_{25} = -\frac{23867281242805}{2005357888} = 11901.7564\ldots \text{ days} \tag{11.30}$$

The same value is given by Oechslin.

The motion of tube 25 is used to rotate the Moon around the Earth. We can compute this motion precisely. We first compute the motion of the Moon

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with respect to the Earth. It is the motion of arbor 26. We have[4]

$$\mathrm{V}_{26}^{14} = \mathrm{V}_{25}^{14} \times \left(-\frac{72}{6}\right) = -\left(\mathrm{V}_{25}^{0} - \mathrm{V}_{14}^{0}\right) \times 12 = \left(\mathrm{V}_{14}^{0} - \mathrm{V}_{25}^{0}\right) \times 12 \tag{11.31}$$

$$= \left(\frac{473632}{172990675} + \frac{2005357888}{23867281242805}\right) \times 12 \tag{11.32}$$

$$= \frac{336758509216}{119336406214025} \times 12 \tag{11.33}$$

$$= \frac{4041102110592}{119336406214025} \tag{11.34}$$

$$\mathrm{P}_{26}^{14} = \frac{119336406214025}{4041102110592} = 29.5306\ldots \text{ days} \tag{11.35}$$

As expected, we find an approximation of the synodic month. This value is not given by Oechslin.

We can also compute the tropical motion of the Moon:

$$\mathrm{V}_{26}^{0} = \mathrm{V}_{26}^{14} + \mathrm{V}_{14}^{0} = \frac{4041102110592}{119336406214025} + \frac{473632}{172990675} = \frac{4367833830368}{119336406214025} \tag{11.36}$$

$$\mathrm{P}_{26}^{0} = \frac{119336406214025}{4367833830368} = 27.3216\ldots \text{ days} \tag{11.37}$$

The same value is given by Oechslin.

11.3.3 The anomalies of the planets

Finally, we examine how the anomalies of the planets Mercury, Mars, Jupiter and Saturn have been obtained. In each case, there is a central wheel fixed to the frame. Oechslin doesn't give the teeth numbers of these wheels, but they are actually irrelevant, as they all mesh with identical wheels pivoting on the mean motion wheels. It therefore suffices to consider the case of Mercury.

The Mercury *satellite* wheel on arbor 10 rotates as the tube 8 rotates with the mean motion of Mercury. In the frame of tube 8, we have

$$\mathrm{V}_{10}^{8} = \mathrm{V}_{9}^{8} \times \left(-\frac{a}{a}\right) = -\mathrm{V}_{9}^{8} = \mathrm{V}_{8}^{9} = \mathrm{V}_{8}^{0} \tag{11.38}$$

$$\mathrm{P}_{10}^{8} = \mathrm{P}_{8}^{0} \tag{11.39}$$

Therefore, on the Mercury tube 8, the eccentric arbor of Mercury oscillates with the same period as the mean tropical motion of Mercury. This causes the acceleration and slowing down of the motion to occur at the same places in time, and therefore will account for the equation of center. Of course, in this simple case, the tropical and anomalistic motions of Mercury have been considered identical, which they are not exactly.

The same constructions are used for Mars, Jupiter and Saturn.

[4]Instead of 72, Oechslin gives the number of teeth of the upper wheel on tube 25 as 30/72. The teeth of this wheel are possibly uneven, but I am not sure that this is what Oechslin meant.

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11.4 The tellurium

The tellurium shows the motion of the Earth around the Sun. The axis of the Earth is tilted and always points to the same direction. The structure of the tellurium is similar to that of the tellurium in a globe clock now kept in Stuttgart (Oechslin 8.5) and constructed in 1770, except that the latter also has the Moon revolving around the Earth.

Like in the Stuttgart globe clock mentioned above, this tellurium has two inputs. The first input is the motion of arbor 29 which is that of a sidereal day. It is counterclockwise as seen from the front of the tellurium:

$$\mathrm{V}_{29}^{0} = -\mathrm{V}_{4}^{0} = \frac{25270}{25201} \tag{11.40}$$

This motion is used to produce the rotation of the Earth with respect to a fixed frame.

The second input is that of arbor 27 stemming from the orrery. This motion is that of the tropical year and it rotates clockwise as seen from the front of the tellurium.

$$\mathrm{V}_{27}^{0} = -\frac{473632}{172990675} \tag{11.41}$$

I am only considering the motion of arbor 27 from the tellurium side, hence the negative sign.

The motion of arbor 27 is used to produce the counterclockwise motion of tube 28:

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{27}^{0} \times \left(-\frac{80}{80}\right) = -\mathrm{V}_{27}^{0} \tag{11.42}$$

Hence, tube 28 makes one turn counterclockwise in a tropical year. This tube supports the Earth frame.

The meridian of the Earth, which supports its axis, is fixed to a wheel on tube 33. The motion of this wheel replicates the motion of the central arbor 31 which is fixed. Therefore, the meridian, and also the axis of the Earth, always keep the same orientation with respect to the fixed frame, no matter where the Earth is located.

Finally, the arbor 35 replicates the motion of tube 30:

$$\mathrm{V}_{35}^{0} = \mathrm{V}_{30}^{0} = \mathrm{V}_{29}^{0} \times \left(-\frac{66}{66}\right) = -\mathrm{V}_{29}^{0} = -\frac{25270}{25201} \tag{11.43}$$

And arbor 36, which is that of the Earth's axis, has the velocity:

$$\mathrm{V}_{36}^{0} = \mathrm{V}_{35}^{0} \times \left(-\frac{30}{30}\right) = -\mathrm{V}_{35}^{0} = \frac{25270}{25201} \tag{11.44}$$

Consequently, the Earth rotates counterclockwise with the velocity of the sidereal day, as it should.

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11.5 The celestial globe

The celestial globe on the top of the clock shows the geocentric motion of the sky, the planets, the Sun and the Moon. The globe is rotating inside a fixed frame representing the meridian. The vertical axis is the axis of the Earth, but the Sun, the Moon and the planets approximately move in the ecliptic, whose axis is tilted by 23.5° with respect to the equator. This axis rotates with the celestial globe.

A comparison with the gears of the globes of the *Weltmaschinen* of Stuttgart (Oechslin 8.1) (1769) and Gotha (Oechslin 8.3) (1780) shows that the structures of these three globes are very similar, and there are mainly small differences in teeth counts, but without altering the ratios.[5] In fact, the structure of the present globe is almost that of the Stuttgart machine, except for Mercury where the ratio is different. Moreover, in the Gotha machine, the motion of the lunar apsides and nodes is fraught with an error (either in the machine itself, or in Oechslin's account).

As mentioned earlier, the input to the globe is the motion of the vertical arbor 4 which makes one turn clockwise (seen from above) in a sidereal day:

$$\mathrm{V}_4^0 = -\frac{25270}{25201} \tag{11.45}$$

The central axis 47 of the globe is tied to three fixed wheels which will be used to obtain some of the motions displayed.

11.5.1 The motions of the Moon and the Sun

Within the celestial globe, all the motions are produced from the motion of tube 39. The velocity of this tube in the globe reference frame 47 is:

$$\mathrm{V}_{39}^{47} = \mathrm{V}_{37}^{47} \times \left(-\frac{71}{92}\right) \times \left(-\frac{7}{148}\right) = \mathrm{V}_{37}^{47} \times \frac{497}{13616} \tag{11.46}$$

where the tube/wheel 37 is a fixed part on the meridian frame of the globe. Now, since

$$\mathrm{V}_{37}^{47} = \mathrm{V}_0^{47} = -\mathrm{V}_{47}^0 = -\mathrm{V}_4^0 = \frac{25270}{25201} \tag{11.47}$$

it follows that

$$\mathrm{V}_{39}^{47} = \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \tag{11.48}$$

$$\mathrm{P}_{39}^{47} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{11.49}$$

[5] A photograph of the gears of the globe is shown in the 1989 exhibition catalogue [3, p. 528].

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The tube 39 actually corresponds to the mean motion of the Moon and the period of its rotation around the globe is one tropical month. The same value is given by Oechslin, also in sidereal days.

This motion is used to obtain the mean motion of the Sun on tube 41:

$$\mathrm{V}_{41}^{47} = \mathrm{V}_{39}^{47} \times \left(-\frac{112}{53}\right) \times \left(-\frac{4}{113}\right) = \mathrm{V}_{39}^{47} \times \frac{448}{5989} \tag{11.50}$$

$$= \frac{6279595}{171568408} \times \frac{448}{5989} = \frac{351657320}{128440399439} \tag{11.51}$$

$$\mathrm{P}_{41}^{47} = \frac{128440399439}{351657320} = 365.2430\ldots \text{ days} \tag{11.52}$$

The same value is given by Oechslin, also in sidereal days.

The actual motion of the Sun is on tube 68. This tube has the same motion as another tube S' (erroneously numbered 41 by Oechslin), being connected through axis 61 and the same gear ratios.

The motion of this tube S' is itself the same as that of an unnumbered mobile frame containing a number of gears and which I am calling S.[6] This frame has an oscillating motion around the position of frame 41 which is the mean Sun. This oscillation is produced by an excentric pin on a wheel on arbor 49 located on frame 41. The oscillation period is the time it takes for this wheel to rotate on frame 41:

$$\mathrm{V}_{49}^{41} = \mathrm{V}_{47}^{41} \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) = \mathrm{V}_{47}^{41} = -\mathrm{V}_{41}^{47} \tag{11.53}$$

Therefore

$$\mathrm{P}_{49}^{41} = -\mathrm{P}_{41}^{47} \tag{11.54}$$

(The negative sign is irrelevant here.)

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. In reality, the apsides are not fixed, but move at a rate of one turn in about 21000 years with respect to the zodiac. That would be the period that Hahn would have had to implement.

Now, the mean motion of the Sun is used to obtain the apsidal precession of the Moon on tube 43:

[6] Oechslin's plate is slightly incorrect as it mistakenly links a 116-teeth wheel on tube S' with the 113-teeth wheel on tube 41. See the Gotha plate which is better in that respect.

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$$\mathrm{V}_{43}^{47} = \mathrm{V}_{41}^{47} \times \left(-\frac{28}{78}\right) \times \left(-\frac{23}{73}\right) \tag{11.55}$$

$$= \mathrm{V}_{41}^{47} \times \frac{322}{2847} \tag{11.56}$$

$$= \frac{351657320}{128440399439} \times \frac{322}{2847} = \frac{4923202480}{15898687704471} \tag{11.57}$$

$$\mathrm{P}_{43}^{47} = \frac{15898687704471}{4923202480} = 3229.3385\ldots \text{ days} \tag{11.58}$$

The same value is given by Oechslin, also in sidereal days.

The actual value is about 3233 days (8.85 years). It should be noted that in the Gotha *Weltmaschine*, a ratio of $\left(-\frac{28}{81}\right)$ is used instead of $\left(-\frac{28}{78}\right)$, and this causes the motions of the lunar apsides and nodes to be wrong.

The motion of the apsides is then used to obtain the apparent motion of the Moon, namely that of the mean Moon plus the equation of center. The equation of center is computed based on the position of the perigee. Now, whereas the equation of center for the Sun rested on a fixed wheel on axis 47, because the perihelion/aphelion were considered fixed on the celestial sphere, the equation of center for the Moon rests on the wheel corresponding to the apsidal precession, because the perigee/apogee are fixed with respect to that wheel. The period of the oscillation caused by the equation of center of the Moon is however not the tropical month:

$$\mathrm{V}_{44}^{39} = \mathrm{V}_{43}^{39} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{43}^{39} = \mathrm{V}_{39}^{43} \tag{11.59}$$

$$= \mathrm{V}_{39}^{47} + \mathrm{V}_{47}^{43} = \frac{6279595}{171568408} - \frac{4923202480}{15898687704471} \tag{11.60}$$

$$= \frac{106165514457065}{2925358537622664} \tag{11.61}$$

$$\mathrm{P}_{44}^{39} = \frac{2925358537622664}{106165514457065} = 27.5546\ldots \text{ days} \tag{11.62}$$

This is the anomalistic month. This value is not given by Oechslin.

As far as the Moon is concerned, the mean motion of the Sun is also used to obtain the precession of the lunar nodes on tube 46:

$$\mathrm{V}_{46}^{47} = \mathrm{V}_{41}^{47} \times \left(-\frac{28}{78}\right) \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{39}\right) \tag{11.63}$$

$$= \mathrm{V}_{41}^{47} \times \left(-\frac{2975}{55341}\right) = \frac{351657320}{128440399439} \times \left(-\frac{2975}{55341}\right) \tag{11.64}$$

$$= -\frac{1046180527000}{7108020145353699} \tag{11.65}$$

This value is negative, because the nodes have a retrograde motion.

$$\mathrm{P}_{46}^{47} = -\frac{7108020145353699}{1046180527000} = -6794.2577\ldots \text{ days} \tag{11.66}$$

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The same value is given by Oechslin. This corresponds to about 18.6 tropical years.

11.5.2 The motions of the planets

The motion of the true Sun is used to obtain the apparent motions of Mercury and Venus.

11.5.2.1 Venus

I am first considering the motion of Venus, as it is the simplest. Hahn merely has Venus oscillating around the position of the Sun with the period of a wheel on arbor 59. The velocity of this wheel is

$$\mathrm{V}_{59}^{S} = \mathrm{V}_{47}^{S} \times \left(-\frac{31}{20}\right) \times \left(-\frac{26}{44}\right) \times \left(-\frac{28}{41}\right) = \mathrm{V}_{47}^{S} \times \left(-\frac{2821}{4510}\right) \tag{11.67}$$

Now, since

$$\mathrm{V}_{47}^{S} \approx \mathrm{V}_{47}^{41} \tag{11.68}$$

we have

$$\mathrm{V}_{59}^{S} \approx \mathrm{V}_{47}^{41} \times \left(-\frac{2821}{4510}\right) \approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{2821}{4510}\right) \tag{11.69}$$

$$\approx \frac{99202529972}{57926620146989} \tag{11.70}$$

and

$$\mathrm{P}_{59}^{S} \approx \frac{57926620146989}{99202529972} = 583.9228\ldots \text{ days} \tag{11.71}$$

which is the synodic period of Venus. This value is not given by Oechslin. The above value is an average, because it is based on the motion of the true Sun. The oscillation of Venus makes it possible to show the retrogradations of this planet.

From this period, we can obtain the tropical orbit period of Venus which is[7]

$$\text{tropical orbit period} = \frac{1}{\frac{1}{\text{tropical year}} + \frac{1}{\text{synodic period}}} \tag{11.72}$$

$$= \frac{1}{\frac{1}{\mathrm{P}_{41}^{47}} + \frac{1}{\mathrm{P}_{59}^{S}}} \tag{11.73}$$

$$\approx \frac{57926620146989}{257799981292} = 224.6959\ldots \text{ days} \tag{11.74}$$

The same value is given by Oechslin, also in sidereal days.

[7]The derivation of this formula can for instance be found in section 22.4.3.1, in the chapter on Klein's Tychonic clock.

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11.5.2.2 Mercury

The motion of Mercury is more complex because Hahn both takes into account the retrogradations of Mercury and the excentricity of its orbit (but not the precession of the apsides). If we do not take the excentricity into account, the position of Mercury is that of arbor 56 and it is also merely oscillating around the true Sun.

For Mercury, Hahn did not use the same ratios as in Stuttgart and Gotha, but the period obtained is very close.

The velocity of Mercury's axis 56 around its average position (tube 54) with respect the mean Sun is

$$\mathrm{V}_{54}^{S} = \mathrm{V}_{47}^{S} \times \left(-\frac{69}{28}\right) \times \left(-\frac{55}{43}\right) \times \left(-\frac{43}{43}\right) = \mathrm{V}_{47}^{S} \times \left(-\frac{3795}{1204}\right) \tag{11.75}$$

As before, since

$$\mathrm{V}_{47}^{S} \approx \mathrm{V}_{47}^{41} \tag{11.76}$$

we have

$$\mathrm{V}_{54}^{S} \approx \mathrm{V}_{47}^{41} \times \left(-\frac{3795}{1204}\right) \approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{3795}{1204}\right) \tag{11.77}$$

$$\approx \frac{188387850}{21829791209} \tag{11.78}$$

and

$$\mathrm{P}_{54}^{S} \approx \frac{21829791209}{188387850} = 115.8768\ldots \text{ days} \tag{11.79}$$

which is the synodic period of Mercury. Like for Venus, the above value is an average, because it is based on the motion of the true Sun. The oscillation of Mercury makes it possible to show the retrogradations of this planet.

Again, we can also deduce the tropical orbit period of Mercury which is

$$\frac{5522937175877}{62783390810} = 87.9681\ldots \text{ days} \tag{11.80}$$

The same value is given by Oechslin, also in sidereal days.

Hahn does however add a second oscillation on top of the first one, in that the pin driving Mercury is excentered from arbor 56. Mercury rotates around the axis 56.

Arbor 56 replicates tube 54 which replicates arbor 47. Consequently, the crank has a fixed orientation with respect to the globe.

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11.5.2.3 Mars

The motions of Mars, Jupiter and Saturn are based on the true motion of the Sun. The motion of the Sun is replicated in three 116-teeth wheels which have the same motion as they are connected with identical gears to arbor 61. The intermediate of these three 116-teeth is the one on tube 68 and is the one carrying the actual Sun, as was already mentioned.

The mean motion of Mars is obtained first on tube 63:

$$\mathrm{V}_{63}^{47} = \mathrm{V}_{S'}^{47} \times \left(-\frac{116}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{12}{32}\right) \times \left(-\frac{32}{119}\right) \tag{11.81}$$

$$\approx \mathrm{V}_{S'}^{47} \times \frac{696}{1309} = \frac{351657320}{128440399439} \times \frac{696}{1309} = \frac{1205682240}{828219127417} \tag{11.82}$$

$$\mathrm{P}_{63}^{47} \approx \frac{828219127417}{1205682240} = 686.9298\ldots \text{ days} \tag{11.83}$$

This is Mars' orbital period. The same value is given by Oechslin, also in sidereal days. Like for Mercury and Venus, the above value is an average, because it is based on the motion of the true Sun.

Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. This is done exactly in the same way as the equation of center for the Sun. There are two wheels of 48 teeth, one of them located on the frame for the mean motion of Mars, the other fixed inside the celestial globe (frame 47). We have

$$\mathrm{V}_{64}^{63} = \mathrm{V}_{47}^{63} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{47}^{63} = \mathrm{V}_{63}^{47} \tag{11.84}$$

Therefore

$$\mathrm{P}_{64}^{63} = \mathrm{P}_{63}^{47} \tag{11.85}$$

The oscillation period is exactly that of the tropical orbit period, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Mars is on a frame that Oechslin doesn't seem to number, but that we will name m.

For the geocentric view of the motion of Mars, Hahn proceeds as for Venus, where he corrected the position of Venus using the elongation with the true Sun S. In the case of Mars, the correction is done with respect to S', and we have

$$\mathrm{V}_{66}^{S'} = \mathrm{V}_{66}^{m} + \mathrm{V}_{m}^{S'} = \mathrm{V}_{S'}^{m} \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{60}\right) + \mathrm{V}_{m}^{S'} \tag{11.86}$$

$$= \mathrm{V}_{S'}^{m} + \mathrm{V}_{m}^{S'} = 0 \tag{11.87}$$

We could also have seen immediately that the motion of arbor 66 replicates that of S', and therefore that $\mathrm{V}_{66}^{S'} = 0$.

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The synodic period of Mars corresponds to the velocity

$$\mathrm{V}^m_{S'} \approx \mathrm{V}^m_{41} \approx \mathrm{V}^{47}_{41} - \mathrm{V}^{47}_{m} \approx \mathrm{V}^{47}_{41} - \mathrm{V}^{47}_{63} \tag{11.88}$$

$$\approx \frac{351657320}{128440399439} - \frac{1205682240}{828219127417} \tag{11.89}$$

$$\approx \frac{30795133880}{24018354695093} \tag{11.90}$$

and therefore

$$\mathrm{P}^m_{S'} \approx \frac{24018354695093}{30795133880} = 779.9399\ldots \text{ days} \tag{11.91}$$

This value is not given by Oechslin.

11.5.2.4 Jupiter

The motion of Jupiter is obtained in a similar way as that of Mars. The mean motion of Jupiter is obtained first on tube 72, from the mean motion of Mars:

$$\mathrm{V}^{47}_{72} = \mathrm{V}^{47}_{63} \times \left(-\frac{119}{31}\right) \times \left(-\frac{31}{31}\right) \times \left(-\frac{5}{34}\right) \times \left(-\frac{34}{121}\right) \tag{11.92}$$

$$= \mathrm{V}^{47}_{63} \times \frac{595}{3751} \approx \frac{1205682240}{828219127417} \times \frac{595}{3751} \tag{11.93}$$

$$\approx \frac{42198878400}{182744114525951} \tag{11.94}$$

$$\mathrm{P}^{47}_{72} \approx \frac{182744114525951}{42198878400} = 4330.5443\ldots \text{ days} \tag{11.95}$$

This is Jupiter's orbital period. The same value is given by Oechslin. Like for Mercury, Venus and Mars, the above value is an average, because it is based on the motion of the true Sun.

Like for Mars, Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. We have

$$\mathrm{V}^{72}_{73} = \mathrm{V}^{72}_{47} \times \left(-\frac{60}{60}\right) = -\mathrm{V}^{72}_{47} = \mathrm{V}^{47}_{72} \tag{11.96}$$

Therefore

$$\mathrm{P}^{72}_{73} = \mathrm{P}^{47}_{72} \tag{11.97}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Jupiter is on a frame that Oechslin doesn't seem to number, but that we will name j. We have

$$\mathrm{V}^S_{75} = \mathrm{V}^j_{75} + \mathrm{V}^S_j = \mathrm{V}^j_S \times \left(-\frac{60}{60}\right) \times \left(-\frac{60}{60}\right) + \mathrm{V}^S_j \tag{11.98}$$

$$= \mathrm{V}^j_S + \mathrm{V}^S_j = 0 \tag{11.99}$$

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In other words

$$\mathrm{V}_{75}^{47} = \mathrm{V}_{75}^{S} + \mathrm{V}_{S}^{47} = \mathrm{V}_{S}^{47} \tag{11.100}$$

We could likewise compute the synodic period of Jupiter, as we did for Mars.

11.5.2.5 Saturn

The motion of Saturn is also obtained similarly. The mean motion of Saturn is obtained first on tube 78, from the mean motion of Jupiter:

$$\mathrm{V}_{78}^{47} = \mathrm{V}_{72}^{47} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) \tag{11.101}$$

$$= \mathrm{V}_{72}^{47} \times \frac{363}{901} \approx \frac{42198878400}{182744114525951} \times \frac{363}{901} \tag{11.102}$$

$$\approx \frac{126596635200}{1360764026346131} \tag{11.103}$$

$$\mathrm{P}_{72}^{47} \approx \frac{1360764026346131}{126596635200} = 10748.8166\ldots \text{ days} \tag{11.104}$$

This is Saturn's orbital period. The same value is given by Oechslin. Like for Mercury, Venus, Mars and Jupiter, the above value is an average, because it is based on the motion of the true Sun.

Like for Mars and Jupiter, Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. We have

$$\mathrm{V}_{79}^{78} = \mathrm{V}_{47}^{78} \times \left(-\frac{60}{60}\right) = -\mathrm{V}_{47}^{78} = \mathrm{V}_{78}^{47} \tag{11.105}$$

Therefore

$$\mathrm{P}_{79}^{78} = \mathrm{P}_{78}^{47} \tag{11.106}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Saturn is on a frame that Oechslin doesn't seem to number, but that we will name s. We have

$$\mathrm{V}_{81}^{S} = \mathrm{V}_{81}^{s} + \mathrm{V}_{s}^{S} = \mathrm{V}_{S}^{s} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) + \mathrm{V}_{s}^{S} \tag{11.107}$$

$$= \mathrm{V}_{S}^{s} + \mathrm{V}_{s}^{S} = 0 \tag{11.108}$$

In other words

$$\mathrm{V}_{81}^{47} = \mathrm{V}_{81}^{S} + \mathrm{V}_{S}^{47} = \mathrm{V}_{S}^{47} \tag{11.109}$$

We could likewise compute the synodic period of Saturn, as we did for Mars.

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11.6 References

[1] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert*. Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[2] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

[3] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 2: Aufsätze*, volume 7 of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

[4] Ernst Zinner. *Deutsche und niederländische astronomische Instrumente des 11.-18. Jahrhunderts*. München: C. H. Beck, 1967. [2nd edition].

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Chapter 12

(Oechslin: 8.9)

Hahn's globe clock in Villingen (1770-1783)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

12.1 Introduction

The clock described here was constructed sometime in 1770-1783 by Philipp Matthäus Hahn (1739-1790)[1]. It was located in Villingen-Schwenningen[2] but is now located in Furtwangen.

This clock is made of three separate parts. The central part shows the hours and minutes on an ordinary dial, and the hours alone on a smaller 24-hour dial below the main dial.

The right part shows the month, the sign of the zodiac and the planetary regents (in the order Mars, Sun, Venus, Mercury, Moon, Saturn and Jupiter). This part is surmounted by a geographical globe. The left part shows the day of the week and the day of the month, and is surmounted by a celestial globe.

The way the two globes are driven is very similar to that used in the double-globe clock of Karlsruhe (Oechslin 8.8) constructed in 1785.

12.2 The geographical globe

The component of the geographical globe gets its input from the clock at the center through arbor 19 which makes one turn in 24 hours. Seen from the

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]See especially the 1989 exhibition catalogue [4, p. 414-415]. Besides in Oechslin's 1996 book, the clock was also illustrated by Abeler [1, pl. 15a]. In 1974, the clock was located in the Kienzle-Uhrenmuseum.

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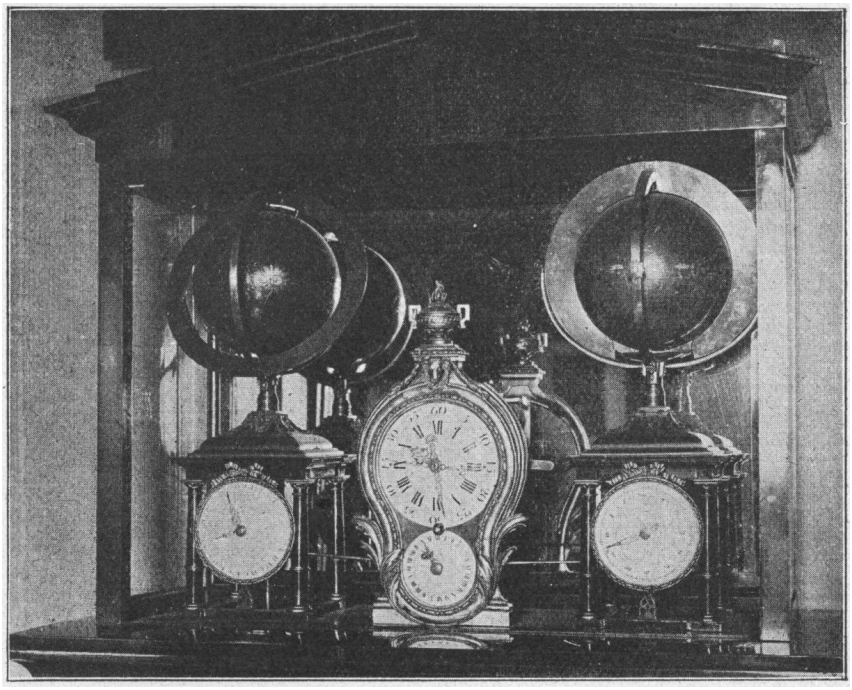


Figure 12.1: Hahn's clock in Villingen. (source: [2])

right, we have

$$\mathrm{V}_{19}^{0} = 1 \tag{12.1}$$

This motion is used to derive the motion of the central vertical arbor 20:

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{19}^{0} \times \frac{57}{57} = \mathrm{V}_{19}^{0} = 1 \tag{12.2}$$

This arbor makes one turn counterclockwise in one day. This corresponds to arbor 28 in the Karlsruhe clock.

Likewise, the motion of arbor 20 is used to produce the motion of the vertical tube 22:

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{20}^{0} \times \left(-\frac{7}{118}\right) \times \left(-\frac{3}{65}\right) = \mathrm{V}_{20}^{0} \times \frac{21}{7670} = \frac{21}{7670} \tag{12.3}$$

$$\mathrm{P}_{22}^{0} = \frac{7670}{21} = 365.2380\ldots \text{ days} \tag{12.4}$$

This is an approximation of the tropical year and the same value is also given by Oechslin. Tube 22 does therefore make one turn counterclockwise (as seen from above) in one year. This tube seems to move the day/night boundary.

The same ratio $\frac{7670}{21}$ was used in the astronomical clock in Basel (Oechslin 8.10) constructed in 1775 and in the double-globe clock of Karlsruhe (Oechslin 8.8).

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As for Karlsruhe, this, however, is not entirely correct. If the limit day/night moves in one year, then the Earth should actually make one turn in one sidereal day, and not 24 jours as it does now (or seems to be doing). This suggests a construction error, and that the sidereal day motion produced for the left globe was initially also meant to be used in the geographical globe.

On tube 22, a 76-teeth wheel meshes with another similar wheel at right angle and is used to display the month.

12.3 The celestial globe

The component of the celestial globe gets its input from the clock at the center through arbor 1 which makes one turn in $\frac{29}{57}$ hours. Seen from above, we have

$$\mathrm{V}_1^0 = \frac{57}{29} \tag{12.5}$$

This motion is used on the one hand to derive the day of the week and the day of the month, and on the other hand to derive the motion of the vertical arbor 10 which is the main input for the motion of the globe. We have

$$\mathrm{V}_{10}^0 = \mathrm{V}_1^0 \times \left(-\frac{35}{33}\right) \times \frac{38}{79} = \frac{57}{29} \times \left(-\frac{1330}{2607}\right) = -\frac{25270}{25201} \tag{12.6}$$

Arbor 10 thus makes one turn clockwise in one sidereal day. This is the usual value used by Hahn for the velocity of the sidereal day.

However, arbor 10 actually moves an entire frame containing gears which are themselves use to produce the motions of the Sun and the Moon. Arbor 10 moves the celestial sphere which rotates around the vertical axis representating the axis of the Earth. But the Sun and the Moon revolve (approximately) around the axis of the ecliptic, which is tilted by 23.5 degrees. The Sun is fixed on tube 18 and the Moon on tube 15. In order to compute the motions of these two tubes, we first compute the motions of tubes 14 (Moon) and 17 (Sun) with respect to the celestial globe (frame 10):

$$\mathrm{V}_{14}^{10} = \mathrm{V}_{11}^{10} \times \left(-\frac{53}{54}\right) \times \left(-\frac{54}{66}\right) \times \left(-\frac{3}{66}\right) = \mathrm{V}_{11}^{10} \times \left(-\frac{53}{1452}\right) \tag{12.7}$$

$$= -\mathrm{V}_{10}^{11} \times \left(-\frac{53}{1452}\right) = \mathrm{V}_{10}^0 \times \frac{53}{1452} \tag{12.8}$$

$$= -\frac{25270}{25201} \times \frac{53}{1452} = -\frac{669655}{18295926} \tag{12.9}$$

$$\mathrm{P}_{14}^{10} = -\frac{18295926}{669655} = -27.3214\ldots \text{ days} \tag{12.10}$$

$$\mathrm{V}_{17}^{10} = \mathrm{V}_{14}^{10} \times \left(-\frac{32}{53}\right) \times \left(-\frac{14}{113}\right) = \mathrm{V}_{14}^{10} \times \frac{448}{5989} \tag{12.11}$$

$$= -\frac{669655}{18295926} \times \frac{448}{5989} = -\frac{2830240}{1033719819} \tag{12.12}$$

$$\mathrm{P}_{17}^{10} = -\frac{1033719819}{2830240} = -365.2410\ldots \text{ days} \tag{12.13}$$

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So, tube 14 makes one turn clockwise (from above) in one tropical month, and tube 17 makes one turn clockwise in one tropical year. The same values are given by Oechslin, also in sidereal days.

The same value for the tropical month was used in the double-globe clock located in Zurich (Oechslin 8.7) constructed that same year in 1785. And the value of the tropical year was used in the globe clock in Darmstadt (Oechslin 8.6), also in 1785. Both were also used in the double-globe clock in Karlsruhe (Oechslin 8.8) in 1785.

Finally

$$\mathrm{V}_{15}^{10} = \mathrm{V}_{14}^{10} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{14}^{10} = \frac{669655}{18295926} \tag{12.14}$$

$$\mathrm{V}_{18}^{10} = \mathrm{V}_{17}^{10} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{17}^{10} = \frac{2830240}{1033719819} \tag{12.15}$$

The Moon and the Sun rotate counterclockwise around the celestial sphere in one tropical month and one tropical year.

12.4 References

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[2] Max Engelmann. *Leben und Wirken des württembergischen Pfarrers und Feintechnikers Philipp Matthäus Hahn.* Berlin: Richard Carl Schmidt & Co., 1923.

[3] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[4] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.

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Chapter 13

(Oechslin: 8.3)

Hahn's *Weltmaschine* in Gotha (1780)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

13.1 Introduction

This *Weltmaschine* (figure 13.1) was constructed in 1780 by Philipp Matthäus Hahn (1739-1790) and his brother David Georg (1747-1814)[1] and is located in the Friedenstein Palace in Gotha (Germany).[2]

The *Württembergische Landesbibliothek* in Stuttgart keeps a drawing (figure 13.2) which is related to the Gotha *Weltmaschine*.[3] This drawing may have been made by Johann Christoph Schuster (1759-1823), Hahn's collaborator and future son-in law. It is possible that this drawing was the basis of several machines, the present one, but also another one of which some elements were auctioned in 2023 (figures 13.3 and 13.4).[4]

The Gotha machine is made of a large base in three parts, with a dial in the front, a tellurium on the left side, and an orrery on the right side. Above the central part, there is a celestial globe with an internal work.

This is a rather complete and complex mechanism, and it is to be noted that the orrery was copied in the "astro-skeleton" clock designed by Mark Frank and completed in 2021.

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] For the history of this machine, see especially the 1989 exhibition catalogue [4, p. 393-395, pl .13]. The Gotha machine is mentioned by Zinner [5, p. 354]. Oechslin also described Hahn's *Weltmaschinen*, see especially [2, p. 55-57].

[3] The drawing is reproduced in colour in the 1989 exhibition catalogue [4, pl .5].

[4] [3]

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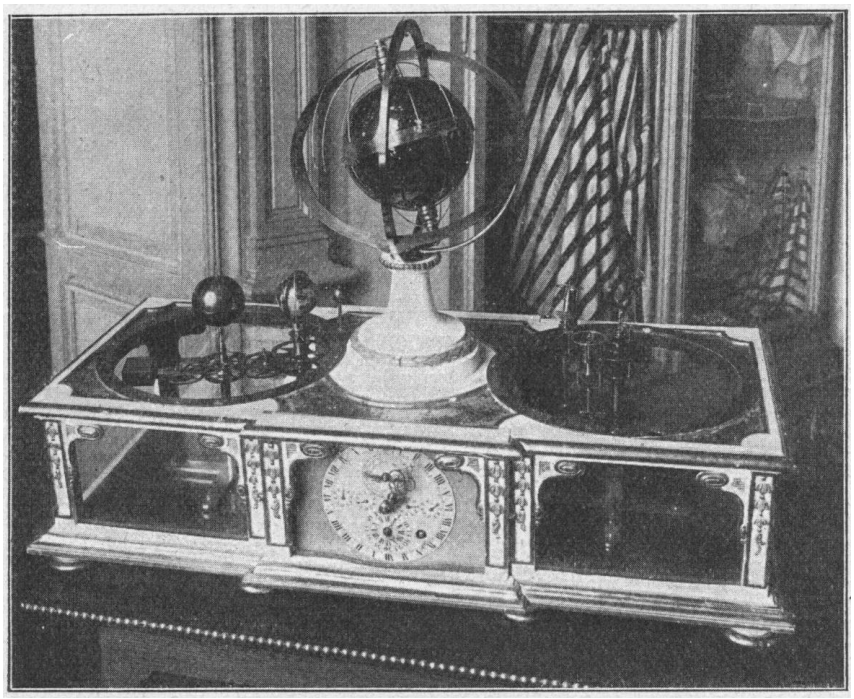


Figure 13.1: Hahn's clock in Gotha. The tellurium is on the left and the orrery on the right. (source: [1])

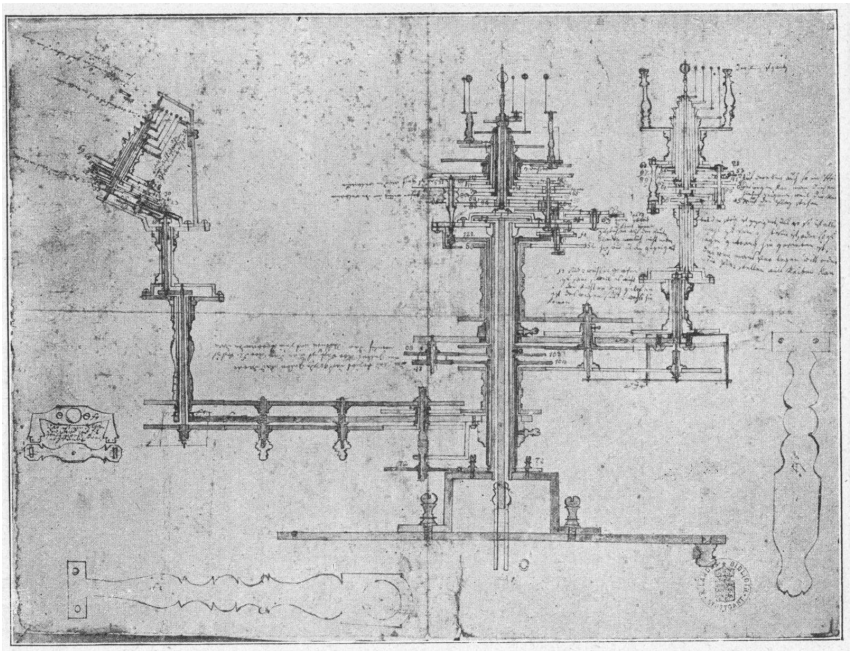


Figure 13.2: Cross-section of a Copernican orrery similar to the Gotha machine, possibly by Johann Christoph Schuster (1759-1823). (source: [1])

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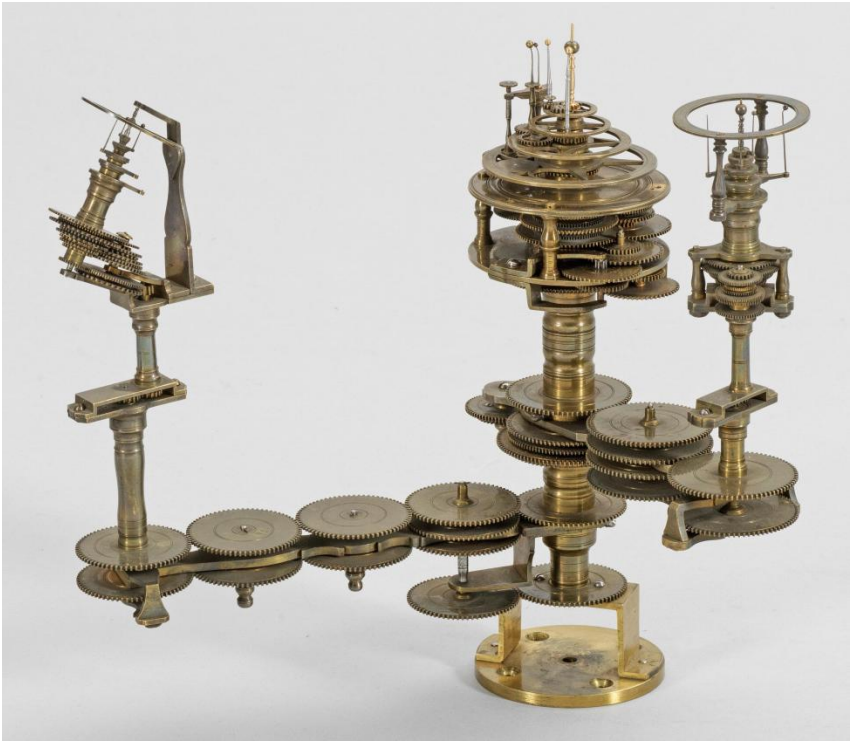


Figure 13.3: The orrery gears auctioned in 2023. (source: Kunstauktionshaus Schloss Ahlden, 6 May 2023, lot 684)

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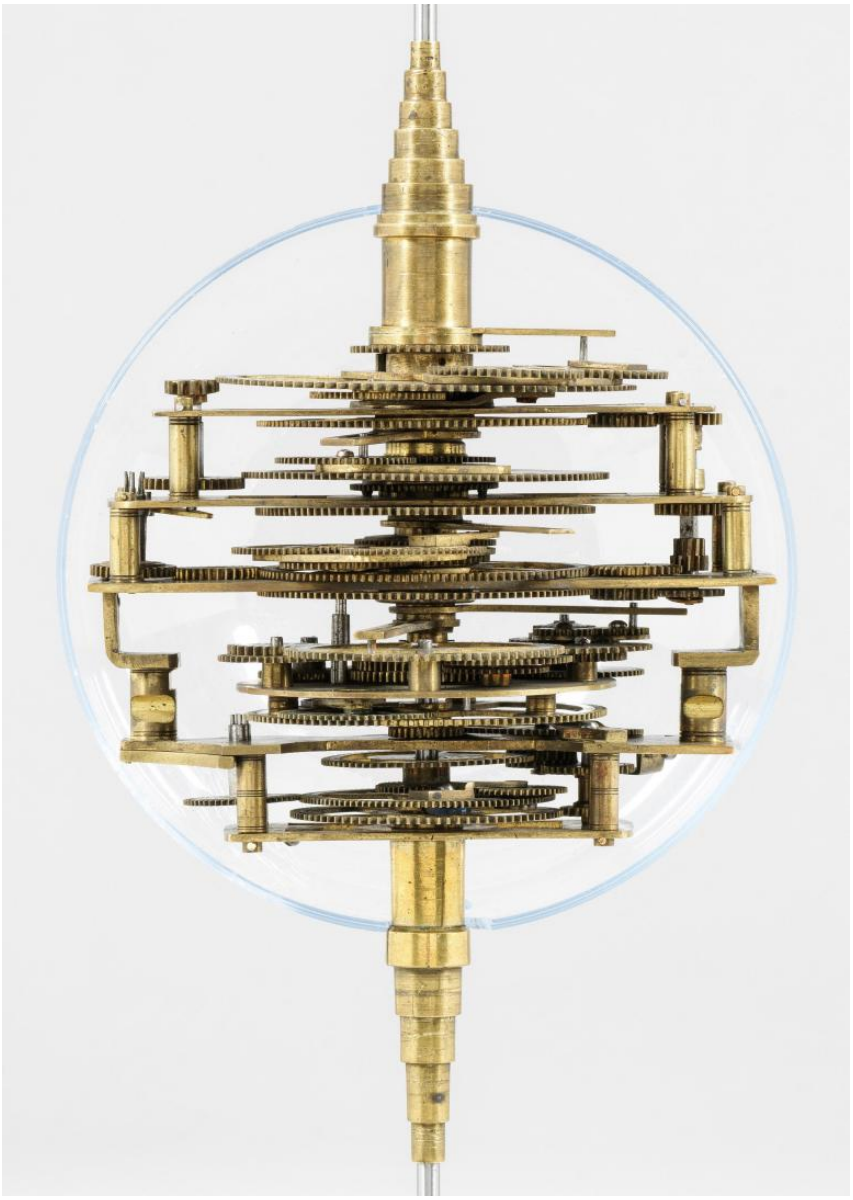


Figure 13.4: The globe gears auctioned in 2023. (source: Kunstauktionshaus Schloss Ahlden, 6 May 2023, lot 684)

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There may be two errors in this machine, namely in the gears for the satellites of Saturn, and in the celestial globe for the lunar apsides and nodes. These errors are detailed below.

Oechslin was the first to describe this machine in 1996, but he actually didn't have the opportunity to disassemble it. He based his analysis on archives provided to him by Helmut Sändig and R. Lang from Dresden [2, p. 5].

13.2 The going work

Oechslin does not show the details of the going work, but his drawing shows that one hand on arbor 1 makes a turn in 24 hours.

$$\mathrm{T}_1^0 = 1 \tag{13.1}$$

The rotation is clockwise seen from the front, but from the left on Oechslin's drawing. This motion is then transfered to the central vertical arbor 5:

$$\mathrm{V}_5^0 = \mathrm{V}_1^0 \times \left(-\frac{76}{79}\right) \times \left(-\frac{57}{28}\right) \times \left(-\frac{28}{58}\right) \times \left(-\frac{35}{33}\right) \tag{13.2}$$

$$= (-1) \times \frac{25270}{25201} = -\frac{25270}{25201} \tag{13.3}$$

$$\mathrm{P}_5^0 = -\frac{25201}{25270} = -0.9972\ldots \text{ days} = -86164.0838\ldots \text{ seconds} \tag{13.4}$$

$$= -23 \text{ h } 56 \text{ m } 4.083 \text{ s} \tag{13.5}$$

This is the velocity of arbor 5 seen from above. This arbor moves clockwise at the rate of the sidereal day. This ratio is the same as the one used on the globe clock located in Darmstadt. The motion of this arbor is transfered to the celestial globe above.

The motion of arbor 5 is also transfered to two other vertical arbors/tubes, namely tube 67 (tellurium) and arbor 7 (orrery). We have

$$\mathrm{V}_{67}^0 = -\mathrm{V}_5^0 \tag{13.6}$$

$$\mathrm{P}_{67}^0 = \frac{25201}{25270} \tag{13.7}$$

$$\mathrm{V}_7^0 = \mathrm{V}_{67}^0 \tag{13.8}$$

$$\mathrm{P}_7^0 = \mathrm{P}_{67}^0 \tag{13.9}$$

These two arbors/tubes rotate counterclockwise in one sidereal day.

We now basically have only vertical arbors to examine, except for the tilted axes of the Earth and of Saturn.

13.3 The tellurium

In the tellurium, the central arbor 70 is fixed and supports the Sun as well as two fixed wheels with 78 and 104 teeth. The rotating tube 67 is used to

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move the rotating frame 71 supporting the Earth-Moon system. This is done indirectly. In the rotating reference frame, we have

$$\mathrm{V}_{70}^{71} = \mathrm{V}_{67}^{71} \times \left(-\frac{119}{118}\right) \times \left(-\frac{4}{85}\right) \times \left(-\frac{6}{104}\right) = -\mathrm{V}_{67}^{71} \times \frac{21}{7670} = \mathrm{V}_{71}^{67} \times \frac{21}{7670} \tag{13.10}$$

This relation is in the form

$$\mathrm{V}_a^c = \mathrm{V}_c^b \times x \tag{13.11}$$

where the motions of a and b are known, the value of x is known, and the motion of c is sought.

From this equation, we easily derive

$$\mathrm{V}_b^c = \mathrm{V}_b^a \times \frac{1}{1+x} \tag{13.12}$$

and

$$\mathrm{V}_a^c = \mathrm{V}_a^b \times \frac{x}{1+x} \tag{13.13}$$

or

$$\mathrm{V}_c^a = \mathrm{V}_b^a \times \frac{x}{1+x} \tag{13.14}$$

In our case, taking $a = 70$, $b = 67$ and $c = 71$, $x = \frac{21}{7670}$, we have

$$\mathrm{V}_{71}^{70} = \mathrm{V}_{71}^{0} = \mathrm{V}_{67}^{70} \times \frac{21}{7691} = \frac{25270}{25201} \times \frac{21}{7691} = \frac{530670}{193820891} \tag{13.15}$$

$$\mathrm{P}_{71}^{70} = \frac{193820891}{530670} = 365.2380\ldots \text{ days} \tag{13.16}$$

This is an approximation of the tropical year. The same value is given by Oechslin.

Now, Hahn obtains the motion of the frame 73 supporting Earth's axis. We expect this axis to be non rotating (only translating) in the absolute frame, so that this axis does always keep the same orientation.

$$\mathrm{V}_{73}^{71} = \mathrm{V}_{70}^{71} \times \left(-\frac{78}{78}\right) \times \left(-\frac{78}{78}\right) = \mathrm{V}_{70}^{71} \tag{13.17}$$

$$\mathrm{V}_{73}^{0} = \mathrm{V}_{73}^{71} + \mathrm{V}_{71}^{0} = \mathrm{V}_{70}^{71} + \mathrm{V}_{71}^{0} = \mathrm{V}_{70}^{0} = 0 \tag{13.18}$$

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The motion of the Earth on its axis 77 in the absolute frame is obtained as follows:

$$\mathrm{V}_{77}^{0} = \mathrm{V}_{75}^{0} \times \left(-\frac{12}{8}\right) \times \left(-\frac{8}{12}\right) = \mathrm{V}_{75}^{0} \tag{13.19}$$

$$\mathrm{V}_{75}^{0} = \mathrm{V}_{75}^{71} + \mathrm{V}_{71}^{0} = \mathrm{V}_{67}^{71} \times \left(-\frac{78}{78}\right) \times \left(-\frac{78}{78}\right) + \mathrm{V}_{71}^{0} \tag{13.20}$$

$$= \mathrm{V}_{67}^{71} + \mathrm{V}_{71}^{0} = \mathrm{V}_{67}^{0} = \frac{25270}{25201} \tag{13.21}$$

$$\mathrm{P}_{75}^{0} = \frac{25201}{25270} \tag{13.22}$$

Hence, the Earth rotates around its axis in one sidereal day.

There is also a frame showing the limit of the lit and unlit parts of the Earth and this frame is fixed on the rotating frame 71. It is always in the same orientation towards the Sun.

We finally come to the motion of the Moon. The direction of the Moon is determined by the frame tied to tube 80. This frame has the following velocity relative to the rotating Earth frame:

$$\mathrm{V}_{80}^{71} = \mathrm{V}_{67}^{71} \times \left(-\frac{78}{78}\right) \times \left(-\frac{12}{87}\right) \times \left(-\frac{41}{41}\right) \times \left(-\frac{41}{167}\right) \tag{13.23}$$

$$= \mathrm{V}_{67}^{71} \times \frac{164}{4843} \tag{13.24}$$

$$= \left(\mathrm{V}_{67}^{0} + \mathrm{V}_{0}^{71}\right) \times \frac{164}{4843} \tag{13.25}$$

$$= \left(\frac{25270}{25201} - \frac{530670}{193820891}\right) \times \frac{164}{4843} = \frac{31786627600}{938674575113} \tag{13.26}$$

$$\mathrm{P}_{80}^{71} = \frac{938674575113}{31786627600} = 29.5304\ldots \text{ days} \tag{13.27}$$

This is an approximation of the synodic month. The same value is given by Oechslin.

The tropical month can be computed as follows:

$$\mathrm{V}_{80}^{0} = \mathrm{V}_{80}^{71} + \mathrm{V}_{71}^{0} = \frac{31786627600}{938674575113} + \frac{530670}{193820891} = \frac{34356662410}{938674575113} \tag{13.28}$$

$$\mathrm{P}_{80}^{0} = \frac{938674575113}{34356662410} = 27.3214\ldots \text{ days} \tag{13.29}$$

The same value is given by Oechslin.

Finally, the Moon is a globe half lit, half unlit, and it should always have the same orientation with respect to frame 71:

$$\mathrm{V}_{83}^{71} = \mathrm{V}_{83}^{80} + \mathrm{V}_{80}^{71} = \mathrm{V}_{81}^{80} \times \left(-\frac{53}{53}\right) \times \left(-\frac{53}{53}\right) + \mathrm{V}_{80}^{71} = \mathrm{V}_{81}^{80} + \mathrm{V}_{80}^{71} = \mathrm{V}_{81}^{71} = 0 \tag{13.30}$$

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13.4 The orrery

The orrery gets its input from arbor 7 mentioned above. This arbor makes one turn counterclockwise as seen from above in one sidereal day:

$$\mathrm{V}_7^0 = \frac{25270}{25201} \tag{13.31}$$

Part of the structure of the orrery is fixed to the reference frame. This includes a tube 20 which carries three wheels and the entire frame housing the gears for Mercury, Venus, the Earth and the Moon, and Mars. The Jupiter and Saturn systems rotate around this central system.

13.4.1 The mean motions

We will first concentrate on the internal system. Like the general orrery, its input is only the motion of arbor 7. The mean motion of Mercury is that of tube 11 and is obtained as follows:

$$\mathrm{V}_{11}^0 = \mathrm{V}_7^0 \times \left(-\frac{52}{52}\right) \times \left(-\frac{6}{53}\right) \times \left(-\frac{32}{77}\right) \times \left(-\frac{20}{83}\right) = \mathrm{V}_7^0 \times \frac{3840}{338723} \tag{13.32}$$

$$= \frac{25270}{25201} \times \frac{3840}{338723} = \frac{13862400}{1219451189} \tag{13.33}$$

$$\mathrm{P}_{11}^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{13.34}$$

This is the tropical revolution of Mercury. The same value is given by Oechslin.

The motion of Mercury is used to obtain the motion of Venus:

$$\mathrm{V}_{13}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) \tag{13.35}$$

$$= \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{13.36}$$

$$\mathrm{P}_{13}^0 = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{13.37}$$

The same value is given by Oechslin.

The motion of Mercury is also used to obtain the motion of the Moon:

$$\mathrm{V}_{19}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{84}{85}\right) \times \left(-\frac{101}{31}\right) \tag{13.38}$$

$$= \frac{13862400}{1219451189} \times \frac{8484}{2635} = \frac{23521720320}{642650776603} \tag{13.39}$$

$$\mathrm{P}_{19}^0 = \frac{642650776603}{23521720320} = 27.3215\ldots \text{ days} \tag{13.40}$$

This is the tropical month (also given by Oechslin) and the value is slightly different from that used in the tellurium.

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The motion of Venus is then used to obtain the motion of the Earth:

$$\mathrm{V}_{15}^0 = \mathrm{V}_{13}^0 \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = \mathrm{V}_{13}^0 \times \frac{3239}{5265} \tag{13.41}$$

$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{13.42}$$

$$\mathrm{P}_{15}^0 = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{13.43}$$

The same value is given by Oechslin.

And the motion of the Earth is used to obtain the motion of Mars:

$$\mathrm{V}_{17}^0 = \mathrm{V}_{15}^0 \times \left(-\frac{116}{33}\right) \times \left(-\frac{18}{119}\right) = \mathrm{V}_{15}^0 \times \frac{696}{1309} \tag{13.44}$$

$$= \frac{473632}{172990675} \times \frac{696}{1309} = \frac{329647872}{226444793575} \tag{13.45}$$

$$\mathrm{P}_{17}^0 = \frac{226444793575}{329647872} = 686.9293\ldots \text{ days} \tag{13.46}$$

The same value is given by Oechslin.

The motion of Mars is used to obtain the motion of Jupiter:

$$\mathrm{V}_{28}^0 = \mathrm{V}_{17}^0 \times \left(-\frac{119}{31}\right) \times \left(-\frac{5}{121}\right) = \mathrm{V}_{17}^0 \times \frac{595}{3751} \tag{13.47}$$

$$= \frac{329647872}{226444793575} \times \frac{595}{3751} = \frac{329647872}{1427553648235} \tag{13.48}$$

$$\mathrm{P}_{28}^0 = \frac{1427553648235}{329647872} = 4330.5410\ldots \text{ days} \tag{13.49}$$

The same value is given by Oechslin.

Finally, the motion of Saturn is obtained from the motion of Jupiter. First, in the rotating frame of Jupiter:

$$\mathrm{V}_{45}^{28} = \mathrm{V}_{20}^{28} \times \left(-\frac{103}{68}\right) \times \left(-\frac{41}{104}\right) = \mathrm{V}_{20}^{28} \times \frac{4223}{7072} \tag{13.50}$$

$$= \mathrm{V}_{0}^{28} \times \frac{4223}{7072} = -\mathrm{V}_{28}^{0} \times \frac{4223}{7072} \tag{13.51}$$

$$= \left(-\frac{329647872}{1427553648235}\right) \times \frac{4223}{7072} = -\frac{43503217608}{315489356259935} \tag{13.52}$$

and then in the absolute frame:

$$\mathrm{V}_{45}^0 = \mathrm{V}_{45}^{28} + \mathrm{V}_{28}^0 = \left(-\frac{43503217608}{315489356259935}\right) + \frac{329647872}{1427553648235} \tag{13.53}$$

$$= \frac{2668087464}{28680850569085} \tag{13.54}$$

$$\mathrm{P}_{45}^0 = \frac{28680850569085}{2668087464} = 10749.5915\ldots \text{ days} \tag{13.55}$$

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The same value is given by Oechslin.

The supports of Jupiter and Saturn actually carry two gear trains each, in order to take care of the eccentric motions of Jupiter and Saturn, and of the motions of the satellites. In the similar orrery of Furtwangen (Oechslin 8.12) constructed in 1774, these two gear trains were replaced by horizontal arbors.

13.4.2 Taking the excentricities into account

The Earth and Venus are shown revolving on circles centered on the Sun, but for the other planets, Hahn took the excentricities of their orbits into account.

Mercury is placed on an excentric axis on wheel 26, whose velocity is

$$\mathrm{V}_{26}^{11} = \mathrm{V}_{20}^{11} \times \left(-\frac{12}{12}\right) \times \left(-\frac{12}{12}\right) = \mathrm{V}_{20}^{11} \tag{13.56}$$

and

$$\mathrm{V}_{26}^{0} = \mathrm{V}_{26}^{11} + \mathrm{V}_{11}^{0} = \mathrm{V}_{20}^{11} + \mathrm{V}_{11}^{0} = \mathrm{V}_{0}^{11} + \mathrm{V}_{11}^{0} = 0 \tag{13.57}$$

In other words, the orientation of the wheel supporting Mercury's axis is fixed in the absolute reference frame and therefore when Mercury is closest to the Sun at some point, it will be farthest half a period later. Mercury does that way go through an excentric cercle, approximating the elliptical motion of the planet.

The situation is exactly the same for Mars:

$$\mathrm{V}_{22}^{17} = \mathrm{V}_{20}^{17} \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{40}\right) = \mathrm{V}_{20}^{17} \tag{13.58}$$

and

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{22}^{17} + \mathrm{V}_{17}^{0} = \mathrm{V}_{20}^{17} + \mathrm{V}_{17}^{0} = \mathrm{V}_{20}^{0} = 0 \tag{13.59}$$

It is also similar for Jupiter:

$$\mathrm{V}_{32}^{28} = \mathrm{V}_{20}^{28} \times \left(-\frac{92}{82}\right) \times \left(-\frac{82}{92}\right) = \mathrm{V}_{20}^{28} \tag{13.60}$$

and

$$\mathrm{V}_{32}^{0} = \mathrm{V}_{32}^{28} + \mathrm{V}_{28}^{0} = \mathrm{V}_{20}^{28} + \mathrm{V}_{28}^{0} = \mathrm{V}_{20}^{0} = 0 \tag{13.61}$$

And also for Saturn:

$$\mathrm{V}_{49}^{45} = \mathrm{V}_{20}^{45} \times \left(-\frac{72}{72}\right) \times \left(-\frac{66}{66}\right) \times \left(-\frac{66}{66}\right) \times \left(-\frac{66}{66}\right) = \mathrm{V}_{20}^{45} \tag{13.62}$$

and

$$\mathrm{V}_{49}^{0} = \mathrm{V}_{49}^{45} + \mathrm{V}_{45}^{0} = \mathrm{V}_{20}^{45} + \mathrm{V}_{45}^{0} = \mathrm{V}_{20}^{0} = 0 \tag{13.63}$$

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13.4.3 The satellites

13.4.3.1 The Moon

The Moon is located on a 30-teeth wheel on tube 24. This tube rotates around an axis located on the Earth frame 15. Its velocity on the Earth frame 15 is

$$\mathrm{V}_{24}^{15} = \mathrm{V}_{19}^{15} \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) = \mathrm{V}_{19}^{15} \tag{13.64}$$

$$= \mathrm{V}_{19}^{0} + \mathrm{V}_{0}^{15} = \frac{23521720320}{642650776603} - \frac{473632}{172990675} = \frac{7072716193376}{208861502395975} \tag{13.65}$$

$$\mathrm{P}_{24}^{15} = \frac{208861502395975}{7072716193376} = 29.5305\ldots \text{ days} \tag{13.66}$$

This is an approximation of the synodic month, different from the one used in the tellurium. This value is not given by Oechslin.

The tropical month is then given by:

$$\mathrm{V}_{24}^{0} = \mathrm{V}_{24}^{15} + \mathrm{V}_{15}^{0} = \frac{7072716193376}{208861502395975} + \frac{473632}{172990675} = \frac{23521720320}{642650776603} \tag{13.67}$$

$$\mathrm{P}_{24}^{0} = \frac{642650776603}{23521720320} = 27.3215\ldots \text{ days} \tag{13.68}$$

which is the same value given by Oechslin. We also have

$$\mathrm{V}_{24}^{0} = \mathrm{V}_{19}^{0} \tag{13.69}$$

which was seen earlier. The tropical month in the orrery, as already mentioned, differs from the tropical month used in the tellurium.

13.4.3.2 Jupiter's satellites

The motion of the satellites of Jupiter is obtained by a number of gears located on the rotating frame 32, which is itself pivoting on frame 28. However, since we saw that $\mathrm{V}_{32}^{0} = 0$, this frame is actually only in translation with respect to the absolute frame 0.

The input to the system of Jupiter's satellites is arbor 35. The motion of arbor 35 eventually goes back to tube 50, which replicates the motion of tube 30. Tube 30, in turn, replicates the motion of arbor 7, but in the opposite direction:

$$\mathrm{V}_{50}^{0} = \mathrm{V}_{30}^{0} = -\mathrm{V}_{7}^{0} = -\frac{25270}{25201} \tag{13.70}$$

We can now compute the velocity of arbor 35. We have

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$$\mathrm{V}_{35}^{0} = \mathrm{V}_{35}^{32} + \mathrm{V}_{32}^{0} = \mathrm{V}_{34}^{32} \times \left(-\frac{12}{12}\right) + \mathrm{V}_{32}^{0} \tag{13.71}$$

$$= -\mathrm{V}_{34}^{32} + \mathrm{V}_{32}^{0} = \mathrm{V}_{32}^{34} = \mathrm{V}_{32}^{28} - \mathrm{V}_{34}^{28} \tag{13.72}$$

$$= \mathrm{V}_{20}^{28} \times \left(-\frac{92}{82}\right) \times \left(-\frac{82}{92}\right) - \mathrm{V}_{50}^{28} \times \left(-\frac{82}{82}\right) \times \left(-\frac{82}{82}\right) \tag{13.73}$$

$$= \mathrm{V}_{20}^{28} - \mathrm{V}_{50}^{28} = \mathrm{V}_{20}^{50} = -\mathrm{V}_{50}^{0} = \mathrm{V}_{7}^{0} = \frac{25270}{25201} \tag{13.74}$$

The motions of the four satellites are then the following:

$$\mathrm{V}_{37}^{0} = \mathrm{V}_{35}^{0} \times \left(-\frac{35}{57}\right) \times \left(-\frac{56}{61}\right) = \mathrm{V}_{35}^{0} \times \frac{1960}{3477} \tag{13.75}$$

$$= \frac{25270}{25201} \times \frac{1960}{3477} = \frac{2606800}{4611783} \tag{13.76}$$

$$\mathrm{P}_{37}^{0} = \frac{4611783}{2606800} = 1.76913\ldots \text{ days} = 1 \text{ d. } 18.45\ldots \text{ h. (Io)} \tag{13.77}$$

$$\mathrm{V}_{39}^{0} = \mathrm{V}_{37}^{0} \times \left(-\frac{67}{68}\right) \times \left(-\frac{45}{89}\right) = \mathrm{V}_{37}^{0} \times \frac{3015}{6052} \tag{13.78}$$

$$= \frac{2606800}{4611783} \times \frac{3015}{6052} = \frac{654958500}{2325875893} \tag{13.79}$$

$$\mathrm{P}_{39}^{0} = \frac{2325875893}{654958500} = 3.55118\ldots \text{ days (Europe)} \tag{13.80}$$

$$\mathrm{V}_{41}^{0} = \mathrm{V}_{39}^{0} \times \left(-\frac{94}{41}\right) \times \left(-\frac{21}{97}\right) = \mathrm{V}_{39}^{0} \times \frac{1974}{3977} \tag{13.81}$$

$$= \frac{654958500}{2325875893} \times \frac{1974}{3977} \tag{13.82}$$

$$= \frac{1292888079000}{9250008426461} \tag{13.83}$$

$$\mathrm{P}_{41}^{0} = \frac{9250008426461}{1292888079000} = 7.15453\ldots \text{ days (Ganymede)} \tag{13.84}$$

$$\mathrm{V}_{43}^{0} = \mathrm{V}_{41}^{0} \times \left(-\frac{66}{53}\right) \times \left(-\frac{21}{61}\right) = \mathrm{V}_{41}^{0} \times \frac{1386}{3233} \tag{13.85}$$

$$= \frac{1292888079000}{9250008426461} \times \frac{1386}{3233} \tag{13.86}$$

$$= \frac{162903897954000}{2718661567522583} \tag{13.87}$$

$$\mathrm{P}_{43}^{0} = \frac{2718661567522583}{162903897954000} = 16.68874\ldots \text{ days (Callisto)} \tag{13.88}$$

When Oechslin computed the orbital period of Io, he mistakenly used the gear count 36 instead of 35, even though his formula uses 35, and this resulted in the incorrect period 1.719993 days. This error then propagated to the other satellites, as each motion is computed from the previous one. The same mistake was made in the analysis of the Furtwangen planetarium, but not in the Nuremberg machine.

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13.4.3.3 Saturn's satellites

Likewise, the motion of the satellites of Saturn is obtained by a number of gears located on the rotating frame 49. However, since we saw that $V^0_{49} = 0$, this frame is actually only in translation with respect to the absolute frame 0. For this reason, the orientation of Saturn's axis as well as the orbits of the five satellites are fixed in the absolute frame.

The input to the system of Saturn's satellites is arbor 55. We can compute its velocity:

$$V^0_{55} = V^{49}_{55} + V^0_{49} = V^{49}_{55} = V^{49}_{54} \times \left(-\frac{16}{16}\right) \tag{13.89}$$

$$= -V^{49}_{54} = V^{54}_{49} = V^{45}_{49} - V^{45}_{54} \tag{13.90}$$

$$= V^{45}_{20} \times \left(-\frac{72}{72}\right) \times \left(-\frac{66}{66}\right) \times \left(-\frac{66}{66}\right) \times \left(-\frac{66}{66}\right) \tag{13.91}$$

$$- V^{45}_{50} \times \left(-\frac{72}{72}\right) \times \left(-\frac{66}{66}\right) \times \left(-\frac{66}{66}\right) \times \left(-\frac{66}{66}\right) \tag{13.92}$$

$$= V^{45}_{20} - V^{45}_{50} = V^{50}_{20} = -V^0_{50} = V^0_7 = \frac{25270}{25201} \tag{13.93}$$

Before computing the motions of the five satellites, we must observe that there are two possible problems. First, Oechslin's plan gives a ratio of 86/33 for Tethys, and if we use this value, we do in fact obtain the incorrect periods of 1.4924, 2.1652, 3.5704, 12.5857, 62.2576 days, which are all wrong by the same factor of about 1.27, because the motion of each satellite is obtained from the previous one.

However, in his computations, Oechslin uses the ratio 68/33. Moreover, this satellite system uses the same ratios as the Furtwangen orrery constructed in 1774 (Oechslin 8.12). Both systems seem to use the ratio 86/33 and Gotha's system was obviously copied from the Furtwangen one, without fixing the problem with the periods.

It seems that Oechslin found that the wheel with 86 teeth should have had 68 teeth and this does indeed yield better values of the periods.[5]

There is however another possibility. In the Furtwangen orrery, the ratio used in the motion of Tethys is $\frac{78}{20}$ and this was replaced by $\frac{39}{10}$ in Gotha. It is possible that the cause of the errors is a typo in the Furtwangen orrery calculations that had the teeth count 98 become 78, hence $\frac{98}{20}$ become $\frac{78}{20}$, or, when we simplify, $\frac{49}{10}$ become $\frac{39}{10}$. Another solution to the present problem would then be to replace the 39-teeth wheel by a 49-teeth wheel.

So, the periods given by Oechslin are those obtained when Tethys uses the ratio 68/33. The motions of the five satellites are then the following, where I have added "(a)" to distinguish this choice from the second one "(b)" below.

[5] See [2, p. 204] on this error.

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For Tethys:

$$\mathrm{V}^0_{57}(a) = \mathrm{V}^0_{55} \times \left(-\frac{10}{39}\right) \times \left(-\frac{68}{33}\right) = \mathrm{V}^0_{55} \times \frac{680}{1287} \tag{13.94}$$

$$= \frac{25270}{25201} \times \frac{680}{1287} = \frac{17183600}{32433687} \tag{13.95}$$

$$\mathrm{P}^0_{57}(a) = \frac{32433687}{17183600} = 1.8874\ldots \text{ days} \tag{13.96}$$

For Dione:

$$\mathrm{V}^0_{59}(a) = \mathrm{V}^0_{57}(a) \times \left(-\frac{22}{46}\right) \times \left(-\frac{49}{34}\right) = \mathrm{V}^0_{57}(a) \times \frac{539}{782} \tag{13.97}$$

$$= \frac{17183600}{32433687} \times \frac{539}{782} = \frac{24764600}{67815891} \tag{13.98}$$

$$\mathrm{P}^0_{59}(a) = \frac{67815891}{24764600} = 2.7384\ldots \text{ days} \tag{13.99}$$

For Rhea:

$$\mathrm{V}^0_{61}(a) = \mathrm{V}^0_{59}(a) \times \left(-\frac{27}{32}\right) \times \left(-\frac{23}{32}\right) = \mathrm{V}^0_{59}(a) \times \frac{621}{1024} \tag{13.100}$$

$$= \frac{24764600}{67815891} \times \frac{621}{1024} = \frac{9286725}{41934464} \tag{13.101}$$

$$\mathrm{P}^0_{61}(a) = \frac{41934464}{9286725} = 4.5155\ldots \text{ days} \tag{13.102}$$

For Titan:

$$\mathrm{V}^0_{63}(a) = \mathrm{V}^0_{61}(a) \times \left(-\frac{40}{36}\right) \times \left(-\frac{12}{47}\right) = \mathrm{V}^0_{61}(a) \times \frac{40}{141} \tag{13.103}$$

$$= \frac{9286725}{41934464} \times \frac{40}{141} = \frac{15477875}{246364976} \tag{13.104}$$

$$\mathrm{P}^0_{63}(a) = \frac{246364976}{15477875} = 15.9172\ldots \text{ days} \tag{13.105}$$

For Iapetus:

$$\mathrm{V}^0_{65}(a) = \mathrm{V}^0_{63}(a) \times \left(-\frac{68}{39}\right) \times \left(-\frac{8}{69}\right) = \mathrm{V}^0_{63}(a) \times \frac{544}{2691} \tag{13.106}$$

$$= \frac{15477875}{246364976} \times \frac{544}{2691} = \frac{526247750}{41435509401} \tag{13.107}$$

$$\mathrm{P}^0_{65}(a) = \frac{41435509401}{526247750} = 78.7376\ldots \text{ days} \tag{13.108}$$

If instead we replace the ratio 10/39 by 10/49, we obtain the following periods, which are marked "(b)" in order to distinguish them from the previous ones:

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For Tethys:

$$\mathrm{V}^0_{57}(b) = \mathrm{V}^0_{55} \times \left(-\frac{10}{49}\right) \times \left(-\frac{86}{33}\right) = \mathrm{V}^0_{55} \times \frac{860}{1617} \tag{13.109}$$

$$= \frac{25270}{25201} \times \frac{860}{1617} = \frac{3104600}{5821431} \tag{13.110}$$

$$\mathrm{P}^0_{57}(b) = \frac{5821431}{3104600} = 1.8750\ldots \text{ days} \tag{13.111}$$

For Dione:

$$\mathrm{V}^0_{59}(b) = \mathrm{V}^0_{57}(b) \times \left(-\frac{22}{46}\right) \times \left(-\frac{49}{34}\right) = \mathrm{V}^0_{57}(b) \times \frac{539}{782} \tag{13.112}$$

$$= \frac{3104600}{5821431} \times \frac{539}{782} = \frac{10866100}{29560773} \tag{13.113}$$

$$\mathrm{P}^0_{59}(b) = \frac{29560773}{10866100} = 2.7204\ldots \text{ days} \tag{13.114}$$

For Rhea:

$$\mathrm{V}^0_{61}(b) = \mathrm{V}^0_{59}(b) \times \left(-\frac{27}{32}\right) \times \left(-\frac{23}{32}\right) = \mathrm{V}^0_{59}(b) \times \frac{621}{1024} \tag{13.115}$$

$$= \frac{10866100}{29560773} \times \frac{621}{1024} = \frac{24448725}{109674752} \tag{13.116}$$

$$\mathrm{P}^0_{61}(b) = \frac{109674752}{24448725} = 4.4859\ldots \text{ days} \tag{13.117}$$

For Titan:

$$\mathrm{V}^0_{63}(b) = \mathrm{V}^0_{61}(b) \times \left(-\frac{40}{36}\right) \times \left(-\frac{12}{47}\right) = \mathrm{V}^0_{61}(b) \times \frac{40}{141} \tag{13.118}$$

$$= \frac{24448725}{109674752} \times \frac{40}{141} = \frac{40747875}{644339168} \tag{13.119}$$

$$\mathrm{P}^0_{63}(b) = \frac{644339168}{40747875} = 15.8128\ldots \text{ days} \tag{13.120}$$

For Iapetus:

$$\mathrm{V}^0_{65}(b) = \mathrm{V}^0_{63}(b) \times \left(-\frac{68}{39}\right) \times \left(-\frac{8}{69}\right) = \mathrm{V}^0_{63}(b) \times \frac{544}{2691} \tag{13.121}$$

$$= \frac{40747875}{644339168} \times \frac{544}{2691} = \frac{13582625}{1062448959} \tag{13.122}$$

$$\mathrm{P}^0_{65}(b) = \frac{1062448959}{13582625} = 78.2211\ldots \text{ days} \tag{13.123}$$

Since this orrery was taken over in Mark Frank's "astro-skeleton," the question arises whether the error on Tethys made its way to Frank's machine or not.

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13.5 The celestial globe

The last part of Hahn's *Weltmaschine* is the celestial globe. This globe rotates and shows the geocentric motion of the planets, the Moon and the motion of the lunar nodes. The motions of the planets are shown with their retrogradations and the motions of the Sun and the Moon are shown with their equation of center.

A comparison with the gears of this globe and the globes of the *Weltmaschine* of Stuttgart (Oechslin 8.1) (1769) and the Aschaffenburg globe clock (Oechslin 8.4) (1776/1777), shows that the structures of these three globes are very similar, and there are mainly small differences in teeth counts, but without altering the ratios. There may however be an error in the motion of the lunar apsides and nodes in the present globe, as I will argue below. The globe of the Nuremberg *Weltmaschine* (Oechslin 8.2) follows the same principles, but uses different ratios.

As described above, the celestial globe of the Gotha machine is driven by the vertical arbor 5 which is making one turn clockwise (viewed from above) in one sidereal day:

$$\mathrm{V}_5^0 = -\frac{25270}{25201} \tag{13.124}$$

This motion rotates the entire globe and gives it a motion through the vertical axis with is the axis of the celestial pole (Earth's axis). This apparent motion goes from East to West, hence must be clockwise as seen from above.

However, the axis of all the planets, as well as those of the Sun and the Moon (when neglecting the inclination of its orbit to the ecliptic), is that going through the poles of the ecliptic and it is inclined at an angle of 23.5° to the axis of the celestial poles.

The central axis 94 of the globe is tied to three fixed wheels which will be used to obtain some of the motions displayed.

13.5.1 The motions of the Moon and the Sun

Within the celestial globe, all the motions are produced from the motion of tube 86. The velocity of this tube in the globe reference frame 94 is:

$$\mathrm{V}_{86}^{94} = \mathrm{V}_{84}^{94} \times \left(-\frac{71}{92}\right) \times \left(-\frac{7}{148}\right) = \mathrm{V}_{84}^{94} \times \frac{497}{13616} \tag{13.125}$$

where the arbor/wheel 84 is a fixed wheel on the meridian frame of the globe. Now, since

$$\mathrm{V}_{84}^{94} = \mathrm{V}_0^{94} = -\mathrm{V}_{94}^0 = -\mathrm{V}_5^0 = \frac{25270}{25201} \tag{13.126}$$

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it follows that

$$\mathrm{V}_{86}^{94} = \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \tag{13.127}$$

$$\mathrm{P}_{86}^{94} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{13.128}$$

This value is given by Oechslin, and also in sidereal days.

The tube 86 actually corresponds to the mean motion of the Moon and the period of its rotation around the globe is one tropical month. This value, incidentally, is different from the two other values already seen for the tropical month, in the tellurium and in the orrery.

This motion is used to obtain the mean motion of the Sun on tube 88:

$$\mathrm{V}_{88}^{94} = \mathrm{V}_{86}^{94} \times \left(-\frac{112}{53}\right) \times \left(-\frac{4}{113}\right) = \mathrm{V}_{86}^{94} \times \frac{448}{5989} \tag{13.129}$$

$$= \frac{6279595}{171568408} \times \frac{448}{5989} = \frac{351657320}{128440399439} \tag{13.130}$$

$$\mathrm{P}_{88}^{94} = \frac{128440399439}{351657320} = 365.2430\ldots \text{ days} \tag{13.131}$$

This value is given by Oechslin, and also in sidereal days.

The actual motion of the Sun is on tube 114. This tube has the same motion as another tube S' (erroneously numbered 88 by Oechslin), being connected through axis 107 and the same gear ratios.

The motion of this tube S' is itself the same as that of an unnumbered mobile frame containing a number of gears and which I am calling S. This frame has an oscillating motion around the position of frame 88 which is the mean Sun. This oscillation is produced by an excentric pin on a wheel on arbor 95 located on frame 88. The oscillation period is the time it takes for this wheel to rotate on frame 88:

$$\mathrm{V}_{95}^{88} = \mathrm{V}_{94}^{88} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{94}^{88} = \mathrm{V}_{88}^{94} \tag{13.132}$$

Therefore

$$\mathrm{P}_{95}^{88} = \mathrm{P}_{88}^{94} \tag{13.133}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. In reality, the apsides are not fixed, but move at a rate of one turn in about 21000 years with respect to the zodiac. That would be the period that Hahn would have had to implement.

Now, the mean motion of the Sun is used to obtain the apsidal precession of the Moon on tube 90:

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$$\mathrm{V}_{90}^{94} = \mathrm{V}_{88}^{94} \times \left(-\frac{28}{81}\right) \times \left(-\frac{23}{73}\right) \tag{13.134}$$

$$= \mathrm{V}_{88}^{94} \times \frac{644}{5913} \tag{13.135}$$

$$= \frac{351657320}{128440399439} \times \frac{644}{5913} = \frac{9846404960}{33020351386209} \tag{13.136}$$

$$\mathrm{P}_{90}^{94} = \frac{33020351386209}{9846404960} = 3353.5439\ldots \text{ days} \tag{13.137}$$

The actual value is about 3233 days (8.85 years).

This might seem like a small discrepancy, but it is in fact likely an error. In his computations, Oechslin replaces the value 81 by 78 and obtains the much better approximation:

$$\mathrm{V(h)}_{90}^{94} = \mathrm{V}_{88}^{94} \times \left(-\frac{28}{78}\right) \times \left(-\frac{23}{73}\right) \tag{13.138}$$

$$= \mathrm{V}_{88}^{94} \times \frac{322}{2847} \tag{13.139}$$

$$= \frac{351657320}{128440399439} \times \frac{322}{2847} = \frac{4923202480}{15898687704471} \tag{13.140}$$

$$\mathrm{P(h)}_{90}^{94} = \frac{15898687704471}{4923202480} = 3229.3385\ldots \text{ days} \tag{13.141}$$

where V(h) and P(h) are hypothetical velocities and periods.

It seems therefore that the actual train contains a 81-teeth wheel, but that a better value would have been 78. In fact, in the Stuttgart *Weltmaschine* and the Aschaffenburg globe clock, the first ratio is also $\left(-\frac{28}{78}\right)$ and not $\left(-\frac{28}{81}\right)$.

This is then used to obtain the apparent motion of the Moon, namely that of the mean Moon plus the equation of center. The equation of center is computed based on the position of the perigee. Now, whereas the equation of center for the Sun rested on a fixed wheel on axis 94, because the perihelion/aphelion were considered fixed on the celestial sphere, the equation of center for the Moon rests on the wheel corresponding to the apsidal precession, because the perigee/apogee are fixed with respect to that wheel. The period of the oscillation caused by the equation of center of the Moon is however not the tropical month:

$$\mathrm{V}_{91}^{86} = \mathrm{V}_{90}^{86} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{90}^{86} = \mathrm{V}_{86}^{90} \tag{13.142}$$

$$= \mathrm{V}_{86}^{94} + \mathrm{V}_{94}^{90} = \frac{6279595}{171568408} - \frac{9846404960}{33020351386209} \tag{13.143}$$

$$= \frac{220567289199775}{6075744655062456} \tag{13.144}$$

$$\mathrm{P}_{91}^{86} = \frac{6075744655062456}{220567289199775} = 27.54599\ldots \text{ days} \tag{13.145}$$

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This is the anomalistic month. This value seems reasonable, but with the corrected value of V_{94}^{90}, we obtain:

$$\mathrm{V(h)}_{91}^{86} = \mathrm{V}_{90}^{86} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{90}^{86} = \mathrm{V}_{86}^{90} \tag{13.146}$$

$$= \mathrm{V}_{86}^{94} + \mathrm{V(h)}_{94}^{90} = \frac{6279595}{171568408} - \frac{4923202480}{15898687704471} \tag{13.147}$$

$$= \frac{106165514457065}{2925358537622664} \tag{13.148}$$

$$\mathrm{P(h)}_{91}^{86} = \frac{2925358537622664}{106165514457065} = 27.5546\ldots \text{ days} \tag{13.149}$$

As far as the Moon is concerned, the mean motion of the Sun is also used to obtain the precession of the lunar nodes on tube 93:

$$\mathrm{V}_{93}^{94} = \mathrm{V}_{88}^{94} \times \left(-\frac{28}{81}\right) \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{86}\right) \tag{13.150}$$

$$= \mathrm{V}_{88}^{94} \times \left(-\frac{5950}{114939}\right) = \frac{351657320}{128440399439} \times \left(-\frac{5950}{114939}\right) \tag{13.151}$$

$$= -\frac{2092361054000}{14762811071119221} \tag{13.152}$$

This value is negative, because the nodes have a retrograde motion.

$$\mathrm{P}_{93}^{94} = -\frac{14762811071119221}{2092361054000} = -7055.5753\ldots \text{ days} \tag{13.153}$$

This corresponds to about $\frac{114939}{5950} = 19.3\ldots$ tropical years, whereas the correct value is about 18.6 years.

Again, if we replace 81 by 78, we obtain:

$$\mathrm{V(h)}_{93}^{94} = \mathrm{V}_{88}^{94} \times \left(-\frac{28}{78}\right) \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{86}\right) \tag{13.154}$$

$$= \mathrm{V}_{88}^{94} \times \left(-\frac{2975}{55341}\right) = \frac{351657320}{128440399439} \times \left(-\frac{2975}{55341}\right) \tag{13.155}$$

$$= -\frac{1046180527000}{7108020145353699} \tag{13.156}$$

$$\mathrm{P(h)}_{93}^{94} = -\frac{7108020145353699}{1046180527000} = -6794.2577\ldots \text{ days} \tag{13.157}$$

The same value is given by Oechslin. We now almost have the correct value of 6798 days for the retrogradation of the lunar nodes.

13.5.2 The motions of the planets

The motion of the true Sun is used to obtain the apparent motions of Mercury and Venus.

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13.5.2.1 Venus

I am first considering the motion of Venus, as it is the simplest. Hahn merely has Venus oscillating around the position of the Sun with the period of a wheel on arbor 105. The velocity of this wheel is

$$\mathrm{V}^S_{105} = \mathrm{V}^S_{94} \times \left(-\frac{62}{40}\right) \times \left(-\frac{52}{44}\right) \times \left(-\frac{28}{82}\right) = \mathrm{V}^S_{94} \times \left(-\frac{2821}{4510}\right) \tag{13.158}$$

Now, since

$$\mathrm{V}^S_{94} \approx \mathrm{V}^{88}_{94} \tag{13.159}$$

we have

$$\mathrm{V}^S_{105} \approx \mathrm{V}^{88}_{94} \times \left(-\frac{2821}{4510}\right) \approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{2821}{4510}\right) \tag{13.160}$$

$$\approx \frac{99202529972}{57926620146989} \tag{13.161}$$

and

$$\mathrm{P}^S_{105} \approx \frac{57926620146989}{99202529972} = 583.92281\ldots \text{ days} \tag{13.162}$$

which is the synodic period of Venus. This value is not given by Oechslin. The above value is an average, because it is based on the motion of the true Sun. The oscillation of Venus makes it possible to show the retrogradations of this planet.

From this period, we can obtain the tropical orbit period of Venus which is[6]

$$\text{tropical orbit period} = \frac{1}{\frac{1}{\text{tropical year}} + \frac{1}{\text{synodic period}}} \tag{13.163}$$

$$= \frac{1}{\frac{1}{\mathrm{P}^{94}_{88}} + \frac{1}{\mathrm{P}^S_{105}}} \tag{13.164}$$

$$\approx \frac{57926620146989}{257799981292} = 224.6959\ldots \text{ days} \tag{13.165}$$

This value is also given by Oechslin.

13.5.2.2 Mercury

The motion of Mercury is more complex because Hahn both takes into account the retrogradations of Mercury and the excentricity of its orbit (but not the

[6]The derivation of this formula can for instance be found in section 22.4.3.1, in the chapter on Klein's Tychonic clock.

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precession of the apsides). If we do not take the excentricity into account, the position of Mercury is that of arbor 102 and it is also merely oscillating around the true Sun.

The velocity of Mercury's axis 102 around its average position (arbor 98) with respect the mean Sun is

$$\mathrm{V}_{98}^{S} = \mathrm{V}_{94}^{S} \times \left(-\frac{58}{39}\right) \times \left(-\frac{58}{34}\right) \times \left(-\frac{41}{33}\right) = \mathrm{V}_{94}^{S} \times \left(-\frac{68962}{21879}\right) \tag{13.166}$$

As before, since

$$\mathrm{V}_{94}^{S} \approx \mathrm{V}_{94}^{88} \tag{13.167}$$

we have

$$\mathrm{V}_{98}^{S} \approx \mathrm{V}_{94}^{88} \times \left(-\frac{68962}{21879}\right) \approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{68962}{21879}\right) \tag{13.168}$$

$$\approx \frac{836241106960}{96901637907789} \tag{13.169}$$

and

$$\mathrm{P}_{98}^{S} \approx \frac{96901637907789}{836241106960} = 115.8776\ldots \text{ days} \tag{13.170}$$

which is the synodic period of Mercury. This value is not given by Oechslin. Like for Venus, the above value is an average, because it is based on the motion of the true Sun. The oscillation of Mercury makes it possible to show the retrogradations of this planet.

Again, we can also deduce the tropical orbit period of Mercury which is

$$\frac{2810147499325881}{31944902606120} = 87.9685\ldots \text{ days} \tag{13.171}$$

This value is also given by Oechslin.

Hahn does however add a second oscillation on top of the first one, in that the pin driving Mercury is excentered from arbor 102. Mercury rotates around the axis 102.

Arbor 102 replicates arbor 100 which replicates arbor 94. Consequently, the crank has a fixed orientation with respect to the globe.

13.5.2.3 Mars

The motions of Mars, Jupiter and Saturn are based on the true motion of the Sun. The motion of the Sun is replicated in three 116-teeth wheels which have the same motion as they are connected with identical gears to arbor 107. The intermediate of these three 116-teeth is the one on tube 114 and is the one carrying the actual Sun, as was already mentioned.

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The mean motion of Mars is obtained first on tube 109:

$$\mathrm{V}_{109}^{94} = \mathrm{V}_{S'}^{94} \times \left(-\frac{116}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{12}{32}\right) \times \left(-\frac{32}{119}\right) \tag{13.172}$$

$$\approx \mathrm{V}_{S'}^{94} \times \frac{696}{1309} = \frac{351657320}{128440399439} \times \frac{696}{1309} = \frac{1205682240}{828219127417} \tag{13.173}$$

$$\mathrm{P}_{109}^{94} \approx \frac{828219127417}{1205682240} = 686.9298\ldots \text{ days} \tag{13.174}$$

This is Mars' orbital period and it is also given by Oechslin. Like for Mercury and Venus, the above value is an average, because it is based on the motion of the true Sun.

Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. This is done exactly in the same way as the equation of center for the Sun. There are two wheels of 52 teeth, one of them located on the frame for the mean motion of Mars, the other fixed inside the celestial globe (frame 94). We have

$$\mathrm{V}_{110}^{109} = \mathrm{V}_{94}^{109} \times \left(-\frac{52}{52}\right) = -\mathrm{V}_{94}^{109} = \mathrm{V}_{109}^{94} \tag{13.175}$$

Therefore

$$\mathrm{P}_{110}^{109} = \mathrm{P}_{109}^{94} \tag{13.176}$$

The oscillation period is exactly that of the tropical orbit period, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Mars is on a frame that Oechslin doesn't seem to number, but that we will name m.

For the geocentric view of the motion of Mars, Hahn proceeds as for Venus, where he corrected the position of Venus using the elongation with the true Sun S. In the case of Mars, the correction is done with respect to S', and we have

$$\mathrm{V}_{112}^{S'} = \mathrm{V}_{112}^{m} + \mathrm{V}_{m}^{S'} = \mathrm{V}_{S'}^{m} \times \left(-\frac{44}{44}\right) \times \left(-\frac{44}{44}\right) + \mathrm{V}_{m}^{S'} \tag{13.177}$$

$$= \mathrm{V}_{S'}^{m} + \mathrm{V}_{m}^{S'} = 0 \tag{13.178}$$

We could also have seen immediately that the motion of arbor 112 replicates that of S', and therefore that $\mathrm{V}_{112}^{S'} = 0$.

The synodic period of Mars corresponds to the velocity

$$\mathrm{V}_{S'}^{m} \approx \mathrm{V}_{88}^{m} \approx \mathrm{V}_{88}^{94} - \mathrm{V}_{m}^{94} \approx \mathrm{V}_{88}^{94} - \mathrm{V}_{109}^{94} \tag{13.179}$$

$$\approx \frac{351657320}{128440399439} - \frac{1205682240}{828219127417} \tag{13.180}$$

$$\approx \frac{30795133880}{24018354695093} \tag{13.181}$$

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and therefore

$$\mathrm{P}^m_{S'} \approx \frac{24018354695093}{30795133880} = 779.9399\ldots \text{ days} \tag{13.182}$$

This value is not given by Oechslin.

13.5.2.4 Jupiter

The motion of Jupiter is obtained in a similar way as that of Mars. The mean motion of Jupiter is obtained first on tube 118, from the mean motion of Mars:

$$\mathrm{V}^{94}_{118} = \mathrm{V}^{94}_{109} \times \left(-\frac{119}{31}\right) \times \left(-\frac{31}{31}\right) \times \left(-\frac{5}{34}\right) \times \left(-\frac{34}{121}\right) \tag{13.183}$$

$$= \mathrm{V}^{94}_{109} \times \frac{595}{3751} \approx \frac{1205682240}{828219127417} \times \frac{595}{3751} \tag{13.184}$$

$$\approx \frac{42198878400}{182744114525951} \tag{13.185}$$

$$\mathrm{P}^{94}_{118} \approx \frac{182744114525951}{42198878400} = 4330.5443\ldots \text{ days} \tag{13.186}$$

This is Jupiter's orbital period. This value is also given by Oechslin. Like for Mercury, Venus and Mars, the above value is an average, because it is based on the motion of the true Sun.

Like for Mars, Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. We have

$$\mathrm{V}^{118}_{119} = \mathrm{V}^{118}_{94} \times \left(-\frac{48}{48}\right) = -\mathrm{V}^{118}_{94} = \mathrm{V}^{94}_{118} \tag{13.187}$$

Therefore

$$\mathrm{P}^{118}_{119} = \mathrm{P}^{94}_{118} \tag{13.188}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Jupiter is on a frame that Oechslin doesn't seem to number, but that we will name j. We have

$$\mathrm{V}^S_{121} = \mathrm{V}^j_{121} + \mathrm{V}^S_j = \mathrm{V}^j_S \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) + \mathrm{V}^S_j \tag{13.189}$$

$$= \mathrm{V}^j_S + \mathrm{V}^S_j = 0 \tag{13.190}$$

In other words

$$\mathrm{V}^{94}_{121} = \mathrm{V}^S_{121} + \mathrm{V}^{94}_S = \mathrm{V}^{94}_S \tag{13.191}$$

We could likewise compute the synodic period of Jupiter, as we did for Mars.

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13.5.2.5 Saturn

The motion of Saturn is also obtained similarly. The mean motion of Saturn is obtained first on tube 124, from the mean motion of Jupiter:

$$\mathrm{V}_{124}^{94} = \mathrm{V}_{118}^{94} \times \left(-\frac{121}{34}\right) \times \left(-\frac{12}{106}\right) \tag{13.192}$$

$$= \mathrm{V}_{118}^{94} \times \frac{363}{901} \approx \frac{42198878400}{182744114525951} \times \frac{363}{901} \tag{13.193}$$

$$\approx \frac{126596635200}{1360764026346131} \tag{13.194}$$

$$\mathrm{P}_{118}^{94} \approx \frac{1360764026346131}{126596635200} = 10748.8166\ldots \text{ days} \tag{13.195}$$

This is Saturn's orbital period. This value is also given by Oechslin. Like for Mercury, Venus, Mars and Jupiter, the above value is an average, because it is based on the motion of the true Sun.

Like for Mars and Jupiter, Hahn then takes the excentricity of the orbit into account, in that he adds the equation of center. We have

$$\mathrm{V}_{125}^{124} = \mathrm{V}_{94}^{124} \times \left(-\frac{50}{50}\right) = -\mathrm{V}_{94}^{124} = \mathrm{V}_{124}^{94} \tag{13.196}$$

Therefore

$$\mathrm{P}_{125}^{124} = \mathrm{P}_{124}^{94} \tag{13.197}$$

The oscillation period is exactly that of the tropical year, so that the perihelion (where the Sun is fastest) and aphelion are at fixed locations on the celestial sphere. This oscillation approximates the equation of center. The resulting true motion of Saturn is on a frame that Oechslin doesn't seem to number, but that we will name s. We have

$$\mathrm{V}_{127}^{S} = \mathrm{V}_{127}^{s} + \mathrm{V}_{s}^{S} = \mathrm{V}_{S}^{s} \times \left(-\frac{54}{54}\right) \times \left(-\frac{54}{54}\right) + \mathrm{V}_{s}^{S} \tag{13.198}$$

$$= \mathrm{V}_{S}^{s} + \mathrm{V}_{s}^{S} = 0 \tag{13.199}$$

In other words

$$\mathrm{V}_{127}^{94} = \mathrm{V}_{127}^{S} + \mathrm{V}_{S}^{94} = \mathrm{V}_{S}^{94} \tag{13.200}$$

We could likewise compute the synodic period of Saturn, as we did for Mars.

13.6 References

[1] Max Engelmann. *Leben und Wirken des württembergischen Pfarrers und Feintechnikers Philipp Matthäus Hahn.* Berlin: Richard Carl Schmidt & Co., 1923.

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[2] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert*. Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[3] Günther Oestmann. A Note on an unknown planetarium (*Weltmaschine*) by Philipp Matthäus Hahn. *Bulletin of the Scientific Instrument Society*, 160:42–43, March 2024.

[4] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

[5] Ernst Zinner. *Deutsche und niederländische astronomische Instrumente des 11.-18. Jahrhunderts*. München: C. H. Beck, 1967. [2nd edition].

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Chapter 14

(Oechslin: 8.11)

Hahn's tellurium in Winterthur (1780)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

14.1 Introduction



Figure 14.1: General view of Hahn's tellurium in Winterthur. (photography by the author)

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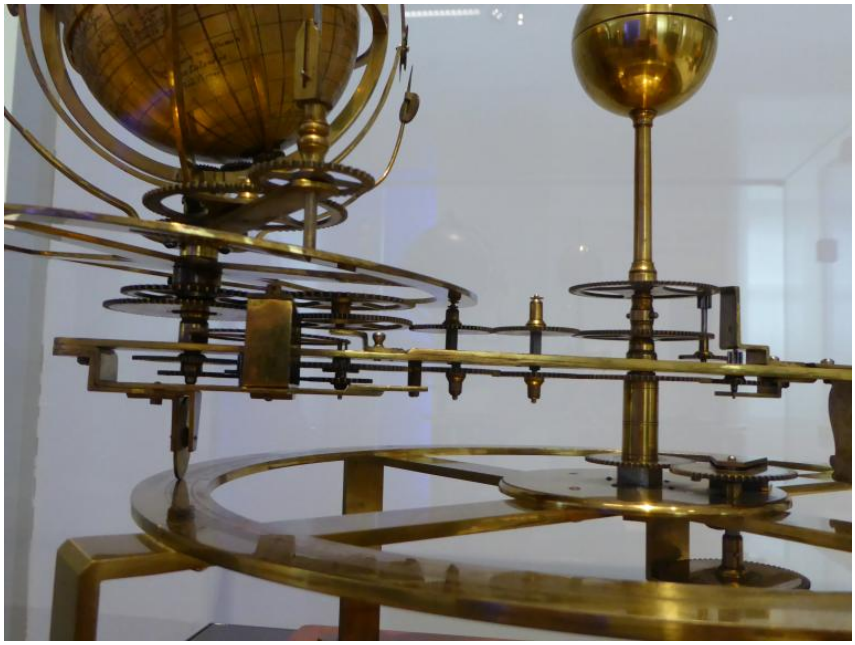


Figure 14.2: Detail of Hahn's tellurium in Winterthur. (photography by the author)

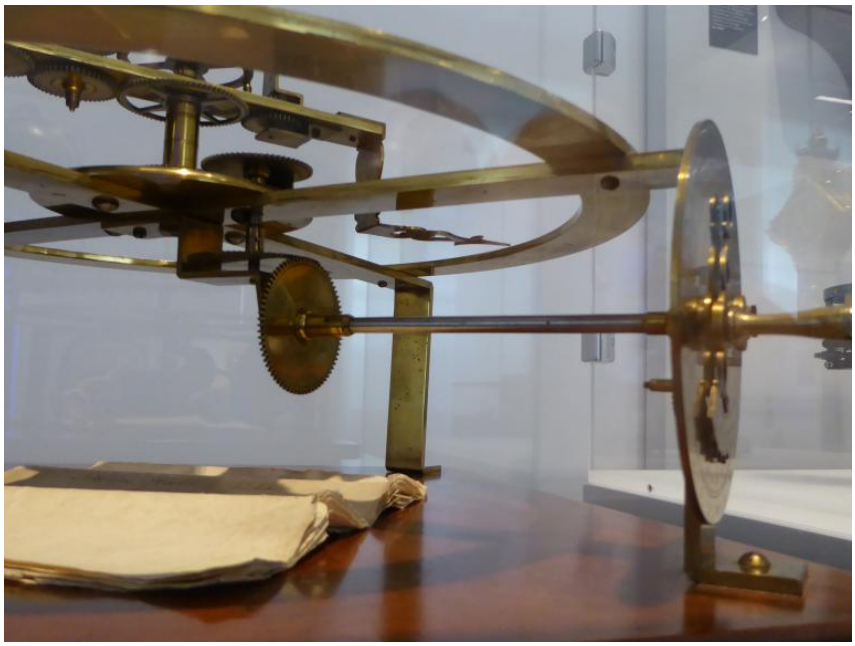


Figure 14.3: Detail of Hahn's tellurium in Winterthur. (photography by the author)

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The tellurium described here was constructed in 1780 by Philipp Matthäus Hahn (1739-1790)[1] and is located in the Winterthur *Uhrenmuseum* (Switzerland).[2] It shows the motion of the Earth around the Sun and of the Moon around the Earth. It does not have a clockwork and is operated by a crank.

It was described by Oechslin who dated it to 1780 and attributed it to Hahn. However, the museum itself dates it to about 1725. The front dial has the inscription "Hahn Echterdingum," so that it was perhaps too quickly attributed to Hahn.

As I will demonstrate below, this tellurium embodies many problems and its periods have been very poorly calculated. I find it therefore hard to believe that this is the work of Philipp Matthäus Hahn, or of Hahn alone.

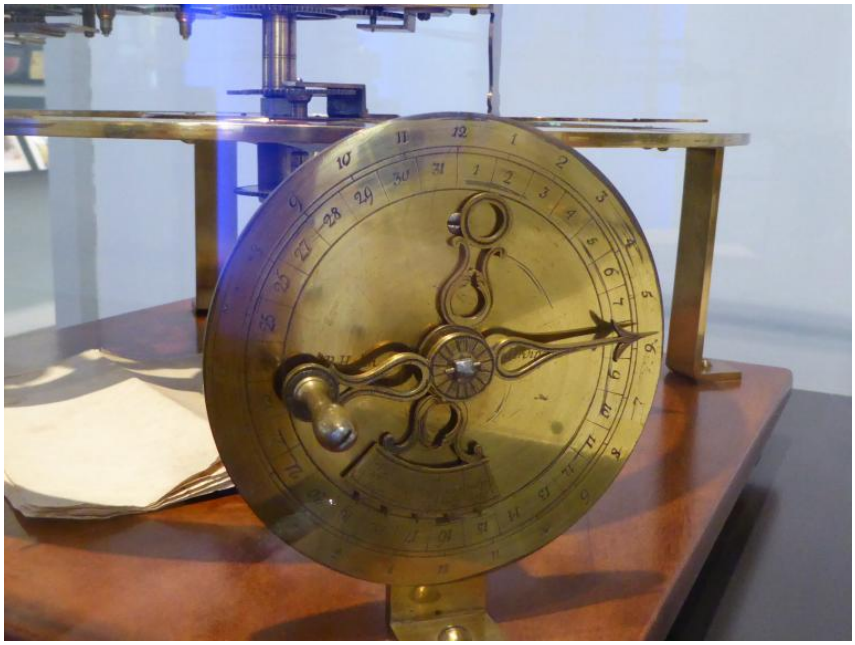


Figure 14.4: The dial of Hahn's tellurium in Winterthur. (photography by the author)

14.2 The driving mechanism

One turn of the crank corresponds to one day and the dial is divided into 24 hours (figure 14.4). When the crank is turned, a pin moves a 31-teeth wheel by one tooth and this wheel is linked to the hand for the day of the month.

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]See especially the 1989 exhibition catalogue [2, p. 454-455] where Oechslin dates the tellurium after 1780. My photographs of the tellurium were taken on July 6, 2024.

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The arbor 1 can then be thought of turning clockwise (seen from the front of the dial) in one day:

$$\mathrm{T}_1^0 = 1 \tag{14.1}$$

This motion is transferred to the vertical tube 5:

$$\mathrm{V}_5^0 = \mathrm{V}_1^0 \times \left(-\frac{b}{b}\right) \times \left(-\frac{61}{61}\right) = \mathrm{V}_1^0 = -1 \tag{14.2}$$

This tube makes one turn clockwise as seen from above.

Tube 5 surrounds a fixed vertical arbor 7 which goes through the Sun.

14.3 The motion of the main support arm

The motion of the main support arm 6 carrying the Earth-Moon system is obtained as follows. A 103-teeth wheel is fixed to the central arbor going to the Sun. This wheel is therefore not moving. Then, there is a gear train on superior part of the support arm, whose first wheel meshes with the above mentioned 103-teeth wheel. There is also another train on the inferior part of the support arm, whose first wheel meshes with a 80-teeth wheel fixed on tube 5. These two trains meet at arbor 12. (Oechslin's drawing is misleading, as the arbors 8 and 14 on one hand, and 9 and 13 on the other hand are distinct.) We can now compute the motion of tube 5 in the reference frame of the support arm 6:

$$\mathrm{V}_{12}^6 = \mathrm{V}_7^6 \times \left(-\frac{103}{f}\right) \times \left(-\frac{f}{f}\right) \times \left(-\frac{f}{103}\right) \times \left(-\frac{103}{6}\right) \times \left(-\frac{64}{3}\right) \tag{14.3}$$

$$= \mathrm{V}_7^6 \times \frac{3296}{9} \tag{14.4}$$

$$\mathrm{V}_5^6 = \mathrm{V}_{12}^6 \times \left(-\frac{80}{c}\right) \times \left(-\frac{c}{c}\right) \times \left(-\frac{c}{80}\right) \tag{14.5}$$

$$= \mathrm{V}_{12}^6 = \mathrm{V}_7^6 \times \frac{3296}{9} \tag{14.6}$$

Then

$$\mathrm{V}_6^0 = \mathrm{V}_6^5 + \mathrm{V}_5^0 = -\mathrm{V}_7^6 \times \frac{3296}{9} + \mathrm{V}_5^0 \tag{14.7}$$

$$= \mathrm{V}_6^0 \times \frac{3296}{9} + \mathrm{V}_5^0 \tag{14.8}$$

Therefore

$$\mathrm{V}_5^0 = \mathrm{V}_6^0 \times \left(1 - \frac{3296}{9}\right) = -\mathrm{V}_6^0 \times \frac{3287}{9} \tag{14.9}$$

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and

$$\mathrm{V}_6^0 = -\mathrm{V}_5^0 \times \frac{9}{3287} = \frac{9}{3287} \tag{14.10}$$

$$\mathrm{P}_6^0 = \frac{3287}{9} = 365.2222\ldots \text{ days} \tag{14.11}$$

This is an approximation of the tropical year, but it is surprisingly a rather bad approximation. The same value is given by Oechslin. This may raise some doubts about the attribution of the clock, but the ratio $\frac{9}{3296}$ and the value of the tropical year are also found in the double-globe astronomical clock of Zurich (Oechslin 8.7), so that things are not so clear.

14.4 The orientation of Earth's axis

It is easy to see that the axis of the Earth (tube 17) keeps a fixed orientation when the Earth revolves around the Sun. We have in the reference frame 6:

$$\mathrm{V}_{17}^6 = \mathrm{V}_7^6 \times \left(-\frac{103}{f}\right) \times \left(-\frac{f}{f}\right) \times \left(-\frac{f}{103}\right) \times \left(-\frac{103}{6}\right) \times \left(-\frac{21}{21}\right)\left(-\frac{6}{103}\right) \tag{14.12}$$

$$= \mathrm{V}_7^6 \tag{14.13}$$

and therefore

$$\mathrm{V}_{17}^0 = \mathrm{V}_7^0 = 0 \tag{14.14}$$

14.5 The motion of the lunar nodes

Let us first assume that the main frame 6 makes one turn counterclockwise in 365.25 days. We can now compute the revolution of the lunar nodes (tube 15) in the reference frame 6:

$$\mathrm{V}_{15}^6 = \mathrm{V}_7^6 \times \left(-\frac{103}{f}\right) \times \left(-\frac{f}{f}\right) \times \left(-\frac{f}{103}\right) \times \left(-\frac{56}{53}\right) = \mathrm{V}_7^6 \times \frac{56}{53} \tag{14.15}$$

$$= -\mathrm{V}_6^0 \times \frac{56}{53} \tag{14.16}$$

$$\mathrm{V}_{15}^0 = \mathrm{V}_{15}^6 + \mathrm{V}_6^0 = \mathrm{V}_6^0 \times \left(1 - \frac{56}{53}\right) = -\mathrm{V}_6^0 \times \frac{3}{53} = -\frac{9}{3287} \times \frac{3}{53} \tag{14.17}$$

$$= -\frac{27}{174211} \tag{14.18}$$

$$\mathrm{P}_{15}^0 = -\frac{174211}{27} = -6452.2592\ldots \text{ days} \tag{14.19}$$

This value is negative because the nodes are retrograding, but it is a very bad approximation of the period of revolution of the nodes which is about 6798 days. The same value is given by Oechslin.

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14.6 The motion of the Moon

Let us now compute the motion of the Moon. The phase of the Moon is merely obtained by a train of gears that ensures that the Moon always keeps the same orientation towards the Sun.

The direction of the Moon is given by tube 22. We can compute the motion of this tube with respect to the moving frame 6:

$$\mathrm{V}_{22}^6 = \mathrm{V}_5^6 \times \left(-\frac{80}{c}\right) \times \left(-\frac{c}{c}\right) \times \left(-\frac{c}{80}\right) \times \left(-\frac{32}{63}\right) \times \left(-\frac{6}{90}\right) \tag{14.20}$$

$$= \mathrm{V}_5^6 \times \left(-\frac{32}{945}\right) = \left(\mathrm{V}_5^0 - \mathrm{V}_6^0\right) \times \left(-\frac{32}{945}\right) \tag{14.21}$$

$$= \left(-1 - \frac{9}{3287}\right) \times \left(-\frac{32}{945}\right) \tag{14.22}$$

$$= \frac{105472}{3106215} \tag{14.23}$$

$$\mathrm{P}_{22}^6 = \frac{3106215}{105472} = 29.4506\ldots \text{ days} \tag{14.24}$$

This value is also rather bad, because the correct value is the synodic month of about 29.53 days.

Oechslin finds a period of 29.531250 days, but his derivation is wrong.

We can also compute the tropical motion of the Moon:

$$\mathrm{V}_{22}^0 = \mathrm{V}_{22}^6 + \mathrm{V}_6^0 = \frac{105472}{3106215} + \frac{9}{3287} = \frac{113977}{3106215} \tag{14.25}$$

$$\mathrm{P}_{22}^0 = \frac{3106215}{113977} = 27.2529\ldots \text{ days} \tag{14.26}$$

This value is also bad, but agrees with the one given by Oechslin.

14.7 The motion of the Earth around its axis

Finally, we can compute the motion of the Earth around its axis (arbor 19). The motion of this axis actually is the opposite of the motion of tube 5, which is making one turn clockwise in 24 hours.

It would actually seem that tube 5 should have the motion of the sidereal day, and not make one turn in 24 hours.

If we instead assume that tube 5 makes one turn in one sidereal day. the motion of the Earth around its axis is correct.

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14.8 The rotation of the Sun

The Sun is mounted on tube 28 and its rotation is merely derived from the motion of tube 5:

$$\mathrm{V}^0_{28} = \mathrm{V}^0_5 \times \left(-\frac{80}{40}\right) \times \left(-\frac{8}{32}\right) \times \left(-\frac{6}{84}\right) = \mathrm{V}^0_5 \times \left(-\frac{1}{28}\right) = \frac{1}{28} \quad (14.27)$$

$$\mathrm{P}^0_{28} = 28 \text{ days} \quad (14.28)$$

The Sun makes one turn in 28 days. This is also a crude approximation of the rotation of the Sun, worse than the one for instance used in the Nuremberg *Weltmaschine*.

Oechslin found a period of 25.94 days which is incorrect.

14.9 Conclusion

This tellurium is obviously rather puzzling, and, as I wrote in the introduction, I find it hard to attribute it to Philipp Matthäus Hahn, or at least to Hahn alone. There are too many errors, the sidereal day is not computed, no well-known ratio of Hahn appears here, the tropical year is bad, the revolution of the nodes is bad, and the synodic month is bad. On the other hand, the ratios used for the tropical year are also found in the double-globe astronomical clock of Zurich (Oechslin 8.7).

In light of the many errors of this mechanism, Oechslin assumed in 1996 that it must have been one of the first works by Hahn [1, p. 198], but this does not fit with the announced construction date of 1780 which he himself put forward in the catalogue for the 1989 Hahn exhibition. This machine is possibly older than 1780, and may have been finalized in 1780. Perhaps Hahn was only partly involved in its construction.

14.10 References

[1] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert*. Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[2] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn*. Stuttgart: Württembergisches Landesmuseum, 1989.

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Chapter 15

(Oechslin: 8.6)

Hahn's globe clock in Darmstadt (1785)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

15.1 Introduction

The clock described here was constructed in 1785 by Philipp Matthäus Hahn (1739-1790)[1] and is located in Darmstadt. It is a cubic case with four sides, three of which have dials, one showing the time, another one giving calendar data, and the third given the years in Johann Albrecht Bengel's chronology of the world.[2]

On top of the base, the Earth-Moon system rotates around the Sun. Over them, a globe represents the motion of the sky, and over it there is another lunar sphere.

15.2 The going work

The clock is driven by a spring and regulated by a pendulum.

One face of the base shows the hours and minutes. These are shown on arbors/tubes 7 and 2. (Note that the central arbor 2 must in fact be linked to

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [10, p. 399-401, pl .14], and Oechslin [9, p. 46]. This clock is also mentioned by Zinner [11, p. 354]. There are several similar clocks by Hahn, one of which was described by Hahn in 1770 [6]. At least two such clocks are kept in the *Landesmuseum* in Stuttgart, including a clock described in this book (Oechslin 8.5). A similar clock was also described by Hahn in 1774 [5]. Another similar clock was also reconstructed by Alfred Leiter [7].

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the 40-teeth wheel which seems to be part of an independent tube. Or perhaps Oechslin meant to show that the two are only connected by friction.) In any case, we have

$$\mathrm{T}_2^0 = \frac{1}{24} \text{ (one rotation in an hour)} \tag{15.1}$$

And given that velocity, we can obtain the half-period of the pendulum:

$$T = 3600 \times \frac{8}{72} \times \frac{8}{56} \times \frac{6}{36} \times \frac{1}{22} = \frac{100}{231} \tag{15.2}$$

Hence the pendulum must be about 60 cm.

(15.3)

The motion of the hour hand, that is a rotation in 12 hours, is transmitted to the other parts of the clock. The rotation in 12 hours is first converted in a rotation in 24 hours on arbors 9, 10, 11 and 12. The arbors 10 and 11 are probably tied by friction.

$$\mathrm{T}_{11}^0 = 1 \tag{15.4}$$

This motion is used for the display of the weeks and months on one side of the base. I will not describe this part in detail, first because I want to focus on the regular astronomical motions, and second because I do not have all the details of that part of the gears. The motion is also used to display the year in Johann Albrecht Bengel's chronology of the world.[3] A similar dial is shown on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

After that, the motion or arbor 11 is transfered on another side of the base, on arbor 20:

$$\mathrm{T}_{20}^0 = \mathrm{T}_{11}^0 \times \left(-\frac{57}{58}\right) \times \left(-\frac{35}{33}\right) \times \left(-\frac{76}{79}\right) \times \left(-\frac{79}{79}\right) = \frac{25270}{25201} \tag{15.5}$$

In certain cases, it may be necessary to change the sign of the result. This is not necessary here, because the wheels on arbor 11 and 20 could be considered flipped to vertical axes and all the rotations seen from above.

$$\mathrm{P}_{20}^0 = \frac{25201}{25270} = 0.9972\ldots \text{ days} = 86164.0838\ldots \text{ seconds} \tag{15.6}$$

This is the sidereal day. The same motion is transfered to the vertical arbor 29 going through the Sun:

$$\mathrm{V}_{29}^0 = -\frac{25270}{25201} \tag{15.7}$$

[3] See in particular Bengel's works [1, 2, 3, 4] and Marini [8].

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Notice that this motion is clockwise as seen from above, hence the negative sign.

Using the sidereal motion of arbor 20, Hahn obtains the motion of arbor 24:

$$\mathrm{T}^0_{24} = \mathrm{T}^0_{20} \times \left(-\frac{32}{32}\right) \times \left(-\frac{32}{32}\right) \times \left(-\frac{6}{64}\right) \times \left(-\frac{3}{103}\right) = \frac{113715}{41531248} \quad (15.8)$$

$$\mathrm{P}^0_{24} = \frac{41531248}{113715} = 365.22224\ldots \text{ days} \quad (15.9)$$

This is the tropical year. The 103-teeth wheel making one turn in one tropical year then advances another 10-teeth wheel on arbor 25 once a year. The latter therefore makes one turn in 10 years, but intermittently. That motion is yet transfered to a wheel on arbor 26 making one turn in 100 years. And finally, it is transfered to a wheel making one turn in 8000 years. Supposedly, the year can be read on this dial as a combination of a value between 0 and 99 (the units and tens), and of another one between 0 and 79 (the hundreds and thousands).

15.3 The tellurium

The part above the base is a tellurium, namely a rendering of the motions of the Earth and the Moon around the Sun. The input of this tellurium is the motion of arbor 29 in one sidereal day mentioned above. This motion is used to produce the motion of the rotating frame 34 of the Earth-Moon system:

$$\mathrm{V}^0_{34} = \mathrm{V}^0_{29} \times \left(-\frac{41}{41}\right) \times \left(-\frac{41}{41}\right) \times \left(-\frac{41}{41}\right) \times \left(-\frac{6}{64}\right) \times \left(-\frac{3}{103}\right) \quad (15.10)$$

$$= \mathrm{T}^0_{24} = \frac{113715}{41531248} \quad (15.11)$$

$$\mathrm{P}^0_{34} = 365.22224\ldots \text{ days} \quad (15.12)$$

The same value is given by Oechslin.

Hence, the velocity of frame 34 is the same as that of arbor 24 seen earlier, but in the opposite direction. The Earth-Moon frame rotates in the direct direction (counterclockwise) in one tropical year.

Now, the moving frame 34 has two inputs. First, there is arbor 35 which is actually tied to the reference frame. This arbor carries a 65-teeth wheel. Second, there is the central arbor 29 whose motion is that of the sidereal day, but clockwise as seen from above:

$$\mathrm{V}^0_{35} = 0 \quad (15.13)$$

$$\mathrm{V}^0_{29} = -\frac{25270}{25201} \quad (15.14)$$

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The tilted axis of the Earth is on tube 39 and its orientation is determined from the wheel on tube 35. The velocity of the tilted axis is

$$\mathrm{V}_{39}^{34} = \mathrm{V}_{35}^{34} \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) = \mathrm{V}_{35}^{34} \tag{15.15}$$

$$\mathrm{V}_{39}^{0} = \mathrm{V}_{39}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{35}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{35}^{0} + \mathrm{V}_{0}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{35}^{0} = 0 \tag{15.16}$$

In other words, the Earth's axis is still in the reference frame and always keeps the same orientation.

The rotation of the Earth around its axis 44 is obtained from the sidereal day:

$$\mathrm{V}_{44}^{0} = \mathrm{V}_{44}^{39} = \mathrm{V}_{43}^{39} \times \left(-\frac{30}{30}\right) \tag{15.17}$$

And

$$\mathrm{V}_{43}^{39} = \mathrm{V}_{43}^{34} + \mathrm{V}_{34}^{39} = \mathrm{V}_{29}^{34} \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) + \mathrm{V}_{34}^{39} \tag{15.18}$$

$$= \mathrm{V}_{29}^{34} + \mathrm{V}_{34}^{39} = \mathrm{V}_{29}^{34} - \mathrm{V}_{39}^{34} = \mathrm{V}_{29}^{34} - \mathrm{V}_{35}^{34} = \mathrm{V}_{29}^{34} + \mathrm{V}_{34}^{35} = \mathrm{V}_{29}^{35} \tag{15.19}$$

$$= \mathrm{V}_{29}^{0} = -\frac{25270}{25201} \tag{15.20}$$

$$\mathrm{V}_{44}^{0} = \frac{25270}{25201} \tag{15.21}$$

As expected, the Earth performs a rotation around its axis in one sidereal day. This motion is counterclockwise as seen from above.

However, we should also notice that the tropical year is inconsistent with the sidereal day. Normally, after one day, the angular motion of the Earth with respect to the Sun should be the difference of its motion with respect to the zodiac and the motion of the Earth around the Sun. We have therefore

$$\frac{1 \text{ day}}{1 \text{ sidereal day}} - \frac{1}{1 \text{ tropical year}} = 1 \tag{15.22}$$

and the tropical year and sidereal day are related.

This translates to

$$\mathrm{V}_{44}^{34} = \mathrm{V}_{44}^{39} + \mathrm{V}_{39}^{34} = \frac{25270}{25201} + \mathrm{V}_{0}^{34} = \frac{25270}{25201} - \mathrm{V}_{34}^{0} \tag{15.23}$$

$$= \frac{25270}{25201} - \frac{113715}{41531248} = \frac{41531245}{41531248} \tag{15.24}$$

This is the mean velocity of wheel/arbor 44, as it is wobbling in frame 34. But most importantly, we can observe that the value of V_{44}^{34} is not 1.

With the modern values, the above corresponds to

$$\frac{86400}{86164.0905} - \frac{1}{365.2422} \approx 1 \tag{15.25}$$

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In Hahn's gears, we should normally have had

$$\frac{25270}{25201} - \frac{25270}{25201} \times \frac{6}{64} \times \frac{3}{103} = 1 \text{ (False)} \tag{15.26}$$

but this is not exactly the case. It is quite possible that Hahn wasn't able to find adequate gears to ensure this equality. On the other hand, if he had used the ratios $\frac{23}{57} \times \frac{6}{76} \times \frac{6}{70}$ instead of $\frac{6}{64} \times \frac{3}{103}$ (and with an additional direction reversal), Hahn would have been able to ensure the above equality.

Hahn's approximation of the tropical year is in fact better than his approximation of the sidereal day. This is easy to see when computing the tropical year from the sidereal year. We then find a tropical year of $\frac{25201}{69} = 365.2318\ldots$ days. In fact, the ratio $\frac{6}{64} \times \frac{3}{103}$ used for the tropical year is quite good, and Hahn should have replaced his ratio $\frac{57}{58} \times \frac{35}{33} \times \frac{76}{79}$ for the sidereal day by the ratio $\frac{3296}{3287} = \frac{32}{173} \times \frac{103}{19}$.

The Moon is located on frame 46. We can compute the velocity of this frame with respect to the rotating frame 34. This motion is derived from the train producing the rotation of the Earth:

$$\mathrm{V}_{46}^{34} = \mathrm{V}_{42}^{34} \times \left(-\frac{12}{41}\right) \times \left(-\frac{12}{104}\right) \tag{15.27}$$

$$\mathrm{V}_{42}^{34} = -\mathrm{V}_{43}^{34} = -\left(\mathrm{V}_{43}^{0} + \mathrm{V}_{0}^{34}\right) = -\mathrm{V}_{43}^{0} - \mathrm{V}_{0}^{34} = -\mathrm{V}_{43}^{0} + \mathrm{V}_{34}^{0} \tag{15.28}$$

$$= -\mathrm{V}_{43}^{39} + \frac{113715}{41531248} \tag{15.29}$$

$$= \frac{25270}{25201} + \frac{113715}{41531248} = \frac{41758675}{41531248} \tag{15.30}$$

We now have

$$\mathrm{V}_{46}^{34} = \frac{375828075}{11068077592} \tag{15.31}$$

And

$$\mathrm{P}_{46}^{34} = \frac{11068077592}{375828075} = 29.4498\ldots \text{ days} \tag{15.32}$$

This value of the synodic month is not very accurate, because the ratios used to compute V_{46}^{34} could have been better.

$$\mathrm{V}_{46}^{0} = \mathrm{V}_{46}^{34} + \mathrm{V}_{34}^{0} = \frac{375828075}{11068077592} + \frac{113715}{41531248} = \frac{812266245}{22136155184} \tag{15.33}$$

$$\mathrm{P}_{46}^{0} = \frac{22136155184}{812266245} = 27.2523\ldots \text{ days} \tag{15.34}$$

This value of the tropical month is also not very accurate, for the same reason. The same value is given by Oechslin, but he didn't give the value of the synodic month.

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The orientation of the lit part of the Moon depends on the relative position of tube 47 with respect to frame 46. The Moon has a permanent lit part whose orientation is defined by the axis 49. We therefore must compute the velocity of axis 49 with respect to frame 34. This velocity is expected to be 0:

$$\mathrm{V}_{49}^{34} = \mathrm{V}_{49}^{46} + \mathrm{V}_{46}^{34} = \mathrm{V}_{47}^{46} \times \left(-\frac{44}{44}\right) \times \left(-\frac{44}{44}\right) + \mathrm{V}_{46}^{34} \tag{15.35}$$

$$= \mathrm{V}_{47}^{46} + \mathrm{V}_{46}^{34} = \mathrm{V}_{47}^{34} = 0 \tag{15.36}$$

15.4 The celestial globe

The celestial globe is located above the base and within a fixed frame. It shows the motions of the Sun, the Moon, the lunar nodes and Venus from a geocentric perspective. It is thus similar to the celestial globe in the globe clock in Stuttgart (Oechslin 8.5) from 1770, although the construction differs. This globe takes as only input the motion of arbor 29 in one sidereal day (clockwise from above). This arbor goes through the Sun which is actually fixed on the moving Earth frame 34. The Sun rotates with this frame.

The virtual vertical axis going through the celestial globe represents Earth's rotation axis. Consequently, the celestial sphere rotates around this axis. It does however do so indirectly. The celestial has its main axis tilted and this axis goes through the poles of the ecliptic. The ecliptic is shown by a strip and the Moon, the Sun, Venus and the lunar nodes are shown by curved hands rotating around these poles. The meridian and the equator planes are shown through the outside frame. The motions of the Sun, the Moon, and Venus are shown as seen from the Earth. For instance, the retrogradations of Venus are shown, but they are artefacts of the motion of the Earth around the Sun. The globe does not show the motion of Venus around the Sun.

The input motion of one turn in a sidereal day causes the tilted axis of the celestial globe to spin clockwise (from above) around the vertical axis. In fact, the entire globe is carried in this motion, and this corresponds to the observable motion of the sky.

At the base of celestial sphere, there is a fixed 41-teeth wheel. This wheel meshes with a similar wheel on which arbor another such wheel meshes with yet a similar wheel on the axis of the poles. The consequence is that the fixed reference frame is transfered inside the celestial globe. We will examine the inside of this globe in a moment, but first, we can study the small lunar globe located over the celestial sphere.

15.4.1 The lunar sphere at the top

The rotation of the celestial sphere is transfered at the top to 12-teeth wheel on an arbor which is actually the continuation of the input arbor 29. The

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velocity of the lunar sphere is

$$\mathrm{V}_{71}^{0} = \mathrm{V}_{29}^{0} \times \left(-\frac{12}{26}\right) \times \left(-\frac{3}{41}\right) = \mathrm{V}_{29}^{0} \times \frac{18}{533} \tag{15.37}$$

$$= \left(-\frac{25270}{25201}\right) \times \frac{18}{533} = -\frac{454860}{13432133} \tag{15.38}$$

$$\mathrm{P}_{71}^{0} = -\frac{13432133}{454860} = -29.5302\ldots \text{ days} \tag{15.39}$$

This value is not given by Oechslin.

This globe makes one turn clockwise (from above) in a synodic month. It is half golden, half black, and half of that globe is hidden. Through its rotation it can show the approximate phases of the Moon, in a similar way as the display of the lunar phases in the current astronomical clock in the Strasbourg cathedral.

This sphere is similar to the one on top of the globe clock in Stuttgart (Oechslin 8.5) from 1770. The same ratio $\frac{18}{533}$ is used in both clocks. However, in Stuttgart the lunar globe turns counterclockwise, which seems incorrect.

15.4.2 The inner workings of the celestial sphere

We now assume as a reference frame that of the celestial globe itself. The only input is that of tube 51 which has the opposite motion as that of arbor 29 with reference to the fixed frame:

$$\mathrm{V}_{51}^{29} = -\mathrm{V}_{29}^{0} = \frac{25270}{25201} \tag{15.40}$$

This is first used to obtain the mean motion of the Moon on tube 60. We first compute the velocity of the arbor 52, as it will be used in other parts of the globe.

$$\mathrm{V}_{52}^{29} = \mathrm{V}_{51}^{29} \times \left(-\frac{60}{60}\right) = -\mathrm{V}_{51}^{29} \tag{15.41}$$

$$\mathrm{V}_{60}^{29} = \mathrm{V}_{52}^{29} \times \left(-\frac{60}{61}\right) \times \left(-\frac{24}{89}\right) \times \left(-\frac{15}{109}\right) \tag{15.42}$$

$$= \mathrm{V}_{51}^{29} \times \frac{21600}{591761} = \frac{545832000}{14912968961} \tag{15.43}$$

$$\mathrm{P}_{60}^{29} = \frac{14912968961}{545832000} = 27.3215\ldots \text{ days} \tag{15.44}$$

This is an approximation of the tropical month. The same value is given by Oechslin, also in sidereal days.

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The motion of the lunar nodes is obtained as follows:

$$\mathrm{V}_{56}^{29} = \mathrm{V}_{52}^{29} \times \left(-\frac{3}{30}\right) \times \left(-\frac{3}{36}\right) \times \left(-\frac{3}{48}\right) \times \left(-\frac{20}{71}\right) = \mathrm{V}_{52}^{29} \times \frac{1}{6816} \quad (15.45)$$

$$= \left(-\frac{25270}{25201}\right) \times \frac{1}{6816} = -\frac{12635}{85885008} \quad (15.46)$$

$$\mathrm{P}_{56}^{29} = -\frac{85885008}{12635} = -6797.3888\ldots \text{ days} \quad (15.47)$$

The same value is given by Oechslin, also in sidereal days. The actual value of the draconic period is 6798.3 days, this is therefore an excellent approximation. This period is negative, because the motion of the nodes is clockwise.

The motion of arbor 52 is also used to produce the apsidal precession of the Moon on tube 57:

$$\mathrm{V}_{57}^{29} = \mathrm{V}_{52}^{29} \times \left(-\frac{3}{30}\right) \times \left(-\frac{3}{36}\right) \times \left(-\frac{3}{81}\right) \quad (15.48)$$

$$= \left(-\frac{25270}{25201}\right) \times \left(-\frac{1}{3240}\right) = \frac{2527}{8165124} \quad (15.49)$$

$$\mathrm{P}_{57}^{29} = \frac{8165124}{2527} = 3231.1531\ldots \text{ days} \quad (15.50)$$

The same value is given by Oechslin, also in sidereal days. The actual value of the apsidal precession is about 3233 days and the motion of the apsides is counterclockwise, hence the positive value.

The angular difference between tube 60 (mean motion of the Moon) and tube 57 (apsides) is the mean anomaly of the Moon. In the reference frame of the mean Moon, tube 60, we have

$$\mathrm{V}_{62}^{60} = \mathrm{V}_{57}^{60} \times \left(-\frac{44}{44}\right) \times \left(-\frac{44}{44}\right) = \mathrm{V}_{57}^{60} \quad (15.51)$$

$$= \mathrm{V}_{57}^{29} - \mathrm{V}_{60}^{29} = \frac{2527}{8165124} - \frac{545832000}{14912968961} \quad (15.52)$$

$$= -\frac{175354187953}{4831801943364} \quad (15.53)$$

$$\mathrm{P}_{62}^{60} = -\frac{4831801943364}{175354187953} = -27.5545\ldots \text{ days} \quad (15.54)$$

The period of this angular difference is the anomalistic month. This value is not given by Oechslin.

Using an excentric pin, the motion of arbor 62 is used to add the equation of center to the mean anomaly, so as to obtain (an approximation of) the true anomaly of the Moon.

The motions of the Sun and Venus are obtained from arbor 63. Its velocity is

$$\mathrm{V}_{63}^{29} = \mathrm{V}_{52}^{29} \times \left(-\frac{3}{33}\right) = \left(-\frac{25270}{25201}\right) \times \left(-\frac{3}{33}\right) = \frac{25270}{277211} \quad (15.55)$$

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The velocity of the Sun on arbor 65 is then

$$\mathrm{V}_{65}^{29} = \mathrm{V}_{63}^{29} \times \left(-\frac{16}{33}\right) \times \left(-\frac{7}{113}\right) = \frac{25270}{277211} \times \frac{112}{3729} = \frac{2830240}{1033719819} \quad (15.56)$$

and the revolution period is

$$\mathrm{P}_{65}^{29} = \frac{1033719819}{2830240} = 365.2410\ldots \text{ days} \quad (15.57)$$

which is yet another value for the tropical year. The same value is given by Oechslin. This value was also used in the double-globe clock in Karlsruhe (Oechslin 8.8), also constructed in 1785, and in that of Villingen (Oechslin 8.9), dated 1770-1783.

The mean motion of Venus on the ecliptic, that is its orbital period, is obtained as follows on arbor 67, from arbor 63:

$$\mathrm{V}_{67}^{29} = \mathrm{V}_{63}^{29} \times \left(-\frac{11}{35}\right) \times \left(-\frac{16}{103}\right) \quad (15.58)$$

$$= \frac{25270}{277211} \times \frac{176}{3605} = \frac{11552}{2595703} \quad (15.59)$$

$$\mathrm{P}_{67}^{29} = \frac{2595703}{11552} = 224.6972\ldots \text{ days} \quad (15.60)$$

The same value is given by Oechslin, also in sidereal days.

The actual orbital period of Venus is about 224.701 days.

Finally, the irregular motion of Venus depends on its elongation to the Sun. The mean elongation is the angular difference between the mean position of Venus and the (mean) position of the Sun (Hahn didn't take into account the equation of center for the Sun), that is between tubes 67 and 65. This elongation is translated into the relative motion of arbor 69 on frame 65:

$$\mathrm{V}_{69}^{65} = \mathrm{V}_{67}^{65} \times \left(-\frac{52}{52}\right) \times \left(-\frac{52}{52}\right) = \mathrm{V}_{67}^{65} = \mathrm{V}_{67}^{29} - \mathrm{V}_{65}^{29} \quad (15.61)$$

$$= \frac{11552}{2595703} - \frac{2830240}{1033719819} = \frac{182336768}{106473141357} \quad (15.62)$$

$$\mathrm{P}_{69}^{65} = \frac{106473141357}{182336768} = 583.9367\ldots \text{ days} \quad (15.63)$$

and its period is that of the synodic revolution of Venus. This period is not given by Oechslin.

The wheel on arbor 69 moves an excentric pin which accelerates or slows down the motion of Venus, producing the well-known retrogradations.

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15.5 References

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Chapter 16

(Oechslin: 8.8)

Hahn's globe clock in Karlsruhe (1785)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

16.1 Introduction

The clock described here was constructed in 1785 by Philipp Matthäus Hahn (1739-1790)[1] and is located in in a private collection in Karlsruhe.[2] It is one of a number of double-globe clocks made by Hahn. This clock is made of two similar and interconnected parts. These two parts have each a cubic base and are topped by globes. The right part contains the going work and is topped by a geographic sphere. The left part has a calendar dial and is topped by a celestial sphere showing the motion of the sky, of the Sun and of the Moon.

The structure of both parts is similar to that if the double-globe clock in Villingen (Oechslin 8.9), except that in Villingen the going work is in the middle of the two globe components, and not part of one of them.

16.2 The going work

The going work is located in the right part of the double globe clock. It displays the hours on a 24-hour dial and the minutes on an smaller central dial. The clock is spring-driven and regulated by a pendulum. The barrel wheel on arbor

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [2, p. 425-426].

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1 makes one turn clockwise (from the front) in 44 hours.

$$\mathrm{T}_1^0 = \frac{24}{44} = \frac{6}{11} \tag{16.1}$$

The going train leads to arbor 3 which is that of the minute hand. It makes one turn in an hour:

$$\mathrm{T}_3^0 = \mathrm{T}_1^0 \times \left(-\frac{72}{12}\right) \times \left(-\frac{66}{9}\right) = \frac{6}{11} \times 44 = 24 \tag{16.2}$$

This motion is also that of a tube surrounding that arbor, and this motion is used to obtain that of the hour hand on tube 7:

$$\mathrm{T}_7^0 = \mathrm{T}_3^0 \times \left(-\frac{12}{48}\right) \times \left(-\frac{10}{60}\right) = 1 \tag{16.3}$$

Tube 7 thus makes one turn clockwise in 24 hours.

The motion of tube 7 is transferred to geographical globe above and to the left component of the double-globe clock as described below.

16.3 The geographic sphere

The geographic sphere is on the right of the double-globe clock and shows the motion of the Earth around a vertical axis. It also shows the limit between day and night, and thus moves a ring according to the motion of the Sun. However, since the orientation of the Earth's axis is not taken into account, this representation is approximative.

The motion of the sphere is derived from the motion of tube 7 in the going work. Tube 7 is coupled to tube 8, and tube 8 carries a 57-teeth which used to transfer its motion to a 57-teeth wheel on arbor 26, whic in turn transfers its motion to a 79-teeth wheel on arbor 27 and finally to another 57-teeth wheel on the vertical arbor 28. That arbor is the central arbor moving the Earth sphere. We have

$$\mathrm{V}_{28}^0 = \mathrm{T}_8^0 \times \frac{57}{57} \times \frac{57}{79}\frac{79}{57} = \mathrm{T}_8^0 = 1 \tag{16.4}$$

Consequently, the vertical arbor 28 makes one turn counterclockwise (seen from above) in one day.

The arbor 28 also carries a 7-leaves pinion which is used to obtain the motion of tube 30:

$$\mathrm{V}_{30}^0 = \mathrm{V}_{28}^0 \times \left(-\frac{7}{118}\right) \times \left(-\frac{3}{65}\right) = \mathrm{V}_{28}^0 \times \frac{21}{7670} = \frac{21}{7670} \tag{16.5}$$

$$\mathrm{P}_{30}^0 = \frac{7670}{21} = 365.2380\ldots \text{ days} \tag{16.6}$$

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This is an approximation of the tropical year and the same value is given by Oechslin. Tube 30 does therefore make one turn counterclockwise (as seen from above) in one year. This tube seems to move the limit day/night.

The same ratio $\frac{7670}{21}$ was used in the astronomical clock in Basel (Oechslin 8.10) constructed in 1775 and in the double-globe clock of Villingen (Oechslin 8.9).

This, however, is not entirely correct. If the limit day/night moves in one year, then the Earth should actually make one turn in one sidereal day, and not 24 jours as it does now (or seems to be doing). This suggests a construction error, and that the sidereal day motion produced for the left globe was initially also meant to be used in the geographical globe. The same problem occurs in the Villingen clock.

16.4 The celestial sphere

The celestial sphere shows the motion of the sky, as well as the motion of the Sun and the Moon. It is located on top of a similar base as the going work, which displays the day of the week and the day of the month.

The motion of the celestial sphere is obtained from that of tube 7 in the going work. That tube is coupled to tube 8, and tube 8 carries a 57-teeth wheel meshing with a 29-teeth wheel on the horizontal arbor 9 connecting the two parts of the clock and visible from the outside.

In the left component, that arbor transmits its motion to arbor 16, and then to the vertical arbor 17 which is the input to the celestial sphere mechanism. We have

$$\mathrm{V}_{17}^{0} = \mathrm{T}_{8}^{0} \times \left(-\frac{57}{29}\right) \times \left(-\frac{35}{33}\right) \times \left(-\frac{38}{79}\right) \tag{16.7}$$

$$= \mathrm{T}_{8}^{0} \times \left(-\frac{25270}{25201}\right) = -\frac{25270}{25201} \tag{16.8}$$

This is the usual value used by Hahn for the velocity of the sidereal day. Hence, arbor 17 makes one turn clockwise (from above) in one sidereal day.

However, arbor 17 actually moves an entire frame containing gears which are themselves use to produce the motions of the Sun and the Moon. Arbor 17 moves the celestial sphere which rotates around the vertical axis representating the axis of the Earth. But the Sun and the Moon revolve (approximately) around the axis of the ecliptic, which is tilted by 23.5 degrees. The Sun is fixed on tube 25 and the Moon on tube 22. In order to compute the motions of these two tubes, we first compute the motions of tubes 21 (Moon) and 24

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(Sun) with respect to the celestial globe (frame 17):

$$\mathrm{V}_{21}^{17} = \mathrm{V}_{18}^{17} \times \left(-\frac{53}{53}\right) \times \left(-\frac{53}{66}\right) \times \left(-\frac{3}{66}\right) = \mathrm{V}_{18}^{17} \times \left(-\frac{53}{1452}\right) \tag{16.9}$$

$$= -\mathrm{V}_{17}^{18} \times \left(-\frac{53}{1452}\right) = \mathrm{V}_{17}^{0} \times \frac{53}{1452} \tag{16.10}$$

$$= -\frac{25270}{25201} \times \frac{53}{1452} = -\frac{669655}{18295926} \tag{16.11}$$

$$\mathrm{P}_{21}^{17} = -\frac{18295926}{669655} = -27.3214\ldots \text{ days} \tag{16.12}$$

$$\mathrm{V}_{24}^{17} = \mathrm{V}_{21}^{17} \times \left(-\frac{32}{53}\right) \times \left(-\frac{14}{113}\right) = \mathrm{V}_{21}^{17} \times \frac{448}{5989} \tag{16.13}$$

$$= -\frac{669655}{18295926} \times \frac{448}{5989} = -\frac{2830240}{1033719819} \tag{16.14}$$

$$\mathrm{P}_{24}^{17} = -\frac{1033719819}{2830240} = -365.2410\ldots \text{ days} \tag{16.15}$$

So, tube 21 makes one turn clockwise (from above) in one tropical month, and tube 24 makes one turn clockwise in one tropical year. The latter is given by Oechslin, and also in sidereal days. The former is also given by Oechslin, in sidereal days, and then in days, hours, minutes, and seconds, but with an incorrect conversion (27.425075) to its decimal representation. The same error appears in the Zurich double globe clock (Oechslin 8.7), but the conversion was correctly made for the double globe clock in Villingen (Oechslin 8.9).

The same value for the tropical month was used in the double-globe clock located in Zurich (Oechslin 8.7) constructed that same year in 1785. And the value of the tropical year was used in the globe clock in Darmstadt (Oechslin 8.6), also in 1785. Both were also used in the double-globe clock in Villingen (Oechslin 8.9).

Finally

$$\mathrm{V}_{22}^{17} = \mathrm{V}_{21}^{17} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{21}^{17} = \frac{669655}{18295926} \tag{16.16}$$

$$\mathrm{V}_{25}^{17} = \mathrm{V}_{24}^{17} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{24}^{17} = \frac{2830240}{1033719819} \tag{16.17}$$

The Moon and the Sun rotate counterclockwise around the celestial sphere in one tropical month and one tropical year.

16.5 References

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Chapter 17

(Oechslin: 8.7)

Hahn's globe clock in Zurich (1785)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

17.1 Introduction

The clock described here was constructed in 1785 by Philipp Matthäus Hahn (1739-1790)[1] and was located in Zurich.[2] It is one of a number of double-globe clocks made by Hahn. This clock has a simple base with a large dial showing the hours on a 24-hours scale, the minutes on on inner dial, and the day of the week and of the month on another inner dial.

Two globes are mounted on top of the base. The globe on the left shows the Earth and its position with respect to the Sun. The globe on the right shows the celestial sphere and is itself surmounted by a small lunar globe.

17.2 The going work

The clock is driven by a spring and regulated by a pendulum. The barrel wheel on arbor 1 makes one turn clockwise (viewed from the front) in 56 hours:

$$\mathrm{T}_1^0 = \frac{24}{56} = \frac{3}{7} \tag{17.1}$$

[1] For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2] See especially the 1989 exhibition catalogue [3, p. 416-417]. I believe that this clock was auctioned by Koller in 2008, as the description of the auction perfectly fits the descriptions by Oechslin, both in 1989 and 1996 [2, p. 51]. This clock was originally located in the Geras monastery in Austria.

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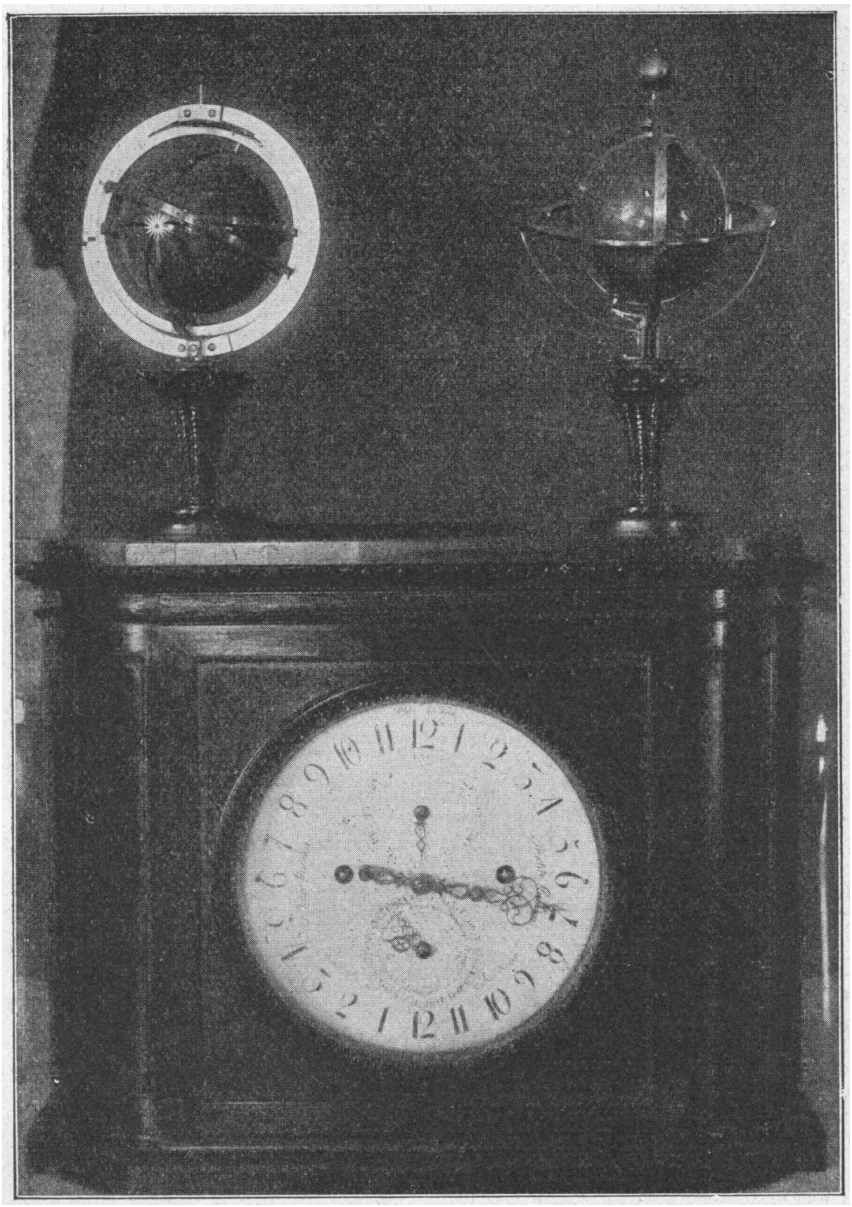


Figure 17.1: Hahn's clock from the Geras monastery in Austria. It is very similar (or identical?) to the clock described here. (source: [1])

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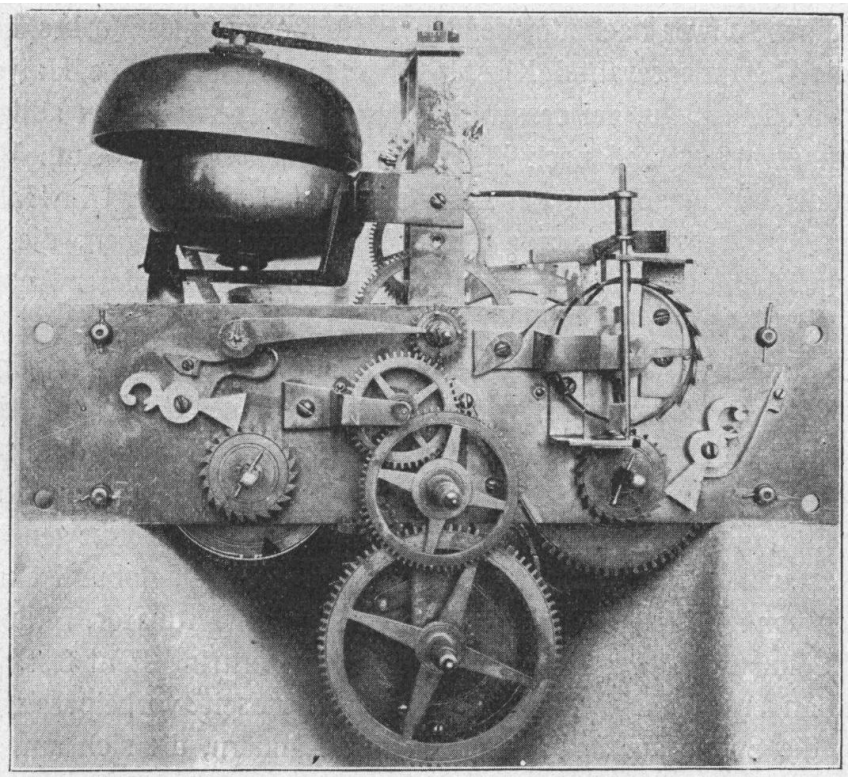


Figure 17.2: The mechanism of Hahn's clock from the Geras monastery in Austria. (source: [1])

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Figure 17.3: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)

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Figure 17.4: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)

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Figure 17.5: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)

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Figure 17.6: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)

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Figure 17.7: The mechanism of Hahn's clock from the Geras monastery in Austria, as sold in 2008. (source: `https://www.kollerauktionen.ch`)

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This leads to arbor 3 which carries the wheel of the minute hand:

$$\mathrm{T}_3^0 = \mathrm{T}_1^0 \times \left(-\frac{70}{10}\right) \times \left(-\frac{72}{9}\right) = \mathrm{T}_1^0 \times 56 = 24 \tag{17.2}$$

Tube 7 carries the hour hand:

$$\mathrm{T}_7^0 = \mathrm{T}_3^0 \times \left(-\frac{24}{48}\right) \times \left(-\frac{6}{72}\right) = \mathrm{T}_3^0 \times \frac{1}{24} = 1 \tag{17.3}$$

This tube makes one turn in one day. It is coupled with the central arbor 8 which also has the same period.

$$\mathrm{T}_8^0 = \mathrm{T}_7^0 = 1 \tag{17.4}$$

This is used to obtain the input motions of the two globes.

17.3 The Earth globe

The Earth globe shows the motion of the Earth with respect to the Sun. The Sun is at a fixed position, the Earth rotates around its axis in 24 hours, and its axis also rotates around the vertical axis. This revolution must be clockwise as seen from above.

The input of the Earth globe is the vertical arbor 17. Its velocity (from above) is

$$\mathrm{V}_{17}^0 = \mathrm{V}_8^0 \times \frac{57}{57} \times \left(-\frac{57}{57}\right) = -\mathrm{V}_8^0 = \mathrm{T}_8^0 = 1 \tag{17.5}$$

(the motion of arbor 8 is still viewed from the front in the above)

The arbor 17 makes one turn counterclockwise in one day as seen from above.

This arbor then moves the arbor 20 which is tilted and the actual axis of the Earth. The motion of arbor 20 is on average the same as that of arbor 17. This is the apparent motion of the Earth with respect to the Sun, resulting in a day of 24 hours.

But at the North pole, arbor 20 carries a pinion which is part of a train ending with the fixed 103-teeth wheel on the frame carrying the day-night boundary. The motion of arbor 17 causes a rotation of arbor 20 around the vertical axis. In order to understand this motion, we can examine it in the reference frame of the Earth's meridian, which I call M, but is also named 18 by Oechslin. This meridian is the circle going through the vertical axis and the axis of the Earth. Since both arbors 17 and 22 are at fixed locations on M, we have

$$\mathrm{V}_{22}^M = \mathrm{V}_{20}^M \times \left(-\frac{3}{64}\right) \times \left(-\frac{6}{103}\right) = \mathrm{V}_{20}^M \times \frac{9}{3296} \tag{17.6}$$

$$\mathrm{V}_{17}^M = \mathrm{V}_{20}^M \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{36}\right) = \mathrm{V}_{20}^M \tag{17.7}$$

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Hence

$$\mathrm{V}_{22}^{M} = \mathrm{V}_{17}^{M} \times \frac{9}{3296} \tag{17.8}$$

$$\mathrm{V}_{M}^{22} = \mathrm{V}_{M}^{0} = \mathrm{V}_{M}^{17} \times \frac{9}{3296} \tag{17.9}$$

The ratio $\frac{9}{3296}$ also appears in the Winterthur tellurium (Oechslin 8.11).

Now, we have

$$\mathrm{V}_{M}^{17} = \mathrm{V}_{M}^{0} - \mathrm{V}_{17}^{0} \tag{17.10}$$

and therefore

$$\mathrm{V}_{M}^{0} = \left(\mathrm{V}_{M}^{0} - \mathrm{V}_{17}^{0}\right) \times \frac{9}{3296} \tag{17.11}$$

$$\mathrm{V}_{M}^{0} \times \left(1 - \frac{9}{3296}\right) = -\mathrm{V}_{17}^{0} \times \frac{9}{3296} \tag{17.12}$$

$$\mathrm{V}_{M}^{0} \times \frac{3287}{3296} = -\mathrm{V}_{17}^{0} \times \frac{9}{3296} \tag{17.13}$$

and thus

$$\mathrm{V}_{M}^{0} = -\mathrm{V}_{17}^{0} \times \frac{9}{3287} = -\frac{9}{3287} \tag{17.14}$$

$$\mathrm{P}_{M}^{0} = -\frac{3287}{9} = -365.2222\ldots \text{ days} \tag{17.15}$$

The meridian circle M makes one turn in one year. The same value is given by Oechslin. The motion is clockwise (from above), as expected. It is however a rather bad approximation of the tropical year.

Incidentally, it is also the same value as the one used in the Winterthur tellurium.

17.4 The celestial globe

The celestial globe as a structure very similar to that of the Earth globe. It shows the motion of the sky, and there are two hands rotating around the globe, one for the Sun and one for the Moon. Moreover, a small lunar globe is fixed on top of the celestial globe.

17.4.1 The general motion of the globe

The input of the celestial globe is the vertical arbor 25. Its velocity (from above) is

$$\mathrm{V}_{25}^{0} = \mathrm{V}_{8}^{0} \times \left(-\frac{57}{58}\right) \times \frac{70}{66} \times \left(-\frac{76}{79}\right) = \mathrm{V}_{8}^{0} \times \frac{25270}{25201} = -\frac{25270}{25201} \tag{17.16}$$

This is the familiar velocity used by Hahn for the sidereal day. Thus, arbor 25 makes one turn clockwise (seen from above) in one sidereal day.

This arbor represents the axis of the Earth and the entire structure of the globe rotates with arbor 25 like the apparent sky.

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17.4.2 The motion of the Moon

A gear train at the bottom of the globe is used to move the hand of the Moon. This motion is obtain using a fixed 53-teeth wheel on the tube 26 supporting the meridian. Arbor 25 goes through this tube. We can compute the motion of the Moon (tube 28) with respect to the globe (frame 25):

$$\mathrm{V}_{28}^{25}? = \mathrm{V}_{26}^{25} \times \left(-\frac{53}{66}\right) \times \left(-\frac{4}{66}\right) = \mathrm{V}_{26}^{25} \times \frac{53}{1089} = -\mathrm{V}_{25}^{26} \times \frac{53}{1089} \tag{17.17}$$

$$= -\mathrm{V}_{25}^{0} \times \frac{53}{1089} = \frac{25270}{25201} \times \frac{53}{1089} = \frac{1339310}{27443889} \tag{17.18}$$

$$\mathrm{P}_{28}^{25}? = \frac{27443889}{1339310} = 20.4910\ldots \text{ days} \tag{17.19}$$

This is obviously wrong. Oechslin observed that the 4-leaves/4-pin pinion should actually have had only 3 pins. In that case, we have

$$\mathrm{V}_{28}^{25} = \mathrm{V}_{26}^{25} \times \left(-\frac{53}{66}\right) \times \left(-\frac{3}{66}\right) = \mathrm{V}_{26}^{25} \times \frac{53}{1452} = -\mathrm{V}_{25}^{26} \times \frac{53}{1452} \tag{17.20}$$

$$= -\mathrm{V}_{25}^{0} \times \frac{53}{1452} = \frac{25270}{25201} \times \frac{53}{1452} = \frac{669655}{18295926} \tag{17.21}$$

$$\mathrm{P}_{28}^{25} = \frac{18295926}{669655} = 27.3214\ldots \text{ days} \tag{17.22}$$

This is now a good approximation of the tropical month, the motion of the Moon with respect to the stars. This ratio was also used in the double-globe clock in Karlsruhe (Oechslin 8.8), also from 1785.

Incidentally, this value is also given by Oechslin, in sidereal days, and then in days, hours, minutes, and seconds, but with an incorrect conversion (27.425075) to its decimal representation. The same error appears in the Karlsruhe double globe clock (Oechslin 8.8), but the conversion was correctly made for the double globe clock in Villingen (Oechslin 8.9).

17.4.3 The motion of the Sun

Similarly, a gear train at the top of the globe is used to move the hand of the Sun. This motion is obtain using a fixed 3-teeth/3-pin wheel on the meridian frame. We can compute the motion of the Sun (tube 30) with respect to the globe (frame 25):

$$\mathrm{V}_{30}^{25} = \mathrm{V}_{26}^{25} \times \left(-\frac{3}{64}\right) \times \left(-\frac{6}{103}\right) = \mathrm{V}_{26}^{25} \times \frac{9}{3296} = -\mathrm{V}_{25}^{26} \times \frac{9}{3296} \tag{17.23}$$

$$= -\mathrm{V}_{25}^{0} \times \frac{9}{3296} = \frac{25270}{25201} \times \frac{9}{3296} = \frac{113715}{41531248} \tag{17.24}$$

$$\mathrm{P}_{30}^{25} = \frac{41531248}{113715} = 365.2222\ldots \text{ days} \tag{17.25}$$

This is an approximation of the tropical year and the Sun does indeed move counterclockwise with respect to the stars. The same value was given by

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Oechslin. Interestingly, though, this value is close but not identical to the one used in the Earth globe.

17.4.4 The motion of the lunar globe

Finally, we consider the motion of the small lunar globe on top of the celestial sphere. This motion is obtained through a 12-teeth pinion located at the top of the meridian ring, but which moves with the entire globe frame. This pinion thus makes one turn clockwise in one sidereal day:

$$\mathrm{V}_{25}^{0} = -\frac{25270}{25201} \tag{17.26}$$

The lunar globe is on arbor 32 and its velocity is

$$\mathrm{V}_{32}^{0} = \mathrm{V}_{25}^{0} \times \left(-\frac{12}{26}\right) \times \left(-\frac{3}{41}\right) = \mathrm{V}_{25}^{0} \times \frac{18}{533} \tag{17.27}$$

$$= -\frac{25270}{25201} \times \frac{18}{533} = -\frac{454860}{13432133} \tag{17.28}$$

$$\mathrm{P}_{32}^{0} = -\frac{13432133}{454860} = -29.5302\ldots \text{ days} \tag{17.29}$$

The lunar globe makes one turn clockwise (from above) in one synodic month. Half of the Moon is hidden by a hemisphere and thus the phases of the Moon are shown.

Oechslin only gave the synodic month in sidereal days, but without clearly saying so.

It should be observed that the ratio $\frac{18}{533}$ and this synodic month value were also used in the globe clocks of Stuttgart (Oechslin 8.5, 1770) and Darmstadt (Oechslin 8.6, 1785).

17.5 References

[1] Max Engelmann. *Leben und Wirken des württembergischen Pfarrers und Feintechnikers Philipp Matthäus Hahn.* Berlin: Richard Carl Schmidt & Co., 1923.

[2] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert.* Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

[3] Christian Väterlein, editor. *Philipp Matthäus Hahn 1739-1790 — Pfarrer, Astronom, Ingenieur, Unternehmer. Teil 1: Katalog*, volume 6 of *Quellen und Schriften zu Philipp Matthäus Hahn.* Stuttgart: Württembergisches Landesmuseum, 1989.

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Chapter 18

(Oechslin: 8.2)

Hahn's *Weltmaschine* in Nuremberg (c1770-1790)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

18.1 Introduction

This "world machine", or *Weltmaschine*, was designed by Philipp Matthäus Hahn (1739-1790)[1] in the 1770s, as attested by Erhard Friedrich Schoenhardt's watercolor paintings from c1779 (figures 18.1, 18.5, 18.6, 18.7, and 18.8), and further developped until Hahn's death. In the 1770s, the machine didn't include the planet Uranus which was only discovered in 1781 by William Herschel.

The current version of the machine does incorporate Uranus, and it must have been extended during Hahn's later years. It is certainly one of the first orreries with Uranus.[2]

A manuscript by Hahn describing this machine is reproduced in section 18.7. This manuscript does not give the details of the gears, but gives a good overview of the machine. It also provides useful informations on Hahn's astronomical sources.

After the death of Hahn, the machine had a complicated history.[3] It first

[1]For biographical details on Hahn, I refer the reader to the chapter on the Ludwigsburg *Weltmaschine* (Oechslin 8.1).

[2]However, I do not believe that it is the first one. An incomplete orrery by Engelbert Seige (Oechslin 10.3) containing a train for Uranus was probably made in 1781 or 1782.

[3]On the history of this machine, see especially the 1989 exhibition catalogue [19, p. 383-392, pl .8-11]. and also Engelmann [4, p. 166-172], Zinner [22, p. 353], King [7, p. 237-239], the Behaim Globe exhibition catalogue [20, p. 556-557] and Schaffer [15]. The gift of Hahn's machine to the Chinese Emperor Qianlong when the first British embassy (Macartney Embassy) was established (1792-1794) is described in a number of sources. The official

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came in possession of Ernest Albert Henry de Mylius (1749-1805), a German general, who published a description of the machine based on Hahn's manuscript [12] (see §18.7). In 1791, the machine was exhibited in London and was bought from Mylius by the East India Company. The machine was then recased and refurbished by the British watchmaker Vulliamy of London[4] and in 1793 the machine was given as a gift from the British King George III (1738-1820) to the Chinese emperor Qianlong (1711-1799) (figure 18.2).[5] But in 1806 it came back through an unknown path to Switzerland in a ruinous condition. In 1878, the *Germanisches Nationalmuseum* in Nuremberg bought the machine, but it could only be reconstructed in 1985. Engelmann illustrated the appearance of the machine in 1923 (figure 18.3). In 1977, the appearance was almost the same, but with some slight simplifications [11, p. 245]. The machine was until recently exhibited in the museum, but it is currently (2025) in storage.

account of the embassy is by Staunton [17] with the description of Hahn's machine. See also [5, p. 25], [16, p. 88], [13, p. 55-57, 233], [14, p. LXII], [6, p. 122-123], [21, p. 85], [1, p. 17], [8, p. 148-151], [18, p. 194], [2, p. 36-37], [9, p. 150]. An orrery is also described by Czennia and Clingham as being that of Fraser, but the description probably applies to Hahn's machine [3, p. 205].

[4][16, p. 88]

[5]Schaffer reproduces Mylius' engraving of Hahn's machine as it was refurbished [15]. Chen reproduces a watercolor of the same machine [2, p. 36-37].

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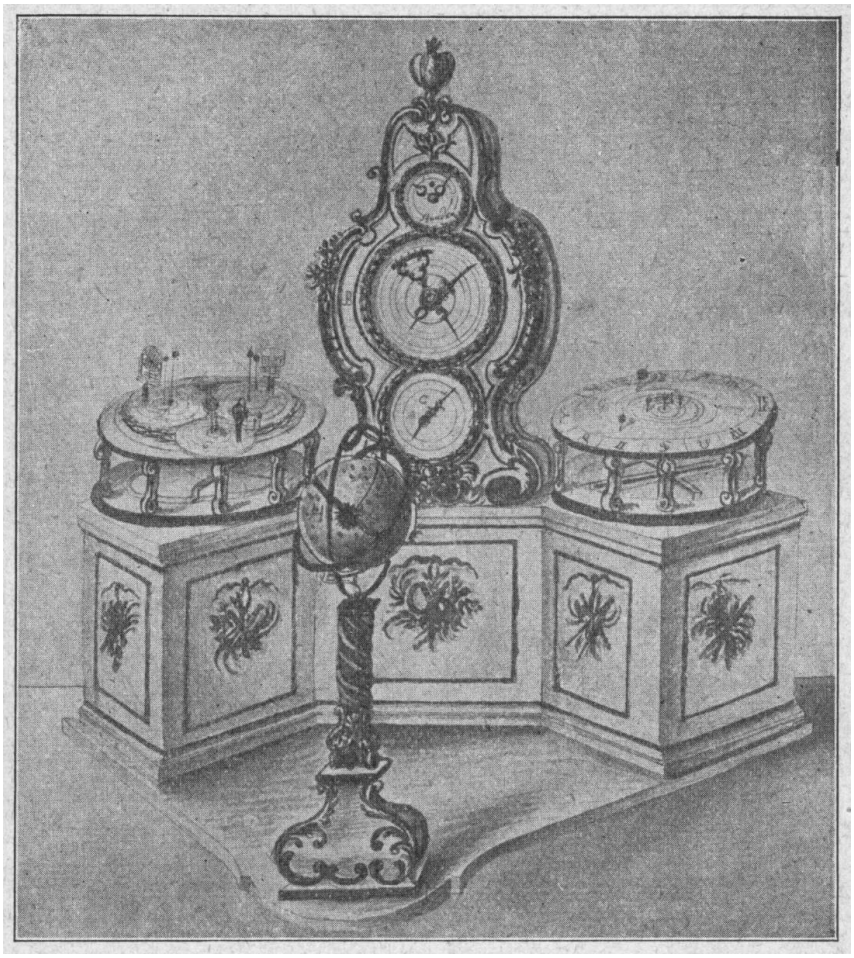


Figure 18.1: Hahn's "Weltmaschine" as it was originally (watercolor painting, Schoenhardt, c1779). (source: [4])

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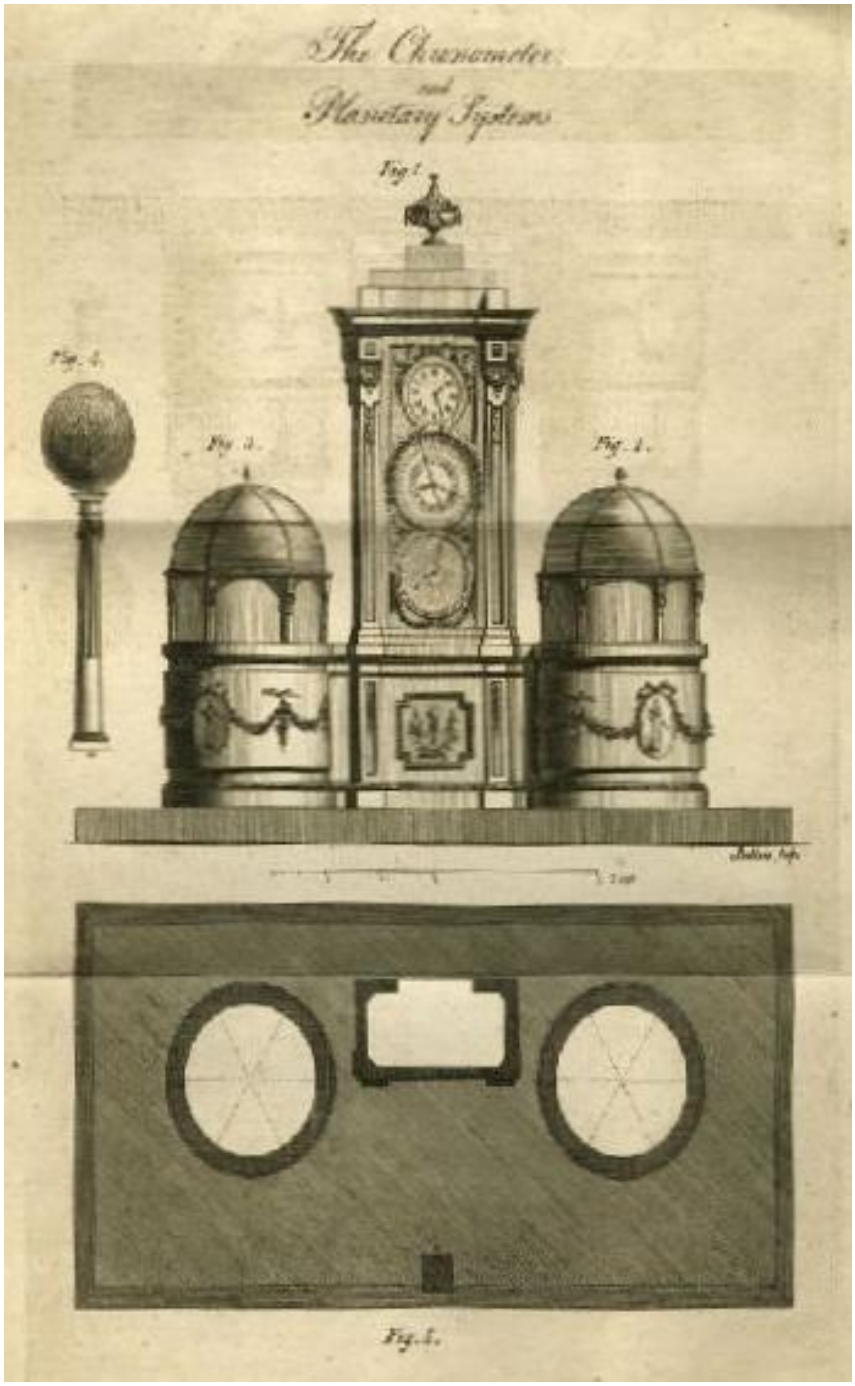


Figure 18.2: Hahn's "Weltmaschine" after it was refurbished for the Chinese Emperor (source: [12])

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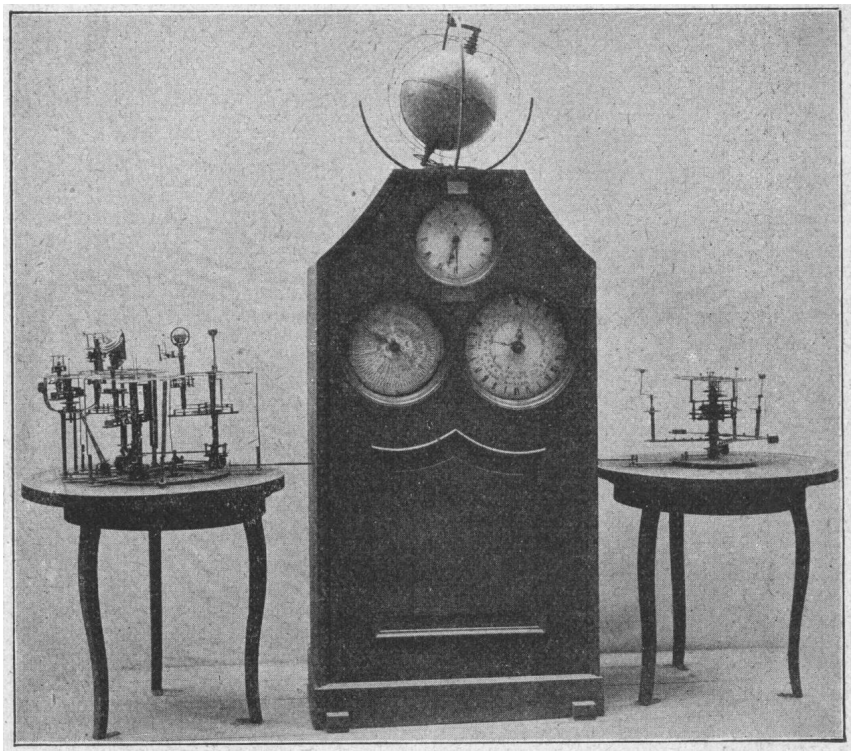
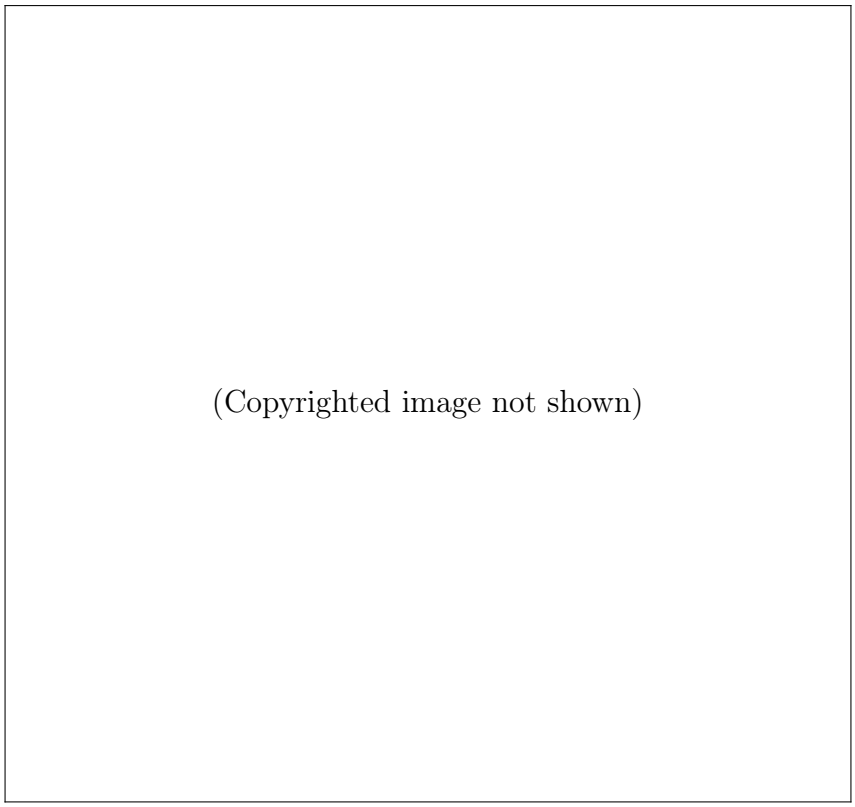


Figure 18.3: The state of Hahn's "Weltmaschine" in 1923. (source: [4])

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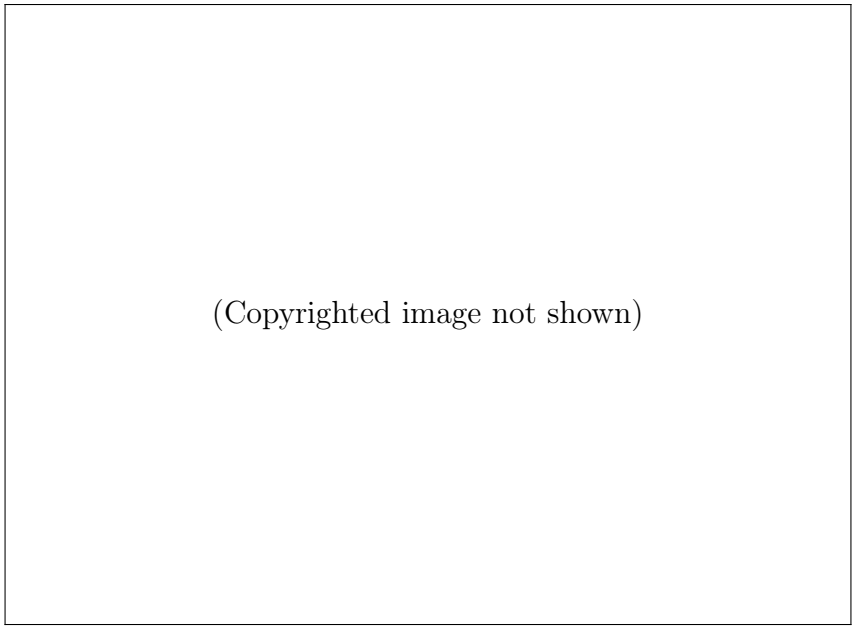


Figure 18.4: Hahn's "Weltmaschine" in Nuremberg as it currently appears (source: `https://www.gnm.de/objekte/weltmaschine`)

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This machine is essentially made of five separate parts which are interconnected. The first two parts are the clockwork and the calendar part which occupy three dials. The third part is a planetarium or orrery. The fourth part is the celestial globe. The fifth part shows separate views of the various satellite systems.

I will describe these parts in that order. This order hasn't been chosen randomly, though. The motion starts with the clockwork and is then led to the calendar part. The three other parts are derived from the motion of the clockwork, but the orrery has the simplest structure. This is why I will examine it first. The celestial globe will be examined next, because its structure has obviously been derived from the orrery, and not the opposite. Finally, I will examine the last part, which exhibits a number of problems.

18.2 The clockwork

The clockwork was spring-driven and regulated by a pendulum. The first wheel (on arbor 1) made one turn in 52.5 hours, a little more than two days. Its velocity (in turns/day) is

$$\mathrm{T}_1^0 = \frac{16}{35} \tag{18.1}$$

and the motion is clockwise. This velocity is determined by the pendulum which used to make one half-oscillation in one second.

The second arbor moves counterclockwise with the velocity

$$\mathrm{T}_2^0 = \mathrm{T}_1^0 \times \left(-\frac{120}{16}\right) = \frac{16}{35} \times \left(-\frac{120}{16}\right) = -\frac{24}{7} \tag{18.2}$$

Then, the third arbor moves again clockwise with the velocity

$$\mathrm{T}_3^0 = \mathrm{T}_2^0 \times \left(-\frac{84}{12}\right) = \left(-\frac{24}{7}\right) \times \left(-\frac{84}{12}\right) = 24 \tag{18.3}$$

In other words, this arbor makes 24 turns in a day, hence one turn per hour. The minute hand is fixed to this arbor.

Using the intermediate arbor 4, the motion is transferred to arbor 5, the escapement wheel arbor. This arbor has a velocity

$$\mathrm{T}_5^0 = \mathrm{T}_3^0 \times \left(-\frac{80}{12}\right) \times \left(-\frac{72}{8}\right) = 1440 \tag{18.4}$$

In other words, this arbor makes one turn in 60 seconds and to it is attached the seconds hand on the small dial within the upper dial. The escape wheel has 30 teeth and the pendulum must have been about 1 meter long.

The arbor of the minute hand is also used to obtain the motion of the hour hand on tube 7:

$$\mathrm{T}_7^0 = \mathrm{T}_3^0 \times \left(-\frac{36}{36}\right) \times \left(-\frac{6}{72}\right) = 2 \tag{18.5}$$

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This arbor thus makes two turns in a day, hence one turn in twelve hours.

The motion of the hour hand is also transferred to the calendar dial, using three pairs of similar wheels at right angles.

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18.3 The calendar work

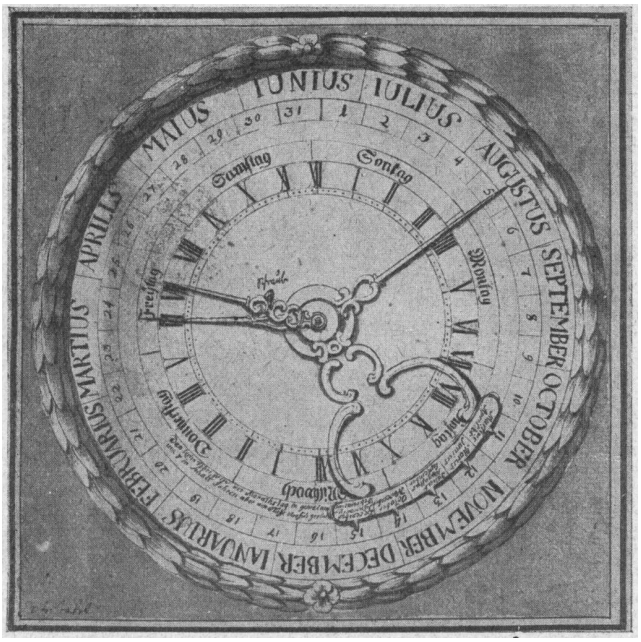


Figure 18.5: The dial for the days and months in Hahn's "Weltmaschine" as it was originally (watercolor painting, Schoenhardt, c1779). (source: [4])

The calendar work actually contains two dials. The input motion of the upper dial is that of arbor 10 originating from the clockwork. It makes one turn in 12 hours. The upper dial of the calendar work had four hands, one for the hours on a 24-hour scale, one for the day of the week, one for the day of the month, and one for the month which made one turn in a year.

The motion of the latter hand was transferred to the lower dial which had apparently one tube (25) making a turn in 100 years, and another (27) making a turn in 1000 years. However, Oechslin's plan for this part is not that clear and may be wrong.

I will not go into the details of this work, as it involves the corrections of the month lengths, and because this part was not sufficiently documented by Oechslin. I also want to focus on the astronomical parts.

However, it is necessary to explicit the transmission from the calendar part to the celestial globe. The central arbor 12 of the upper calendar dial makes one turn clockwise in one day:

$$\mathrm{T}^0_{12} = -\mathrm{V}^0_{12} = 1 \tag{18.6}$$

This motion is transferred to the vertical arbor 30 having the following velocity

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(measured positively counterclockwise from above):

$$\mathrm{V}_{30}^{0} = \mathrm{V}_{12}^{0} \times \left(-\frac{38}{44}\right) \times \left(-\frac{76}{79}\right)\left(-\frac{35}{29}\right) \tag{18.7}$$

$$= (-1) \times \left(-\frac{38}{44}\right) \times \left(-\frac{76}{79}\right)\left(-\frac{35}{29}\right) \tag{18.8}$$

$$= \frac{25270}{25201} \tag{18.9}$$

$$\mathrm{P}_{30}^{0} = \frac{25201}{25270} = 0.997269\ldots \text{ days} = 23 \text{ days } 56 \text{ mn } 4.0838\ldots \text{ s} \tag{18.10}$$

This period is also given by Oechslin. Arbor 30 therefore makes one turn in one sidereal day. The same value had also been used on the clocks of Aschaffenburg, Stuttgart, Darmstadt, and Gotha.

This motion is transferred to the other three parts of the machine. It is transferred vertically to the globe, through several pairs of similar gears at right angle. It is also transferred to the complete orrery but with an arbor 32 running horizontally at twice this speed. Finally, it is transferred to the set of satellite systems, using the velocity of the sidereal day.

I will now first describe the orrery, because its structure is relatively simple, and because must have been the basis for the other two parts of the machine.

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18.4 The orrery

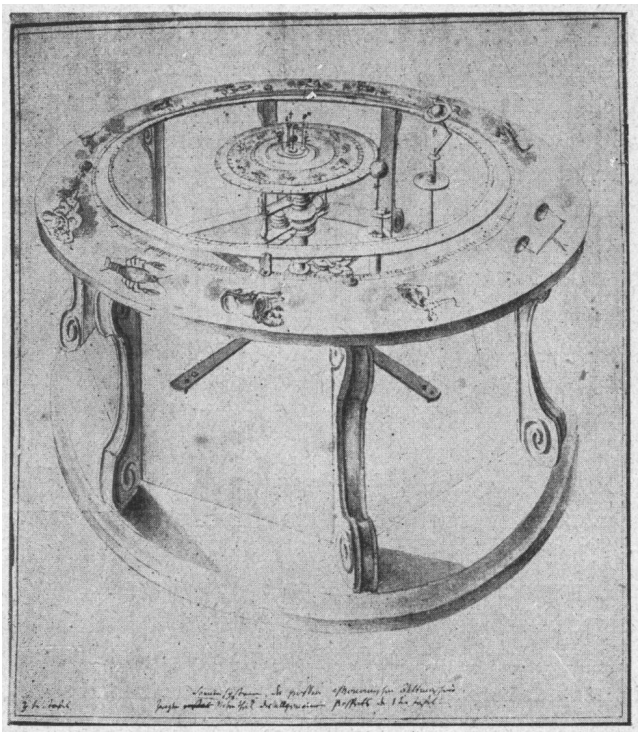


Figure 18.6: The orrery in Hahn's "Weltmaschine" as it was originally (watercolor painting, Schoenhardt, c1779). (source: [4])

The orrery is an heliocentric representation of the solar system, up to Uranus, with the Moon, but without the satellites of Jupiter, Saturn and Uranus. All the planets are given an eccentric motion, except for Venus and the Earth.

As mentioned earlier, the input of the orrery is the horizontal arbor 32 making one turn in half a sidereal day. This motion is clockwise as seen from the right on Oechslin's drawing:

$$\mathrm{V}_{32}^{0} = -2\mathrm{V}_{30}^{0} = -\frac{50540}{25201} \tag{18.11}$$

In fact, Oechslin shows the arbor with two different motions, one clockwise, and another one counterclockwise. I am assuming that the one part of the arbor reaching the orrery moves clockwise as seen from the right.

This motion is again slowed down and transferred to the vertical arbor 34 which makes one turn counterclockwise in one sidereal day:

$$\mathrm{V}_{34}^{0} = \mathrm{V}_{30}^{0} = \frac{25270}{25201} \tag{18.12}$$

The orrery is set in a frame which is not moving and to which several wheels are attached. The gear trains appear very contrived, with the motion

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transmitted to the upper stages, before descending again to Jupiter, then going up again to Saturn and Uranus. I am therefore following the train in order.

18.4.1 The mean motions of the planets and the Moon

First, the central arbor 34 goes up inside the fixed frame and reaches a 24-teeth wheel. This wheel meshes with a 53-teeth wheel on tube 35. The train is continued until the Mercury tube 38, whose velocity is

$$\mathrm{V}_{38}^0 = \mathrm{V}_{34}^0 \times \left(-\frac{24}{53}\right) \times \left(-\frac{16}{77}\right) \times \left(-\frac{45}{45}\right) \times \left(-\frac{10}{83}\right) \tag{18.13}$$

$$= \mathrm{V}_{34}^0 \times \frac{3840}{338723} = \frac{13862400}{1219451189} \tag{18.14}$$

$$\mathrm{P}_{38}^0 = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{18.15}$$

$$= 87 \text{ d } 23 \text{ h } 14 \text{ m } 17.5491\ldots \text{ s} \tag{18.16}$$

The same period is given by Oechslin. And we will see later that the same ratio is used in the celestial globe. The period is also given in Hahn's description of the machine (section 18.7).

The motion of Mercury is then used to obtain the motion of the Moon (sidereal month):

$$\mathrm{V}_{40}^0 = \mathrm{V}_{38}^0 \times \left(-\frac{101}{31}\right) \times \left(-\frac{84}{85}\right) = \mathrm{V}_{38}^0 \times \frac{8484}{2635} = \frac{23521720320}{642650776603} \tag{18.17}$$

$$\mathrm{P}_{40}^0 = \frac{642650776603}{23521720320} = 27.3215\ldots \text{ days} \tag{18.18}$$

$$= 27 \text{ d } 7 \text{ h } 43 \text{ m } 5.2949\ldots \text{ s} \tag{18.19}$$

The same period is given by Oechslin. This is also the same ratio as the one used in the celestial globe. The period is also given in Hahn's description of the machine (section 18.7).

However, tube 40 doesn't carry the Moon directly, but the Moon replicates the motion of this tube.

The motion of Mercury is also used to obtain the motion of Venus:

$$\mathrm{V}_{42}^0 = \mathrm{V}_{38}^0 \times \left(-\frac{27}{40}\right) \times \left(-\frac{29}{50}\right) = \mathrm{V}_{38}^0 \times \frac{783}{2000} \tag{18.20}$$

$$= \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{18.21}$$

$$\mathrm{P}_{42}^0 = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{18.22}$$

$$= 224 \text{ d } 16 \text{ h } 41 \text{ m } 24.6722\ldots \text{ s} \tag{18.23}$$

The same period is given by Oechslin. And this is again the same ratio as the one used in the celestial globe. The period is also given in Hahn's description of the machine (section 18.7).

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The motion of Venus is then used to obtain the one of the Earth:

$$\mathrm{V}^0_{44} = \mathrm{V}^0_{42} \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) = \mathrm{V}^0_{42} \times \frac{3239}{5265} \tag{18.24}$$

$$= \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{18.25}$$

$$\mathrm{P}^0_{44} = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{18.26}$$

$$= 365 \text{ d } 5 \text{ h } 49 \text{ m } 37.4001\ldots \text{ s} \tag{18.27}$$

The same period is given by Oechslin. And this is again the same ratio as the one used in the celestial globe. A slightly different period is given in Hahn's description of the machine (section 18.7).

Next the motion of the Earth is used to obtain the motion of Mars:

$$\mathrm{V}^0_{46} = \mathrm{V}^0_{44} \times \left(-\frac{77}{27}\right) \times \left(-\frac{11}{59}\right) = \mathrm{V}^0_{44} \times \frac{847}{1593} \tag{18.28}$$

$$= \frac{473632}{172990675} \times \frac{847}{1593} = \frac{3315424}{2277472275} \tag{18.29}$$

$$\mathrm{P}^0_{46} = \frac{2277472275}{3315424} = 686.9324\ldots \text{ days} \tag{18.30}$$

$$= 686 \text{ d } 22 \text{ h } 22.7036\ldots \text{ m} \tag{18.31}$$

The same period is given by Oechslin. And this is again the same ratio as the one used in the celestial globe. A slightly different period is given in Hahn's description of the machine (section 18.7).

The motion of Mars is then used to obtain the motion of Jupiter:

$$\mathrm{V}^0_{48}? = \mathrm{V}^0_{46} \times \left(-\frac{60}{68}\right) \times \left(-\frac{17}{89}\right) = \mathrm{V}^0_{46} \times \frac{15}{89} \tag{18.32}$$

$$= \frac{3315424}{2277472275} \times \frac{15}{89} = \frac{3315424}{13513002165} \tag{18.33}$$

$$\mathrm{P}^0_{48}? = \frac{13513002165}{3315424} = 4075.7991\ldots \text{ days} \tag{18.34}$$

This appears to be wrong, but Oechslin indicated that the 17-teeth pinion should actually have had 16 teeth.[6] In that case, we would have had

$$\mathrm{V}^0_{48} = \mathrm{V}^0_{46} \times \left(-\frac{60}{68}\right) \times \left(-\frac{16}{89}\right) = \mathrm{V}^0_{46} \times \frac{240}{1513} \tag{18.35}$$

$$= \frac{3315424}{2277472275} \times \frac{240}{1513} = \frac{53046784}{229721036805} \tag{18.36}$$

$$\mathrm{P}^0_{48} = \frac{229721036805}{53046784} = 4330.5365\ldots \text{ days} \tag{18.37}$$

$$= 4330 \text{ d } 12 \text{ h } 52.6274\ldots \text{ m} \tag{18.38}$$

[6] See [13, p. 204-205] on this error.

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The same period is given by Oechslin. And this is then again the same ratio as the one used in the celestial globe. A slightly different period is given in Hahn's description of the machine (section 18.7). Incidentally, in the globe, we will see below that there is no 16-teeth pinion to obtain this motion, but a 8-teeth pinion.

The motion of Jupiter is then used to obtain the motion of Saturn:

$$\mathrm{V}^0_{50} = \mathrm{V}^0_{48} \times \left(-\frac{61}{41}\right) \times \left(-\frac{26}{96}\right) = \mathrm{V}^0_{48} \times \frac{793}{1968} \tag{18.39}$$

$$= \frac{53046784}{229721036805} \times \frac{793}{1968} = \frac{4932704}{53012546955} \tag{18.40}$$

$$\mathrm{P}^0_{50} = \frac{53012546955}{4932704} = 10747.1575\ldots \text{ days} \tag{18.41}$$

$$= 10747 \text{ d } 3 \text{ h } 46.8484\ldots \text{ m} \tag{18.42}$$

The same period is given by Oechslin. And this is also the same ratio as the one used in the celestial globe. A slightly different period is given in Hahn's description of the machine (section 18.7).

Finally, the motion of Saturn is used to obtain the motion of Uranus:

$$\mathrm{V}^0_{52} = \mathrm{V}^0_{50} \times \left(-\frac{54}{54}\right) \times \left(-\frac{30}{83}\right) = \mathrm{V}^0_{50} \times \frac{30}{83} \tag{18.43}$$

$$= \frac{4932704}{53012546955} \times \frac{30}{83} = \frac{9865408}{293336093151} \tag{18.44}$$

$$\mathrm{P}^0_{52} = \frac{293336093151}{9865408} = 29733.8025\ldots \text{ days} \tag{18.45}$$

The same period is given by Oechslin. And this is also the same ratio as the one used in the celestial globe. However this period is somewhat too short, the actual tropical orbital period of Uranus being 30589 days (about 84 years), but this approximation may be explained by the fact that Uranus had only recently been discovered and that in the 1780s its revolution period was thought to be about 82 years. In Hahn's description of the machine (section 18.7), a longer period of about 82 years and 321 days is given.

It is clear here that all the motions have been obtained in sequence. This is not at all clear on the celestial globe, where the design is rather opaque. It is in fact likely that the design of the orrery preceded that of the globe, the globe being essentially a geocentric representation of the orrery.

18.4.2 The motion of the Moon

The Moon is also revolving around the Earth. We can compute the velocity of arbor 57 carrying the lunar arm, with respect to tube 44, which carries the

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Earth.

$$V_{57}^{44} = V_{40}^{44} \times \left(-\frac{60}{30}\right) \times \left(-\frac{15}{30}\right) = V_{40}^{44} \tag{18.46}$$

$$= V_{40}^{0} - V_{44}^{0} = \frac{23521720320}{642650776603} - \frac{473632}{172990675} \tag{18.47}$$

$$= \frac{7072716193376}{208861502395975} \tag{18.48}$$

$$P_{57}^{44} = \frac{208861502395975}{7072716193376} = 29.5305\ldots \text{ days} \tag{18.49}$$

This is, as expected, the synodic period of the Moon.

On the other hand, we could have noticed that arbor 57 replicates the motion of tube 40, as mentioned earlier, so that the above result is not unexpected.

Oechslin doesn't give the value of the synodic month, only that of the tropical month. We can also compute it:

$$V_{57}^{0} = V_{57}^{44} + V_{44}^{0} = \frac{7072716193376}{208861502395975} + \frac{473632}{172990675} \tag{18.50}$$

$$= \frac{23521720320}{642650776603} \tag{18.51}$$

$$P_{57}^{0} = \frac{642650776603}{23521720320} = 27.3215\ldots \text{ days} \tag{18.52}$$

18.4.3 The eccentric motions of the planets

Each planet in the orrery, except Venus and the Earth, is given an eccentric motion. To that purpose, each planet is offset on an arbor which is rotating at a certain speed.

First, Mercury's arbor carries a 21-teeth wheel which meshes with another wheel which meshes with another 21-teeth wheel on the central axis. This axis is fixed to the frame and also carries the Sun. Therefore, the arbor with the offset Mercury actually always points in the same direction. So, what Hahn achieves here is to offset the entire circular orbit by a certain amount.

The same is achieved with Mars, albeit with an additional wheel, but the result is the same.

This is still the case for Jupiter, Saturn and Uranus. In all these cases, there is a train on the planet's arm and ending with a wheel which is fixed to the fixed frame. In the case of Jupiter, there are three intermediate arbors between the arbor carrying Jupiter and the fixed arbor. In the case of Saturn, there are five intermediate arbors. And in the case of Uranus, there are seven intermediate arbors. In all these cases, the first and last wheel has the same number of teeth and this merely causes the planet's orbit to be offset, because the cranks always keep the same orientations.

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18.5 The celestial globe



Figure 18.7: The celestial globe of Hahn's "Weltmaschine" as it was originally (watercolor painting, Schoenhardt, c1779). (source: [4])

The celestial globe shows the motions of the known planets, the Sun, the Moon and the lunar nodes from a geocentric perspective, and it must therefore take into account the motion of the Earth around the Sun. The motions depicted cannot merely be the mean or true motions of the planets in longitude, but these motions must sometimes be offset because of the motion of the Earth. This makes the construction of the globe much more complex than that of a simple orrery, and it also complicates its analysis. However, as we will see, the ratios found in the globe are the same as those used in the orrery, so that it is likely that the orrery was designed first, and the heliocentric representation was transformed into a geocentric one.

Moreover, a comparison with the *Weltmaschine* of Stuttgart (Oechslin 8.1) (1769), the globe clock in Aschaffenburg (Oechslin 8.4) (1776/1777), and the *Weltmaschine* of Gotha (Oechslin 8.3) (1780), shows that the Nuremberg celestial globe greatly differs from these three and both the orrery and the celestial globe of the Nuremberg *Weltmaschine* must have been made after the previous

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three machines/clocks. Even though the construction principles are the same, the gear ratios are too different to view the Nuremberg machine as a variant of the other three. Oechslin observed that the current globe mechanism is certainly not the original one [13, p. 207-208].

The input of the celestial globe is the vertical arbor 191 making one turn in one sidereal day. This motion is clockwise as seen from above:

$$\mathrm{V}_{191}^{0} = -\mathrm{V}_{30}^{0} = -\frac{25270}{25201} \tag{18.53}$$

This is the only common base with the three machines mentioned above.

This vertical arbor also represents the axis of the Earth. The motion of the Earth is counterclockwise as seen from above, and therefore the sky seems to rotate clockwise.

The globe is made of many components, but is tied to a central arbor which is inclined by 23.5°. The axis of the globe is therefore the axis of the poles of the ecliptic. What Hahn did was to have the planets, the Sun, the Moon and the lunar nodes move around this axis, and at the time to have the entire structure rotate around the axis of the Earth. This then results in a geocentric view of the planets, the Sun, and the Moon.

18.5.1 The input motion

In order to do so, Hahn needed an additional input. On the fixed meridian frame 192, there is a small 36-teeth wheel which meshes with another 72-teeth wheel whose axis is on the rotating frame. The arbor of that axis carries another 24-teeth wheel meshing with a 106-teeth wheel on a tube 194 entering the globe. The motion of this tube with respect to the globe produces all the other motions. In the sequel, I am calling the globe reference frame G, but it is the same reference frame as arbor 191 or tube 201 which will be mentioned later. We have

$$\mathrm{V}_{194}^{G} = \mathrm{V}_{192}^{G} \times \left(-\frac{36}{72}\right) \times \left(-\frac{24}{106}\right) \tag{18.54}$$

$$= \mathrm{V}_{192}^{G} \times \frac{6}{53} = \left(-\mathrm{V}_{191}^{192}\right) \times \frac{6}{53} = \left(-\mathrm{V}_{191}^{0}\right) \times \frac{6}{53} \tag{18.55}$$

$$= \frac{25270}{25201} \times \frac{6}{53} = \frac{151620}{1335653} \tag{18.56}$$

$$\mathrm{P}_{194}^{G} = \frac{1335653}{151620} = 8.809\ldots \text{ days} \tag{18.57}$$

This motion doesn't have any particular significance.

I will compute all the velocities and periods with respect to the globe.

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18.5.2 The tropical orbital periods of Mercury and Venus

The motion of tube 194 is first transmitted to tubes 196 and 198:

$$\mathrm{V}^G_{196} = \mathrm{V}^G_{194} \times \left(-\frac{32}{77}\right) \times \left(-\frac{20}{83}\right) \tag{18.58}$$

$$= \frac{151620}{1335653} \times \frac{640}{6391} = \frac{13862400}{1219451189} \tag{18.59}$$

$$\mathrm{P}^G_{196} = \frac{1219451189}{13862400} = 87.9682\ldots \text{ days} \tag{18.60}$$

$$\mathrm{V}^G_{198} = \mathrm{V}^G_{196} \times \left(-\frac{54}{50}\right) \times \left(-\frac{29}{80}\right) \tag{18.61}$$

$$= \frac{13862400}{1219451189} \times \frac{783}{2000} = \frac{935712}{210250205} \tag{18.62}$$

$$\mathrm{P}^G_{198} = \frac{210250205}{935712} = 224.6954\ldots \text{ days} \tag{18.63}$$

These two periods are good approximations of the orbital tropical periods of Mercury and Venus. Oechslin also gives the periods in sidereal days. The actual motions of Mercury and Venus will however be corrected, as shown below, and this will require the computation of the motion of the Sun.

18.5.3 The motion of the Sun

The mean motion of the Sun is that of the 81-teeth wheel on tube 200:

$$\mathrm{V}^G_{200} = \mathrm{V}^G_{198} \times \left(-\frac{79}{65}\right) \times \left(-\frac{41}{81}\right) \tag{18.64}$$

$$= \mathrm{V}^G_{198} \times \frac{3239}{5265} = \frac{935712}{210250205} \times \frac{3239}{5265} = \frac{473632}{172990675} \tag{18.65}$$

$$\mathrm{P}^G_{200} = \frac{172990675}{473632} = 365.2427\ldots \text{ days} \tag{18.66}$$

This motion is counterclockwise with respect to the globe which represents the celestial sphere. It is an approximation, albeit not a very good one, of the tropical year. Moreover, given that the motions are shown with respect to the actual celestial sphere, we should have obtained the sidereal year. If the result of the computation is the tropical year, that means that the features of the sky are not used, only the coordinate system tied to the vernal equinox.

Oechslin doesn't give this period, but computes the motion of the Sun with respect to the globe in sidereal days. We can also compute it:

$$\frac{\mathrm{V}^G_{200}}{\mathrm{V}^{191}_0} = \frac{1502896}{550424875} \tag{18.67}$$

which gives a period of 366.242823... sidereal days, and this agrees with Oechslin's result.

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The true motion of the sun, which is the mean motion of the sun corrected by the equation of center, is given by another tube, also named 200 by Oechslin, but which I will name 200′. This tube has an oscillating motion with respect to tube 200. This motion is obtained by a crank on arbor 202 which pivots on the structure making a turn during one tropical year. Moreover, this arbor carries a 48-teeth wheel which meshes with a similar wheel fixed on the globe's frame. The motion of arbor 202 with respect to 200 can easily be computed:

$$\mathrm{V}_{202}^{200} = \mathrm{V}_{G}^{200} \times \left(-\frac{48}{48}\right) = -\mathrm{V}_{G}^{200} \tag{18.68}$$

Arbor 202 does therefore rotate with respect to 200 with exactly the same period as 200 with respect to G. The oscillation of tube 200′ does therefore have the same period of a tropical year, and given the right dimension of the crank, this gives an approximation of the equation of center. The sign of the above velocity is not relevant, since the oscillation would produce the same effect no matter the rotation direction of the crank. It is however necessary to set the wheels appropriately so that the greatest equation of center takes place at the right moment.

The motion of the Sun is transferred to a hand fixed in the lower part of the globe.

18.5.4 The lunar nodes

We can next compute the motion of the lunar nodes which correspond to tube 207. This motion is obtained by Hahn from the true motion of the Sun, it does therefore have an oscillating component, which it actually should not have. However, this oscillating component is very small and can be neglected. We have

$$\mathrm{V}_{207}^{G} = \mathrm{V}_{200'}^{G} \times \left(-\frac{144}{24}\right) \times \left(-\frac{6}{36}\right) \times \left(-\frac{14}{39}\right) \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{86}\right) \tag{18.69}$$

$$= \mathrm{V}_{200'}^{G} \times \left(-\frac{2975}{55341}\right) \approx \mathrm{V}_{200}^{G} \times \left(-\frac{2975}{55341}\right) \tag{18.70}$$

$$\approx \frac{473632}{172990675} \times \left(-\frac{2975}{55341}\right) = -\frac{56362208}{382939077807} \tag{18.71}$$

$$\mathrm{P}_{207}^{G} \approx -\frac{382939077807}{56362208} = -6794.2525\ldots \text{ days} \tag{18.72}$$

This is an approximation of the period of revolution of the lunar nodes. Oechslin also gives this period in sidereal days. The period is negative, because there is a retrogradation of the nodes.

We can observe that this value is close, but not identical to the one used for the Stuttgart machine.

The motion of the lunar nodes is transferred to a hand fixed in the lower part of the globe.

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18.5.5 The motion of the Moon

The mean motion of the Moon is that of tube 211:

$$\mathrm{V}^G_{211} = \mathrm{V}^G_{196} \times \left(-\frac{101}{31}\right) \times \left(-\frac{84}{85}\right) \tag{18.73}$$

$$= \mathrm{V}^G_{196} \times \frac{8484}{2635} = \frac{13862400}{1219451189} \times \frac{8484}{2635} = \frac{23521720320}{642650776603} \tag{18.74}$$

$$\mathrm{P}^G_{211} = \frac{642650776603}{23521720320} = 27.3215\ldots \text{ days} \tag{18.75}$$

This is a good approximation of the tropical month, that is the revolution of the Moon with respect to the zodiac. Oechslin also gives this period in sidereal days.

The actual motion of the Moon is obtained by adding an oscillation whose period is that of tube 208 with respect to tube 211. We therefore first compute the velocity of tube 208. This motion is also based on the corrected motion of the Sun:

$$\mathrm{V}^G_{208} = \mathrm{V}^G_{200'} \times \left(-\frac{144}{24}\right) \times \left(-\frac{6}{36}\right) \times \left(-\frac{14}{39}\right) \times \left(-\frac{23}{73}\right) \tag{18.76}$$

$$= \mathrm{V}^G_{200'} \times \frac{322}{2847} \approx \mathrm{V}^G_{200} \times \frac{322}{2847} \tag{18.77}$$

$$\approx \frac{473632}{172990675} \times \frac{322}{2847} = \frac{152509504}{492504451725} \tag{18.78}$$

$$\mathrm{P}^G_{208} = \approx \frac{492504451725}{152509504} = 3229.3361\ldots \text{ days} \approx 8.84 \text{ years} \tag{18.79}$$

This is an approximation of the tropical motion of the lunar apsides. This period is also given in sidereal days by Oechslin.

We can now compute the velocity of tube 208 with respect to tube 211:

$$\mathrm{V}^{211}_{208} = \mathrm{V}^G_{208} - \mathrm{V}^G_{211} \approx \frac{152509504}{492504451725} - \frac{23521720320}{642650776603} \tag{18.80}$$

$$\approx -\frac{21579926351867072}{594628697321340825} \tag{18.81}$$

$$\mathrm{P}^{211}_{208} \approx -\frac{594628697321340825}{21579926351867072} = -27.5547\ldots \text{ days} \tag{18.82}$$

This is an approximation of the anomalistic month, which is the period of the equation of center. It is not given by Oechslin.

The motion of the Moon is transferred to a hand fixed in the lower part of the globe.

18.5.6 The corrected motions of Mercury and Venus

We can now return to the motions of Mercury and Venus. We have seen above that tube 196 has the mean orbital motion of Mercury, and tube 198 the mean orbital motion of Venus.

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In the fourth compartment (from the bottom) of the globe, these two motions are combined with the true motion of the Sun. The motion of tube 200′ is transferred to that of tube 212 by the use of the auxiliary arbor 203. Now this tube 212 actually moves an entire cage containing all the gears for the motions of Mercury and Venus. This cage will be called the solar frame S below.

These two planets are actually oscillating around a mean position which is that of the Sun, mostly because of the revolution of the Earth around the Sun. However, for Venus whose orbit is almost circular, there is a simple oscillation. But for Mercury, whose orbit is very eccentric, Hahn compounded the oscillation due to the motion of the Earth with that due to Mercury's eccentric orbit.

For Venus, the oscillation is obtained from a crank on arbor 220. The velocities of this arbor with respect to the solar frame S and the globe are

$$\mathrm{V}^S_{220} = \mathrm{V}^S_{198} \times \left(-\frac{72}{36}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}^S_{198} \tag{18.83}$$

$$\mathrm{V}^G_{220} = \mathrm{V}^S_{220} + \mathrm{V}^G_S = \mathrm{V}^S_{198} + \mathrm{V}^G_S \tag{18.84}$$

$$\approx \mathrm{V}^G_{198} + \mathrm{V}^S_G + \mathrm{V}^G_S = \mathrm{V}^G_{198} - \mathrm{V}^G_S + \mathrm{V}^G_S = \mathrm{V}^G_{198} \tag{18.85}$$

Hence, Venus's motion with respect to the celestial sphere has an oscillation whose period is that of the tropical orbital motion. This is correct.

The motion of Venus with respect to the Sun, that is, the synodic motion of Venus, is

$$\mathrm{V}^S_{220} = \mathrm{V}^S_{198} = \mathrm{V}^G_{198} + \mathrm{V}^S_G \tag{18.86}$$

$$\approx \frac{935712}{210250205} - \frac{473632}{172990675} = \frac{23404352}{13666263325} \tag{18.87}$$

$$\mathrm{P}^S_{220} \approx \frac{13666263325}{23404352} = 583.9197\ldots \text{ days} \tag{18.88}$$

This is the synodic period of Venus.

Likewise, for Mercury, the main oscillation is obtained from a crank on arbor 218. The gear counts are exactly the same as for Venus. The velocities of this arbor with respect to the solar frame S and the globe are therefore

$$\mathrm{V}^S_{218} = \mathrm{V}^S_{196} \times \left(-\frac{72}{36}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}^S_{196} \tag{18.89}$$

$$\mathrm{V}^G_{218} = \mathrm{V}^S_{218} + \mathrm{V}^G_S = \mathrm{V}^S_{196} + \mathrm{V}^G_S \tag{18.90}$$

$$\approx \mathrm{V}^G_{196} + \mathrm{V}^S_G + \mathrm{V}^G_S = \mathrm{V}^G_{196} - \mathrm{V}^G_S + \mathrm{V}^G_S = \mathrm{V}^G_{196} \tag{18.91}$$

Hence, Mercury's motion with respect to the celestial sphere has an oscillation whose period is that of the tropical orbital motion. This is correct.

The main motion of Mercury with respect to the Sun, that is, the synodic

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motion of Mercury, is

$$\mathrm{V}^S_{218} = \mathrm{V}^S_{196} = \mathrm{V}^G_{196} + \mathrm{V}^S_G \tag{18.92}$$

$$\approx \frac{13862400}{1219451189} - \frac{473632}{172990675} = \frac{3420189088}{396321636425} \tag{18.93}$$

$$\mathrm{P}^S_{218} \approx \frac{396321636425}{3420189088} = 115.8771\ldots \text{ days} \tag{18.94}$$

This is the synodic period of Mercury.

Finally, the main frame of Mercury is itself given an oscillatory motion in order to account for Mercury's elliptic orbit. Mercury will move faster when it is at its perihelion and slower when it is at its aphelion. For this purpose, a second crank is located on arbor 216 and this arbor replicates the motion of arbor 214 which goes through the tube 218. And the arbor 214 does itself replicate the motion of tube 201 which is fixed on the globe. Consequently, the direction of the second crank is fixed with respect to the globe, and this corresponds to the assumption that Mercury's perihelion is fixed.

Eventually, Mercury and Venus are fixed on their tubes at the upper ecliptic pole of the globe and they move around the sphere with their mean tropical motions, with a small additional oscillation in the case of Mercury, and with a correction for the geocentric perspective.

18.5.7 The motion of Mars

The compartment devoted to the motion of Mars has three input motions. The first input is actually that of a 50-teeth wheel fixed to the globe (frame 201). A second input is that of the corrected motion of the Sun (tube 212). The third input is that of tube 222 which is obtained from the corrected motion of the Sun.

$$\mathrm{V}^G_{222} = \mathrm{V}^G_{212} \times \left(-\frac{77}{27}\right) \times \left(-\frac{22}{118}\right) = \mathrm{V}^G_{212} \times \frac{847}{1593} = \mathrm{V}^G_{200'} \times \frac{847}{1593} \tag{18.95}$$

$$\approx \mathrm{V}^G_{200} \times \frac{847}{1593} = \frac{473632}{172990675} \times \frac{847}{1593} = \frac{3315424}{2277472275} \tag{18.96}$$

$$\mathrm{P}^G_{222} \approx \frac{2277472275}{3315424} = 686.9324\ldots \text{ days} \tag{18.97}$$

This is a good approximation of Mars' tropical orbit period. Oechslin also gives this period in sidereal days.

Tube 222 actually carries an arbor 223 which causes itself the main frame $222'$ supporting Mars to oscillate. We can compute the velocity of arbor 223 with respect to tube 222, and also with respect to the globe:

$$\mathrm{V}^{222}_{223} = \mathrm{V}^{222}_{201} \times \left(-\frac{50}{50}\right) = -\mathrm{V}^{222}_{201} = \mathrm{V}^{201}_{222} = \mathrm{V}^G_{222} \tag{18.98}$$

and

$$\mathrm{V}^G_{223} = \mathrm{V}^{222}_{223} + \mathrm{V}^G_{222} = 2\mathrm{V}^G_{222} \tag{18.99}$$

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As a consequence of this construction, the frame 222′ actually accelerates during half of Mars' tropical orbit period, and slows down during the other half. Mars moves faster when it is at its perihelion and slower when it is at its aphelion.

The motion of Mars is now again corrected by an oscillatory motion in order to obtain the geocentric position of Mars which corresponds to tube 222″ (merely named Mars by Oechslin). For that purpose, the frame 222′ carries an arbor 225 with a crank. This arbor replicates the motion of tube 212 which is that of the corrected motion of the Sun. Consequently, the oscillation has the period of Mars' synodic motion. We can check this:

$$\mathrm{V}_{225}^{222'} = \mathrm{V}_{212}^{222'} = \mathrm{V}_{212}^{G} - \mathrm{V}_{222'}^{G} \approx \mathrm{V}_{212}^{G} - \mathrm{V}_{222}^{G} \approx \mathrm{V}_{200}^{G} - \mathrm{V}_{222}^{G} \tag{18.100}$$

$$\approx \frac{473632}{172990675} - \frac{3315424}{2277472275} = \frac{353329472}{275574145275} \tag{18.101}$$

$$\mathrm{P}_{225}^{222'} \approx \frac{275574145275}{353329472} = 779.9353\ldots \text{ days} \tag{18.102}$$

Like Mercury and Venus, Mars is fixed on its tube at the upper ecliptic pole of the globe and it moves around the sphere with its mean tropical motion, corrected for the elliptic motion, and for the geocentric perspective.

18.5.8 Input motions for Jupiter and Saturn

The motions of Jupiter and Saturn are obtained in similar ways. Like for Mars, their motions are corrected for their eccentricity and for the geocentric perspective.

There are two input motions. The first is that of arbor 226 and it is obtained from the true motion of Mars in longitude. We have

$$\mathrm{V}_{226}^{G} = \mathrm{V}_{222'}^{G} \times \left(-\frac{60}{30}\right) \tag{18.103}$$

$$\approx -\mathrm{V}_{222}^{G} \times 2 = -\frac{3315424}{2277472275} \times 2 = -\frac{6630848}{2277472275} \tag{18.104}$$

The second input motion is that of arbor 227 and it is derived from arbor 226:

$$\mathrm{V}_{227}^{G} = \mathrm{V}_{226}^{G} \times \left(-\frac{30}{34}\right) \approx \frac{6630848}{2581135245} \tag{18.105}$$

The motions of tubes 237 and 239 are next obtained:

$$\mathrm{V}_{237}^{G}? = \mathrm{V}_{239}^{G} = \mathrm{V}_{226}^{G} \times \left(-\frac{54}{42}\right) \times \left(-\frac{59}{77}\right) \tag{18.106}$$

$$= \mathrm{V}_{226}^{G} \times \frac{531}{539} \tag{18.107}$$

$$\approx \left(-\frac{6630848}{2277472275}\right) \times \frac{531}{539} = -\frac{947264}{330254925} \tag{18.108}$$

$$\mathrm{P}_{237}^{G}? = \mathrm{P}_{239}^{G} \approx -\frac{330254925}{947264} = -348.6408\ldots \text{ days} \tag{18.109}$$

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This period seems of no significance, but it seems that there is an error in Hahn's construction. Oechslin seems to indicate that the 42-teeth wheel on arbor 236 should actually have had 44 teeth.[7] If we replace 42 by 44, we obtain

$$\mathrm{V}^G_{237} = \mathrm{V}^G_{239} = \mathrm{V}^G_{226} \times \left(-\frac{54}{44}\right) \times \left(-\frac{59}{77}\right) \tag{18.110}$$

$$= \mathrm{V}^G_{226} \times \frac{1593}{1694} \tag{18.111}$$

$$\approx \left(-\frac{6630848}{2277472275}\right) \times \frac{1593}{1694} = -\frac{473632}{172990675} \tag{18.112}$$

$$\mathrm{P}^G_{237} = \mathrm{P}^G_{239} \approx -\frac{172990675}{473632} = -365.2427\ldots \text{ days} \tag{18.113}$$

and this is the exact same ratio as that obtained for P^G_{200} above, the only difference being that the motion is clockwise here.

18.5.9 The motion of Jupiter

The mean motion of Jupiter in longitude is that of tube 228 and it is obtained from arbor 227. We have

$$\mathrm{V}^G_{228} = \mathrm{V}^G_{227} \times \left(-\frac{8}{89}\right) \approx \frac{6630848}{2581135245} \times \left(-\frac{8}{89}\right) \tag{18.114}$$

$$\approx -\frac{53046784}{229721036805} \tag{18.115}$$

$$\mathrm{P}^G_{228} \approx -\frac{229721036805}{53046784} = -4330.5365\ldots \text{ days} \tag{18.116}$$

Oechslin also gives this period in sidereal days. The value is negative, because tube 228 moves clockwise with respect to the globe, but the 90-teeth wheel on arbor 228″, and whose mean motion is that of tube 228, meshes with a similar wheel on tube 244, which is that of Jupiter. Jupiter therefore has a mean counterclockwise motion of period about 4330.5365 days. This is Jupiter's tropical orbit period.

Hahn first adds an oscillation in order to take into account Jupiter's elliptic orbit. He does so as he did for Mars, with a crank moving on a wheel on arbor 243, and this wheel meshes with a similar wheel 242 fixed on the globe. The crank has the support 228′, similar to 222′ for Mars, move back and forth with respect to wheel 228. Consequently, the motion of Jupiter is accelerated during half of Jupiter's tropical orbit period, and slowed down during the other half. We can also compute this motion:

$$\mathrm{V}^G_{243} = \mathrm{V}^{228}_{243} + \mathrm{V}^G_{228} = \mathrm{V}^{228}_{242} \times \left(-\frac{35}{35}\right) + \mathrm{V}^G_{228} \tag{18.117}$$

$$= -\mathrm{V}^{228}_{242} + \mathrm{V}^G_{228} = -(\mathrm{V}^G_{242} + \mathrm{V}^{228}_G) + \mathrm{V}^G_{228} \tag{18.118}$$

$$= 2\mathrm{V}^G_{228} \tag{18.119}$$

[7] See [13, p. 204-205] on this error.

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This is similar to the motion of arbor 223 in the case of Mars.

Finally, Hahn adds a second oscillation in order to take into account the geocentric view. He does this as follows. We have seen above that the tube 239 has the motion of the tropical year, but clockwise. A 24-teeth wheel on that tube meshes with a similar wheel on tube 240, which again meshes with another such wheel on arbor 241. The latter carries a crank which causes the 90-teeth wheel on arbor 228″ to oscillate. The orientation of arbor 241 replicates that of tube 239.

We can compute the velocity of arbor 241 with respect to the support 228′, as we did compute the velocity of arbor 225 with respect to the frame 222′ in the case of Mars:

$$\begin{aligned}
\mathrm{V}_{241}^{228'} &= \mathrm{V}_{239}^{228'} \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) = \mathrm{V}_{239}^{228'} && (18.120)\\
&= \mathrm{V}_{239}^{G} - \mathrm{V}_{228'}^{G} \approx \mathrm{V}_{239}^{G} - \mathrm{V}_{228}^{G} && (18.121)\\
&\approx -\frac{473632}{172990675} + \frac{53046784}{229721036805} && (18.122)\\
&\approx -\frac{348424065376}{138981227267025} && (18.123)\\
\mathrm{P}_{241}^{228'} &\approx -\frac{138981227267025}{348424065376} = -398.8852\ldots \text{ days} && (18.124)
\end{aligned}$$

This is the synodic period of Jupiter.

If we compute the motion of arbor 241 with respect to G, we find

$$\begin{aligned}
\mathrm{V}_{241}^{G} &= \mathrm{V}_{241}^{228'} + \mathrm{V}_{228'}^{G} \approx \mathrm{V}_{241}^{228'} + \mathrm{V}_{228}^{G} && (18.125)\\
&\approx -\frac{348424065376}{138981227267025} - \frac{53046784}{229721036805} && (18.126)\\
&\approx -\frac{473632}{172990675} && (18.127)
\end{aligned}$$

that is exactly the mean motion of the Sun.

The crank on arbor 241 then moves the wheel on arbor 228″ back and forth and this wheel meshes with Jupiter's final tube 244.

In summary, wheel 228 represents the mean motion of Jupiter in longitude, but with the opposite sign (clockwise instead of counterclockwise). Then this motion is corrected for the elliptic motion, resulting in the motion of frame 228′. Finally, the motion is corrected a second time for the motion of the Earth, resulting in the motion of wheel 228″. This motion is then flipped into that of tube 244 which is counterclockwise.

The hand of Jupiter is fixed on its tube at the northern pole of the ecliptic.

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18.5.10 The motion of Saturn

The mean motion of Saturn in longitude is that of tube 232. It is obtained from arbor 228. We have

$$\mathrm{V}^G_{232} = \mathrm{V}^G_{228} \times \left(-\frac{61}{41}\right) \times \left(-\frac{41}{41}\right) \times \left(-\frac{26}{26}\right) \times \left(-\frac{26}{96}\right) \tag{18.128}$$

$$= \mathrm{V}^G_{228} \times \frac{793}{1968} \tag{18.129}$$

$$\approx \left(-\frac{53046784}{229721036805}\right) \times \frac{793}{1968} = -\frac{4932704}{53012546955} \tag{18.130}$$

$$\mathrm{P}^G_{232} \approx -\frac{53012546955}{4932704} = -10747.1575\ldots \text{ days} \tag{18.131}$$

This is an approximation of Saturn's tropical orbit period. Oechslin also gives this period in sidereal days.

Hahn first adds an oscillation in order to take into account Saturn's elliptic orbit. He does so as he did for Mars and Jupiter, with a crank moving on a wheel on arbor 248, and this wheel meshes with a similar wheel 247 fixed on the globe. Consequently, the motion of the support 232′ is accelerated during half of Saturn's tropical orbit period, and slowed down during the other half. There is here also a construction error, namely that one of the wheels has 42 teeth, whereas the other one has 44 teeth. They should have the same number of teeth. This was not marked by Oechslin on his plan.

Finally, Hahn adds a second oscillation in order to take into account the geocentric view. This is done like for Jupiter. A crank on arbor 246 replicates the motion of tube 237.

We can compute the velocity of arbor 246 with respect to the support 232′, as we did compute the velocity of arbor 225 with respect to the frame 222′ in the case of Mars, and that of arbor 241 with respect to the frame 228′ in the case of Jupiter:

$$\mathrm{V}^{232'}_{246} = \mathrm{V}^{232'}_{237} \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) = \mathrm{V}^{232'}_{237} \tag{18.132}$$

$$= \mathrm{V}^G_{237} - \mathrm{V}^G_{232'} \approx \mathrm{V}^G_{237} - \mathrm{V}^G_{232} \tag{18.133}$$

$$\approx -\frac{473632}{172990675} + \frac{4932704}{53012546955} \tag{18.134}$$

$$\approx -\frac{1102756392128}{416943681801075} \tag{18.135}$$

$$\mathrm{P}^{228'}_{246} \approx -\frac{416943681801075}{1102756392128} = -378.0922\ldots \text{ days} \tag{18.136}$$

This is the synodic period of Saturn.

The crank on arbor 246 then moves the wheel 232″ back and forth and this wheel meshes with Saturn's final tube 249.

In summary, wheel 232 represents the mean motion of Saturn in longitude, but with the opposite sign (clockwise instead of counterclockwise). Then this

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motion is corrected for the elliptic motion, resulting in the motion of frame 232′. Finally, the motion is corrected a second time for the motion of the Earth, resulting in the motion of wheel 232″. This motion is then flipped into that of tube 249 which is counterclockwise.

The hand of Saturn is fixed on its tube at the northern pole of the ecliptic.

18.5.11 The motion of Uranus

The motion of Uranus is simpler and obtained from the mean motion of Saturn on tube 232. That motion is replicated in the last compartment on arbor 234. Therefore

$$\mathrm{V}^G_{234} = \mathrm{V}^G_{232} \tag{18.137}$$

and

$$\mathrm{V}^G_{235} = \mathrm{V}^G_{234} \times \left(-\frac{30}{83}\right) \approx \left(-\frac{4932704}{53012546955}\right) \times \left(-\frac{30}{83}\right) \tag{18.138}$$

$$\approx \frac{9865408}{293336093151} \tag{18.139}$$

$$\mathrm{P}^G_{235} = \frac{293336093151}{9865408} = 29733.8025\ldots \text{ days} \tag{18.140}$$

This is an approximation of Uranus' tropical orbit period, but, as mentioned earlier, it is somewhat too short, because at that time the revolution was thought to be about 82 years, instead of 30589 days (about 84 years). Oechslin also gives the period in sidereal days.

Hahn did not add an oscillation for the geocentric perspective, perhaps because the oscillation due to the motion of the Earth is very small. Oechslin suggested that the second oscillation is missing because Uranus is a late addition.

The hand of Uranus is fixed on its tube at the northern pole of the ecliptic.

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18.6 The satellite systems

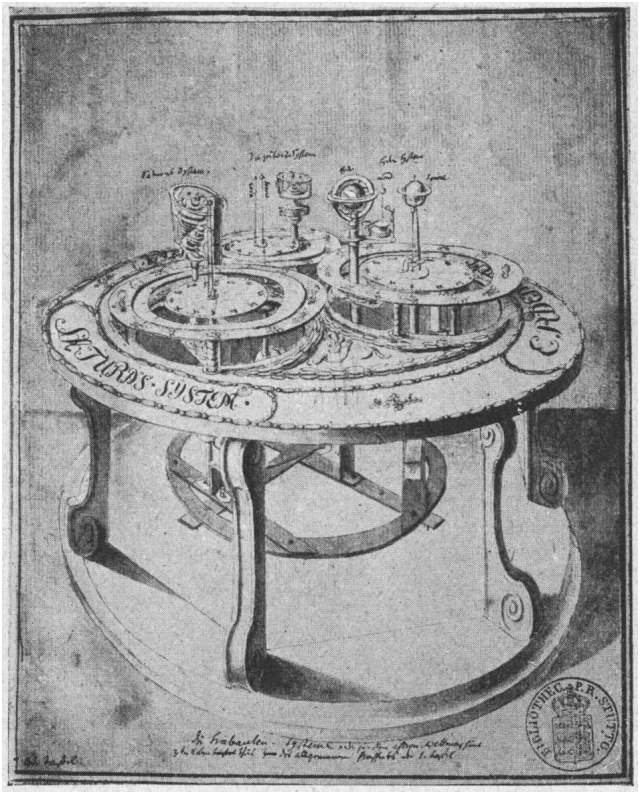


Figure 18.8: The satellite systems in Hahn's "Weltmaschine" as it was originally (watercolor painting, Schoenhardt, c1779). (source: [4])

The satellite systems show the motions of the satellites of the Earth, Jupiter, Saturn and Uranus.

The input of the satellite systems is the horizontal arbor 78 making one turn in one sidereal day. This motion is counterclockwise as seen from the right on Oechslin's drawing (the same remark as above applies here, and we might have adopted a different vantage point):

$$\mathrm{V}_{78}^{0} = \mathrm{V}_{30}^{0} = \frac{25270}{25201} \tag{18.141}$$

This motion is transferred to the four satellite systems, namely those of the Earth (tellurium), of Jupiter, Saturn and Uranus. The tellurium has only one input, but the three other systems have two inputs. One of the inputs is a rotation in one sidereal day, and the other input stems from the previous satellite system.

I will describe these four systems in order. However, it will appear below that there are errors in the connections from the tellurium to Jupiter, from Jupiter to Saturn, and from Saturn to Uranus.

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18.6.1 The tellurium

The tellurium represents the motions of the Earth and the Moon around the Sun, but also the rotation of the Sun around its axis. As I will show below, it is set in a fixed frame which represents the zodiac, not the fixed stars.

The input to the tellurium is a motion of one turn per sidereal day. It is transferred to the vertical arbor 89 which is turning clockwise as seen from above:

$$\mathrm{V}_{89}^{0} = -\mathrm{V}_{30}^{0} = -\frac{25270}{25201} \tag{18.142}$$

This motion is transmitted inside the rotating cage 94 representing the motion of the Earth around the Sun.

But the motion of the arbor 89 is also used to move the rotating cage:

$$\mathrm{V}_{94}^{0} = \mathrm{V}_{89}^{0} \times \left(-\frac{32}{32}\right) \times \left(-\frac{6}{25}\right) \times \left(-\frac{25}{25}\right) \times \left(-\frac{7}{59}\right)\left(-\frac{7}{73}\right) \tag{18.143}$$

$$= \mathrm{V}_{89}^{0} \times \left(-\frac{294}{107675}\right) = \frac{25270}{25201} \times \frac{294}{107675} = \frac{1485876}{542703535} \tag{18.144}$$

$$\mathrm{P}_{94}^{0} = \frac{542703535}{1485876} = 365.2414\ldots \text{ days} \tag{18.145}$$

Cage 94 represents the mean tropical motion of the Earth. The same period is given by Oechslin. Another wheel on tube 94 is used to transfer that same motion to the system of Jupiter.

The rotating cage 94 has two inputs: one is the motion of the central arbor 89, the other is that of a wheel on the fixed frame 107. These motions are used for the rotation of the Sun around an inclined axis, for the eccentric motion of the Earth, and for the motion of the Moon around the Earth.

18.6.1.1 The motion of the Sun and the precession of the equinoxes

The Sun has actually two motions. One is a motion around its axis, and the other is the motion of its axis around the ecliptic, or so it seems. The first is obtained by the motion of arbor 106, and the second by the motion of tube 111. We have

$$\mathrm{V}_{106}^{94} = \mathrm{V}_{89}^{94} \times \left(-\frac{37}{69}\right) \times \left(-\frac{6}{77}\right) = \mathrm{V}_{89}^{94} \times \frac{74}{1771} \tag{18.146}$$

$$= \left(\mathrm{V}_{89}^{0} - \mathrm{V}_{94}^{0}\right) \times \frac{74}{1771} = \left(-\frac{25270}{25201} - \frac{1485876}{542703535}\right) \times \frac{74}{1771} \tag{18.147}$$

$$= \left(-\frac{545675326}{542703535}\right) \times \frac{74}{1771} = -\frac{5768567732}{137303994355} \tag{18.148}$$

$$\mathrm{V}_{106}^{0} = \mathrm{V}_{106}^{94} + \mathrm{V}_{94}^{0} = -\frac{5768567732}{137303994355} + \frac{1485876}{542703535} = -\frac{5392641104}{137303994355} \tag{18.149}$$

$$\mathrm{P}_{106}^{0} = -\frac{137303994355}{5392641104} = -25.4613\ldots \text{ days} \tag{18.150}$$

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The same period is given by Oechslin.

$$\mathrm{V}_{111}^{94} = \mathrm{V}_{107}^{94} \times \left(-\frac{90}{90}\right) \times \left(-\frac{90}{46}\right) \times \left(-\frac{24}{23}\right) \times \left(-\frac{24}{49}\right) \tag{18.151}$$

$$= \mathrm{V}_{107}^{94} \times \frac{25920}{25921} = -\mathrm{V}_{94}^{107} \times \frac{25920}{25921} = -\frac{1485876}{542703535} \times \frac{25920}{25921} \tag{18.152}$$

$$= -\frac{157199616}{57418034003} \tag{18.153}$$

$$\mathrm{P}_{111}^{94} = -\frac{57418034003}{157199616} = -365.2555\ldots \text{ days} \tag{18.154}$$

This is an approximation of the sidereal year and it was not given by Oechslin. We can also compute the motion of tube 111 with respect to the absolute frame:

$$\mathrm{V}_{111}^{0} = \mathrm{V}_{111}^{94} + \mathrm{V}_{94}^{0} = -\frac{157199616}{57418034003} + \frac{1485876}{542703535} = \frac{30324}{287090170015} \tag{18.155}$$

$$\mathrm{P}_{111}^{0} = \frac{287090170015}{30324} = 9467424.1529\ldots \text{ days} \approx 26000 \text{ years} \tag{18.156}$$

This is the period of the precession of the equinoxes.

So, what Hahn has been doing is to have the frame 94 rotate with the period of the tropical year (or an approximation thereof). The fixed frame therefore represents the reference tropical frame. But because of the precession of the equinoxes, the tropical frame actually has a motion of precession with respect to the fixed stars. Conversely, if the equinoxes are fixed, the fixed stars must move counterclockwise, and this is the motion of tube 111. As we will see below, Hahn set the motions of the three other satellite systems in the tropical frame, and he did not distinguish the sidereal frame.

Hahn fixed the support of the Sun's axis on that moving frame 111, so that the orientation of the Sun's axis is assumed to be fixed with respect to the stars. And in this frame, the rotation of the Sun is given by

$$\mathrm{V}_{112}^{111} = \mathrm{V}_{106}^{111} \times \left(-\frac{25}{24}\right) = \left(\mathrm{V}_{106}^{94} - \mathrm{V}_{111}^{94}\right) \times \left(-\frac{25}{24}\right) \tag{18.157}$$

$$= \left(-\frac{5768567732}{137303994355} + \frac{157199616}{57418034003}\right) \times \left(-\frac{25}{24}\right) \tag{18.158}$$

$$= -\frac{124031078956}{3157991870165} \times \left(-\frac{25}{24}\right) = \frac{155038848695}{3789590244198} \tag{18.159}$$

$$\mathrm{P}_{112}^{111} = \frac{3789590244198}{155038848695} = 24.4428\ldots \text{ days} \tag{18.160}$$

This is the period of rotation of the Sun with respect to the stars chosen by Hahn. In fact, it is close to the period of rotation of the Sun at the equator which is about 24.47 days. Oechslin, however, obtains the slightly different value of 24.667464 days.

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We can also compute the rotation of the Sun in the tropical frame, but it is almost the same:

$$\mathrm{V}^0_{112} = \mathrm{V}^{111}_{112} + \mathrm{V}^0_{111} = \frac{155038848695}{3789590244198} + \frac{30324}{287090170015} \tag{18.161}$$

$$= \frac{775196244859}{18947951220990} \tag{18.162}$$

$$\mathrm{P}^0_{112} = \frac{18947951220990}{775196244859} = 24.4427\ldots \text{ days} \tag{18.163}$$

18.6.1.2 The elliptic motion of the Earth

As I mentioned above, the motion of frame 94 is the mean motion of the Earth in the zodiac, and its period is the tropical year. Arbor 116 is then the mean position of the Earth's axis. The actual axis of the Earth, or rather the vertical axis going through the actual center of the Earth, is arbor 118, and it is offset from arbor 116.

This arbor 118 is located on frame 114, and this frame rotates around arbor 116. We can compute the period of rotation of this frame with respect to frame 94:

$$\mathrm{V}^{94}_{114} = \mathrm{V}^{94}_{107} \times \left(-\frac{90}{90}\right) \times \left(-\frac{60}{60}\right) = \mathrm{V}^{94}_{107} \tag{18.164}$$

Since wheel 107 is fixed on the absolute frame, this means that the frame 114 has a fixed orientation. Hahn therefore has the center of the Earth move on an offset circle with the line of apsides having a fixed orientation in the zodiac. This is not entirely correct, because there is a difference between the tropical and anomalistic years, but it is a reasonable approximation. It does moreover simplify the analysis of the remaining part of the tellurium.

We can also observe that the meridian circle on which the Earth's axis is pivoting is tied to frame 114, and therefore the Earth's axis has a fixed orientation in space.

18.6.1.3 The rotation of the Earth

The frame 114 contains a number of gears and three motions are output. The central arbor 118 is the one that causes the rotation of the Earth. This motion is easy to compute:

$$\mathrm{V}^{114}_{118} = \mathrm{V}^0_{118} = \mathrm{V}^{114}_{116} \times \left(-\frac{14}{48}\right) \times \left(-\frac{48}{14}\right) = \mathrm{V}^{114}_{116} \tag{18.165}$$

Moreover, the motion of arbor 116 actually replicates the motion of arbor 89 and we saw that

$$\mathrm{V}^0_{89} = -\frac{25270}{25201} \tag{18.166}$$

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Consequently, arbor 118 has a clockwise motion (from above) whose period is a sidereal day.

The actual inclined arbor 119 of the Earth has the same motion, but counterclockwise. The Earth is making one turn with respect to the zodiac in one sidereal day.

18.6.1.4 The motion of the Moon

The second and third outputs from frame 114 are those for the Moon and the lunar nodes. The direction of the mean motion of the Moon is that of frame 121. We have

$$\mathrm{V}^0_{121} = \mathrm{V}^0_{116} \times \left(-\frac{14}{48}\right) \times \left(-\frac{71}{46}\right) \times \left(-\frac{6}{74}\right) = \mathrm{V}^0_{116} \times \left(-\frac{497}{13616}\right) \tag{18.167}$$

$$= \mathrm{V}^0_{89} \times \left(-\frac{497}{13616}\right) = \left(-\frac{25270}{25201}\right) \times \left(-\frac{497}{13616}\right) \tag{18.168}$$

$$= \frac{6279595}{171568408} \tag{18.169}$$

$$\mathrm{P}^0_{121} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{18.170}$$

This is an approximation of the tropical month, the revolution of the Moon with respect to the zodiac. The same value is given by Oechslin.

The direction of the lunar nodes is given by tube 125:

$$\mathrm{V}^0_{125} = \mathrm{V}^0_{122} \times \left(-\frac{38}{31}\right) \times \left(-\frac{19}{51}\right) \times \left(-\frac{8}{68}\right) = \mathrm{V}^0_{122} \times \left(-\frac{1444}{26877}\right) \tag{18.171}$$

$$= \mathrm{V}^0_{94} \times \left(-\frac{1444}{26877}\right) \tag{18.172}$$

$$= \frac{1485876}{542703535} \times \left(-\frac{1444}{26877}\right) = -\frac{715201648}{4862080970065} \tag{18.173}$$

$$\mathrm{P}^0_{125} = -\frac{4862080970065}{715201648} = -6798.1959\ldots \text{ days} \tag{18.174}$$

This is an excellent approximation of the period of precession of the lunar nodes. The value is negative, because it is a clockwise precession. The same value is given by Oechslin.

The motion of frame 125 is replicated on arbor 127 which carries a tilted plane causing the axis of the Moon to move up or down as it revolves around the Earth.

The Moon's mean direction is that of arbor 128. The actual Moon is offset from that arbor and is sometimes closer, sometimes farther from the Earth, in order to account for its elliptic motion. We can compute the motion of arbor

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128 with respect to the mean direction of the Moon:

$$\mathrm{V}_{128}^{121} = \mathrm{V}_{125}^{121} \times \left(-\frac{53}{23}\right) \times \left(-\frac{24}{56}\right) = \mathrm{V}_{125}^{121} \times \frac{159}{161} \tag{18.175}$$

$$= \left(\mathrm{V}_{125}^{0} - \mathrm{V}_{121}^{0}\right) \times \frac{159}{161} \tag{18.176}$$

$$= \left(-\frac{715201648}{4862080970065} - \frac{6279595}{171568408}\right) \times \frac{159}{161} \tag{18.177}$$

$$= \left(-\frac{1216404323533259}{33101047244202520}\right) \times \frac{159}{161} \tag{18.178}$$

$$= -\frac{27629755348826883}{761324086616657960} \tag{18.179}$$

$$\mathrm{P}_{128}^{121} = -\frac{761324086616657960}{27629755348826883} = -27.5544\ldots \text{ days} \tag{18.180}$$

This is an approximation of the anomalistic month. It was not given by Oechslin.

We can also compute it in the absolute frame:

$$\mathrm{V}_{128}^{0} = \mathrm{V}_{128}^{121} + \mathrm{V}_{121}^{0} = \mathrm{V}_{125}^{121} \times \frac{159}{161} + \mathrm{V}_{121}^{0} \tag{18.181}$$

$$= \left(\mathrm{V}_{125}^{0} - \mathrm{V}_{121}^{0}\right) \times \frac{159}{161} + \mathrm{V}_{121}^{0} \tag{18.182}$$

$$= \left(-\frac{715201648}{4862080970065} - \frac{6279595}{171568408}\right) \times \frac{159}{161} + \frac{6279595}{171568408} \tag{18.183}$$

$$= \left(-\frac{1216404323533259}{33101047244202520}\right) \times \frac{159}{161} + \frac{6279595}{171568408} \tag{18.184}$$

$$= \frac{10707043526711}{34605640300757180} \tag{18.185}$$

$$\mathrm{P}_{128}^{0} = \frac{34605640300757180}{10707043526711} = 3232.0444\ldots \text{ days} \tag{18.186}$$

In that case, we see that the offset of the Moon moves very slowly and this period of 3232 days, about 8.85 years, is that of the apsidal precession which is counterclockwise. The same value is given by Oechslin.

18.6.2 The system of Jupiter

The system of Jupiter is simpler than that of the Earth (tellurium). It shows the motions of Jupiter and its four satellites Io, Europa, Ganymede and Callisto (all discovered by Galileo and Simon Marius in 1610), together with the Sun and the Earth, but without the Moon, and assuming a circular orbit for the Earth.

This system has two inputs. One input is that of a rotation in one sidereal day on the central arbor 82. This motion is clockwise as seen from above:

$$\mathrm{V}_{82}^{0} = -\frac{25270}{25201} \tag{18.187}$$

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It is similar to the input of the tellurium.

The other input of Jupiter's system is originating from the tellurium. That input is that of arbor 95

$$\mathrm{V}^0_{95} = \mathrm{V}^0_{94} = \frac{1485876}{542703535} \tag{18.188}$$

This motion is counterclockwise as seen from the right.

The structure of the Jupiter system is somewhat similar to that of the tellurium. There is a rotating frame 97 for the mean tropical motion of Jupiter. Then, on this frame, another frame 137 is rotating to account for Jupiter's eccentric orbit. The system also moves the four satellites of Jupiter, as well as the Earth around the Sun.

We can thus first compute the mean motion of Jupiter:

$$\mathrm{V}^0_{97} = \mathrm{V}^0_{95} \times \left(-\frac{11}{69}\right) \times \left(-\frac{27}{54}\right) = \mathrm{V}^0_{95} \times \frac{11}{138} = \frac{1485876}{542703535} \times \frac{11}{138} \tag{18.189}$$

$$= \frac{247646}{1134743755} \tag{18.190}$$

$$\mathrm{P}^0_{97} = \frac{1134743755}{247646} = 4582.1202\ldots \text{ days} \tag{18.191}$$

This period is a bad approximation of Jupiter's tropical orbit period which should be about 4330 days, but the same value was obtained by Oechslin. It is likely that the train to Jupiter's mean motion was not correctly completed after Hahn's death.

The Earth moves with the Sun on arbor 132. Its motion around the Sun is derived from the mean motion of Jupiter.

$$\mathrm{V}^{97}_{132} = \mathrm{V}^{97}_{129} \times \left(-\frac{69}{25}\right) \times \left(-\frac{30}{15}\right) \times \left(-\frac{59}{30}\right) \tag{18.192}$$

$$= \mathrm{V}^{97}_{129} \times \left(-\frac{1357}{125}\right) = -\mathrm{V}^0_{97} \times \left(-\frac{1357}{125}\right) \tag{18.193}$$

as tube 129 is actually tied to the fixed frame 0. Incidentally, some of the wheels of this train seem to have been missing, as Oechslin put them between parentheses.

Therefore, we have

$$\mathrm{V}^{97}_{132} = \mathrm{V}^0_{97} \times \frac{1357}{125} = \frac{247646}{1134743755} \times \frac{1357}{125} = \frac{247646}{104526875} \tag{18.194}$$

$$\mathrm{P}^{97}_{132} = \frac{104526875}{247646} = 422.0818\ldots \text{ days} \tag{18.195}$$

If we had started with 4330 days, we would have obtained a period of about 398.9 days, that is the synodic period of Jupiter. This seems to confirm that the period of revolution of Jupiter is wrong.

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Moreover, the motion of the Earth around the Sun in a fixed frame would have been

$$\mathrm{V}^0_{132} = \mathrm{V}^{97}_{132} + \mathrm{V}^0_{97} \tag{18.196}$$

$$= \frac{247646}{104526875} + \frac{247646}{1134743755} = \frac{367011372}{141842969375} \tag{18.197}$$

$$\mathrm{P}^0_{132} = \frac{141842969375}{367011372} = 386.4811\ldots \text{ days} \tag{18.198}$$

This period is also incorrect, as a consequence of the incorrect motion of frame 97. The correct period should have been about 365.242 days, that is the tropical year. Oechslin also gives the same incorrect value, but does not comment upon it.

Given this obvious construction error, there is no real point computing the exact ratios for the other motions on such a basis. But since Hahn has used the same ratios in the orrery and the globe, one could assume that the correct period for the motion of the Jupiter is the one that would have been obtained from the motion of the Earth using the ratio $\frac{1593}{847} \times \frac{1513}{240} = \frac{803403}{67760}$, namely the ratio from the Earth to Mars, compounded with the one from Mars to Jupiter.

However, even this is questionable, because the tropical year in the tellurium is not the same as in the orrery and the globe. One might in fact wonder if the tellurium was made at the same time as the other two parts. Perhaps Hahn has merely tried to gather a number of separate constructions that were not initially designed to be together.

With the above assumption, we would now have

$$\mathrm{V(h)}^0_{97} = \mathrm{V}^0_{95} \times \frac{67760}{803403} = \frac{1485876}{542703535} \times \frac{67760}{803403} = \frac{610199744}{2642482715937} \tag{18.199}$$

$$\mathrm{P(h)}^0_{97} = \frac{2642482715937}{610199744} = 4330.5208\ldots \text{ days} \tag{18.200}$$

where V(h) and P(h) represent hypothetical velocities and periods.

It is however best not to make any assumption and to compute the various velocities and periods based on the two inputs taken as variables. I will therefore introduce e for the velocity of arbor 95 (normally an approximation of one turn in a tropical year), and d for the velocity of arbor 81 (normally an approximation of one turn in a sidereal day). Both of these velocities are measured positively counterclockwise as seen from the right. We therefore have

$$\mathrm{V}^0_{81} = d \tag{18.201}$$

$$\mathrm{V}^0_{95} = e \tag{18.202}$$

Moreover, the velocity of the mean motion of Jupiter will be named j, and we currently have

$$\frac{j}{e} = \left(-\frac{11}{69}\right) \times \left(-\frac{27}{54}\right) = \frac{11}{138} \tag{18.203}$$

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Therefore

$$\mathrm{V}^0_{97} = j \tag{18.204}$$

$$\mathrm{P}^0_{97} = \frac{1}{j} \tag{18.205}$$

We can now compute the motion of the Earth. As mentioned above, the Earth moves with the Sun on arbor 132. Its motion around the Sun is derived from the mean motion of Jupiter.

$$\mathrm{V}^{97}_{132} = \mathrm{V}^{97}_{129} \times \left(-\frac{69}{25}\right) \times \left(-\frac{30}{15}\right) \times \left(-\frac{59}{30}\right) \tag{18.206}$$

$$= \mathrm{V}^{97}_{129} \times \left(-\frac{1357}{125}\right) = -\mathrm{V}^0_{97} \times \left(-\frac{1357}{125}\right) \tag{18.207}$$

as tube 129 is actually tied to the fixed frame 0.

Therefore, we have

$$\mathrm{V}^{97}_{132} = \mathrm{V}^0_{97} \times \frac{1357}{125} = j \times \frac{1357}{125} \tag{18.208}$$

$$\mathrm{P}^{97}_{132} = \frac{1}{j} \times \frac{125}{1357} \tag{18.209}$$

The motion of the Earth around the Sun in a fixed frame is

$$\mathrm{V}^0_{132} = \mathrm{V}^{97}_{132} + \mathrm{V}^0_{97} = j \times \frac{1357}{125} + j = j \times \frac{1482}{125} \tag{18.210}$$

$$\mathrm{P}^0_{132} = \frac{1}{j} \times \frac{125}{1482} \tag{18.211}$$

This shows that the system of Jupiter, like those of Saturn and Uranus, incorporates a ratio for the tropical periods of Jupiter and the Earth, but because this is only an approximation, it can be misleading. For instance, if $j \approx 1/4330.5$, we have $\mathrm{P}^0_{132} \approx 365.2580$. This period would appear to be an approximation of the sidereal year. It therefore would seem that the fixed frame is that of the fixed stars. Moreover, the central frame 134 is not moving. It is made to have the same motion as tube 129 which is still.

However, except in the tellurium, it would be more natural to have the fixed frame represent the zodiac, and therefore to have the Earth revolve in a tropical year. I will therefore assume that this is what Hahn meant, the fixed frame being the zodiac. The approximation of the tropical year actually depends not only on the value of j, but also on the internal ratio 125/1482.

The eccentric motion of Jupiter is obtained through frame 137. We can compute the motion of this frame with respect to frame 97:

$$\mathrm{V}^{97}_{137} = \mathrm{V}^{97}_{129} \times \left(-\frac{69}{69}\right) \times \left(-\frac{45}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{22}{45}\right) = \mathrm{V}^{97}_{129} \tag{18.212}$$

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This frame therefore replicates the motion of tube 129 and this tube is fixed on the fixed frame. Consequently, Jupiter is always offset from its mean position towards the same direction. This is of course a simplification, and Hahn considered that Jupiter's line of apsides is fixed in the zodiac, which is not entirely correct.

Finally, we have to compute the motion of the satellites of Jupiter. The four satellites are rotating on an arbor and tubes which are fixed on frame 137, which, as we have seen, has a fixed orientation in space. These motions are obtained from a number of wheels located on a parallel axis which is also fixed on frame 137. The only input of this system is arbor 144. We can compute the motion of this arbor with respect to frame 137:

$$\begin{aligned}
\mathrm{V}_{144}^{137} &= \mathrm{V}_{141}^{137} \times \left(-\frac{27}{27}\right) \times \left(-\frac{22}{22}\right) = \mathrm{V}_{141}^{137} && (18.213)\\
&= \mathrm{V}_{141}^{97} - \mathrm{V}_{137}^{97} && (18.214)\\
&= \mathrm{V}_{82}^{97} \times \left(-\frac{45}{45}\right) \times \left(-\frac{45}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{22}{45}\right) && (18.215)\\
&\quad - \mathrm{V}_{129}^{97} \times \left(-\frac{69}{69}\right) \times \left(-\frac{45}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{22}{45}\right) && (18.216)\\
&= \mathrm{V}_{82}^{97} - \mathrm{V}_{129}^{97} = \mathrm{V}_{82}^{129} = \mathrm{V}_{82}^{0} && (18.217)
\end{aligned}$$

In other words, the motion of arbor 144 merely replicates that of the input arbor 82.

But it is easy to see that since arbor 82 moves clockwise as seen from above, arbor 144 also moves clockwise, and consequently all of Jupiter's satellites move clockwise, which is incorrect. It is however easy to fix this problem, or at least to understand its origin. The motion of the satellites is only derived from that of arbor 82, and conversely, arbor 82 is only used to produce the motion of the satellites. Consequently, the problem of the satellites is merely due to arbor 82 turning in the wrong direction. This error may have been introduced when the different satellite systems have been assembled.

Nevertheless, we can compute the periods of the satellites, and we only

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have to remember that they should turn in opposite directions. We have:

$$\mathrm{V}_{146}^{137} = \mathrm{V}_{144}^{137} \times \left(-\frac{35}{57}\right) \times \left(-\frac{56}{61}\right) = -d \times \frac{1960}{3477} \tag{18.218}$$

$$\mathrm{P}_{146}^{137} = -\frac{1}{d} \times \frac{3477}{1960} \tag{18.219}$$

$$\mathrm{V}_{148}^{137} = \mathrm{V}_{146}^{137} \times \left(-\frac{68}{68}\right) \times \left(-\frac{45}{89}\right) = -d \times \frac{1960}{3477} \times \frac{45}{89} \tag{18.220}$$

$$= -d \times \frac{29400}{103151} \tag{18.221}$$

$$\mathrm{P}_{148}^{137} = -\frac{1}{d} \times \frac{103151}{29400} \tag{18.222}$$

$$\mathrm{V}_{150}^{137} = \mathrm{V}_{148}^{137} \times \left(-\frac{94}{41}\right) \times \left(-\frac{21}{97}\right) = \mathrm{V}_{148}^{137} \times \frac{1974}{3977} \tag{18.223}$$

$$= -d \times \frac{29400}{103151} \times \frac{1974}{3977} = -d \times \frac{58035600}{410231527} \tag{18.224}$$

$$\mathrm{P}_{150}^{137} = -\frac{1}{d} \times \frac{410231527}{58035600} \tag{18.225}$$

$$\mathrm{V}_{152}^{137} = \mathrm{V}_{150}^{137} \times \left(-\frac{66}{53}\right) \times \left(-\frac{21}{61}\right) = \mathrm{V}_{150}^{137} \times \frac{1386}{3233} \tag{18.226}$$

$$= -d \times \frac{58035600}{410231527} \times \frac{1386}{3233} = -d \times \frac{80437341600}{1326278526791} \tag{18.227}$$

$$\mathrm{P}_{152}^{137} = -\frac{1}{d} \times \frac{1326278526791}{80437341600} \tag{18.228}$$

Taking $d = \frac{25270}{25201}$, we obtain

$$\mathrm{P}_{146}^{137} = -\frac{25201}{25270} \times \frac{3477}{1960} = -\frac{4611783}{2606800} = -1.7691\ldots \text{ days} \tag{18.229}$$

$$\mathrm{P}_{148}^{137} = -\frac{25201}{25270} \times \frac{103151}{29400} = -\frac{136816229}{39102000} = -3.4989\ldots \text{ days} \tag{18.230}$$

$$\mathrm{P}_{150}^{137} = -\frac{25201}{25270} \times \frac{410231527}{58035600} = -\frac{544118142733}{77187348000} = -7.0493\ldots \text{ days} \tag{18.231}$$

$$\mathrm{P}_{152}^{137} = -\frac{25201}{25270} \times \frac{1326278526791}{80437341600} = -\frac{159921268677799}{9725605848000} \tag{18.232}$$

$$= -16.4433\ldots \text{ days} \tag{18.233}$$

These are good approximations of the periods of the satellites Io, Europa, Ganymede, and Callisto. However, it should be noted that the periods used in the Furtwangen orrery constructed in 1774 (Oechslin 8.12) are more accurate than those found here.

In summary, the system of Jupiter mainly exhibits two problems. The first problem is that the train for the mean motion of Jupiter is incorrect. The second problem is that the input with the sidereal day has the opposite motion as the one it should have. If these problems were fixed, the system

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would behave reasonably well, although with different ratios as in the orrery and the celestial globe, as far as the tropical motions of Jupiter and the Earth are concerned.

18.6.3 The system of Saturn

The system of Saturn shows the motions of the five satellites Titan (discovered by Christiaan Huygens in 1655), Tethys, Dione, Rhea, and Iapetus (discovered by Giovanni Domenico Cassini between 1671 and 1684). Mimas and Enceladus, discovered by Herschel in 1789, were not included.

We can expect the system of Saturn to be fraught with the same error as the system of Jupiter, because one of that system's input is originating from Jupiter. Indeed the frame 100 corresponding to the mean motion of Saturn has the following motion:

$$\mathrm{V}^0_{100} = \mathrm{V}^0_{98} \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{58}\right) = \mathrm{V}^0_{98} \times \frac{12}{29} = \mathrm{V}^0_{97} \times \frac{12}{29} \tag{18.234}$$

$$= \frac{247646}{1134743755} \times \frac{12}{29} = \frac{2971752}{32907568895} \tag{18.235}$$

$$\mathrm{P}^0_{100} = \frac{32907568895}{2971752} = 11073.4573\ldots \text{ days} \tag{18.236}$$

This value is also given by Oechslin and is obviously false. The correct value should be about 10747 days.

If we take $\mathrm{V}^0_{97} \approx 4330.5$, we find a mean tropical orbit period of 10465 days. This is also not very good, but somehow to be expected from such a simple ratio as 12/29.

Here too, there is no real point computing the exact ratios for the other motions.

If we assume that the ratio between the periods of Jupiter and Saturn is 793/1968, namely the same as in the orrery and celestial globe, with the same caveat as the one given for the Jupiter system, we then would have

$$\mathrm{V(h)}^0_{100} = \mathrm{V(h)}^0_{98} \times \frac{793}{1968} = \mathrm{V(h)}^0_{98} \times \frac{793}{1968} = \mathrm{V(h)}^0_{97} \times \frac{793}{1968} \tag{18.237}$$

$$= \frac{610199744}{2642482715937} \times \frac{793}{1968} = \frac{30243024812}{325025374060251} \tag{18.238}$$

$$\mathrm{P(h)}^0_{100} = \frac{325025374060251}{30243024812} = 10747.1185\ldots \text{ days} \tag{18.239}$$

It is however best not to make any assumption and to compute the various velocities and periods based on the two inputs taken as variables, as I did for Jupiter. I will therefore introduce j for the velocity of arbor 98 (normally an approximation of Jupiter's tropical orbit period), and $-d$ for the velocity of arbor 87 (normally an approximation of one turn in a sidereal day). Both of these velocities are measured positively counterclockwise as seen from the right

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(therefore, arbor 87 rotates clockwise). We therefore have

$$\mathrm{V}_{87}^{0} = -d \tag{18.240}$$
$$\mathrm{V}_{98}^{0} = j \tag{18.241}$$

Moreover, the velocity of the mean motion of Saturn is named s, and we currently have

$$\frac{s}{j} = \left(-\frac{24}{24}\right) \times \left(-\frac{24}{58}\right) = \frac{12}{29} \tag{18.242}$$

Therefore

$$\mathrm{V}_{100}^{0} = s \tag{18.243}$$
$$\mathrm{P}_{100}^{0} = \frac{1}{s} \tag{18.244}$$

The analysis of the system of Saturn is pretty similar to that of Jupiter. There is a similar frame 159 with a fixed orientation for the eccentric motion of Saturn. The central part of the zodiac is also fixed, and the input arbor 163 to the five satellites also replicates the motion of the input arbor 88 and rotates clockwise as seen from above. Like for Jupiter, this causes all the satellites to rotate in the wrong direction. The main difference in this system is that the satellites all move in inclined planes, but these inclined planes have a fixed direction in space.

We can first compute the motion of the Earth. The Earth moves with the Sun on arbor 156. Its motion around the Sun is derived from the mean motion of Saturn.

$$\mathrm{V}_{156}^{100} = \mathrm{V}_{153}^{100} \times \left(-\frac{65}{18}\right) \times \left(-\frac{29}{14}\right) \times \left(-\frac{57}{15}\right) = \mathrm{V}_{153}^{100} \times \left(-\frac{7163}{252}\right) \tag{18.245}$$
$$= -\mathrm{V}_{100}^{0} \times \left(-\frac{7163}{252}\right) \tag{18.246}$$

as tube 153 is actually tied to the fixed frame 0.

Therefore, we have

$$\mathrm{V}_{156}^{100} = \mathrm{V}_{100}^{0} \times \frac{7163}{252} = s \times \frac{7163}{252} \tag{18.247}$$
$$\mathrm{P}_{156}^{100} = \frac{1}{s} \times \frac{252}{7163} \tag{18.248}$$

The motion of the Earth around the Sun in a fixed frame is

$$\mathrm{V}_{156}^{0} = \mathrm{V}_{156}^{100} + \mathrm{V}_{100}^{0} = s \times \frac{7163}{252} + s = s \times \frac{7415}{252} \tag{18.249}$$
$$\mathrm{P}_{156}^{0} = \frac{1}{s} \times \frac{252}{7415} \tag{18.250}$$

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This shows that the system of Saturn, like those of Jupiter and Uranus, incorporates a ratio for the tropical periods of Saturn and the Earth, but because this is only an approximation, it can be misleading. For instance, if $s \approx 1/10747.1$, we have $\mathrm{P}^0_{156} \approx 365.2419$. This period would appear to be an approximation of the tropical year.

Moreover, the central frame 158 is not moving. It is made to have the same motion as tube 153 which is still.

Like for Jupiter, I will assume that Hahn meant to have a fixed zodiac. The approximation of the tropical year actually depends not only on the value of s, but also on the internal ratio 252/7415.

The eccentric motion of Saturn is obtained through frame 159. We can compute the motion of this frame with respect to frame 100:

$$\mathrm{V}^{100}_{159} = \mathrm{V}^{100}_{153} \times \left(-\frac{65}{65}\right) \times \left(-\frac{61}{61}\right) = \mathrm{V}^{100}_{153} \tag{18.251}$$

This frame therefore replicates the motion of tube 153 and this tube is fixed on the fixed frame. Consequently, Saturn is always offset from its mean position towards the same direction. This is of course a simplification, and Hahn considered that Saturn's line of apsides is fixed in the zodiac, which is not entirely correct.

Finally, we have to compute the motion of the satellites of Saturn. The five satellites are rotating on an arbor and tubes which are fixed on frame 159, which, as we have seen, has a fixed orientation in space. These motions are obtained from a number of wheels located on a parallel axis which is also fixed on frame 159. The only input of this system is arbor 163. We can compute the motion of this arbor with respect to frame 159:

$$\begin{aligned}
\mathrm{V}^{159}_{163} &= \mathrm{V}^{159}_{161} \times \left(-\frac{36}{36}\right) \times \left(-\frac{36}{36}\right) = \mathrm{V}^{159}_{161} && (18.252)\\
&= \mathrm{V}^{100}_{161} - \mathrm{V}^{100}_{159} && (18.253)\\
&= \mathrm{V}^{100}_{88} \times \left(-\frac{65}{65}\right) \times \left(-\frac{65}{65}\right) - \mathrm{V}^{100}_{153} \times \left(-\frac{65}{65}\right) \times \left(-\frac{61}{61}\right) && (18.254)\\
&= \mathrm{V}^{100}_{88} - \mathrm{V}^{100}_{153} = \mathrm{V}^{153}_{88} = \mathrm{V}^{0}_{88} && (18.255)
\end{aligned}$$

In other words, the motion of arbor 163 merely replicates that of the input arbor 88.

But it is easy to see that since arbor 88 moves clockwise as seen from above, arbor 163 also moves clockwise, and consequently all of Saturn's satellites move clockwise, which is incorrect. It is however easy to fix this problem, or at least to understand its origin. The motion of the satellites is only derived from that of arbor 88, and conversely, arbor 88 is only used to produce the motion of the satellites. Consequently, the problem of the satellites is merely due to arbor 88 turning in the wrong direction. This error may have been introduced when the different satellite systems have been assembled.

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Nevertheless, we can compute the periods of the satellites, and we only have to remember that they should turn in opposite directions. We have:

$$\mathrm{V}_{165}^{159} = \mathrm{V}_{163}^{159} \times \left(-\frac{10}{39}\right) \times \left(-\frac{68}{33}\right) \tag{18.256}$$

$$= \mathrm{V}_{88}^{0} \times \frac{680}{1287} = -d \times \frac{680}{1287} \tag{18.257}$$

$$\mathrm{P}_{165}^{159} = -\frac{1}{d} \times \frac{1287}{680} \tag{18.258}$$

$$\mathrm{V}_{167}^{159} = \mathrm{V}_{165}^{159} \times \left(-\frac{22}{46}\right) \times \left(-\frac{49}{34}\right) \tag{18.259}$$

$$= -d \times \frac{680}{1287} \times \frac{539}{782} = -d \times \frac{980}{2691} \tag{18.260}$$

$$\mathrm{P}_{167}^{159} = -\frac{1}{d} \times \frac{2691}{980} \tag{18.261}$$

$$\mathrm{V}_{169}^{159} = \mathrm{V}_{167}^{159} \times \left(-\frac{27}{32}\right) \times \left(-\frac{23}{32}\right) \tag{18.262}$$

$$= -d \times \frac{980}{2691} \times \frac{621}{1024} = -d \times \frac{735}{3328} \tag{18.263}$$

$$\mathrm{P}_{169}^{159} = -\frac{1}{d} \times \frac{3328}{735} \tag{18.264}$$

$$\mathrm{V}_{171}^{159} = \mathrm{V}_{169}^{159} \times \left(-\frac{40}{36}\right) \times \left(-\frac{12}{47}\right) \tag{18.265}$$

$$= -d \times \frac{735}{3328} \times \frac{40}{141} = -d \times \frac{1225}{19552} \tag{18.266}$$

$$\mathrm{P}_{171}^{159} = -\frac{1}{d} \times \frac{19552}{1225} \tag{18.267}$$

$$\mathrm{V}_{173}^{159} = \mathrm{V}_{171}^{159} \times \left(-\frac{68}{39}\right) \times \left(-\frac{8}{69}\right) \tag{18.268}$$

$$= -d \times \frac{1225}{19552} \times \frac{544}{2691} = -d \times \frac{20825}{1644201} \tag{18.269}$$

$$\mathrm{P}_{173}^{159} = -\frac{1}{d} \times \frac{1644201}{20825} \tag{18.270}$$

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Taking $d = \frac{25270}{25201}$, we obtain

$$\mathrm{P}^{159}_{165} = -\frac{25201}{25270} \times \frac{1287}{680} = -\frac{32433687}{17183600} = -1.8874\ldots \text{ days (Tethys)} \quad (18.271)$$

$$\mathrm{P}^{159}_{167} = -\frac{25201}{25270} \times \frac{2691}{980} = -\frac{67815891}{24764600} = -2.7384\ldots \text{ days (Dione)} \quad (18.272)$$

$$\mathrm{P}^{159}_{169} = -\frac{25201}{25270} \times \frac{3328}{735} = -\frac{41934464}{9286725} = -4.5155\ldots \text{ days (Rhea)} \quad (18.273)$$

$$\mathrm{P}^{159}_{171} = -\frac{25201}{25270} \times \frac{19552}{1225} = -\frac{246364976}{15477875} = -15.9172\ldots \text{ days (Titan)} \quad (18.274)$$

$$\mathrm{P}^{159}_{173} = -\frac{25201}{25270} \times \frac{1644201}{20825} = -\frac{41435509401}{526247750} = -78.7376\ldots \text{ days (Iapetus)} \quad (18.275)$$

These are good approximations of the periods of the satellites Tethys, Dione, Rhea, Titan and Iapetus.

In summary, the system of Saturn mainly exhibits the same two problems as the system of Jupiter. The first problem is that the train for the mean motion of Saturn is incorrect. The second problem is that the input with the sidereal day has the opposite motion as the one it should have. If these problems were fixed, the system would behave reasonably well, although with different ratios as in the orrery and the celestial globe, as far as the tropical motions of Saturn and the Earth are concerned.

18.6.4 The system of Uranus

The system of Uranus shows the motions of the two satellites Titania and Oberon, which were discovered by William Herschel on January 11, 1787. But this system contains the same problems as those of Jupiter and Saturn. The mean motion of Uranus is that of frame 104. We have

$$\mathrm{V}^0_{104} = \mathrm{V}^0_{98} \times \left(-\frac{24}{58}\right) \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) \times \left(-\frac{30}{63}\right) \quad (18.276)$$

$$= \mathrm{V}^0_{98} \times \frac{40}{203} = \mathrm{V}^0_{97} \times \frac{40}{203} \quad (18.277)$$

$$= \frac{247646}{1134743755} \times \frac{40}{203} = \frac{283024}{6581513779} \quad (18.278)$$

$$\mathrm{P}^0_{104} = \frac{6581513779}{283024} = 23254.2603\ldots \text{ days} \quad (18.279)$$

Oechslin also gives this period and it is of course false. It corresponds to a period of about 64 years. Moreover, even if Saturn had had the right motion, we would have obtained a revolution for Uranus of about 149 years. Both are far from the value of 82 years that was thought to be the period in the 1780s.

For these reasons too, it does not make much sense to compute the motions of Uranus' satellites with the original ratios.

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One should also observe that the input from the Jupiter system to the Saturn system is first slowed down by a ratio 24/58 for the mean motion of Saturn, but that this same ratio is used to create the input to Uranus. So, we could actually consider that Uranus' mean motion is obtained from the mean motion of Saturn and slowing it down by the ratio 30/63, and not from the above ratio 40/203.

If we assume that the ratio between the periods of Saturn and Uranus is 30/83, namely the same as in the orrery and celestial globe, with the same caveat as the one given for the Jupiter system, we then would have

$$\mathrm{V(h)}^0_{104} = \mathrm{V(h)}^0_{100} \times \frac{30}{83} = \mathrm{V(h)}^0_{100} \times \frac{30}{83} \tag{18.280}$$

$$= \frac{30243024812}{325025374060251} \times \frac{30}{83} = \frac{302430248120}{8992368682333611} \tag{18.281}$$

$$\mathrm{P(h)}^0_{100} = \frac{8992368682333611}{302430248120} = 29733.6947\ldots \text{ days} \tag{18.282}$$

It is however best not to make any assumption and to compute the various velocities and periods based on the two inputs taken as variables. I will therefore introduce $-s$ for the velocity of arbor 101 (normally an approximation of Saturn's tropical orbit period), and d for the velocity of arbor 84 (normally an approximation of one turn in a sidereal day). Arbor 84 rotates counterclockwise as seen from above, and arbor 101 rotates clockwise as seen from the right. We therefore have

$$\mathrm{V}^0_{84} = d \tag{18.283}$$

$$\mathrm{V}^0_{101} = -s \tag{18.284}$$

Moreover, the velocity of the mean motion of Uranus is named u, and we currently have

$$\frac{u}{s} = -\left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) \times \left(-\frac{30}{63}\right) = \frac{30}{63} \tag{18.285}$$

Therefore

$$\mathrm{V}^0_{104} = u \tag{18.286}$$

$$\mathrm{P}^0_{104} = \frac{1}{u} \tag{18.287}$$

The analysis of the system of Uranus is similar to those of Jupiter and Saturn. There is a similar frame 180 with a fixed orientation for the eccentric motion of Uranus. The central part of the zodiac is also fixed, and the input arbor 184 to the two satellites also replicates the motion of the input arbor 85 and rotates clockwise as seen from above. Like for Jupiter and Saturn, this causes the two satellites to rotate in the wrong direction.

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We can first compute the motion of the Earth. The Earth moves with the Sun on arbor 177. Its motion around the Sun is derived from the mean motion of Uranus.

$$\mathrm{V}_{177}^{104} = \mathrm{V}_{174}^{104} \times \left(-\frac{48}{24}\right) \times \left(-\frac{50}{10}\right) \times \left(-\frac{58}{7}\right) = \mathrm{V}_{174}^{104} \times \left(-\frac{580}{7}\right) \tag{18.288}$$

$$= -\mathrm{V}_{104}^{0} \times \left(-\frac{580}{7}\right) \tag{18.289}$$

as tube 174 is actually tied to the fixed frame 0.

Therefore, we have

$$\mathrm{V}_{177}^{104} = \mathrm{V}_{104}^{0} \times \frac{580}{7} = u \times \frac{580}{7} \tag{18.290}$$

$$\mathrm{P}_{177}^{104} = \frac{1}{u} \times \frac{7}{580} \tag{18.291}$$

The motion of the Earth around the Sun in a fixed frame is

$$\mathrm{V}_{177}^{0} = \mathrm{V}_{177}^{104} + \mathrm{V}_{104}^{0} = u \times \frac{580}{7} + u = u \times \frac{587}{7} \tag{18.292}$$

$$\mathrm{P}_{177}^{0} = \frac{1}{u} \times \frac{7}{587} \tag{18.293}$$

If $u \approx 1/29733.8$, we have $\mathrm{P}_{177}^{0} \approx 354.5768$. The reason why this period is so different from the expected tropical year is that $u \times \mathrm{P}_{177}^{0}$ is supposed to be the ratio between the tropical year and the tropical orbit period of Uranus. For instance, in the case of Jupiter, 125/1482 is an approximation of the ratio 365.242/4330. And in the case of Saturn, 252/7415 is an approximation of the ratio 365.242/10747. This means that this ratio actually tells us what tropical orbit period was assumed.

In the case of Uranus, if we find such an unexpected value for the motion of the Earth, it is because the ratio between the tropical year and the tropical period of Uranus was assumed to be about 7/587. It should be clear here that the ratio between the tropical periods appears in two different places. First, the mean motion of Uranus is obtained from that of Saturn, and second there is a built-in ratio. The first of these ratios may have been $\frac{\mathrm{P}_{200}^{G}}{\mathrm{P}_{235}^{G}} = 33583550/2733980409$ as in the celestial globe and the orrery. And the second is 7/587. But if we compute these ratios, we find:

$$\frac{33583550}{2733980409} = 0.01228\ldots \tag{18.294}$$

$$\frac{7}{587} = 0.01192\ldots \tag{18.295}$$

But, better, if we compute the tropical period of Uranus from these two ratios

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and the tropical year, we obtain

$$365.242 \times \frac{2733980409}{33583550} \approx 29733.7 \tag{18.296}$$

$$365.242 \times \frac{587}{7} \approx 30628.1 \tag{18.297}$$

$$\tag{18.298}$$

Now, recall that 29733 days is about 82 years, the period that was thought to be that of Uranus in the 1780s. But the actual period is 30589 days. So, we can see that Hahn's machine actually contains two different periods for Uranus. There is the one which is part of the globe and the orrery, and which is the period that was supposed to be the most accurate. But now we see that within the system of Uranus, and even though the connection between the various systems is inaccurate, there is a much more accurate period of Uranus.

This period, which is "encoded" by the ratio 7/587, then only makes sense if we take $u \approx 1/30628.1$. This suggest, in any case, that the system of Uranus was designed very late by Hahn, even after the celestial globe and the orrery, but that he was not able to finalize the connection between the various satellite systems.

We can try to find out when a period nearing 30628 days was given. One possible source is that of the *Astronomisches Jahrbuch für das Jahr 1787* published in Berlin in 1784, where the period of Uranus is given as 83 common years and 292 days, that is 30587 days. This value is also given in other places, such as Johann von Kosteleztky's *Biblia Sacra* published in 1789.

So, the conclusion of this investigation is that the orrery and the celestial globe use a knowledge of the motion of Uranus that was that of the early 1780s, but that the separate system of Uranus uses more accurate data, that Hahn may have found in the Berlin ephemerides, or in a later source. This source, or another, may also have been used for the periods of Jupiter and Saturn in the systems of Jupiter and Saturn, and we have seen that these systems do not use the same ratios as the ones used in the celestial globe and the orrery.

Now, moving on with the analysis of the system of Uranus, we can see that like for Jupiter and Saturn, the central frame 179 is not moving. It is made to have the same motion as tube 174 which is still.

Like for Jupiter and Saturn, I will assume that Hahn meant to have a fixed zodiac. The approximation of the tropical year actually depends not only on the value of u, but also on the internal ratio 7/587.

The eccentric motion of Uranus is obtained through frame 180. We can compute the motion of this frame with respect to frame 104:

$$\mathrm{V}_{180}^{104} = \mathrm{V}_{174}^{104} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) = \mathrm{V}_{174}^{104} \tag{18.299}$$

This frame therefore replicates the motion of tube 174 and this tube is fixed on the fixed frame. Consequently, Uranus is always offset from its mean position towards the same direction. This is of course a simplification, and Hahn

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considered that Uranus's line of apsides is fixed in the zodiac, which is not entirely correct.

Finally, we have to compute the motion of the satellites of Uranus. The two satellites are rotating on an arbor and a tube which are fixed on frame 180, which, as we have seen, has a fixed orientation in space. These motions are obtained from a number of wheels located on a parallel axis 185 which is also fixed on frame 180. The only input of this system is arbor 184. We can compute the motion of this arbor with respect to frame 180:

$$\mathrm{V}_{184}^{180} = \mathrm{V}_{182}^{180} \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) = \mathrm{V}_{182}^{180} \tag{18.300}$$

$$= \mathrm{V}_{182}^{104} - \mathrm{V}_{180}^{104} \tag{18.301}$$

$$= \mathrm{V}_{85}^{104} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) - \mathrm{V}_{174}^{104} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) \tag{18.302}$$

$$= \mathrm{V}_{85}^{104} - \mathrm{V}_{174}^{104} = \mathrm{V}_{85}^{174} = \mathrm{V}_{85}^{0} \tag{18.303}$$

In other words, the motion of arbor 184 merely replicates that of the input arbor 85.

But it is easy to see that since arbor 85 moves clockwise as seen from above, arbor 184 also moves clockwise, and consequently all of Uranus' satellites move clockwise, which is incorrect. It is however easy to fix this problem, or at least to understand its origin. The motion of the satellites is only derived from that of arbor 85, and conversely, arbor 85 is only used to produce the motion of the satellites. Consequently, the problem of the satellites is merely due to arbor 85 turning in the wrong direction. This error may have been introduced when the different satellite systems have been assembled.

Nevertheless, we can compute the periods of the satellites, and we only have to remember that they should turn in opposite directions. We have:

$$\mathrm{V}_{186}^{180} = \mathrm{V}_{184}^{180} \times \left(-\frac{10}{40}\right) \times \left(-\frac{16}{35}\right) \tag{18.304}$$

$$= \mathrm{V}_{85}^{0} \times \frac{4}{35} = -d \times \frac{4}{35} \tag{18.305}$$

$$\mathrm{P}_{186}^{180} = -\frac{1}{d} \times \frac{35}{4} \tag{18.306}$$

$$\mathrm{V}_{187}^{180} = \mathrm{V}_{184}^{180} \times \left(-\frac{10}{40}\right) \times \left(-\frac{13}{44}\right) \tag{18.307}$$

$$= \mathrm{V}_{85}^{0} \times \frac{13}{176} = -d \times \frac{13}{176} \tag{18.308}$$

$$\mathrm{P}_{187}^{180} = -\frac{1}{d} \times \frac{176}{13} \tag{18.309}$$

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Taking $d = \frac{25270}{25201}$, we obtain

$$\mathrm{P}_{186}^{180} = -\frac{25201}{25270} \times \frac{35}{4} = -\frac{25201}{2888} = -8.7261\ldots \text{ days (Titania)} \tag{18.310}$$

$$\mathrm{P}_{187}^{180} = -\frac{25201}{25270} \times \frac{176}{13} = -\frac{2217688}{164255} = -13.5014\ldots \text{ days (Oberon)} \tag{18.311}$$

These are good approximations of the periods of the satellites Titania and Oberon.

In summary, the system of Uranus mainly exhibits the same two problems as the systems of Jupiter and Saturn. The first problem is that the train for the mean motion of Uranus is incorrect. The second problem is that the input with the sidereal day has the opposite motion as the one it should have. If these problems were fixed, the system would behave reasonably well, although with different ratios as in the orrery and the celestial globe, as far as the tropical motions of Uranus and the Earth are concerned. Indeed, as I have shown above, the system of Uranus makes the assumption of a more accurate tropical period of Uranus than the orrery and the celestial globe, and the connection between the tellurium and the other satellite systems should not merely mimic the ratios implemented in the orrery and the celestial globe.

18.6.5 The possible source of the problems

The above analysis has shown that there are several problems in the satellite systems. There are two different sets of problems.[8] A first type of problem is that the revolutions of Jupiter, Saturn and Uranus are wrong, and they are wrong because the revolution of the Earth in the tellurium has not correctly been transferred to the other three satellite systems.

If the entire construction had been by Hahn and consistent, and given that we find the same ratios in the orrery and the globe, it would be somewhat surprising to find different ratios between the satellite systems. Indeed, in the orrery and the celestial globe, the ratios are $847/1593 \times 240/1513 = \frac{67760}{803403}$ from the Earth to Jupiter (via Mars), 793/1968 from Jupiter to Saturn, and 30/83 from Saturn to Uranus. But between the satellite systems, we have the ratios 11/138, 12/29 and 30/63. The first two ratios can be viewed as bad approximations of the ratios used in the orrery and the globe, but the third ratio must be a gross error. One is tempted to assume that 83 was mistakenly read as 63.

The second type of problem is that of the motions of the satellites of Jupiter, Saturn and Uranus. Although the periods are reasonably good, it appears that all these satellites (except the Moon) rotate clockwise, whereas they should rotate counterclockwise. There is a common cause to these problems, namely that the sidereal day input velocity should have been reversed.

[8] On these problems and their likely causes, see [13, p. 205-207].

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These two sets of problems are related to the connections between the satellite systems and it is likely that these connections were only added after Hahn's death, and were computed incorrectly.

Another important observation is that the orrery and celestial globe on the one hand, and the satellite systems on the other hand, use different ratios. I have in particular shown above that in the celestial globe and the orrery, Uranus has a tropical orbit period of 82 years, but that in the separate Uranus system, a period of almost 84 years was assumed, and that independently of the connections between the satellite systems. It would therefore seem that the satellite systems are constructions made later than the other two parts, but that Hahn died before these systems could be interconnected. It is therefore possible that the systems have been interconnected based on ratios used in the orrery and the celestial globe, which they should not have.

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18.7 Hahn's description of the clock

The following description of the clock is given in Engelmann's book [4] (appendix III, p. 216-233) and is a copy of a manuscript that was kept in the Königliche Staatssammlung Vaterländischer Altertümer *in 1923 and may be kept now in the* Landesmuseum Württemberg *in Stuttgart. As the reader will notice, there are some slight differences between the periods given here and those used in the machine. A number of periods given here are in fact Hahn's sources, and not the periods used in the machine. Mylius' description [12] is an English translation of this manuscript.*

Beschreibung

Einer Astronomischen Maschine

welche

1. das ganze Copernicanische Welt-System,
2. die Trabanten-Systeme der Erde, des Jupiter, Saturn und Uranus und
3. die Bewegung der Planeten und Fixsterne, wie solche den Zuschauern von der Erde in die Augen fällt, vorstellt;

und

Nach der Anweisung und Berechnung M. Hahns, Pfarrer in Echterdingen und Mitgliede der Academie der Wissenschaften in Erfurt von dessen 2 Brüdern

Georg David Hahn
und
Gottfried Hahn

in der Herzogl. Würtembergischen Residenz-Stadt Ludwigsburg von 1787 bis 1791 verfertigt worden ist.

I.
Die äußerliche Gestalt der Maschine.

Dieses Werk hat ein hölzernes Gestell von sauberer Bildhauer Arbeit; ist im Grund weiß lackiert und mit gut vergoldeten Verzierungen versehen. Man kann dessen Theile auseinander legen und wieder zusammensetzen.

Der Fußboden hat die Gestalt eines Dreyecks dessen längste Seite 8 Schu, die 2. übrigen jede 5 Schu lang sind. Es ist 3 Zoll vom Boden erhoben, und mit einem Scharnier von Eisen versehen, den man zur Bequemlichkeit des Transports zusammen legen kann.

Auf seiner längsten Seite ist ein hohles Fußgestell 2 Schu hoch, und 2 Schu breit. Es ist nach Architektonischen Regeln gearbeitet, mit vergoldeten Tropheen in seinen Füllungen. Auch dieses kann man in 2 Theile zerlegen. Auf der

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horizontalen Fläche dieses Fuß Gestells steht in der Mitte ein zierlicher Uhrkasten von 6 Schu in der Höhe, auf welchem 3 weiß-lackierte Uhrtafeln unter einander angebracht sind. Die erste von 8 Zoll zeigt die gewöhnliche Stunden und Minuten, die zweite von 15 Zoll, enthält die beweglichen Calender; die 3te und unterste aber von 10. Zoll den Jahrzähler.

Diesen Uhrkasten zu beeden Seiten stehen auf eben diesem Fußgestell rechts und links 2 achteckige Glas-Kästen von 2½ Schu im Durchmesser und 1 Schu tief. Diese Glas-Kästen haben zwischen den Gläsern schön vergoldete Trag-Steine; oben aber sind sie mit einem 3 Zoll breiten Ring bedeckt, in dessen 2 Schu weiten Oeffnungen zur Rechten das Sonnensystem; zur Linken das Trabanten-System der Erde, des Jupiters, des Saturns und des Uranus befindlich sind.

Auf der gegenseitigen Skize des 3 eckigten Fußbodens steht eine schön geschnizte und vergoldete 3½ Schu hohe Pyramide, auf welcher eine bewegliche Himmelskugel von 1 Schu im Durchmesser zwischen einen versilberten Meridian und Horizont ruhet; welche die scheinbare Bewegung der Fix-Sterne und Planeten enthält, deren Bewegung, wie die Bewegung des Ganzen astronomischen Werks von der oben in dem Uhrkasten befindlichen Uhr bewürket wird.

Diese Theile sind also gestellt, daß man, wenn man auf dem Fuß Boden von dem Uhrkasten und beweglichen Kalender steht, den generalen und specialen Theil des Himmels-Baues zur rechten und Linken, die scheinbare Bewegung der Himmelskugel aber auf dem Rucken hat, die man bey einer kleinen Wendung des Leibes ebenfalls leicht übersehen kann.

II.
Die Uhr,
welche alles in Bewegung setzt.

Diese Uhr ist, wie alle astronomische Werke, von Stahl und Messing; hat einen Pendul, der Secunden vibriert; wir von einem Gewicht in Bewegung gesetzt, und alle 8 Tage aufgezogen.

III.
Der bewegliche Calender.

Dieser wird von der Uhr getrieben, und wenn man die Minuten-Zeiger an der Uhr treibt, so empfinden es nebst dem Calender alle astronomischen Werke. Es enthält aber der Calender 1., einen Zeiger, der in 24 Stunden seinen Umlauf vollendet. 2., zeigt er den Wochentag. 3., den Monats Tag, welcher den rechten Tag oder Datum immer angibt, der Monat mag 30 oder 31 Tage enthalten. Z.B. im Februar, der nur 28 Tage hat, überspringt der Zeiger 4 Tage, und zeigt, wie er soll den 1. Merz an. Auch ist in Schaltjahren vor die Richtigkeit gesorgt. 4., den Jahrzeiger, der die laufenden Monate weiset. 5., den Jahrzehler auf der untersten Tafel, wo ein Zeiger in 100. ein anderer in 8000 Jahren einmal seinen Umlauf vollendet. Dieser letztere weiset die erste 2 Ziffern, der erstere aber die 2 letzte.

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Dieser Zeiger dienet, die astronomischen Werke in der Ordnung zu erhalten. Denn wenn man vergeßen sollte, die Uhr aufzuziehen, so bleiben gleich bey dem Stillstand der Uhr nicht nur alle astronomischen Werke, sondern auch die die Zeit bestimmenden Zeiger des Calenders stehen, an welchem man also gleich sehen kann, in welchem Jahr, Monat, Tag, Stunde und Minute stehen geblieben sey. Nun bestehet der Stunden Zeiger des Kalenders aus 2 Zeigern, welche durch eine Schraube von außen mit einander verbunden sind. Der äußere steht mit den astronomischen Werken, der innere mit der Uhr in Verbindung. Wenn man nun durch Herumdrehung der Schraube den äußern Zeiger von den inneren los macht, un den hierzu eingerichteten Triebel in den Viereckigten, aus der Mitte der Zeiger herausgehenden Wellbaum einsteckt, so kann man den äußern Tag-Zeiger so lang herumführen, biß alle Zeit bestimmende Zeiger des Calenders auf dem gegenwärtigen Jahr, Tag und Stunde stehen. Als dann wird der äußere Zeiger wieder auf den innern angeschraubt, und der obere Minuten Zeiger wieder auf die gegenwärtige Minute gestellt, worauf alle astronomischen Werke der gegenwärtigen angenblicklichen Himmels Gestalt wieder völlig gemäß stehen. Ebenso können auch während dem Fortgang der Uhr alle astronomischen Werke vermittelst dieser Schraube von der Uhr abgelöset werden, und man kann sie nun 10. 50. 100. Jahre in die vergangene oder zukünftige Zeiten versetzen, und Beobachtungen anstellen, und sie alsdann wieder in die gegenwärtige Zeit bringen, und sie durch die Schraube mit der Quelle ihrer Bewegung verbinden. Bey dergleichen Versuchen darf man nicht die mindeste Furcht haben, als wenn etwas verdorben werden möchte, wann gleich die astronomischen Werke Tage u. Monate stille stünden. Bey allen Bewegungen der himmlischen Körper wird dadurch keine Minute verloren gehen. Man rücke nur vermittelst des abgelösten Tag-Zeigers, die übrigen Zeiger auf das gegenwärtige Jahr, Monat, Tag u. Stunde; schraube beede Zeiger wieder vest auf einander; gebe diesen verbundenen Zeigern, die etwa noch übrigen Minuten der gegenwärtigen Zeit auf der obern Tafel, so ist wieder alles in seiner vorigen Ordnung.

IV.
Das Sonnen-System.

Innerhalb eines horizontalen Ringes von 2 Schu im Durchschnitt, welcher die Ekliptik vorstellt, und auf welchem die 12 himmlischen Zeichen mit ihren Eintheilungen aufgetragen sind, steht die Sonne in der Mitte. In der Weite von beinahe 4 Theilen, wann nämlich von der Erde biß zur Sonne 10 Theile angenommen werden, bewegt sich um dieselbe der Merkur in 87 Tagen, 23 Stunden,[9] 14 Minuten, 17½ Sekunden.

In der Weite von 7 Teilen die Venus in 224 Tagen, 16 Stunden 41 Minuten, 25 Sek.

In der Weite von 10 die Erde in 365 Tg. 5 Stunden 48 Min. 54½ Sek. um die Erde der Mond in 27 Tg. 7 St. 43 Min, 5 Sek.

[9]Engelmann's transcription had "33 Stunden," which is an obvious typo.

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In der Weite von 15 der Mars in 686 Tg. 22 St. 21 Min.

In der Weite von 52 der Jupiter in 4330 Tg. 12 Std. 47 Min.

Den noch übrigen Raum nehmen Saturn und Uranus ein. Saturn bewegt sich um die Sonne in 10 747 Tg. 3 Std. 23 Min. Uranus nach Herrn Herschel in 82 Jahren und 321 Tagen. Diese Zeit-Bestimmung findet sich bis auf die Minuten und Sekunden, ja sogar bis auf die Brüche in Rad und Getrieb. Dem Zweifler ist man erbötig, in seiner Gegenwart die Maschine auf 10 und mehrere Jahre hinauszutreiben, um die Richtigkeit derselben prüfen zu können.

Man ist meistens den Herren De la Lande,[10] Hell[11] und Röhl[12] gefolgt, und da die Astronomen in der Zeit-Bestimmung des Laufs mehrerer Planeten nicht irrig sind, so hat man den Mittelweg erwählt; zumal weil die Planeten in sehr langer Zeit durch ihre zerschiedene gegenseitige Anziehung in ihrem Lauf sich um etwas befördern oder langsamer werden.

Die Räder sind so berechnet, daß die Anzahl ihrer Zähne niemals über 100 steiget, folglich sie wegen ihrer mäßigen Größe und Leichtigkeit der Räder noch überflüssige Stärke und Dauerhaftigkeit haben. Die Ordnung ist ferner so, daß der hurtigere Planet allemal den langsameren in Bewegung setzt, folglich die Zahl der Räder nicht vervielfältigt werden durfte, wodurch die ganze Maschine eine leiße Bewegung erhält.

Das oben angezeigte Zeit-Maß der Planetischen Umläufe ist nach ihrer mittleren Bewegung zu verstehen, nach welcher sie gleiche Stücke ihrer Bahn in gleichen Zeiten beschrieben. Es ist aber diese mittlere Bewegung auch in die wahre verwandelt, besonders bey dem Merkur, Mars, Jupiter und Saturn, welche starke Ellipsen haben. Bey der Venus, der Erde und dem Monde wurden solches vor überflüssig gehalten, theils wegen ihren ziemlich Zirkel-förmigen Laufbahnen, theils wegen dem kleinen Durchmesser ihrer Bahnen, wo der Unterschied der mittleren und wahren Bewegung unter solchen Umständen nicht merklich sichtbar ist. Bey den ersteren aber ist ihre genaue Abgleichung des Mittelpunktes, welche nach Cassini bey dem Merkur 24 Grad, bey dem Mars $10^1/_2$, bei dem Jupiter $5^1/_2$, bey dem Saturn $6^1/_2$ Grad beträgt, wohl beobachtet, und in ihre Bewegung eingebracht worden; so daß man deutlich sehen kann, wann einer dieser Planeten in dem Ort seiner Sonnen-Ferne oder Sonnen-Nähe befindlich ist. Die Beobachtungen werden es auch deutlich zu erkennen geben, da die Zusammenkünfte der Planeten nach der in den astronomischen Tag-Büchern berechneten Zeit richtig eintreffen, welches sich sonsten mit einem Unterschied von vielen Tagen hätte zuweilen zutragen können.

Die Bahnen der Planeten sind schmale versilberte Ringe, welche unter ihren gehörigen Winkeln nach dem Verhältnis der Erdbahn, die ganz horizontal ist, schief liegen, jene an 2 entgegen gesetzten Orten durschschneiden, und die sogenannte auf und absteigende Knoten der Planeten darstellen; wodurch also die Bewegung der Planeten nach seiner Breite und Länge, auch die Zeit, wann er in seine Knoten tritt, bemerkt werden kann.

[10] Jérôme Lalande (1732-1807).

[11] Maximilian Hell (1720-1792).

[12] Lambert Heinrich Röhl (1724-1790).

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Die Planeten bewegen sich als kleine Kügelein, welche auf zarten stählenen Stänglein, die zwischen den Bahnen aus dem horizontalen Werk senkrecht hervorkommen, angesteckt sind, und also um die Sonne getragen werden. Die Größe ihrer Körper ist gegeneinander mit Ausschluß der Sonne, die gar zu groß worden wäre, in Verhältnis gesetzt; und zwar hat die Erde 1 1/2 Linie im Durchmesser. Da diese vorerst angenommen worden so hat der Mond beinahe 1/3; Merkur etwas mehr als 1/3, Venus 8/9, Mars 3/4, Jupiter 10 1/2, Saturn 9 1/2 und Uranus 4 1/2 Linien zu ihren Durchmesser erhalten. Nach diesem Verhältniß wäre der Durchmesser der Sonne 107 der obigen Theile oder ein Schu. Man mußte sich aber nach dem Raum richten. Der Planet Saturn hat seinen wunderbaren Ring um sich her, welcher in gehöriger Schiefe von 30 Graden also um die Sonne getragen wird, daß er nich einerley Gestalt gegen die Erde zeiget, sondern bald seine Henkel, bald gar nichts von sich sehen läßt, weil während seiner periodischen revolution um die Sonne seine Axe sich selbst parallel bleibt.

Bei dieser Darstellung des Sonnen-Systems ist um der einfachen Einrichtung willen gefilßentl. weggelassen worden; 1., die Schiefe der Axen der Sonne, der Venus und die Erde, weil die Schiefe der Sonnen und der Erd-Axe in dem Erden-System vorkömt.

2. Die Umdrehung der Planeten um ihre Axen, weil ihre Cörper zu klein sind, daß man ohnehin nichts dabei observiren könnte, auch weil die Umwälzung des Sonnen- und Erden-Körpers auf dem besondern Erdensystem vorkommt.

3. Die Bewegung der Trabanten des Jupiters, Saturns und Uranus, weil diese ihre besondere Systeme auf dem gegenüberstehenden Theil der Maschine haben.

4. Die Bewegung der Apsiden Linie oder Vorrückung des Orts der Sonnen-Ferne und der Knoten; weil die Astronomen in der Zeit-Bestimmung ihrer Bewegung nicht einig sind, solche auch ungemein langsam ist. Es ist aber dennoch dafür gesorgt worden, daß man sowohl den Ort der Sonnenferne als der Knoten alle 100 Jahre vorrücken kann, dann bälder macht es doch noch keinen sichtbaren Unterschied aus. Hier kann man also das Sonnensystem nach seinen Haupt Verhältnißen übersehen und die verschiedene Stellungen der Planeten auf jede gegebene Zeit ohne Rechnung erfahren, auch die Erscheinungen des Himmels, wie wir solche von der Erde wahrnehmen erklären.

V.
Die besondere Systeme der Planeten welche Trabanten haben.

1.
Das Erden-System.

Hier ist die Sonne in der Mitte, hat ihre schiefe Axe von 7 Graden, unter welchem Wnkel sich ihr Körper in 25 Tagen und 14 Stunden nach De la Lande um sich selbst drehet. Die Sonne umgibt ein schmaler Ring, welcher die Fläche der Erdenbahn vorstellt. Beide Ringe durchschneiden sich an 2 Orten unter

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einem Winkel von 7 Grad, und stellen die Sonnen-Knoten vor. Hierdurch kann man immer sehen, wann die Erde in den Ort der Sonnen Knoten zu stehen kommt, und folglich ihre allenfalsige Fleken gerade Linie über die Sonnen-Scheibe beschreiben; ingleichen, wenn sie zu einer andern Zeit, da die Erde 3 Zeichen von dem Knoten entfernt ist, mehr elliptisch erscheinen. Ungeachtet aber die Flecken der Sonne nach oben angemerkte Zeit ihren periodischen Umlauf vollenden, so kommen sie doch erst in 27 Tagen 12 Stunden wieder an denjenigen Ort der Sonnen Scheibe, wo man sie vor 27 Tagen gesehen hat, weil die Erde unterdessen auch in ihrer Bahn fortgerückt ist, weßwegen solches die Synodische Zeit der Umwälzung des Sonnen Körpers genannt wird. In einiger Entfernung gehet die Erde in der Zeit eines Tropischen Umlaufs von 365 Tg, 5 St. 8¾ Min. um die Sonne. Vermittelst einer besonderen Bewegung der Fläche ihrer Bahn, welche in 26 000 Jahren ihren Kreislauf einmal vollendet, ist die Vorrückung der Nachtgleichen, oder anscheinende Bewegung der Fixsterne von Abend gegen Morgen angebracht, wodurch zugleich der tropische Umlauf der Erde in den Syderischen von 365 Tg. 6 Std. 49 Min. verändert wird.

Die Erde ist ein Zoll dik. Die 4 sogenannten Welttheile und vornehmsten Königreiche sind darauf gestochen. Ihre Axe ist 23½ Grad von der senkrechten Linie abgelegen, unter welchem Winkel sie sich in 23 Std. 56 Min, 4 Sek. und 5 Terzen einmal herum wälzet, sie behält auch in ihrem jährlichen Umlauf um die Sonne vermittelst einer besonderen Gegenbewegung ihre Axe allezeit der einmal gegebenen Richtung gegen den Welt-Pol parallel, sie mag an einem Ort ihrer Bahn stehen, wo sie will. Hierdurch können die Jahreszeiten, die zerschiedene Länge der Tage und Nächte, und der Auf- und Niedergang der Sonne augenscheinlich erklärt werden. Man sieht ferner, was für Königreiche und Länder gerade Mittag haben, wann es bey uns Morgen, Abend oder Mitternacht ist; wie unsere Erde sich täglich um sich selbst, und in einem Jahre um die Sonne bewege, und ihren Bewohnern eine solche Himmelsaussicht verschaffe, daß sie meynen, die Sonne gehe auf und nieder, und bewege sich durch die 12 himmlische Zeichen p. p.

Der Mond geht in 27 Tg. 7 Std. 23 Min. 5 Sek. nach seiner mittleren Bewegung um die Erde; es ist aber dieselbe durch eine Gegen-Bewegung in eine exentrische verwandelt. Seine apsiden Linien oder das Apogeum beweget sich in 3231 Tagen 8 Std. 35 Min. durch alle Zeichen. Deßwegen tritt er jedesmal in 27 Tg. 13 Std. 18 Min. 34 Sek. wieder in den Ort seiner Erd-Ferne.

Weil der Körper des Monds sehr nahe an den äußersten Ende eines Ringes, der seine schiefe Bahn vorstellt, herumgeführt wird, so sieht man ganz deutlich, wie er sich der Erde nähert und wieder von ihr entfernt. Seine Bahn durchschneidet die Erdbahn unter einem Winkel von 5 Graden.

Der Mond erhebt sich auch selbst von seinen Knoten über die Erdbahn hinauf, und unter dieselbe hinab, indem er beständig der Schiefe seiner Bahn folgt, welche sich auch selbst in einer Zeit von 6798 Tg. 4 Std. 53 Min. nach De la Lande von Morgen gegen Abend durch die 12 Zeichen bewegt, aber daß zu jeder Zeit die in und nahe bey den Knoten sich erreigende Finsternissen bemerkt werden können.

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Der Synodische Monat, da der Mond wieder in die Zusammenkunft mit der Sonne trifft, ergibt sich von selbst in 29 Tg. 12 Std. 44 Min. $2^{9}/_{10}$ Sek., so daß die Neu- und Vollmonde das erste und letzte Viertel, samt allen übrigen Erscheinungen des Mondes daraus deutlich erklärt werden könne.

2.
Das Jovialische System.

Dieses System hat die Sonne wieder in der Mitte, und die Erde gehet um sie herum, aber in einem sehr engen Kreis, der sich zum Abstand des Jupiters wie 10 zu 52 verhält. Der Jupiter bewegt sich um die Sonne und die Erde in 4330 Tg. 12 Std. nach seiner wahren excentrischen Bewegung. Um denselben bewegen sich seine vier Trabanten nach der genauen periodischen und synodischen Zeitbestimmung des De la Lande.

Der	I.	Trabant	1	Tg.	18	Std.	27	Min.	33	Sek.	period.
			1	"	18	"	28	"	36	"	synod.
	II.	"	3	"	13	"	13	"	42	"	period.
			3	"	13	"	17	"	54	"	synod.
	III.	"	7	"	3	"	42	"	33	"	period.
			7	"	3	"	59	"	36	"	synod.
	IV.	"	16	"	16	"	32	"	8	"	period.
			16	"	18	"	5	"	7	"	synod.

Alle diese Bewegungen sind bis auf die Sekunde hinaus in Rad und Getrieb verfaßt. Die Erden-Bewegung mußte diesem System abermal zugewendet werden, damit die Ungleichheit der Zusammenkunft der Trabanten mit ihrem Hauptplaneten, welche von der sicherlichen Parall. Axe der Erde herkommt, und von der Erde aus betrachtet, für den

I.	Trabanten	1	Std.	25	Min.
II.	"	2	"	50	"
III.	"	5	"	44	"
IV.	"	13	"	24	"

höchstens beträgt, gehoben werde; denn von der Sonne aus betrachtet, würde sich diese Ungleichheit der Zusammenkünfte mit dem Haupt-Planeten nicht zeigen. Indem man aber sein Augen-Maaß auf die Stellung der Erde in ihrer Bahn nimmt und von der aus die Zusammenkünfte der Trabanten betrachtet, so treffen sie jederzeit mit denen am Himmel zu. Wenn auch ferner der Haupt-Planet seine mit der Elliptischen Linie seiner Bahn proportionierte excentrische Bewegung nicht zugleich in dessen System empfangen hätte, so würden folgende Ungleichheiten, nach welchen die Synodischen Umläufe die periodischen wegen der ungleich hurtigern Bewegung des Jupiters übertreffen, unvermeidlich worden sein: näml.:

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für den	I.	Trabanten			39	Min.	22	Sek.
	II.	"	1	Std.	19	"	13	"
	III.	"	2	"	39	"	42	"
	IV.	"	6	"	12	"	59	"

Um welchen Zeitunterschied die Trabanten auf der Maschine denen am Himmel selbst hätten zuvorkommen oder zurückbleiben können. Ein Sachverständiger sieht leicht, wie viel Nachdenken diese Systeme erfordert haben, biß die excentrische Bewegung der Trabanten der excentrischen Bewegung der Haupt Planeten auf eine bequeme Art zu geordnet wurde, biß man dem Auge die Erdbewegung wegen der Parall Axe darstellte, damit die Erscheinungen der Maschine überall mit der Zeit-Bewegung des Himmels zutreffen könnten.

Die Trabanten selbst haben ihre verhältnismäßige Entfernung von dem Haupt-Planeten auf folgende Art: Wenn der halbe Durchmesser des Jupiter Körpers 1 ist, so ist die Entfernung des

I.	Trabanten	5½
II.	"	9
III.	"	14
IV.	"	25

Sie bewegen sich nach den Verhältniß der Bahn des Jupiters schief unter einem Winkel von 3 Graden.

Die Bahn des Jupiters liegt selbst schief um 1 Grad 19 Min. in Absicht auf die Erdbahn, aus welcher Einrichtung sich auch die Knoten der Trabanten dem Auge darstellen, so daß man zu jederzeit sehen kann, ob die Zusammenkünfte, von der Sonne aus gesehen, in oder nahe bei dem Knoten sich ereignen, und also die sich hierdurch ergebende Finsternisse und Durchgänge der Trabanten auch den Schatten des Jupiters eine lange oder kurze Dauer haben werden, ingleichen ob der 4. Trabant verfinstert werden könne oder nicht, welches 55 Grad vom Knoten nicht mehr möglich ist. Ebenso läßt sich auch die Zeit des Vorübergangs eines jeden Trabanten vor dem Körper des Jupiters von der Erde aus gesehen, oder die Stellung der Trabanten untereinander, wie sie für jede Zeit durch ein Fernrohr erscheinen, wahrnehmen, daß, wenn man sie hernach am Himmel selbst sucht, leicht bestimmt werden kann, was des zur rechten oder linken des Jupiters wahrgenommene helle Punkt für ein Trabant sey.

3.
Das Saturnische System

hat alle Vollständigkeit des Jovialischen, nämlich die Sonne in der Mitte, die jährliche Bewegung der Erde um die Sonne in solcher Entfernung, welche gegen die Entfernung des Saturns von der Sonne wie 10 zu 95 sich verhält. Die Elliptische Bewegung des Saturns in 10 747 Tg. 3 St. 23 Min. um die Sonne. Die Schiefe seiner Bahn gegen die Erdbahn unter einem Winkel von 2½ Graden. Die periodische und synodische Bewegung seiner Trabanten unter einem Winkel von 30 Graden: Und zwar haben sie nach Mr. Proas auf welchem sich De la

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Lande beruft, und dessen Zeitbestimmung für die richtigste erkannt, folgende revolutionen:

I.	Trabant	1	Tg.	21	Std.	18	Min.	$26^1/_2$	Sek.	period.
		1	"	21	"	18	"	$55^1/_{10}$	"	synod.
II.	"	2	"	17	"	44	"	$51^1/_2$	"	period.
		2	"	17	"	45	"	52	"	wenig. $^1/_{10}$ synod.
III.	"	4	"	12	"	15	"	$11^1/_{10}$	"	period.
		4	"	12	"	27	"	$53^1/_2$	"	synod.
IV.	"	15	"	22	"	41	"	23	"	period.
		15	"	23	"	4	"	$12^1/_{10}$	"	synod.
V.	"	79	"	7	"	49	"	$10^6/_{10}$	"	period.
		79	"	21	"	51	"	$35^7/_{10}$	"	synod.

Sein Ring hat nicht nur hier im Kleinen, sondern auch in dem Sonnen-System nach größerm Maaßstab seine gehörige Schiefe gegen die Bahn des Saturns von 30 Graden, und verhältnismäßige Größe, die Trabanten selbst aber ihren gehörigen Abstand und Entfernung von dem Hauptplaneten, den letzten ausgenommen, welcher gar zu weit hinausgefallen wäre. Wann näml. der halbe Durchmesser des Körpers des Saturn 1 ist, so ist die Schiefe innerhalb seines Ringes $1^1/_2$ die äußere 2 des 1ten Trabanten $4^4/_5$ des 2ten Trabanten, $6^1/_4$ des 3ten, $8^3/_4$ des 4ten $7.20^1/_4$ der 5. sollte 59 haben. Aber wegen Enge des Raumes war man gezwungen, bey dem 4. so nahe als möglich zu bleiben.

Da sich die 4 ersten Trabanten in der Fläche des Rings des Saturns drehen, so haben sie mit ihm auch einerley Lage der Knoten, nämlich den 19. Grad der Jungfrau. Also kann man an der Bewegung der Trabanten jederzeit sehen, wann sie in den Schatten des Saturns eintreten, oder nach der Augenlinie unserer Erde sich über den Körper des Saturns bewegen: oder wieviel, die Parallaxe der Erdebahn in Ansehung der Zeit ihrer Zusammenkunfft mit dem Saturn Unterschied betragen würde, wenn sie nicht würklich vorhanden wäre.

4.
Das System des Uranus.

Die Sonne steht wie bei den 3 vorigen Systemen in der Mitte, und die Erde bewegt sich um die Sonne, aber in einem sehr engen Kreis. Der Uranus mit seinen Trabanten braucht zu seinem Umlauf um die Sonne 82 Jahre 321 Tg. So bestimmt Herr Herschel die Zeit seines Umlaufs. Die Neigung seiner Bahn ist $45^1/_2$ Min. Um ihn bewegen sich 2 Trabanten, der erste in 8 Tagen 18 Std. der 2te in 13 Tg. 12 Std. Weil man aber von ihrer Größe noch nichts gewisses hat, so gab man ihnen die Helffte der Größe von den Trabanten des Jupiters.

VI.
Die bewegliche Himmelskugel oder das von der Erde aus betrachtete Sonnensystem.

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Dieses besteht in einer kupfernen, blau lakkirten, mit den gewöhnlichen Sternbildern und Fixsternen bemalten Kugel, die 1 Schu im ⌀ hat, und innerhalb einer versilberten mit den gehörigen Eintheilungsgraden versehenen Meridians und Horizonts auf einer schön vergoldeten Pyramide beweglich aufgehängt ist, um welche die Ekliptik als ein schmales 1 Zoll breites, mit den 12 himmlischen Zeichen versehenes Band so herum liegt, daß sich die Kugel mit den Fixsternen innerhalb derselben drehen, und nach der Praecessione acquinoctiorum verändert werden kann. Innerhalb der Kugel sind gegen 100 Räder eingeschlossen, von welchen 8 Röhren in dem Pol der Ecliptik oben und unten hervorgehen und sich nach dem von der Erde aus gesehenen Lauf der Planeten bewegen, woran lange, stählerne, polirte Nadeln angeschraubt und um den 4ten Theil der Kugel biß auf die Ecliptik eine über die andere nach der Zirkel-Linie herabgebogen sind, und die Planeten in Gestalt vergoldeter Sterne mit ihren astronomischen Kennzeichen an sich tragen.

Folgende nähere Beschreibung ihrer Theile kann ihre Anordnung und Wirkung in ein hinlängliches Licht setzen.

a) Diese Himmelskugel mit ihren $1^1/_2$ tausend Fixsternen und der Milchstraße bewegt sich nach dem Maaß der täglichen Umwälzung unseres Erdkörpers in 23 Std. 56 Min. 4 Sek. 5 Terz. von Morgen gegen Abend einmal um ihre Axe und gibt dadurch einen jeden Fixstern seinen wahren täglichen Auf- und Untergang, und seine Stellung auf jede Tag und Nacht-Stunde am Himmel, so daß man die Fixsterne und Sternbilder sehr leicht dadurch kann kennen lernen. Denn wenn die Maschine nach der Mittag-Linie gestellt ist, so darf ich mir nur eine gerade Linie vom Mittelpunkt der Kugel bis an den Himmel vorstellen, als dann kann ich den nämlichen Stern oder Sternbild, das auf der Kugel aufgehet, oder in seiner höchsten Erhöhung stehet, aus der Ähnlichkeit der Kugel und des Himmels bald finden. Ferner kann man wahrnehmen, wie die Fixsterne täglich in ihrer scheinbaren Bewegung um 3 Min. 55 Sek. 54 Terz. vorrücken, und in einem halben Jahr ganz andere Sterne sich über den Horizont darstellen, auch wie sie sich in der Zeit ihres Auf- und Unterganges täglich verändern, bis sie in Jahresfrist wieder ihre vorige Stellung erhalten.

b) Die Sonne ist auf der Kugel vorgestellt, wie man sie zu mahlen pflegt, mit vergoldeten Strahlen, die allenthalben von ihrem Mittelpunkt ausgehen. Weil es alsdann erst völlig Nacht wird, wenn sich die Sonne 15 biß 18 Grad eines Vertical-Zirkels unter den Horizont begeben hat, so hat man, den Strahlen der Sonne vom Mittelpunkt aus gerechnet, die Länge von 15 Graden eines Vertical-Zirkels der Kugel gegeben, um den Anfang und die Währung der Morgen und Abenddämmerung auch die Zeit der Verbergung der Planeten und Fixsternen unter die Sonnen Strahlen zu bestimmen.

Da sich die Sonne täglich mit den Fixsternen von Morgen gegen Abend hingegen nach ihrer eigenen Bewegung in 365 Tg. 5 Std. 49 Min. von Abend gegen Morgen bewegt, so kommt sie in Jahresfrist bey allen Planeten und Fixsternen, die sie auf ihrem Weg antrifft, vorbey, und bedekt sie mit ihren Strahlen, daß sie von uns zu solcher Zeit, weil sie mit der Sonne auf- und untergehen, nicht gesehen werden können. Man sieht also auf dieser Kugel immer, wann

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sich ein Stern den Sonnenstrahlen nähert, und sich in der Abenddämmerung zu verbergen, und nach einer kurzen Zeit am Morgen-Horizont wieder vor der Morgen Dämmerung aufzugehen und sichtbar zu werden anfängt. Weiter sieht man nicht nur, wann ihr Mittelpunkt zu jeder Jahreszeit unter dem Horizont hinabgehe, oder über denselben hinaufsteige, sondern auch, wie es nach Unterschied des Zeichens, in welchem sich die Sonne aufhält, länger oder kürzer anstehe, biß sich ihre Strahlen am Gesichts-Kreis vollends entziehen, oder wie sie des Morgens von ihrem wirklichen Aufgang an dem Horizont hinaufzusteigen beginnen, und des Tages Anbruch, wie jenes dessen völligen Abschied verkündigen.

Diese Himmelskugel zeigt also Phänomene der scheinbaren Bewegung. Man sieht sie alle 24 Std., jedoch mit dem Unterschied von etlichen Minuten, welche die Gleichungen der Zeiten zeigen, unter den Meridian tretten. Die Ursache dieses Unterschieds ist diese: Sie wird mit den Fixsternen tägl. in 23 St. 56 Min. 4″ 5‴ herumgeführt, weil deren scheinbare Bewegung von der Umdrehung unseres Erdkörpers entsteht, der sich in dieser Zeit umwälzt und den Beobachter auf der Oberfläche der Erde mit sich herumwälzt. Wenn also die Sonne keine eigene Bewegung hätte, so schiene sie uns immer in oben angezeigter Zeit, und also nicht völlig in 24 Std. wieder unter den Meridian zu kommen. Da sie sich aber von Abend gegen Morgen alle Tage beinahe nur einen Grad der Ekliptik hinter sich bewegt, so machte dieser Schritt, wenn ihre eigene jährliche Bewegung zu aller Zeit gleich hurtig wäre, genau 3′ 55″ 54‴, um welche sie sich durch ihre hinter sich gehende Bewegung verspätet, daß sie nicht mehr gleich den Fixsternen, in 23 Std. 56 Min. 4″ 5‴ sondern nunmehr gerad in 24 Std. unter den Meridian kommen muß, da sich nun die Erde nicht in einer zirkelrunden Bahn um die Sonne bewegt, so kann auch die daraus entspringende scheinbare Bewegung der Sonne nicht zu allen Zeiten gleich hurtig seyn, sondern muß an demjenigen Ort ihrer Bahn, der von dem Mittelpunkt am weitesten entfernt ist, am langsamsten gehen, welches im Anfang des Julius geschieht. Da nun diese eigene Bewegung der Sonne und der Erde bald langsamer bald hurtiger wird, so wird der tägliche Schritt um welchen sich die Sonne täglich verspätet, und welche den Sterntag zu einem rechten Tag verändert, bald kleiner, bald größer, daß sie folglich nicht alle Tage genau in 24 Std. unter den Meridian eintreffen kann. Da nun diese elliptische Sonnenbahn auch innerhalb der Kugel nachgeahmt ist, so zeiget die Sonne auf der Kugel die nähmliche Erscheinungen, und macht ihren täglichen scheinbaren Umlauf von Morgen gegen Abend bald hurtiger bald langsamer, soviel nähmlich die Excentricität ihrer Bahn inwendig beträgt, und wie es die bekannten astronomischen Tafeln, in welchen der Unterschied der wahren und mittleren Zeit für jeden Tag berechnet ist, vor Augen legen. Kraft dieser ungleich hurtigen Bewegung in ihrer Bahn vollendet sie auch die 6 südlichen Zeichen um 8 Tage bälder, als die 6 nördlichen und verharret nicht gleich lang in jedem Zeichen. Durch eben diese eigene jährliche Bewegung der Sonne von Abend gegen Morgen, deren Mittelpunkt der Pol der Ekliptik ist, der von dem Weltpol, um welchen sich die Kugel mit allen Fixsternen, folglich auch selbst der Pol der Ekliptik mit einer gegenseitigen Bewegung

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drehet, 23 Grad 30 Min. entfernt liege, erfolget auch diese Erscheinung, daß die Sonne in dem nördlichen Zeichen ebenso, wie am Himmel selbst sich höher überm Horizont erhebt als in den südlichen und in Schrauben-Gängen von den Winter-Wendepunkt hinauf bis zum Sommer-Wendepunkt eilet; von der wieder umkehret und dort die kürzesten, wie hier die längsten Tage mit täglicher Ab- und Zunahme derselben verursacht.

c) Der Mond hat 1., innerhalb der Kugel seine völlige periodische revolution von 27 Tg. 7 Std. 43′ 5″, so daß er in 1000 Jahren, weder vorläuft noch zurückbleibt, wie in andern Maschinen geschieht, indem deren Verfasser seine revolution nicht bis auf die letzte Secunde in Rad und Getriebe zu verfaßen im Stande waren. 2. ist seine mittlere Bewegung eines excentrischen Scheibleins, welches auf dem Rad seiner mittleren revolution angebracht ist, und seinen Umlauf in 27 Tg. 13 St. 18 Min. 34 S. näml. in der Zeit eines anomalischen Monats hinter sich vollendet. Sein ⌀ ist 6 Grad; hierdurch wird gehalten, daß er an dem Ort seiner Erd-Ferne langsamer, und an dem Ort seiner Erd-Nähe sich hurtiger bewegt, so daß dieser Ort seiner Erd-Ferne selbst in 3231 T. 8 St. 35 Min. seinen Lauf durch den ganzen Thierkreis von Abend gegen Morgen vollendet. Wann diese excentrische Bewegung in der Regel nicht wäre, so könnte der Mond bißweilen um 5 biß 6 Grad des Thier-Kreises von dem Ort, wo er stehen soll, abweichen, wie wir auch bey 10 Std. bälder oder später den Vollmond und Neumond anzeigen, und doch manchmal wieder genau mit dem Himmels-Lauf eintreffen. Der Mond ist also auf der Himmelskugel in so weit mit dem Himmels-Lauf in Ähnlichkeit gesetzt, daß die übrigen Ungleichheiten, denen der Mond aus verschiedenen Ursachen unterworfen ist, niemals biß auf einen ganzen Grad aufwachsen.

Da nun dieser sein Lauf auf einer Kugel entworfen ist, welche die tägliche scheinbare Bewegung der Fixsterne hat, so ergibt sich leicht, daß man seinen täglichen Auf- und Untergang, wie er nach der verschiedenen Zeit seiner Entfernung von der Sonne und seines Standes im Thierkreis sich ändert, sein mit dem Himmel übereinkommendes ungleiches Fortrücken in den Zeichen; seine scheinbare und ungleiche Schrauben Gänge, näml.: seine unterschiedene nach dem Stand des Thierkreises sich verändernde Mittag-Höhe über den Horizont; seine Zusammenkünfte und Gegenscheine mit der Sonne; seine Neu- und Vollmonde; seine monatliche Zusammenkünfte mit den Fixsternen und Planeten, die er in seiner monatlichen Reise um den Thierkreis antrifft, allezeit bemerken kann. Wir hätten ihm auch die Bewegung seiner Breite gegeben, das ist, wie er von den Sonnenweg ab, und um 5 Grade theils gegen Norden, theils gegen Süden über diejenige Punkte des Himmels, wo man die Sonne d. 21. Juni und Dezember Mittags um 12 Uhr stehen sieht, sich zuweilen hinausbegibt, wann wir hätten von der uns vorgesetzten Einfachheit und Dauerhaftigkeit der Maschine abgehen wollen.

Wann die Bewegungen der Sonne und des Monds ohne die übrigen Planeten nur allein vorzustellen wären, so könnte solches leicht geschehen; wie es auch leicht wäre, alle übrige kleine Ungleichheiten des Monds, einige wenige die von gar kurzer Dauer sind, ausgenommen, mechanisch darzustellen.

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Es ist aber anstatt der Breite die Bewegung des Monds Knoten angebracht, das ist, die 2 Punkte, wo die 5 Grade schief liegende Mondsbahn, die Sonnen- und Erdenbahn durchschneidet, und in welchen der Mond in seiner monatlichen revolution in die Erde oder Sonnenbahn 2 × eintritt, sind beweglich gemacht worden. Sie vollenden, vermöge der astronomischen Beobachtungen und Berechnungen von Morgen gegen Abend, und also indem sie hinter sich gehen, in 6798 Tagen und 5 Std. ihren Umlauf durch alle Zeichen. An dem Ende ihrer Zeichen sind 2 Scheiblein angesteckt, mit dem Zeichen ☊ ☋, deren erstes den aufsteigenden, das andere den absteigenden Knoten anzeigt, so daß, wenn der Mond bey dem ersten vorübergeht, ersichtlich ist, daß er nun in der Sonnen-Bahn gegen Norden sich ein wenig abzulenken anfange. Ist der Mond 90 Grad von seinem Knoten entfernt, so weiß man, daß er dorten den Punkt seiner höchsten nördlichen oder südlichen Ausschweifung von 5 Grad erreicht habe, und sich nun wieder der Sonnen-Bahn nähern werde. Und weil man die Gränzen seines Abstandes von dem Knoten weiß, wo noch Sonnen und Monds Finsternisse entstehen können, so ist in jedem Knoten-Zeichen ein schmaler Arm, der sich längs der Ekliptik auf beiden Seiten biß auf 15 Grade ausdehnet, damit man, wenn Neu- oder Vollmonde innerhalb dieses Maaßes einfallen, bestimmen könne, ob eine Sonn- oder Mond-Finsterniß möglich sey, deren Größe daraus abzunehmen ist, wann die Zusammenkunft oder der Gegenschein der Sonne und des Monds sehr nahe ist, oder sich im Knoten selbst ereignet; die Sichtbarkeit aber und die Unsichtbarkeit der Finsternisse zeigt sich, wann die Zusammenkunft oder der Gegenschein über oder unter unsern Horizon geschieht.

Die Knoten der übrigen Planeten sind auf dem Thierkreis durch kleine Scheiblein angezeigt, an welchen man wahrnehmen kann, wann der Mond einen Planeten bedekt, und ob solche Bedekung sichtbar sey, welches sich ereignet, wann er bey Nacht über dem Horizont an dem Ort des Planeten Knoten in die Zusammenkunft mit demselben tritt. Diese Planeten-Knoten haben auf der Maschine keine eigene Bewegung erhalten, weil die Zeit ihrer revolution nicht nur sehr ungewiß, sondern auch sehr langsam ist.

Indessen kann man nach den Bestimmungen des Cassini, durch einen er es erlebt

der	Knoten	des	Merkurs	in	100	Jahren	um	1	Grad	24	Min.
"	"	der	Venus	"	"	"	"	1	"	56	"
"	"	der	Mars	"	"	"	"	1	"	56	"
"	"	der	Jupiters	"	"	"	"	1	"	40	"
"	"	der	Saturns	"	"	"	"	1	"	35	"

wenn man anders biß dahin nicht nähere Bestimmungen findet, hinter sich von Morgen gegen Abend gerückt werden, welcher aber, man rücke oder rücke sie nicht, dem genauen Lauf der Planeten selbst nach ihrer Länge keine Hindernis bringt.

d) Merkurius und Venus, welche ihre Kreise innerhalb der Erd-Bahn um die Sonne vollenden, werden von einem Beobachter der Erde, welcher mit der-

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selben in einer Jahres-Frist um die Sonne herumgetragen wird, als beständige Begleiter, der Sonne gesehen. Sie sezen ihren Weg bald von Abend gegen Morgen biß auf einen gewissen Grad ihrer Entfernung von der Sonne fort. Alsdann sind sie des Abends nach der Sonnen-Untergang einige Zeit sichtbar. Hierauf aber kehren sie um, und eilen mit entgegen gesetzter Richtung ihres Laufes wieder zur Sonne; rücken auch auf der andern Seite der Sonne immer weiter gegen Abend biß auf einen gewißen Grad ihrer Entfernung fort, da sie dann als Morgen-Sterne vor der Sonnen Aufgang erscheinen. Und zwar hat man beobachtet, daß Merkur ungefähr in 93 Tagen sinen Weg von Abend gegen Morgen vollbringe, einen halben Tag still stehe, ehe er wieder umkehrt, und alsdann seinen vorigen Weg in viel kürzerer Zeit, näml. in 22 Tagen von Morgen gegen Abend zurücklege. Fernder, daß das weiteste Ziel seiner Entfernung von der Sonne nicht immer einerley, sondern zuweilen 18, ein andermal 28 Grad sey, dieses höchste Ziel er aber einmal überschreite.

Ebenso hat man bey der Venus angemerkt, daß sie in ungefähr 542 Tagen ihren geraden Weg von Abend gegen Morgen vollende, einen einige still stehe, alsdann umkehre, und eben diesen Weg hinter sich von Morgen gegen Abend in 42 Tagen zurücklege, sich auch einmal über 47 Grad von der Sonne entferne.

In Anschauung dieser Zeit-Bestimmung zeigt vornämlich Merkur wegen seiner großen Excentricität eine große Ungleichheit. Denn eine Zusammenkunft des Merkurs mit der Sonne biß zum folgenden kann manchmal sogar gegen 12 Tagen eine längere Zwischenzeit haben, als sonst gewöhnlich ist.

Es ist aber eine obere Zusammenkunft des Merkurs und der Venus mit der Sonne nach beeder mittlere Bewegung berechnet, und zwar des Merkurs 15 Tg. 21 Std. 3 Min. 22 S. Der Venus aber 583 Tg. 22 Std. 7 Min. 6 Sek.

Diese Erscheinungen auf unserer Himmelskugel in ihren richtigen Stand und in ihre gehörig abwechselnde Veränderungen zu sezen, mußten wir zur Quelle dieser veränderlichen Bewegungen zurückgehen, und die hieher gehörigen Stücke des Kopernicanischen Welt-Systems zum Grund legen. Demnach setzten wir auf das Rad der Excentrischen Erd-Bewegung die mittlere revolution des Merkurs und der Venus, wie sie schon oben angegeben ist, machten diese revolutionen in ihrem gehörigen Maaße excentrisch; gaben dem Abstand des Merkurs 4 und der Venus 7 Theile, ließen den Stifft eines jeden Planeten in den Spalt eines Armes einhenken, welcher nach der Dicke der Stifte in gerader Linie, so lang es der Durchmesser der Planeten-Bahn erforderte, ausgebrochen wurde. Diese Arme an 2 Röhren, die oben zur Kugel hinausgehen, befestiget, geben den daselbst eingestellten 2 Planeten, dem Merkur und der Venus ihre gehörige, oben beschriebene und von der Erde aus beobachtete Bewegung.

e) Die 4 oberen Planeten, Mars, Jupiter, Saturn und Uranus sind in ihren Bewegungen von den 2 unteren unterschieden, weil sie ihre Kreise nicht innerhalb der Erde-Bahn haben. Sie erscheinen also, indem wir sie von der Erde aus beobachten, ganz anders in ihren Bewegungen.

Unerachtet Mars in 686 Tg. 22 Std. 20 Min. seine eigene mittlere revolution endigt, so sieht man ihn dennoch von der Erde aus beinahe 705 Tg. in dem Thier-Kreis gerade fortgehen, am Ende dieser Zeit 2 Tage stille stehen, daß

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er seinen Ort in dem Thierkreis nicht zu verändern scheint, und endlich 75 Tage im Thierkreis zurückgehen. Man findet auch, daß er 10 bis 12 Grad zurücktretten kann, und dieß allemal, wann er im Gegenschein mit der Sonne stehet. Sein Zurückgehen aber geschiehet bey weitem nicht so schnell, als sei geradlaufender Gang. Dieser beträgt in einem Tage 47 Min. jener 24.

Seine Zusammenkünffte mit der Sonne geschehen nach beider mittlerer Bewegung in 779 Tg. 22 Std. 28 Min. 26 Sek. wieder, welche Zeit aber wegen der Excentricität des Mars und der Sonne das einemal zieml. größer als das anderemal seyn kann.

Von den Jupiter und Saturn beobachtet man von der Erde aus folgende Veränderung ihrer Bewegungen für ihren synodischen Umlauf. Von einer Zusammenkunft der Sonne zu der andern zählt man für den Jupiter 398 Tg. 21 Std. 15 Min. 44 Sek. für den Saturn 578 Tg. 2 Std. 8 Min. 7 Sek. Unter diesem Zeitraum ist Jupiter 284 Tg. und Saturn 244 Tg. rechtläufig, ersterer 4, letzterer 8 Tg. stillstehend; ersterer aber 119 Tg. Letzterer 136 Tg. rückgängig. Jupiter geht beynahe 10, Saturn beynahe 7 Grade des Thierkreises zurück. Bey dem Uranus ist das vor- und zurückgehen weggelassen worden, weil es wenig beträgt und an einer so kleinen Kugel nicht beobachtet werden könnte, welches der übrigen Genauigkeit der Maschine ganz unbeschadet geschehen konnte.

Wenn man nun alle die Veränderungen beobachtet, die sich von der Erde aus von den Bewegungen der Himmels Körper machen lassen, und dabei erwäget, daß

Merkur	erst	in	13	Jahren	2	Tagen
Venus	"	"	8	"	2	"
Mars	"	"	15	"	19	"
Jupiter	"	"	13	"	1	"
Saturn	"	"	59	"	2	"

nur wieder beynahe in ihren alten Platz in Ansehung der Sonne, genau betrachtet aber niemal in ihrer ersten Stellung gegen einander kommen: Wenn man, sagen wir alles dieses erweget, und doch findet, daß in unserer Maschine alle diese Bewegungen so nachgeahmt sind, daß sie ihre Ähnlichkeit mit den Himmels-Körpern und ihrer Bewegung über 1000 Jahre behalten, so wird ein jeder die Schwierigkeiten fühlen die wir zu überwinden gehabt haben.

Sollten indeßen in der Folge der Zeit die Planeten durch ihre gegenseitige Anziehung in ihren Bewegungen etwas hurtiger oder langsammer werden, so wird doch solches nicht soviel betragen, daß der Unterschied auf einer gegen den ungeheuer großen Himmels-Bau so kleinen Maschine zu einer merklichen Sichtbarkeit anwachsen könnte.

In Ansehung der Dauerhaftigkeit, Reibung und Abnuzung der Theile hat man sich um so weniger Sorge zu machen, als die langsamme Bewegung der Räder mit dem Aufwand einer geringen Kraft bewirket wird, und von keiner Seite her einen Zwang auszustehen hat. Das Gewicht, welches die Uhr, und durch diese das ganze Werk in Bewegung sezt, ist zwar 12 Pfund. Weil aber die Uhr auf 8 Tage übersezt ist, so kommt auf das Stunden-Rad der Uhr, von

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welchem die übrigen Bewegungen ausgehen, kaum $1^1/_2$ ℔. Die Ursache, warum eine so kleine Krafft zu der Bewegung so vieler Theile hinreichend ist, ist aus der horizontalen Anordnung und Lage aller Theile zu begreifen, da nirgend eine Last in die Höhe gehoben, sondern nur um den Mittelpunkt bewegt werden darf.

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Chapter 19

(Oechslin: 9.1)

Johann's globe clock in Mainz (1796?)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

19.1 Introduction

The clock described here was constructed in c1796 by Nicolaus Alexius Johann (1753-1826) and is located in Mainz. It is Johann's first big astronomical clock.

Johann was not a clockmaker, but a professor, musician and composer. He was born in Steinach an der Saale, now part of Bad Bocklet, about 10km from Bad Kissingen, in Germany. His father was mason. He entered the orders of the Augustinians in 1770 and became Pater Alexius in 1774. He then moved to Freiburg im Breisgau. After studying theology in Freiburg, he became priest in 1777. He has composed pieces of church music, including some operas. It was probably in Freiburg that Johann became acquainted with clockmaking. He remained in Freiburg until 1781 and he was then transferred to the Augustinian monastery in Mainz. It was there that he constructed his first great astronomical clock which was possibly completed in 1796 and is described here.[1]

From 1802 to 1809, Johann was priest in Mainz and during this time he built his second great astronomical clock for the pastor Josef Mangold. This clock was destroyed in 1942 (figure 19.1 and 19.2). It was described by Arentz and Kraetzer.[2]

[1] There is still some doubt about the date of completion of the clock and different dates have been given. I have therefore put a question mark in the title. See Gauly about this matter [5, p. 52-53].

[2] See [4], [7], as well as [2, 3] and [1, p. 200-201].

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In 1809, Johann took the parish of Heidesheim, a few kilometers West of Mainz. He spent his last years again in Mainz.

His brother Baptist Johann (1765-1826) also entered the order of the Augustinians. He seems to have come to Mainz around 1794. The two brothers lived in the same place in 1825. After the death of Nicolaus Alexius Johann, Baptist Johann returned to Steinach and died there only a few months later.

The two brothers are in fact the authors of at least eight astronomical clocks, of which several survive in private collections.[3]

Abeler has tried to find out where the Johann brothers got their interest and knowledge in the construction of clocks. It is for instance possible that Alexius Johann was in touch with Thaddäus Rinderle in Freiburg, although Johann left Freiburg before Rinderle built his astronomical clock (Oechslin 7.1).[4] Abeler also assumes that Johann knew of the descriptions published by Pater Aurelius, Frater David and Philipp Matthäus Hahn, but concludes that Johann devised independent solutions and did not copy those of other clockmakers, although I believe that he was influenced by at least the external appearance of some clocks he knew of.

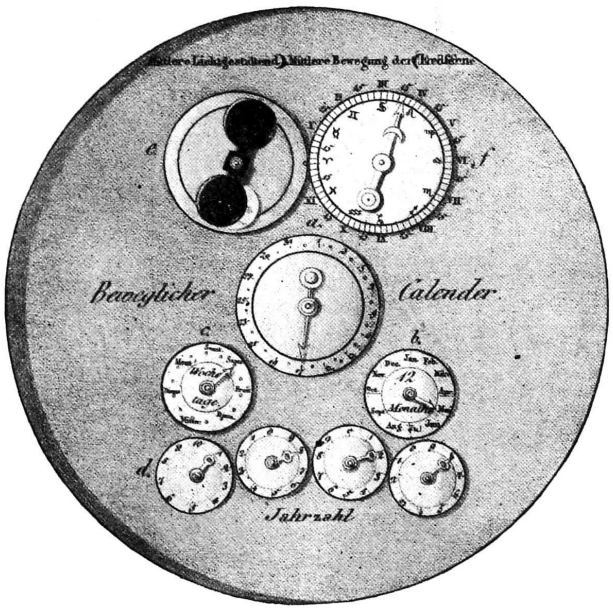


Figure 19.1: The calendar dial of Johann's second big astronomical clock, destroyed by fire in 1942. (source: [2], taken from [4])

[3]For more information on the Johann brothers, see Abeler's article [1] and Gauly's book [5]. On Johann's clocks, see also Maurice [8, v.1, p. 279-280] and King [6, p. 242, 244].

[4]See [1, p. 203].

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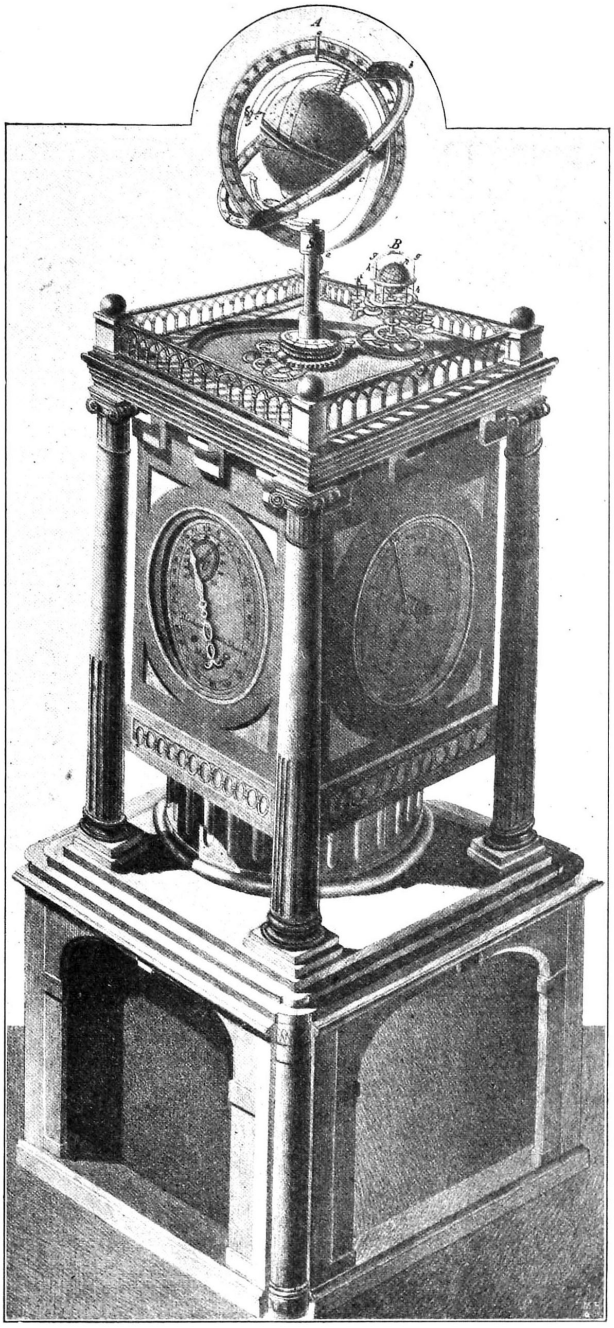


Figure 19.2: Johann's second big astronomical clock, destroyed by fire in 1942. (source: [2], taken from [4])

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19.2 Description of the clock

The clock described here is a globe clock, having a cubic base topped by a celestial globe containing an intricate set of gears.[5] It bears similarities with some of Philipp Matthäus Hahn's globe clocks. Johann described this clock in a manuscript which has been transcribed by Abeler.[6]

The clock was donated by will to the priest seminar in Mainz. In 1857, this clock was damaged during an explosion in the bishop's palace in Mainz and was again put in working order by Clemens Kissel (1849-1911).[7] Then, during WWII, the clock was walled under a staircase of the palace, and thus survived the burning of the palace.

The main side of the base shows the time on a 24-hour dial and the year on four small horizontally aligned dials, one per digit. This is reminiscent of Hahn's clock in Aschaffenburg constructed in 1776/1777 (Oechslin 8.4). A second dial shows the motions of Mercury, Venus and the Earth. A third dial shows the motions of the Earth, Mars, Jupiter and Satur. A small tellurium is also fitted on top of the base, but below the celestial sphere.

I will first describe the going work, then the two parts of the orrery, then the tellurium, then the celestial globe.

19.2.1 The going work

The clock is a weight-driven pendulum clock. The arbor 3 makes one turn in two hours, and we can therefore deduce that the drum wheel on arbor 1 makes one turn clockwise (from the front of the time dials) in $\frac{95}{28}$ days, hence about 81.4 hours:

$$\mathrm{T}_3^0 = 12 \tag{19.1}$$

$$\mathrm{T}_1^0 = \mathrm{T}_3^0 \times \left(-\frac{12}{76}\right) \times \left(-\frac{14}{90}\right) = \frac{28}{95} \tag{19.2}$$

The motion of arbor 3 is transferred to the escape wheel on arbor 5:

$$\mathrm{T}_5^0 = \mathrm{T}_3^0 \times \left(-\frac{72}{6}\right) \times \left(-\frac{60}{6}\right) = \mathrm{T}_3^0 \times 120 = 12 \times 120 = 1440 \tag{19.3}$$

Consequently, the escape wheel makes one turn in 60 seconds. It carries 30 pins or teeth and the pendulum must therefore make a half-swing in one second and be about one meter long.

The motion of arbor 3 is transferred to arbor 6:

$$\mathrm{T}_6^0 = \mathrm{T}_3^0 \times \left(-\frac{72}{72}\right) = -12 \tag{19.4}$$

[5] This clock was exhibited in the 1989 Hahn exhibition catalogue [11, p. 68].
[6] [1, p. 198-200]
[7] See [10, p. 107] and [5, p. 52, 56].

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It is used to obtain the motion of the minute hand on arbor 7:

$$\mathrm{T}_7^0 = \mathrm{T}_6^0 \times \left(-\frac{40}{20}\right) = 24 \tag{19.5}$$

It is also used to obtain the motion of tube 8 which is that of the hour hand on the 24-hour dial:

$$\mathrm{T}_8^0 = \mathrm{T}_6^0 \times \left(-\frac{8}{96}\right) = (-12) \times \left(-\frac{1}{12}\right) = 1 \tag{19.6}$$

This tube makes one turn clockwise in 24 hours. It is coupled with the central arbor 9 whose motion is used to drive the calendar work which is not described here. It is also used to derive the motion of the vertical arbor 22 which drives the celestial globe, here seen from above:

$$\mathrm{V}_{22}^0 = \mathrm{T}_9^0 \times \left(-\frac{76}{79}\right) \times \left(-\frac{76}{58}\right) \times \left(-\frac{35}{44}\right) = \mathrm{T}_9^0 \times \left(-\frac{25270}{25201}\right) = -\frac{25270}{25201} \tag{19.7}$$

$$\mathrm{P}_{22}^0 = -\frac{25201}{25270} = -23 \text{ h } 56 \text{ mn } 4.08\ldots \text{ s.} \tag{19.8}$$

The same value is given by Oechslin.

This, interestingly, is the same ratio as the one used by Hahn for the sidereal day velocity.

19.2.2 The two orreries

The orrery on Johann's clock is divided in two separate orreries. There is the inner orrery for Mercury, Venus, and the Earth, and there is the outer orrery for the Earth, Mars, Jupiter and Saturn.

In the inner orrery, the motion of arbor 22 is used to derive that of arbor 24, here also seen from above:

$$\mathrm{V}_{24}^0 = \mathrm{V}_{22}^0 \times \left(-\frac{19}{47}\right) \times \left(-\frac{6}{65}\right) = \mathrm{V}_{22}^0 \times \frac{114}{3055} = -\frac{25270}{25201} \times \frac{114}{3055} \tag{19.9}$$

$$= -\frac{576156}{15397811} \tag{19.10}$$

$$\mathrm{P}_{24}^0 = -\frac{15397811}{576156} = -26.7250\ldots \text{ days} \tag{19.11}$$

The same ratio is given by Oechslin. The motion of this arbor is used to drive the tellurium described below.

But it is also used to derive the motion of arbor 101 which links the two orreries. Here we measure the velocity of arbor 101 from the side of the *outer* orrery:

$$\mathrm{V}_{101}^0 = \mathrm{V}_{24}^0 \times \left(-\frac{6}{41}\right) = \left(-\frac{576156}{15397811}\right) \times \left(-\frac{6}{41}\right) = \frac{3456936}{631310251} \tag{19.12}$$

The same ratio is given by Oechslin.

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19.2.2.1 The inner orrery

The input of the inner orrery is arbor 108, whose velocity is measured from the side of the *inner* orrery:

$$\mathrm{V}^0_{108} = \mathrm{V}^0_{101} \times \frac{24}{24} = \frac{3456936}{631310251} \tag{19.13}$$

$$\mathrm{P}^0_{108} = \frac{631310251}{3456936} = 182.6213\ldots \text{ days} \tag{19.14}$$

The same ratio is given by Oechslin. The period of this arbor is in fact exactly half of the tropical year, as we will see shortly.

The motion of arbor 108 is used to obtain the motion of tube 113 which drives Mercury:

$$\mathrm{V}^0_{113} = \mathrm{V}^0_{108} \times \left(-\frac{45}{11}\right) \times \left(-\frac{34}{67}\right) = \mathrm{V}^0_{108} \times \frac{1530}{737} \tag{19.15}$$

$$= \frac{3456936}{631310251} \times \frac{1530}{737} = \frac{5289112080}{465275654987} \tag{19.16}$$

$$\mathrm{P}^0_{113} = \frac{465275654987}{5289112080} = 87.9685\ldots \text{ days} \tag{19.17}$$

This is the mean orbital period of Mercury. The same value is given by Oechslin.

Tube 113 is actually excentered from that of Mercury, so that the motion of Mercury is made irregular, although it describes a circular path. Johann has however drawn excentric circles, so that one has to find the intersection between the hand of the planet and the corresponding circle.

The motion of tube 108 is also used to derive those of tube 111 (Venus) and 110 (Earth):

$$\mathrm{V}^0_{111} = \mathrm{V}^0_{108} \times \left(-\frac{47}{22}\right) \times \left(-\frac{35}{92}\right) = \mathrm{V}^0_{108} \times \frac{1645}{2024} = \frac{15124095}{3398329649} \tag{19.18}$$

$$\mathrm{P}^0_{111} = \frac{3398329649}{15124095} = 224.6963\ldots \text{ days} \tag{19.19}$$

$$\mathrm{V}^0_{110} = \mathrm{V}^0_{108} \times \left(-\frac{47}{22}\right) \times \left(-\frac{22}{94}\right) = \mathrm{V}^0_{108} \times \frac{1}{2} = \frac{3456936}{631310251} \times \frac{1}{2} \tag{19.20}$$

$$= \frac{1728468}{631310251} \tag{19.21}$$

$$\mathrm{P}^0_{110} = \frac{631310251}{1728468} = 365.2426\ldots \text{ days} \tag{19.22}$$

We thus have obtained the orbital periods of Venus and the Earth. The same two values are given by Oechslin.

In the case of the Earth, we have the same construction as for Mercury and the tube 110 is in fact offset from the center of the Earth hand. Likewise, the drawn orbit is an eccentric one.[8]

[8] See Oechslin on these eccentric orbits [9, p. 142-143, 155].

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19.2.2.2 The outer orrery

The input of the outer orrery is arbor 101, whose period is

$$\mathrm{P}^0_{101} = \mathrm{P}^0_{108} = \frac{631310251}{3456936} = 182.6213\ldots \text{ days} \tag{19.23}$$

which is half a tropical year.

This arbor is used to obtain the motion of tube 106 which carries the Earth:

$$\mathrm{V}^0_{106} = \mathrm{V}^0_{101} \times \left(-\frac{29}{34}\right) \times \left(-\frac{34}{58}\right) = \mathrm{V}^0_{108} \times \frac{1}{2} = \frac{3456936}{631310251} \times \frac{1}{2} \tag{19.24}$$

$$= \frac{1728468}{631310251} \tag{19.25}$$

$$\mathrm{P}^0_{106} = \frac{631310251}{1728468} = 365.2426\ldots \text{ days} \tag{19.26}$$

We find again the tropical year. The same value is given by Oechslin.

The arbor 101 is also used to obtain the motion of tube 107 which carries Mars:

$$\mathrm{V}^0_{107} = \mathrm{V}^0_{101} \times \left(-\frac{29}{34}\right) \times \left(-\frac{24}{77}\right) = \mathrm{V}^0_{101} \times \frac{348}{1309} \tag{19.27}$$

$$= \frac{3456936}{631310251} \times \frac{348}{1309} = \frac{5926176}{4070862653} \tag{19.28}$$

$$\mathrm{P}^0_{107} = \frac{4070862653}{5926176} = 686.9290\ldots \text{ days} \tag{19.29}$$

The same value is given by Oechslin. This is a good approximation of the orbital period of Mars. The same value is used in the celestial sphere.

The arbor 101 is also used to obtain the motion of tube 104 which carries Jupiter:

$$\mathrm{V}^0_{104}? = \mathrm{V}^0_{101} \times \left(-\frac{12}{48}\right) \times \left(-\frac{14}{74}\right) = \mathrm{V}^0_{101} \times \frac{7}{148} \tag{19.30}$$

$$= \frac{3456936}{631310251} \times \frac{7}{148} = \frac{6049638}{23358479287} \tag{19.31}$$

$$\mathrm{P}^0_{104}? = \frac{23358479287}{6049638} = 3861.1366\ldots \text{ days} \tag{19.32}$$

However, this period (also given by Oechslin) is obviously wrong, since the correct orbital period of Jupiter is about 4330 days. Oechslin does not fix the gears in the drawing (and neither in his calculations), but it is likely that the 74-teeth wheel should actually have had 83 teeth.[9] In that case, we have

$$\mathrm{V}^0_{104} = \mathrm{V}^0_{101} \times \left(-\frac{12}{48}\right) \times \left(-\frac{14}{83}\right) = \mathrm{V}^0_{101} \times \frac{7}{166} \tag{19.33}$$

$$= \frac{3456936}{631310251} \times \frac{7}{166} = \frac{12099276}{52398750833} \tag{19.34}$$

$$\mathrm{P}^0_{104} = \frac{52398750833}{12099276} = 4330.7344\ldots \text{ days} \tag{19.35}$$

[9] See [9, p. 205] on this error.

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which is much better. And moreover, the same value is used in the celestial sphere.

Finally, the arbor 101 is used to obtain the motion of tube 103 which carries Saturn:

$$\mathrm{V}^0_{103} = \mathrm{V}^0_{101} \times \left(-\frac{12}{48}\right) \times \left(-\frac{7}{103}\right) = \mathrm{V}^0_{101} \times \frac{7}{412} \tag{19.36}$$

$$= \frac{3456936}{631310251} \times \frac{7}{412} = \frac{6049638}{65024955853} \tag{19.37}$$

$$\mathrm{P}^0_{103} = \frac{65024955853}{6049638} = 10748.5697\ldots \text{ days} \tag{19.38}$$

The same value is given by Oechslin. This is an approximation of the orbital period of Saturn. And again, the same value is used in the celestial sphere.

Mars, Jupiter and Saturn are also having an irregular motion, like Mercury and the Earth in the inner orrery.

19.2.3 The tellurium

The tellurium is located above the base of the clock, but below the celestial globe. It shows the motion of the Earth around the Sun (not pictured, but on the central vertical axis), and of the Moon around the Earth. The horizontal plane is the plane of the ecliptic. The Earth rotates around a tilted axis which always keeps the same orientation. The Moon also has an irregular motion and moves up and down in order to account for its tilted orbital plane.

19.2.3.1 Main motion

The main motion of the tellurium is obtained from the motion of arbor 24 described above. A 9-leaves pinion on arbor 24 drives the main frame of the tellurium on tube 25:

$$\mathrm{V}^0_{25} = \mathrm{V}^0_{24} \times \left(-\frac{9}{123}\right) = \mathrm{V}^0_{24} \times \left(-\frac{3}{41}\right) \tag{19.39}$$

$$= \left(-\frac{576156}{15397811}\right) \times \left(-\frac{3}{41}\right) = \frac{1728468}{631310251} \tag{19.40}$$

$$\mathrm{P}^0_{25} = \frac{631310251}{1728468} = 365.2426\ldots \text{ days} \tag{19.41}$$

The same value is given by Oechslin. This is the same approximation of the tropical year as in the orreries. The frame supporting the Earth and the Moon makes one turn counterclockwise (seen from above) in one tropical year.

19.2.3.2 The internal motions

For the internal motions within the moving frame 25, two trains are used. One is based on a fixed 80-teeth wheel on the frame 30 supporting the vertical arbor

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22 leading to the globe. The other train is based on the motion of tube 27 which is derived from the motion of the central vertical arbor 22. We have

$$\mathrm{V}^0_{27} = \mathrm{V}^0_{22} \times \left(-\frac{44}{68}\right) \times \left(-\frac{68}{44}\right) = \mathrm{V}^0_{22} \tag{19.42}$$

In other words, tube 27 merely replicates the motion of the internal arbor 22 and makes one turn clockwise in a sidereal day.

$$\mathrm{V}^0_{27} = -\frac{25270}{25201} \tag{19.43}$$

19.2.3.3 The tilt of the axis of the Earth

The tilted arbor 49 around which the Earth rotates pivots on frame 32. This frame is linked to a 80-teeth wheel which is the last wheel of the first train mentioned above. Consequently, we have (with respect to the moving frame 25)

$$\mathrm{V}^{25}_{32} = \mathrm{V}^{25}_{30} \times \left(-\frac{80}{38}\right) \times \left(-\frac{38}{80}\right) = \mathrm{V}^{25}_{30} \tag{19.44}$$

and

$$\mathrm{V}^0_{32} = \mathrm{V}^0_{30} = 0 \tag{19.45}$$

The frame 32 has a fixed orientation in space, and therefore the axis of the Earth always keeps the same orientation.

19.2.3.4 The rotation of the Earth around its axis

The central arbor proving the rotation of the Earth is arbor 29. The train of wheels going from tube 27 to arbor 29 is such that arbor 29 replicates the motion of tube 27:

$$\mathrm{V}^{25}_{29} = \mathrm{V}^{25}_{27} \times \left(-\frac{80}{38}\right) \times \left(-\frac{38}{80}\right) = \mathrm{V}^{25}_{27} \tag{19.46}$$

and therefore also

$$\mathrm{V}^0_{29} = \mathrm{V}^0_{27} \tag{19.47}$$

The velocity of arbor 29 with respect to a fixed frame is that of the sidereal day, but clockwise. This motion is transferred to the tilted arbor 49, which has a fixed orientation, as mentioned above. Consequently, we also have

$$\mathrm{V}^0_{49} = \mathrm{V}^0_{29} \times \left(-\frac{19}{19}\right) = -\mathrm{V}^0_{29} = -\mathrm{V}^0_{27} = \frac{25270}{25201} \tag{19.48}$$

The axis of the Earth rotates counterclockwise in one sidereal day, as it should.

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19.2.3.5 The day/night boundary

There is a frame 35 around the Earth and the motion of this frame replicates that of tube 33 which is fixed on the main rotating frame 25. Therefore:

$$\mathrm{V}_{35}^{25} = \mathrm{V}_{33}^{25} \times \left(-\frac{27}{27}\right) \times \left(-\frac{27}{27}\right) = \mathrm{V}_{33}^{25} \tag{19.49}$$

This frame 35 does therefore always keep the same orientation with respect to the Sun, and it is used to show the day/night boundary.

19.2.3.6 The mean motion of the Moon

The central arbor 29 leading to the Earth also carries a small 6-leaves pinion. This pinion is used to produce the mean motion of the Moon on tube 38, using a train located on frame 32. This frame has a fixed orientation in space. Therefore

$$\mathrm{V}_{38}^{32} = \mathrm{V}_{38}^{0} = \mathrm{V}_{29}^{0} \times \left(-\frac{6}{69}\right) \times \left(-\frac{71}{32}\right) \times \left(-\frac{14}{74}\right) = \mathrm{V}_{29}^{0} \times \left(-\frac{497}{13616}\right) \tag{19.50}$$

$$= -\frac{25270}{25201} \times \left(-\frac{497}{13616}\right) = \frac{6279595}{171568408} \tag{19.51}$$

$$\mathrm{P}_{38}^{0} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{19.52}$$

The same value is given by Oechslin, also in sidereal days (but with a slightly incorrect decimal value). This is a good approximation of the tropical month, the period of the motion of the Moon in the zodiac. The same period is used in the celestial sphere.

19.2.3.7 The orientation of the Moon and the lunar nodes

The Moon is fixed on the arbor going through tube 46 and pivots on a frame (numbered 38 by Oechslin, but I will number it $38'$ as it is not identical with the mean motion of the Moon) containing a number of other gears. This frame is set at the *corrected* position of the Moon. The gears that are contained in it are used for the orientation of the lit part of the Moon towards the Sun, and for the vertical motion of the arbor holding the Moon, to account for the tilt of the lunar orbit.

The orientation of the Moon with respect to the Sun is obtained very easily. The frame 35 which, as we have seen above, keeps the same orientation towards the Sun, carries a 52-teeth wheel which meshes with an intermediate wheel on arbor 45, which in turn meshes with another 52-teeth wheel on tube 46. This wheel therefore has the same orientation as the first 52-teeth wheel and it keeps the same orientation with respect to the Sun. The arbor holding the Moon moves up and down through tube 46, but the orientation of the Moon is determined by the tube.

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Within frame 38′, a gear train causes an oscillation of a plate for the inclination of the lunar orbit. The velocity of this oscillation is that of arbor 48:

$$\mathrm{V}_{48}^{38'} = \mathrm{V}_{35}^{38'} \times \left(-\frac{52}{66}\right) \times \left(-\frac{73}{35}\right) \times \left(-\frac{35}{53}\right) = \mathrm{V}_{35}^{38'} \times \frac{1898}{1749} \tag{19.53}$$

$$= \left(\mathrm{V}_{35}^{0} - \mathrm{V}_{38'}^{0}\right) \times \frac{1898}{1749} = \left(\mathrm{V}_{25}^{0} - \mathrm{V}_{38'}^{0}\right) \times \frac{1898}{1749} \tag{19.54}$$

$$\approx \left(\mathrm{V}_{25}^{0} - \mathrm{V}_{38}^{0}\right) \times \frac{1898}{1749} \approx \left(\frac{1728468}{631310251} - \frac{6279595}{171568408}\right) \times \frac{1898}{1749} \tag{19.55}$$

$$\approx \left(-\frac{145542724201}{4297960188808}\right) \times \frac{1898}{1749} = -\frac{10624618866673}{289120475777892} \tag{19.56}$$

$$\mathrm{P}_{48}^{38'} \approx -\frac{289120475777892}{10624618866673} = -27.2123\ldots \text{ days} \tag{19.57}$$

This value is not given by Oechslin. It is an approximation of the draconic month. The sign of the period is irrelevant here, because we have a periodic and symmetric motion. The ratio used here is slightly different from the one used in the celestial globe.

19.2.3.8 The corrected motion of the Moon

The frame 38 carries a tube 40 whose velocity with respect to frame 38 is

$$\mathrm{V}_{40}^{38} = \mathrm{V}_{35}^{38} \times \left(-\frac{40}{27}\right) \times \left(-\frac{34}{47}\right) = \mathrm{V}_{35}^{38} \times \frac{1360}{1269} \tag{19.58}$$

$$= \left(\mathrm{V}_{35}^{0} - \mathrm{V}_{38}^{0}\right) \times \frac{1360}{1269} = \left(-\frac{145542724201}{4297960188808}\right) \times \frac{1360}{1269} \tag{19.59}$$

$$= -\frac{24742263114170}{681763934949669} \tag{19.60}$$

$$\mathrm{P}_{40}^{38} = -\frac{681763934949669}{24742263114170} = -27.5546\ldots \text{ days} \tag{19.61}$$

This value is not given by Oechslin. It is the anomalistic month. This causes the main oscillation of the lunar motion, namely the equation of center. The ratio used in the celestial sphere is slightly different.

If we compute the velocity of tube 40 in an absolute frame, we obtain

$$\mathrm{V}_{40}^{0} = \mathrm{V}_{40}^{38} + \mathrm{V}_{38}^{0} = -\frac{24742263114170}{681763934949669} + \frac{6279595}{171568408} \tag{19.62}$$

$$= \frac{1688455570445}{5454111479597352} \tag{19.63}$$

$$\mathrm{P}_{40}^{0} = \frac{5454111479597352}{1688455570445} = 3230.2368\ldots \text{ days} \tag{19.64}$$

The same value is given by Oechslin. It is the period of precession of the lunar apsides.

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However, Johann added a second correction to the motion of the Moon. This second oscillation is that of arbor 44.

The 47-teeth wheel on tube 40 carries an excentric pin. The position of this pin is that of the Moon corrected for the anomaly. But the 47-teeth wheel carries the arbor 44 which also carries a wheel with an eccentric pin, which is used to add a second correction. We have

$$\mathrm{V}_{44}^{40}? = \mathrm{V}_{41}^{40} \times \left(-\frac{17}{18}\right) \times \left(-\frac{10}{19}\right) \times \left(-\frac{7}{26}\right) = \mathrm{V}_{41}^{40} \times \left(-\frac{595}{4446}\right) \tag{19.65}$$

$$= -\mathrm{V}_{40}^{38} \times \left(-\frac{595}{4446}\right) = \frac{24742263114170}{681763934949669} \times \left(-\frac{595}{4446}\right) \tag{19.66}$$

$$= -\frac{387411751392925}{79766380389111273} \tag{19.67}$$

$$\mathrm{P}_{44}^{40}? = -\frac{79766380389111273}{387411751392925} = -205.8956\ldots \text{ days} \tag{19.68}$$

and

$$\mathrm{V}_{44}^{38}? = \mathrm{V}_{44}^{40}? + \mathrm{V}_{40}^{38} = -\frac{387411751392925}{79766380389111273} - \frac{24742263114170}{681763934949669} \tag{19.69}$$

$$= -\frac{3282256535750815}{79766380389111273} \tag{19.70}$$

$$\mathrm{P}_{44}^{38}? = -\frac{79766380389111273}{3282256535750815} = -24.3022\ldots \text{ days} \tag{19.71}$$

All these values are obviously meaningless and do not correspond to any well known lunar anomaly. Oechslin has also had to gripple with similar mysterious values, but he did not compute the same ratios as above.[10] He computed for instance V_{44}^{0}, although this is of little use, and found the value $\mathrm{P}_{44}^{0} = -219.9128$ which he could not interpret.

There is however a simple explanation to these erroneous values. If we change the rotation of arbor 44 by adding another set of wheels, we obtain

$$\mathrm{V}_{44}^{40} = \mathrm{V}_{41}^{40} \times \left(-\frac{17}{18}\right) \times \left(-\frac{10}{19}\right) \times \left(-\frac{7}{26}\right) \times \left(-\frac{x}{x}\right) = \mathrm{V}_{41}^{40} \times \frac{595}{4446} \tag{19.72}$$

$$= -\mathrm{V}_{40}^{38} \times \frac{595}{4446} = \frac{24742263114170}{681763934949669} \times \frac{595}{4446} \tag{19.73}$$

$$= \frac{387411751392925}{79766380389111273} \tag{19.74}$$

$$\mathrm{P}_{44}^{40} = \frac{79766380389111273}{387411751392925} = 205.8956\ldots \text{ days} \tag{19.75}$$

and

$$\mathrm{V}_{44}^{38} = \mathrm{V}_{44}^{40} + \mathrm{V}_{40}^{38} = \frac{387411751392925}{79766380389111273} - \frac{24742263114170}{681763934949669} \tag{19.76}$$

$$= -\frac{2507433032964965}{79766380389111273} \tag{19.77}$$

$$\mathrm{P}_{44}^{38} = -\frac{79766380389111273}{2507433032964965} = -31.8119\ldots \text{ days} \tag{19.78}$$

[10] See [9, p. 208-209].

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The first period hasn't changed, but the period of arbor 44 with respect to frame 38 is now about 31.8 days, which is exactly the period of the second lunar anomaly, the evection.

The same problem occurs (but not with the exact same ratios) in the celestial globe, and the same solution applies. This is obviously a construction error by Johann.

These two corrections, the anomaly and the evection, are added, and their sum is used to produce the oscillation of frame 38′ with respect to frame 38.

19.2.4 The celestial sphere

The celestial globe rotates about a vertical axis representing the axis of the Earth, and the planets, the Moon and the Sun rotate around the globe. However, Johann does not have all the tubes come out of the poles of the ecliptic, like in Hahn's globe clocks. Instead, Johann's globe is constructed as two hemispheres and the planets, the Sun and the Moon move on disks that protrude at the junction of the two hemispheres. The gears inside the globe are divided in two groups, those in the Northern hemisphere (Mars, Jupiter and Saturn), and those in the Southern hemisphere (Moon, Sun, Mercury and Venus).

We have seen above that the input to the celestial globe is arbor 22 and that it makes one turn clockwise (seen from above) in one sidereal day. This arbor moves the entire globe. The motions within the globe are obtained using two wheels fixed on the outer (non moving) frame, one with 41 teeth at the bottom of the globe, and another with 10 teeth at the top of the globe.

The two inputs *on* the globe are then those of tube 50 at the bottom, and of tube 82 at the top. We have

$$\mathrm{V}_{50}^{22} = \mathrm{V}_{30}^{22} \times \left(-\frac{41}{74}\right) = -\mathrm{V}_{22}^{30} \times \left(-\frac{41}{74}\right) \tag{19.79}$$

$$= \frac{25270}{25201} \times \left(-\frac{41}{74}\right) = -\frac{518035}{932437} \tag{19.80}$$

$$\mathrm{V}_{82}^{22} = \mathrm{V}_{30}^{22} \times \left(-\frac{10}{20}\right) \times \left(-\frac{6}{65}\right) = -\mathrm{V}_{22}^{30} \times \frac{3}{65} = \frac{25270}{25201} \times \frac{3}{65} \tag{19.81}$$

$$= \frac{15162}{327613} \tag{19.82}$$

where 22 is the globe frame.

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19.2.4.1 The mean motion of the Moon

The mean motion of the Moon is obtained on tube 53:

$$\mathrm{V}_{53}^{22} = \mathrm{V}_{50}^{22} \times \left(-\frac{71}{23}\right) \times \left(-\frac{8}{32}\right) \times \left(-\frac{7}{82}\right) = \mathrm{V}_{50}^{22} \times \left(-\frac{497}{7544}\right) \tag{19.83}$$

$$= \frac{518035}{932437} \times \frac{497}{7544} = \frac{6279595}{171568408} \tag{19.84}$$

$$\mathrm{P}_{53}^{22} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{19.85}$$

The same value is given by Oechslin. It is the tropical month, the period of the mean motion of the Moon with respect to the zodiac. It is the same period as the one used in the tellurium.

19.2.4.2 The lunar nodes

The motion of tube 53 is then used to obtain that of tube 58:

$$\mathrm{V}_{58}^{22} = \mathrm{V}_{53}^{22} \times \left(-\frac{82}{26}\right) \times \left(-\frac{6}{30}\right) \times \left(-\frac{6}{36}\right) \times \left(-\frac{6}{42}\right) \times \left(-\frac{19}{71}\right) \tag{19.86}$$

$$= \mathrm{V}_{53}^{22} \times \left(-\frac{779}{193830}\right) = -\frac{6279595}{171568408} \times \frac{779}{193830} \tag{19.87}$$

$$= -\frac{1968533}{13382335824} \tag{19.88}$$

$$\mathrm{P}_{58}^{22} = -\frac{13382335824}{1968533} = -6798.1262\ldots \text{ days} \tag{19.89}$$

The same value is given by Oechslin. It is the period of precession of the lunar nodes in the zodiac. The period is negative, because there is a retrogradation of the nodes.

We can also compute the motion of tube 53 with respect to tube 58:

$$\mathrm{V}_{53}^{58} = \mathrm{V}_{53}^{22} - \mathrm{V}_{58}^{22} = \frac{6279595}{171568408} + \frac{1968533}{13382335824} \tag{19.90}$$

$$= \frac{491776943}{13382335824} \tag{19.91}$$

$$\mathrm{P}_{53}^{58} = \frac{13382335824}{491776943} = 27.2122\ldots \text{ days} \tag{19.92}$$

This value is not given by Oechslin. It is the draconic month. This ratio is slightly different from the one used in the tellurium.

Johann moreover added an arbor 66 which is used to shift the Moon vertically with the period of the draconic month. Since arbor 66 is located on the frame of the mean Moon (or almost), the period of that motion is the above one of tube 53 with respect to tube 58.

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19.2.4.3 The corrected motion of the Moon

The frame attached to tube 53 carries a number of gears aimed at obtaining the corrected motion of the Moon. When that frame rotates, a wheel on arbor 60 meshes with a fixed wheel on the central arbor 59 of the globe. This causes the rotation of a plate $62'$ which is that of the main correction to the Moon.[11] We have

$$\mathrm{V}_{62'}^{53} = \mathrm{V}_{59}^{53} \times \left(-\frac{67}{22}\right) \times \left(-\frac{14}{43}\right) = \mathrm{V}_{59}^{53} \times \frac{469}{473} = -\mathrm{V}_{53}^{22} \times \frac{469}{473} \tag{19.93}$$

$$= -\frac{6279595}{171568408} \times \frac{469}{473} = -\frac{2945130055}{81151856984} \tag{19.94}$$

$$\mathrm{P}_{62'}^{53} = -\frac{81151856984}{2945130055} = -27.5545\ldots \text{ days} \tag{19.95}$$

This value is not given by Oechslin. It is the anomalistic month. The ratio is not the same as the one used in the tellurium. Like above, the negative sign has no significance here.

The 43-teeth wheel on tube $62'$ carries an excentric arbor 65. The position of this arbor is that of the Moon corrected for the anomaly. But this arbor carries another wheel which also carries an eccentric pin, which is used to add a second correction. We have

$$\mathrm{V}_{65}^{62'}? = \mathrm{V}_{62}^{62'} \times \left(-\frac{17}{18}\right) \times \left(-\frac{10}{26}\right) \times \left(-\frac{7}{19}\right) = \mathrm{V}_{62}^{62'} \times \left(-\frac{595}{4446}\right) \tag{19.96}$$

$$= -\mathrm{V}_{62'}^{53} \times \left(-\frac{595}{4446}\right) = \frac{2945130055}{81151856984} \times \left(-\frac{595}{4446}\right) \tag{19.97}$$

$$= -\frac{92229072775}{18989534534256} \tag{19.98}$$

and

$$\mathrm{V}_{65}^{53}? = \mathrm{V}_{65}^{62'}? + \mathrm{V}_{62'}^{53} = -\frac{92229072775}{18989534534256} - \frac{2945130055}{81151856984} \tag{19.99}$$

$$= -\frac{781389505645}{18989534534256} \tag{19.100}$$

$$\mathrm{P}_{65}^{53}? = -\frac{18989534534256}{781389505645} = -24.3022\ldots \text{ days} \tag{19.101}$$

These values are incorrect, as they were in the tellurium, although the tellurium has slightly different ratios. Here too, Oechslin noticed that something was wrong, but he did not come up with an explanation. The most likely reason is that Johann forgot a pair of gears to implement the second lunar anomaly. The solution is the same as the one given in the tellurium. With

[11] Oechslin names the arbor 62, but does not give a name to the rotating plate, which I thus call $62'$.

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that fix, we have

$$\mathrm{V}_{65}^{62'} = \mathrm{V}_{62}^{62'} \times \left(-\frac{17}{18}\right) \times \left(-\frac{10}{26}\right) \times \left(-\frac{7}{19}\right) \times \left(-\frac{x}{x}\right) = \mathrm{V}_{62}^{62'} \times \frac{595}{4446} \tag{19.102}$$

$$= -\mathrm{V}_{62'}^{53} \times \frac{595}{4446} = \frac{2945130055}{81151856984} \times \frac{595}{4446} \tag{19.103}$$

$$= \frac{92229072775}{18989534534256} \tag{19.104}$$

and

$$\mathrm{V}_{65}^{53} = \mathrm{V}_{65}^{62'} + \mathrm{V}_{62'}^{53} = \frac{92229072775}{18989534534256} - \frac{2945130055}{81151856984} \tag{19.105}$$

$$= -\frac{596931360095}{18989534534256} \tag{19.106}$$

$$\mathrm{P}_{65}^{53} = -\frac{18989534534256}{596931360095} = -31.8119\ldots \text{ days} \tag{19.107}$$

We now have the period of the evection, the second lunar anomaly.

19.2.4.4 The mean motion of the Sun

The mean motion of the Moon is then used to obtain the mean motion of the Sun on tube 68:

$$\mathrm{V}_{68}^{22} = \mathrm{V}_{53}^{22} \times \left(-\frac{82}{63}\right) \times \left(-\frac{10}{174}\right) = \mathrm{V}_{53}^{22} \times \frac{410}{5481} \tag{19.108}$$

$$= \frac{6279595}{171568408} \times \frac{410}{5481} = \frac{183902425}{67169031732} \tag{19.109}$$

$$\mathrm{P}_{68}^{22} = \frac{67169031732}{183902425} = 365.2427\ldots \text{ days} \tag{19.110}$$

The same value is given by Oechslin. It is an approximation of the tropical year.

An eccentric pin on arbor 70 is used to produce the corrected motion of the Sun. We have

$$\mathrm{V}_{70}^{68} = \mathrm{V}_{59}^{68} \times \left(-\frac{64}{32}\right) \times \left(-\frac{16}{32}\right) = \mathrm{V}_{59}^{68} \tag{19.111}$$

Consequently, the arbor 70 replicates the central arbor of the globe, and is thus still. The pin therefore has a fixed direction with respect to the stars, and this direction is at right angle with the line of apsides.

The frame corresponding to the corrected motion of the Sun is not named by Oechslin, but I will name it 68′. This frame carries the gear trains of Mercury and Venus.

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19.2.4.5 The motion of Venus

For Venus, Johann assumes that its orbit is circular and he only considers the retrogradations due to the revolution of the Earth. In his astronomical clocks, Hahn also considered Venus' orbit as circular.

Johann merely implemented an oscillation of Venus around the Sun. The pin causing this oscillation is located excentrically on tube 75. This oscillation should have the period of the synodic revolution of Venus. We can compute the velocity of tube 75 with respect to the frame $68'$ which is that of the corrected Sun. But since the gear train involves an arbor which is set on frame 72 which is not fixed on frame $68'$, we first compute the velocity of tube 75 with respect to frame 72. Note that frame 72 actually replicates the motion of the entire globe.

$$\mathrm{V}_{75}^{72} = \mathrm{V}_{73}^{72} \times \left(-\frac{47}{23}\right) \times \left(-\frac{35}{44}\right) = \mathrm{V}_{73}^{72} \times \frac{1645}{1012} \tag{19.112}$$

$$= -\mathrm{V}_{72}^{73} \times \frac{1645}{1012} = -\mathrm{V}_{72}^{68'} \times \frac{1645}{1012} \tag{19.113}$$

Then

$$\mathrm{V}_{75}^{68'} = \mathrm{V}_{75}^{72} + \mathrm{V}_{72}^{68'} = \mathrm{V}_{72}^{68'} \times \left(1 - \frac{1645}{1012}\right) = -\mathrm{V}_{72}^{68'} \times \frac{633}{1012} \tag{19.114}$$

$$= -\mathrm{V}_{59}^{68'} \times \left(-\frac{64}{24}\right) \times \left(-\frac{24}{64}\right) \times \frac{633}{1012} = -\mathrm{V}_{59}^{68'} \times \frac{633}{1012} \tag{19.115}$$

$$\approx -\mathrm{V}_{59}^{68} \times \frac{633}{1012} = \mathrm{V}_{68}^{59} \times \frac{633}{1012} = \mathrm{V}_{68}^{22} \times \frac{633}{1012} \tag{19.116}$$

$$\approx \frac{183902425}{67169031732} \times \frac{633}{1012} = \frac{38803411675}{22658353370928} \tag{19.117}$$

$$\mathrm{P}_{75}^{68'} \approx \frac{22658353370928}{38803411675} = 583.9268\ldots \text{ days} \tag{19.118}$$

This is indeed a good approximation of the synodic period of Venus. The value is not given by Oechslin.

We can also compute the motion of Venus with respect to the globe:

$$\mathrm{V}_{75}^{22} = \mathrm{V}_{75}^{68'} + \mathrm{V}_{68'}^{22} \approx \mathrm{V}_{75}^{68'} + \mathrm{V}_{68}^{22} \approx \frac{38803411675}{22658353370928} + \frac{183902425}{67169031732} \tag{19.119}$$

$$\approx \frac{302519489125}{67975060112784} \tag{19.120}$$

$$\mathrm{P}_{75}^{22} \approx \frac{67975060112784}{302519489125} = 224.6964\ldots \text{ days} \tag{19.121}$$

The same value is given by Oechslin, also in sidereal days.

19.2.4.6 The motion of Mercury

The motion of Mercury is more complex, because Johann not only took into account its retrogradations, but also the fact that its orbit is highly elliptical.

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For the retrogradation of Mercury, Johann's construction is similar to that used for Venus. There is an eccentric pin on tube 80, which rotates on the frame 68′ of the corrected sun. In this case, tube 77 plays a similar role to tube 72 and also replicates the motion of the entire globe. We have

$$\mathrm{V}_{80}^{77} = \mathrm{V}_{78}^{77} \times \left(-\frac{77}{17}\right) \times \left(-\frac{44}{48}\right) = \mathrm{V}_{78}^{77} \times \frac{847}{204} \tag{19.122}$$

$$= -\mathrm{V}_{77}^{78} \times \frac{847}{204} = -\mathrm{V}_{77}^{68'} \times \frac{847}{204} \tag{19.123}$$

Then

$$\mathrm{V}_{80}^{68'} = \mathrm{V}_{80}^{77} + \mathrm{V}_{77}^{68'} = \mathrm{V}_{77}^{68'} \times \left(1 - \frac{847}{204}\right) = -\mathrm{V}_{77}^{68'} \times \frac{643}{204} \tag{19.124}$$

$$= -\mathrm{V}_{59}^{68'} \times \left(-\frac{64}{24}\right) \times \left(-\frac{24}{64}\right) \times \frac{643}{204} = -\mathrm{V}_{59}^{68'} \times \frac{643}{204} \tag{19.125}$$

$$\approx -\mathrm{V}_{59}^{68} \times \frac{643}{204} = \mathrm{V}_{68}^{59} \times \frac{643}{204} = \mathrm{V}_{68}^{22} \times \frac{643}{204} \tag{19.126}$$

$$\approx \frac{183902425}{67169031732} \times \frac{643}{204} = \frac{118249259275}{13702482473328} \tag{19.127}$$

$$\mathrm{P}_{80}^{68'} \approx \frac{13702482473328}{118249259275} = 115.8779\ldots \text{ days} \tag{19.128}$$

This is indeed a good approximation of the synodic period of Mercury. This value is not given by Oechslin.

We can also compute the motion of Mercury with respect to the globe:

$$\mathrm{V}_{80}^{22} = \mathrm{V}_{80}^{68'} + \mathrm{V}_{68'}^{22} \approx \mathrm{V}_{80}^{68'} + \mathrm{V}_{68}^{22} \approx \frac{118249259275}{13702482473328} + \frac{183902425}{67169031732} \tag{19.129}$$

$$\approx \frac{14160486725}{1245680224848} \tag{19.130}$$

$$\mathrm{P}_{80}^{22} \approx \frac{1245680224848}{14160486725} = 87.9687\ldots \text{ days} \tag{19.131}$$

The same value is given by Oechslin, also in sidereal days.

In addition, the fixed arbor 78 carries an eccentric pin whose purpose seems to be to take into account the elliptical orbit of Mercury. However, this pin seems to be at a fixed position with respect to the corrected Sun, and it isn't clear how this correction can be working.

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19.2.4.7 The motion of the Sun in the upper part

In the upper part of the globe, the motion of the Sun is obtained on tube 84 which is part of the upper inner frame fixed to the globe:[12]

$$\mathrm{V}_{84}^{22} = \mathrm{V}_{82}^{22} \times \left(-\frac{38}{47}\right) \times \left(-\frac{6}{82}\right) = \mathrm{V}_{82}^{22} \times \frac{114}{1927} \tag{19.132}$$

$$= \frac{15162}{327613} \times \frac{114}{1927} = \frac{1728468}{631310251} \tag{19.133}$$

$$\mathrm{P}_{84}^{22} = \frac{631310251}{1728468} = 365.2426\ldots \text{ days} \tag{19.134}$$

This is the same value of the tropical year that we have met earlier. It is also given by Oechslin, also in sidereal days.

The corrected motion of the Sun is obtained by a wheel on a frame I will name 84′ (Oechslin also names it 84). A pin on the wheel on frame 84 drives the corrected motion of the Sun, but this motion is irregular because the axes of the two frames are offset.

This corrected motion of the Sun is replicated on frames 86 and 87 using the intermediate arbor 85 and identical ratios.

19.2.4.8 The mean motions of Mars, Jupiter and Saturn

The mean motion of the Sun in the upper part is also used to obtain the mean motions of Mars (tube 98), Jupiter (tube 94) and Saturn (tube 90). We have

$$\mathrm{V}_{98}^{22} = \mathrm{V}_{84}^{22} \times \left(-\frac{82}{6}\right) \times \left(-\frac{6}{41}\right) \times \left(-\frac{29}{22}\right) \times \left(-\frac{24}{119}\right) \tag{19.135}$$

$$= \mathrm{V}_{84}^{22} \times \frac{696}{1309} = \frac{1728468}{631310251} \times \frac{696}{1309} = \frac{5926176}{4070862653} \tag{19.136}$$

$$\mathrm{P}_{98}^{22} = \frac{4070862653}{5926176} = 686.9290\ldots \text{ days} \tag{19.137}$$

$$\mathrm{V}_{94}^{22} = \mathrm{V}_{84}^{22} \times \left(-\frac{82}{6}\right) \times \left(-\frac{6}{41}\right) \times \left(-\frac{6}{24}\right) \times \left(-\frac{14}{83}\right) \tag{19.138}$$

$$= \mathrm{V}_{84}^{22} \times \frac{7}{83} = \frac{1728468}{631310251} \times \frac{7}{83} = \frac{12099276}{52398750833} \tag{19.139}$$

$$\mathrm{P}_{94}^{22} = \frac{52398750833}{12099276} = 4330.7344\ldots \text{ days} \tag{19.140}$$

$$\mathrm{V}_{90}^{22} = \mathrm{V}_{84}^{22} \times \left(-\frac{82}{6}\right) \times \left(-\frac{6}{41}\right) \times \left(-\frac{6}{24}\right) \times \left(-\frac{7}{103}\right) \tag{19.141}$$

$$= \mathrm{V}_{84}^{22} \times \frac{7}{206} = \frac{1728468}{631310251} \times \frac{7}{206} = \frac{6049638}{65024955853} \tag{19.142}$$

$$\mathrm{P}_{90}^{22} = \frac{65024955853}{6049638} = 10748.5697\ldots \text{ days} \tag{19.143}$$

The same values are given by Oechslin.

[12] The fact that this frame is fixed is not readily apparent in Oechslin's drawing.

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We can see here that Johann took the ratios of the orbital periods of Mars, Jupiter and Saturn to the Earth to be $\frac{1309}{696}$, $\frac{83}{7}$ and $\frac{206}{7}$ and he obtained excellent approximations of the orbital periods. The exact same orbital periods were used in the outer orrery on the base of the clock.

19.2.4.9 The eccentric motions of Mars, Jupiter and Saturn

The eccentric motions of Mars, Jupiter and Saturn are obtained like in the orreries on the base of the clock. Each "base" wheel has its axis offset from the axis of the globe. The base wheels are those on tube 98 (Mars), 94 (Jupiter) and 90 (Saturn), which move according to the mean motions of the planets. Each of these wheels has a pin driving another wheel immediately below it and which is not offset. I am naming these wheels 98′, 94′ and 90′. These three wheels therefore have an irregular motion which approximates that of the elliptic motion of the planet with respect to the stars.

19.2.4.10 The retrogradations of Mars, Jupiter and Saturn

Johann superposes the retrogradations of the planets to the motions of the corrected wheels 98′, 94′ and 90′. This is done in the same way for the three planets. Consider for instance the case of Saturn. Wheel 90′ carries a 40-teeth wheel which meshes with a 24-teeth wheel, which itself meshes with a 40-teeth wheel on the central tube 84′. That tube (named 84 by Oechslin) has the motion of the corrected Sun. Consequently, the first 40-teeth wheel on arbor 92 replicates the motion of the corrected Sun. This wheel carries an eccentric pin which drives the tube 90″ which is linked to the Saturn hand. This tube moves back and forth depending on the angle between the longitude of Saturn (given by tube 90′) and that of the Sun (given by tube 84′). In other words, this back-and-forth motion has the period of the synodic revolution of Saturn, which is what it should be.

In the case of Mars, the design is the same, but the intermediate wheel is an horizontal pinion and the two extreme wheels are contrate wheels.

19.3 References

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Chapter 20

(Oechslin: 2.2)

Klein's geographic clock in Prague (c1732-1737)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

20.1 Introduction

The clock described here was completed probably around 1732-1737 by Jan Klein (1684-1762) (figure 20.2).[1]

Klein was born in Česká Kamenice, now in the Czech Republic.[2] After having been in a Jesuit school, he entered the order of the Jesuits in 1703. He studied and taught mathematics, astronomy and mechanics. In 1732, he became director of the mathematical museum in the Clementinum College and remained in this position until his death.

His biographer Pelcl also mentions that besides his astronomical and geographical clocks, Klein has constructed a large number of other works of art, in particular a moving sphere, an elliptic clock, a quadrant, etc. He also made automata, for instance an automatic harp, a drum, a moving tortoise, etc.

[1] This clock has been dated from 1753 by Böhm [2], but Engelmann [7] believes that it was made around 1732-1737, and that it is the clock which was copied for the Dresden geographical clock made in 1738. I have decided to follow Engelmann, in particular because the Dresden clock has a simpler mechanism than the one from Prague. Oechslin also dated the clock to 1753 [12], and was apparently unaware of Engelmann's article. Pařízek [15] dates this clock around 1760.

[2] Biographical information on Klein is scarce. Almost all that we know comes from Pelcl's 1782 account [16, p. 137-141]. In 1751, Stepling didn't give any biographical details on Klein [20]. In his 1791 account, Strnadt borrows from Stepling and Pelcl [21]. Böhm [2] gives a very brief one-page biographical notice on Klein, which is mostly drawn from Pelcl [16]. See also Maurice [10, v.1, p. 267-268], Abeler [1], Michal [11] and Šíma [19, p. 84-85, 93-95, 113].

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Plassmeyer claims that Klein has also made clocks for the Court of the Chinese emperor in Beijing, but I do not know what his sources are.[3]

Klein died in Prague in 1762.

20.2 Description of the clock

This clock is a geographical clock and belongs to a group of three similar clocks kept in the Clementinum in Prague. It is similar, but slightly more complex, than Klein's geographical clock kept in Dresden and which is dated from 1738 (Oechslin 2.1).[4] Like the clock in Dresden, this clock is inspired by Johann Baptist Homann (1663-1724)'s design of a geographical clock which was constructed in 1705 by Zacharias Landteck (1670-1740) [5, 13] (figure 20.1).

This clock was first described in detail by Böhm in 1863 [2, 3] and in 1910 by Engelmann [7].[5] It was again described by Oechslin in 1996,[6] but Oechslin did not have the opportunity to disassemble the clock and based his description only on that published by Böhm.

It is a table clock with two sides. One side shows the time on a 12-hour dial with additional openings for the day of the month, the sunrise and sunset. The other side shows a fixed the Northern hemisphere, but with a rotating transparent shell representing the part of the Earth which is not lit by the Sun. The Sun rotates around the Earth and there is a ring for the ecliptic.

The common arbor to both sides is arbor 1 which makes one turn in one hour. Measured from the time side, we have

$$\mathrm{V}_1^0 = -24 \tag{20.1}$$

$$\mathrm{P}_1^0 = -\frac{1}{24} = -1 \text{ h} \tag{20.2}$$

[3] [17, p. 94]

[4] See Schardin [18, p. 21], Dolz [6, p. 112-113], Michal [11] and Plaßmeyer [17, p. 94-95]. Pelcl also describes Klein's three clocks in Prague, and mentions that a copy of the geographical clock in Prague had to be made for the King August of Poland and is now in Dresden [16, p. 137-141]. Maurice, incidentally, claims that Klein must have made a third geographical clock [10, v.1, p. 268], because Klein's portrait shows a geographical clock which differs both from the Prague and from the Dresden one.

[5] Engelmann slightly adapted Böhm's drawings, but I am not including them here.

[6] [12, p. 38, 49-50]

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Figure 20.1: Johann Baptist Homann's geographical clock (1705).

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Figure 20.2: Left: Portrait of Jan Klein with his geographic clock from 1753/1754. (source: [2]) Right: portrait in Pelcl's biographical notice [16].

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20.2.1 The time side of the clock

This side of the clock (figure 20.3) gets its input from arbor 1 which carries the minute hand. This motion is derived from the going work which is not described by Oechslin (see figure 20.4 for Böhm's drawing).

The motion of arbor 1 is certainly transferred to tube 10 which makes a turn in 12 hours, but the details are not given by Oechslin. We thus have

$$\mathrm{V}_{10}^{0} = -2 \tag{20.3}$$

$$\mathrm{P}_{10}^{0} = -\frac{1}{2} = -12 \text{ h} \tag{20.4}$$

The motion of tube 10 is first transferred to arbor 11 which makes a turn in a day:

$$\mathrm{V}_{11}^{0} = \mathrm{V}_{10}^{0} \times \left(-\frac{36}{72}\right) = 1 \tag{20.5}$$

This arbor carries a finger which moves a 31-teeth wheel by one position and this will serve to indicate the day of the month in the lower part of the 12-hour dial, just above the figure VI (figure 20.5). The lengths of the months do not seem to be accounted for.

The motion of arbor 11 is used to obtain the motion of arbor 14:

$$\mathrm{V}_{14}^{0} = \mathrm{V}_{11}^{0} \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{73}\right) = \mathrm{V}_{11}^{0} \times \frac{1}{365} = \frac{1}{365} \tag{20.6}$$

$$\mathrm{P}_{14}^{0} = 365 \text{ days} \tag{20.7}$$

This arbor carries a cam and this cam is used to move the hours of sunrise and sunset back and forth. These hours are written on a horizontal slider, with the hour of sunrise on the left and the hours of sunset on the right (see figure 20.5). The figures on the left part are given in opposite order as the figures on the right, so that when the sunrise is one hour later, the sunset is also given one hour earlier.

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Figure 20.3: The front side of Klein's clock. (source: [2])

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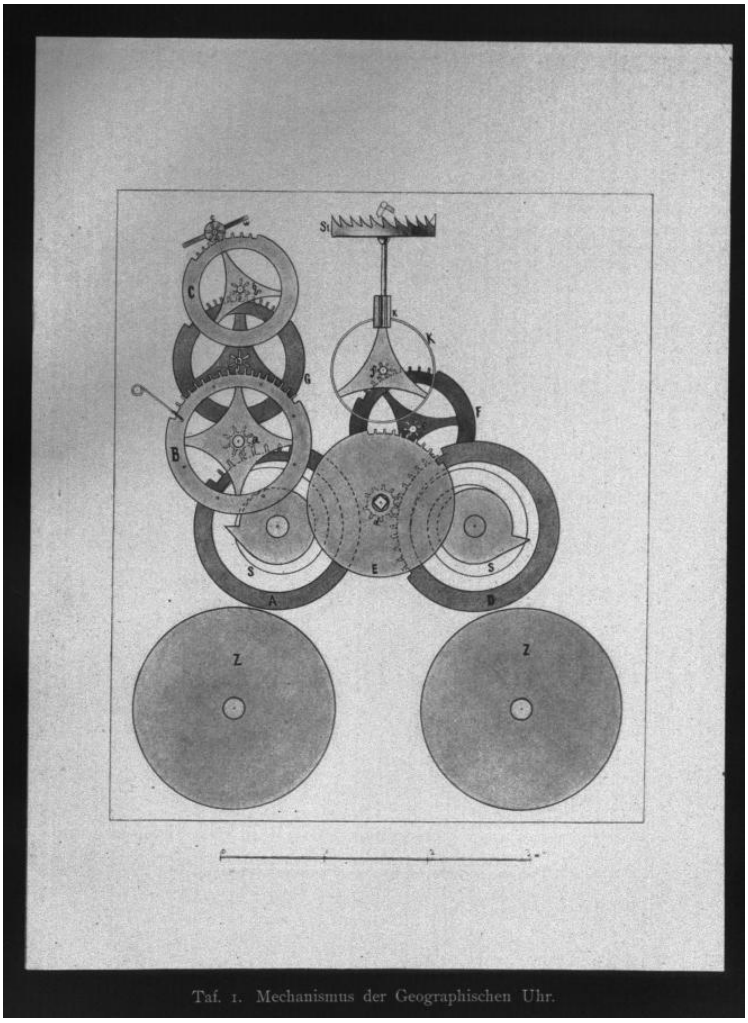


Figure 20.4: Details of the going and striking works of Klein's clock. (source: [2])

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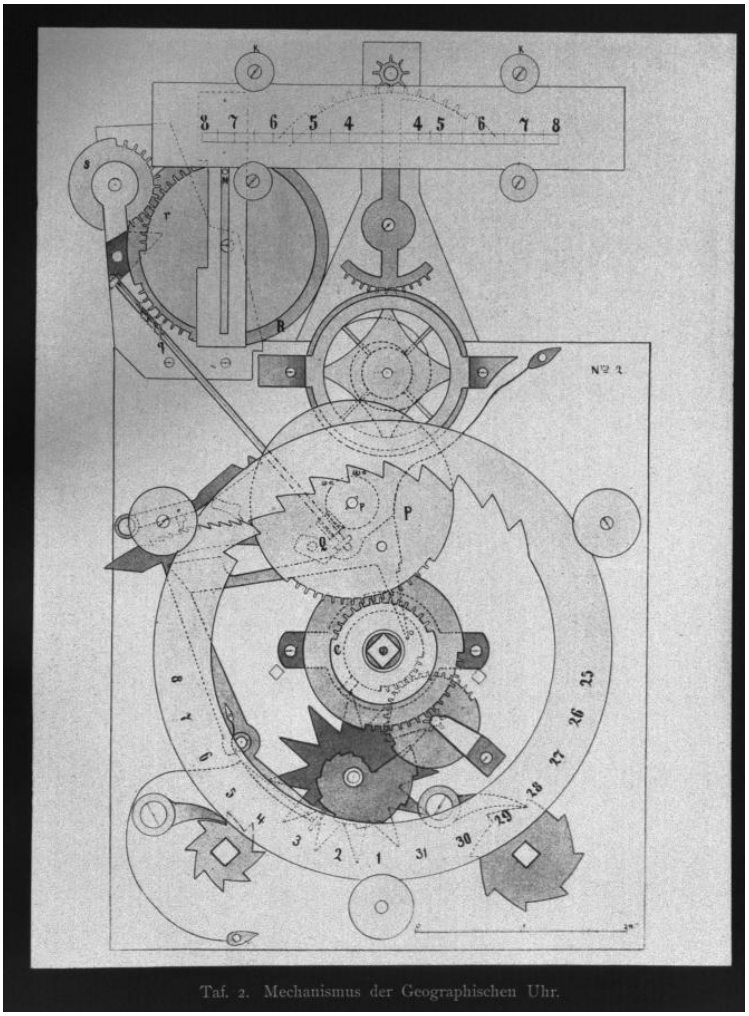


Figure 20.5: Details of the day of the month and sunrise/sunset mechanisms in Klein's clock. (source: [2])

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20.2.2 The geographic side of the clock

The geographic side (figure 20.6) of the clock shows a fixed Northern hemisphere and a moving shell shows which part of the globe is lit, and which one is not, taking into account both the diurnal motion of the Sun, and the inclination of Earth's axis on the ecliptic.

The input motion of that side is also arbor 1 which makes one turn clockwise in one hour as seen from the other side:

$$\mathrm{V}_1^0 = -24 \tag{20.8}$$

$$\mathrm{P}_1^0 = -\frac{1}{24} = -1 \text{ h} \tag{20.9}$$

20.2.2.1 The rotating motion of the shell and the Sun

The arbor 1 carries a 3-pin pinion which meshes with a 72-teeth wheel on tube 2, which is on the central axis of the geographic side. The center of this side is therefore higher than the center of the opposite side which is at the level of arbor 1. Tube 2 has the velocity (measured from the geographic side)

$$\mathrm{V}_2^0 = \mathrm{V}_1^0 \times \frac{3}{72} = \mathrm{V}_1^0 \times \frac{1}{24} = -1 \tag{20.10}$$

This tube makes a turn clockwise in one day and carries the transparent shell representing the shadow. This shell is not a hemisphere, but must extend at least $90° + 23.5°$. The clockwise motion is the expected motion, because the apparent motion of the Sun is clockwise.

The shell is also mobile around an horizontal axis (in fact, horizontal on the drawing, but parallel to the dial), but this axis moves with frame 2. The Sun is at a fixed position on frame 2, at right angle with the horizontal axis of the shell, so that the Sun is always in the middle of the lit part of the Earth.

20.2.2.2 The motion of the ecliptic

The ecliptic is represented by a ring 9 located around the Earth, outside of the Sun (part of frame 2), but inside of the 24-hour dial (frame 6). This ring carries an internal 73-teeth wheel, which meshes with a 6-leaves pinion on frame 2 (figure 20.7). The arbor 8 of this pinion carries a 30-teeth wheel which meshes with another 6-leaves pinion on arbor 7, which is also part of the frame 2. Finally, this arbor 7 carries a 6-pointed star wheel which is advanced (meshes) by a finger on the 24-hour dial. In order to compute the motion of the ecliptic, it is best to consider first the motions relative to frame 2. We have

$$\mathrm{V}_9^2 = \mathrm{V}_6^2 \times \frac{1}{6} \times \left(-\frac{6}{30}\right) \times \frac{6}{73} = \mathrm{V}_6^2 \times \left(-\frac{1}{365}\right) = \mathrm{V}_2^0 \times \frac{1}{365} \tag{20.11}$$

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Figure 20.6: The geographic side of Klein's clock. (source: [2])

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and

$$V_9^0 = V_9^2 + V_2^0 = V_2^0 \times \left(\frac{1}{365} + 1\right) = V_2^0 \times \frac{366}{365} = -\frac{366}{365} \tag{20.12}$$

$$P_9^0 = -\frac{365}{366} = -23 \text{ h } 56 \text{ m } 3.9344\ldots \text{ s} \tag{20.13}$$

The ecliptic ring, or the zodiac, makes a turn clockwise in a sidereal day. This is the expected motion.

20.2.2.3 The oscillating motion of the shell

The shell has a back and forth motion between the two solstices. In Winter, the shell covers the North pole, but in Summer the North pole is lit by the Sun. This is done by having a cam make a turn in one year with respect to the frame 2 (figure 20.7).[7] This motion is introduced within this frame by a finger located on the frame 6 of the 24-hour dial. When frame 2 rotates, a 5-pointed star on arbor 4 meets this finger and rotates the arbor. This motion is then transferred to the vertical arbor 5 which carries the cam. We have

$$V_5^2 = V_6^2 \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{73}\right) = V_6^2 \times \frac{1}{365} = -V_2^6 \times \frac{1}{365} \tag{20.14}$$

$$= -V_2^0 \times \frac{1}{365} = \frac{1}{365} \tag{20.15}$$

$$P_5^2 = 365 \text{ days} \tag{20.16}$$

The cam acts on a lever which is connected to the shell and causes its oscillation.

In the 1738 clock kept in Dresden (Oechslin 2.1), the motions of the ecliptic and of the shell are obtained by the same gears, which seems a simpler solution than the one used here.

[7] See also [12, p. 148-149] on this construction.

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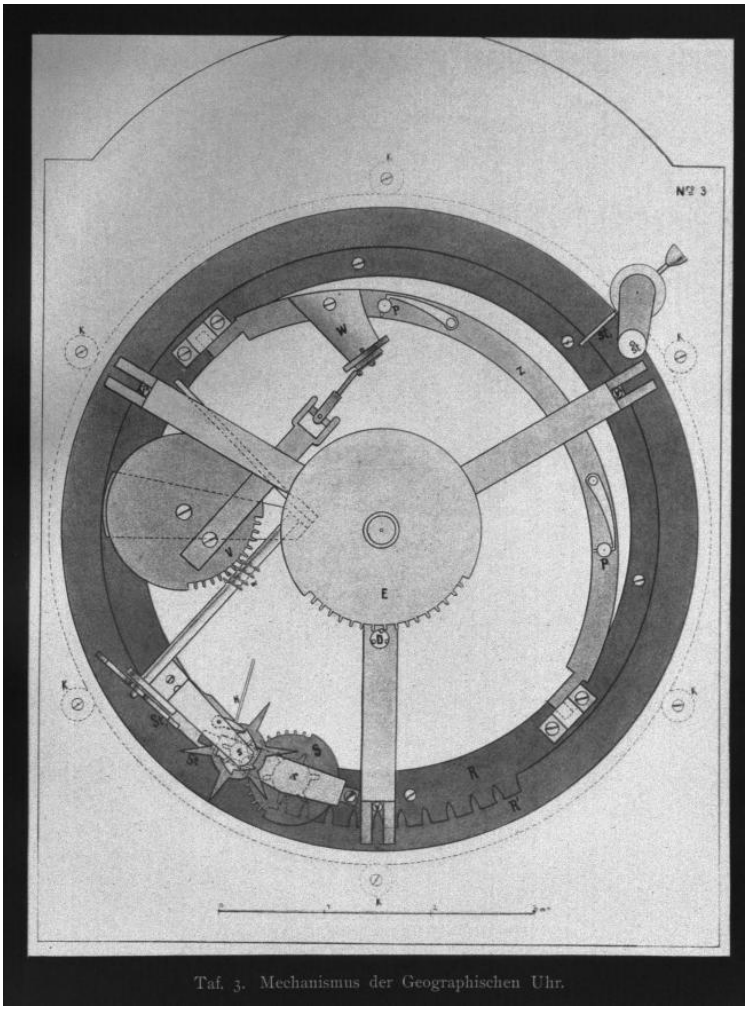


Figure 20.7: Details of the mechanism of the globe in Klein's clock. At the bottom, we can see the internal 73-teeth wheel (R') and the 6-pointed star wheel St. Towards the left, there is the 73-teeth wheel V which carries the cam. This wheel is moved by a worm. The wheel at the center is the 72-teeth wheel E which is part of tube 2. Under it, we can see the 3-pin pinion D making a turn in an hour. (source: [2])

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Chapter 21

(Oechslin: 2.1)

Klein's geographic clock in Dresden (1738)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

21.1 Introduction

The clock described here was constructed in 1738 by Jan Klein (1684-1762).[1]

This clock was an order of the elector of Saxony Frederick Augustus II (Augustus III of Poland) (1696-1763). The year 1738 is inscribed on the mock pendulum.[2]

The clock is a geographical clock inspired by Johann Baptist Homann (1663-1724)'s design of a geographical clock which was constructed in 1705 by Zacharias Landteck (1670-1740) [4, 12] (figure 21.1).

It is similar to Klein's geographical clock kept in the Clementinum in Prague (Oechslin 2.2) which may be the earlier one.[3] In any case, the Dresden clock has a simpler mechanism than that of Prague which uses three versions of the annual motion, instead of two here. The Dresden clock is topped by the Polish crown.

[1] For biographical information on Klein, see the chapter on the geographic clock in Prague.

[2] For brief descriptions of this clock, see Schardin [17, p. 21], Oechslin [11, p. 38, 49-50], Michal [10] and Pařízek [14] who dates it around 1760. This clock was also shown at the 1989 Hahn exhibition [22, p. 63-64].

[3] Pelcl also describes Klein's three clocks in Prague, and mentions that a copy of the geographical clock in Prague had to be made for the King August of Poland and is now in Dresden [15, p. 137-141]. Although the Prague geographical clock has been dated around 1753 by Böhm [1], I prefer to follow Engelmann [6] who believes that the Prague clock was made sometime between 1732 and 1737. Maurice, incidentally, claims that Klein must have made a third geographical clock, because Klein's portrait shows a geographical clock which is neither that of Dresden, nor that of Prague [9, v.1, p. 268].

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The globe of the clock was made by Pater Sichelbarth, but Schardin writes that he was born in 1690 and died in 1747.[4] I believe that there is a confusion with Pater Ignatius Sichelbarth (1708-1780), a Jesuit who was a painter and went to China around 1745 where he became a mandarin.



Figure 21.1: Johann Baptist Homann's geographical clock (1705).

[4] [17, p. 21]

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Figure 21.2: Left: Portrait of Jan Klein with his geographic clock from 1753/1754. (source: [1]) Right: portrait in Pelcl's biographical notice [15].

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Figure 21.3: Klein's 1738 clock on a 1975 stamp from the DDR.

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Although Oechslin describes the gear trains of the clock, I believe that he did not have the opportunity to disassemble the clock. He may have used some unpublished source, or only examined the works from the outside, comparing it with the geographical clock located in Prague.

This clock is a table clock with two sides. The clock is spring-driven with a fusee and regulated by a short pendulum.[5] There are two striking works and six bells. The clock works 8 days. One side shows the time on a 12-hour dial with additional openings for the day of the month, the sunrise and sunset. There is also an aperture for a mock pendulum. The other side shows a fixed the Northern hemisphere, but with a rotating transparent shell representing the part of the Earth which is not lit by the Sun. The Sun rotates around the Earth and there is a ring for the ecliptic. For the study of this clock, it is best to first read the description of the geographical clock located in Prague, as it is supplemented by Böhm's drawings [1].

The common arbor to both sides is arbor 2 which makes one turn in one hour. Measured from the time side, we have

$$\mathrm{V}_2^0 = -24 \tag{21.1}$$

$$\mathrm{P}_2^0 = -\frac{1}{24} = -1 \text{ h} \tag{21.2}$$

[5] [5, p. 113]

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21.2 The time side of the clock

This side of the clock gets its input from arbor 2 which carries the minute hand. This motion is derived from the going work which is not described by Oechslin.

The motion of arbor 2 is transferred to tube 8 which makes a turn in 12 hours, but the teeth counts are not given by Oechslin. We thus have

$$\mathrm{V}_8^0 = -12 \tag{21.3}$$

$$\mathrm{P}_8^0 = -\frac{1}{2} = -12 \text{ h} \tag{21.4}$$

The motion of tube 8 is first transferred to arbor 9 which makes a turn in a day, but Oechslin doesn't give the details of the gears:

$$\mathrm{V}_9^0 = 1 \tag{21.5}$$

This arbor carries a finger which moves a 31-teeth wheel by one position and this will serve to indicate the day of the month in the lower part of the 12-hour dial, just above the figure VI. The lengths of the months do not seem to be accounted for.

The motion of arbor 9 is used to obtain the motion of arbor 13:

$$\mathrm{V}_{13}^0 = \mathrm{V}_9^0 \times \left(-\frac{a}{a}\right) \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{73}\right) = \mathrm{V}_9^0 \times \left(-\frac{1}{365}\right) = -\frac{1}{365} \tag{21.6}$$

$$\mathrm{P}_{13}^0 = -365 \text{ days} \tag{21.7}$$

This arbor carries a cam and this cam is used to move the hours of sunrise and sunset back and forth. These hours are written on a horizontal slider, with the hour of sunrise on the left and the hours of sunset on the right. The figures on the left part are given in opposite order as the figures on the right, so that when the sunrise is one hour later, the sunset is also given one hour earlier.

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21.3 The geographic side of the clock

The geographic side of the clock shows a fixed Northern hemisphere and a moving shell shows which part of the globe is lit, and which one is not, taking into account both the diurnal motion of the Sun, and the inclination of Earth's axis on the ecliptic. Just above the main dial, there is a small dial which can be used to regulate the clock.

The input motion of that side is also arbor 2 which makes one turn clockwise in one hour as seen from the other side:

$$\mathrm{V}_2^0 = -24 \tag{21.8}$$

$$\mathrm{P}_2^0 = -\frac{1}{24} = -1 \text{ h} \tag{21.9}$$

21.3.1 The rotating motion of the shell and the Sun

The arbor 2 carries a 3-pin pinion which meshes with a 72-teeth wheel on tube 3, which is on the central axis of the geographic side. The center of this side is therefore higher than the center of the opposite side which is at the level of arbor 2. Tube 3 has the velocity (measured from the geographic side)

$$\mathrm{V}_3^0 = \mathrm{V}_2^0 \times \frac{3}{72} = \mathrm{V}_2^0 \times \frac{1}{24} = -1 \tag{21.10}$$

This tube makes a turn clockwise in one day and carries the transparent shell representing the shadow. This shell is not a hemisphere, but must extend at least $90° + 23.5°$. The clockwise motion is the expected motion, because the apparent motion of the Sun is clockwise.

The shell is also mobile around an horizontal axis (on the drawing, but in fact parallel to the dial), but this axis moves with frame 3. The Sun is at a fixed position on frame 3, at right angle with the horizontal axis of the shell, so that the Sun is always in the middle of the lit part of the Earth.

21.3.2 The motion of the ecliptic and the oscillation of the shell

The ecliptic is represented by a ring located around the Earth and this ring is part of frame 6. The ring is located outside of the Sun (part of frame 3), but inside of the 24-hour dial (frame 4).

The ecliptic ring, or the zodiac, makes a turn clockwise in a sidereal day. This motion is obtained as follows. A finger is located on the frame 4 of the 24-hour dial. When frame 3 rotates, a 5-pointed star on arbor 5 meets this finger and rotates the arbor. This motion is then transferred to the vertical

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arbor 6 which carries the ecliptic ring. We have

$$V_6^3 = V_4^3 \times \frac{1}{5} \times \left(-\frac{1}{73}\right) = V_4^3 \times \left(-\frac{1}{365}\right) = -V_3^4 \times \left(-\frac{1}{365}\right) \tag{21.11}$$

$$= V_3^0 \times \frac{1}{365} = -\frac{1}{365} \tag{21.12}$$

$$P_6^3 = -365 \text{ days} \tag{21.13}$$

The frame 6 makes a turn clockwise in 365 days.

There is a back and forth motion of the shell between the two solstices. In Winter, the shell covers the North pole, but in Summer the North pole is lit by the Sun. This is done by having a tilted rail on frame 6 and this rail guides the shell, having it oscillate with the period P_6^3.

In the Prague clock (Oechslin 2.2), there is another similar gear train which is used to produce the oscillation of the shell around an axis parallel to the dial, but it is a more complex solution.

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Chapter 22

(Oechslin: 2.3)

Klein's Tychonic clock in Prague (1751)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

22.1 Introduction

The clock described here was completed in 1751 by Jan Klein (1684-1762).[1] This clock displays the Tychonic system and belongs to a group of three similar clocks kept in the Clementinum in Prague.

This clock was first described by Stepling in 1751 [17], mentioned by Pelcl [13], and described in detail by Böhm in 1863 [1, 2]. Böhm wrote about the "very complex mechanism of this clock" [1, p. 7], but this clock is in fact a rather simple clock. It is a table clock with two sides, one for the time and calendar indications, the other one for the Tychonic system.

Oechslin did not have the opportunity to disassemble the clock and based his description only on that published by Böhm.[2]

[1] For biographical information on Klein, see the chapter on the geographic clock in Dresden.

[2] This clock was shown at the 1989 Hahn exhibition [20, p. 65]. See also Oechslin [10, p. 32-33, 49-50] and Michal [9].

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Figure 22.1: Left: Portrait of Jan Klein with his geographic clock from 1753/1754. (source: [1]) Right: portrait in Pelcl's biographical notice [13].

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22.2 The going work

The going work and the striking work are described neither by Böhm [1] nor by Oechslin, but we know that the going work drives the central tube 2, which is the common input to the two faces of the clock. I will measure the motion of this tube from the Tychonic side only. On that side, tube 2 makes a turn clockwise in 24 hours:

$$\mathrm{V}_2^0 = -1 \tag{22.1}$$
$$\mathrm{P}_2^0 = -1 \text{ day} \tag{22.2}$$

22.3 The front side of the clock

This side of the clock (figure 22.2) shows the time on a 12-hour dial, as well as the digits of the year (in two parts), the dominical letter, the golden number, the solar cycle, the epact, the indiction, the Easter full Moon, the day with which the year ends, and data for the current month (fixed feasts, month, signs of the zodiac, lengths of days and nights, and sunrises and sunsets).

This side of the clock takes its input from tube 2 which provides the base motion of the two sides of the clock. Using two worms, this motion is used to obtain the motion of arbor 20 (all motions measured from this side of the clock, except for V_2^0):

$$\mathrm{V}_{20}^0 = \mathrm{V}_2^0 \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{73}\right) = -\frac{1}{365} \tag{22.3}$$
$$\mathrm{P}_{20}^0 = -365 \text{ days} \tag{22.4}$$

This arbor makes a turn clockwise in a year (but with a crude approximation). This arbor also carries a finger which acts on a lever, causing the arbor 22 which carries a 6-pointed star to make a sixth of a turn. This arbor also carries a 12-leaves pinion which meshes with a 200-teeth wheel on arbor 23 which carries a disk with the year, the dominical letter and the day of the week with which the year ends. We thus have

$$\mathrm{V}_{22}^0 = -\frac{1}{365} \times \frac{1}{6} \tag{22.5}$$
$$\mathrm{P}_{22}^0 = -6 \text{ years} \tag{22.6}$$
$$\mathrm{V}_{23}^0 = \mathrm{V}_{22}^0 \times \left(-\frac{12}{200}\right) = \mathrm{V}_{22}^0 \times \left(-\frac{6}{100}\right) = \frac{1}{365} \times \frac{1}{100} \tag{22.7}$$
$$\mathrm{P}_{23}^0 = 100 \text{ years} \tag{22.8}$$

Finally, the arbor 22 also carries a 24-teeth pinion which meshes with a

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Figure 22.2: The front side of Klein's clock. (source: [1])

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Figure 22.3: The front side of Klein's clock. (source: [17])

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Figure 22.4: The front side of Klein's clock. (source: [18])

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76-teeth wheel on arbor 24. We therefore have

$$\mathrm{V}^0_{24} = \mathrm{V}^0_{22} \times \left(-\frac{24}{76}\right) = \mathrm{V}^0_{22} \times \left(-\frac{6}{19}\right) \tag{22.9}$$

$$\mathrm{P}^0_{24} = \mathrm{P}^0_{22} \times \left(-\frac{19}{6}\right) = 19 \text{ years} \tag{22.10}$$

This arbor carries the figures for the golden numbers.

The motion of arbor 24 is also transferred to the arbors 25, 26 and 27:

$$\mathrm{V}^0_{25} = \mathrm{V}^0_{24} \times \left(-\frac{76}{60}\right) = \mathrm{V}^0_{24} \times \left(-\frac{19}{15}\right) \tag{22.11}$$

$$\mathrm{P}^0_{25} = \mathrm{P}^0_{24} \times \left(-\frac{15}{19}\right) = -15 \text{ years} \tag{22.12}$$

$$\mathrm{V}^0_{26} = \mathrm{V}^0_{25} \times \left(-\frac{60}{76}\right) = \mathrm{V}^0_{25} \times \left(-\frac{15}{19}\right) \tag{22.13}$$

$$\mathrm{P}^0_{26} = \mathrm{P}^0_{25} \times \left(-\frac{19}{15}\right) = 19 \text{ years} \tag{22.14}$$

$$\mathrm{V}^0_{27} = \mathrm{V}^0_{25} \times \left(-\frac{60}{112}\right) = \mathrm{V}^0_{25} \times \left(-\frac{15}{28}\right) \tag{22.15}$$

$$\mathrm{P}^0_{27} = \mathrm{P}^0_{25} \times \left(-\frac{28}{15}\right) = 28 \text{ years} \tag{22.16}$$

These arbors carry the figures for the indiction (15 year cycle), the epact and Easter full Moons (19 year cycle) and the solar cycle (28 year cycle).

Finally, the arbor 20 also carries a disk with 12 pins (see figure 22.5), which acts on a lever which moves the arbor 21 by a twelfth of a turn each year. These twelve pins are shown as a wheel by Oechslin. This arbor carries the data for each month, in particular the signs of the zodiac, the times of sunrise and sunset, and the durations of the day and night.

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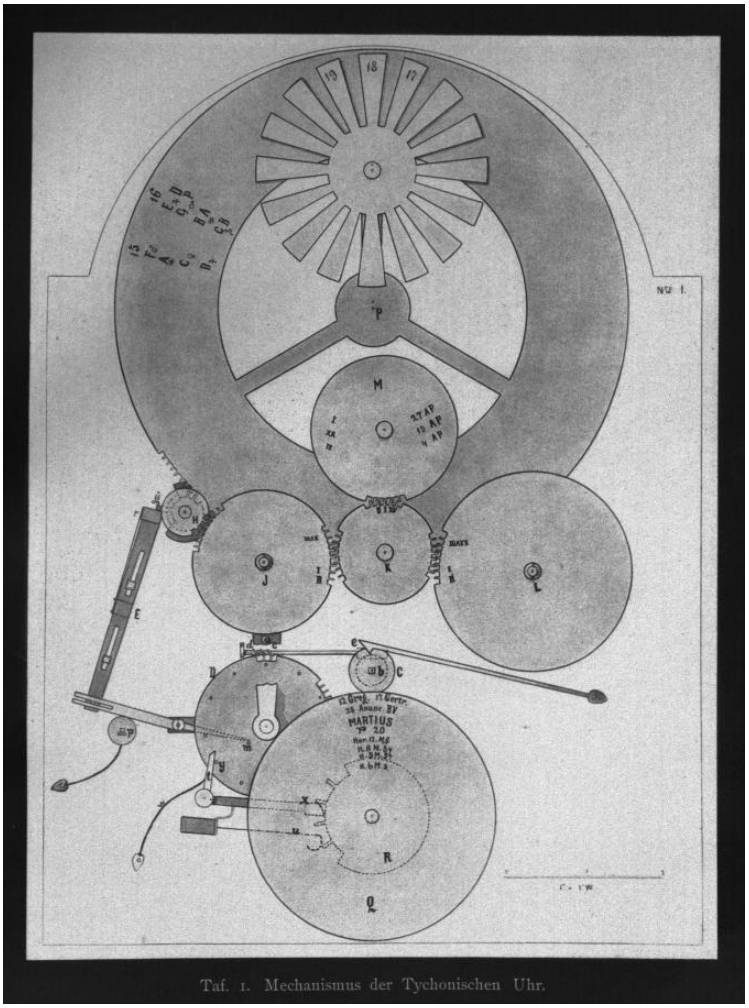


Figure 22.5: The gears for the computus and calendar elements in Klein's clock. (source: [1])

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22.4 The Tychonic system

This side of the clock shows the Tychonic system of the world, with the Earth at the center, the Moon around the Earth, and further the Sun, and all the planets rotating around the Sun. The Sun, with all the planets, rotates around the Earth. This system was published by Tycho Brahe in 1588 (figure 22.6).

There is also a hand for the lunar nodes, but which rotates around the Sun.

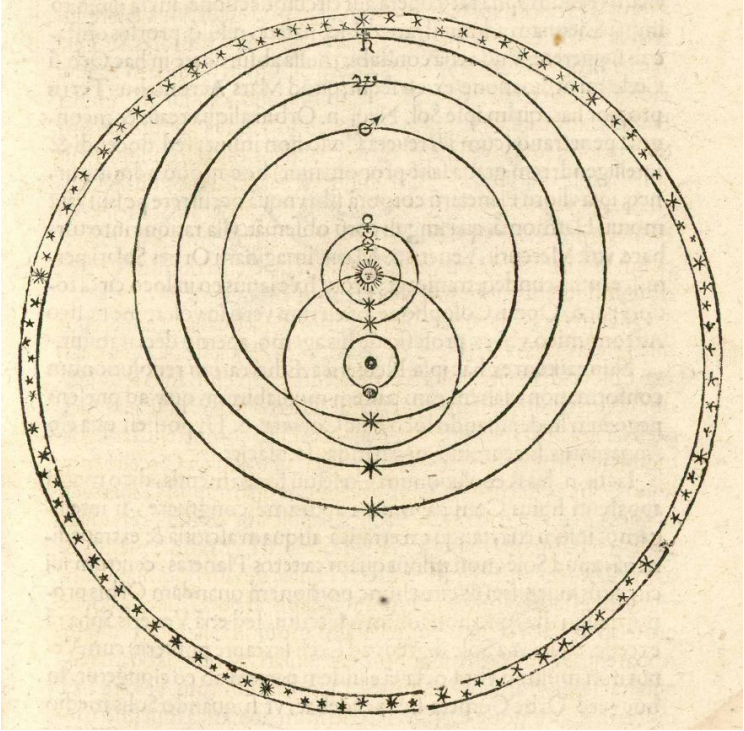


Figure 22.6: The Tychonic system (1588) (source: [3])

22.4.1 The general diurnal motion

As I have mentioned above, tube 2 makes a turn in 24 hours:

$$V_2^0 = -1 \tag{22.17}$$

$$P_2^0 = -1 \text{ day} \tag{22.18}$$

This tube carries an entire frame which rotates clockwise in one day. This frame carries the system around the Sun, so that the Sun rotates around the Earth (center) in 24 hours.

On the central axis, there are two arbors. The arbor in the front is the one carrying the figure of the Earth in the center of the dial. Another arbor 3 is located in the back, on the same axis, but is fixed. The latter carries a 6-teeth

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Figure 22.7: The Tychonic system on Klein's clock. (source: [1])

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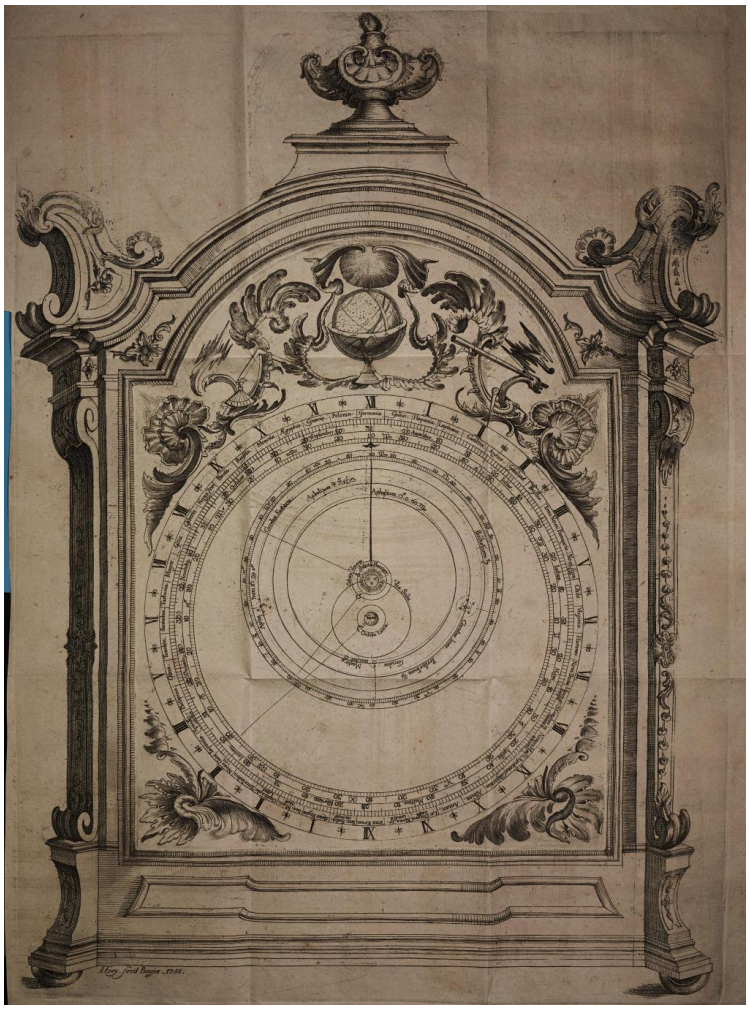


Figure 22.8: The Tychonic system on Klein's clock. (source: [17])

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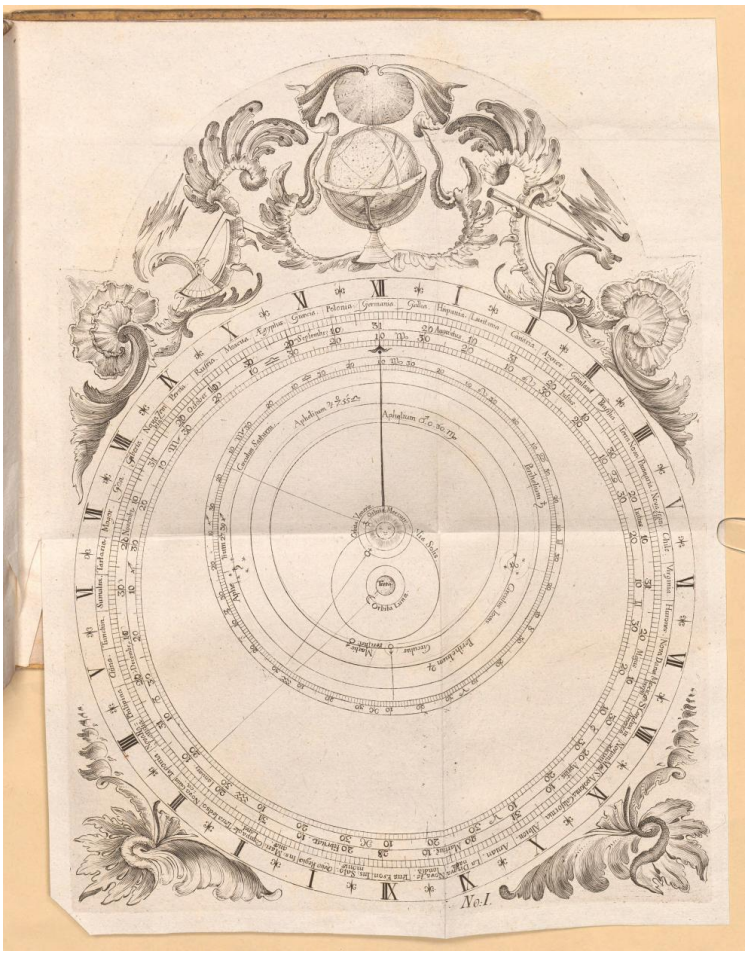


Figure 22.9: The Tychonic system on Klein's clock. (source: [18])

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pinion and it is the rotation of the frame 2 which introduces motions inside this frame, using this fixed pinion.

The frame 2 can be thought of the synodic frame of the Earth. The motions of the planets will all be relative to the Sun, and we can expect to find the synodic periods of the planets.

22.4.2 The motion of the Earth

The Earth is carried by the arbor 6 mentioned earlier and we can compute the motion of this arbor with respect to the general moving frame 2 (see also figure 22.10):

$$\mathrm{V}_3^2 = -\mathrm{V}_2^3 = -\mathrm{V}_2^0 = 1 \tag{22.19}$$

$$\mathrm{V}_6^2 = \mathrm{V}_3^2 \times \left(-\frac{6}{60}\right) \times \left(-\frac{15}{60}\right) \times \left(-\frac{8}{73}\right) = \mathrm{V}_3^2 \times \left(-\frac{1}{365}\right) = -\frac{1}{365} \tag{22.20}$$

In the absolute frame, we have

$$\mathrm{V}_6^0 = \mathrm{V}_6^2 + \mathrm{V}_2^0 = -\frac{1}{365} - 1 = -\frac{366}{365} \tag{22.21}$$

$$\mathrm{P}_6^0 = -\frac{365}{366} = -23\text{ h }56\text{ m }3.9344\ldots\text{ s} \tag{22.22}$$

It isn't clear from the photograph of the clock if the figure of the Earth shows a planisphere. If it does, then the Earth actually shouldn't be moving and be still, with only the Sun rotating clockwise around the Earth.

In any case, the motion of arbor 6 will serve as an input for the motion of all the other planets, for the line of nodes and for the Moon.

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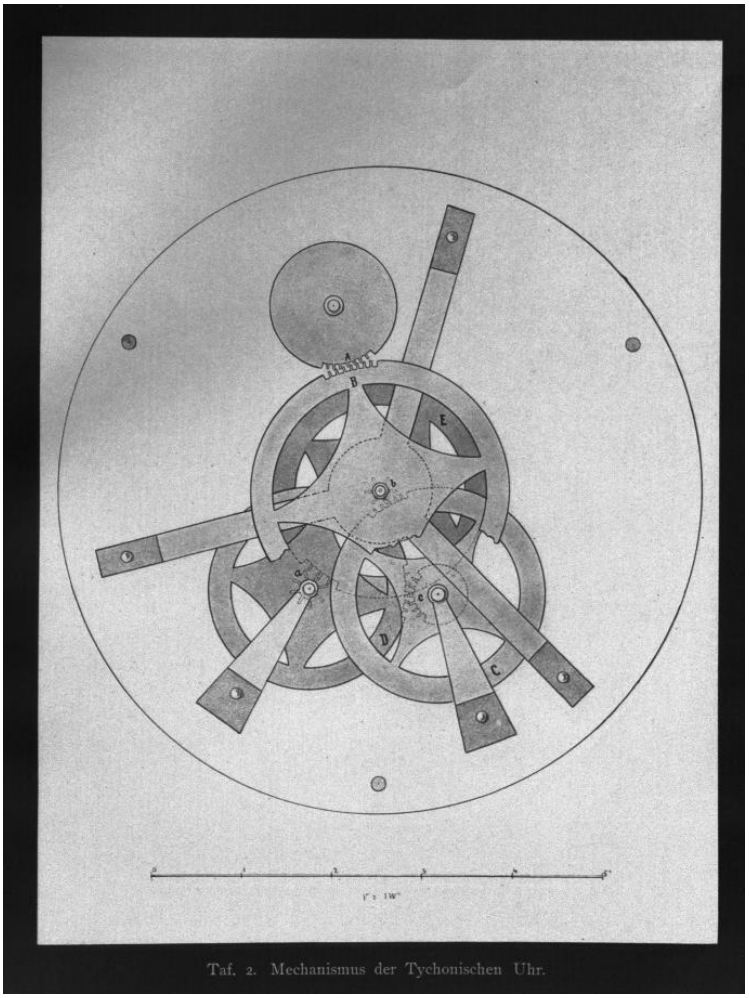


Figure 22.10: The gears for the motion of arbor 6 (center) in Klein's clock. (source: [1])

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22.4.3 The motions of the planets

22.4.3.1 The theoretical motions

Before examining how the planets move on Klein's clock, it is useful to study first how they should move, and to see what are the properties of the motions. As mentioned above, in the Tychonic system the Earth is at the center and the Sun revolves around the Earth, in the same time the Earth makes a turn around the Sun in the Copernican system. Moreover, the other planets rotate around the Sun. There is in fact a simple relation between the Copernican system and the Tychonic system.

In kinematics, we can describe the motion $z(t)$ of a point around a circle as a function of the time using Euler's notation:

$$z(t) = re^{i\omega t} = r\cos(\omega t) + ir\sin(\omega t) \tag{22.23}$$

where t is the time (for instance expressed in days), r is the radius of the circle and ω is the angular velocity, expressed in radians per unit of time. If P is the period of rotation, then $\omega = 2\pi/P$.

In the Copernican system, there is such a relation for each planet. For the Earth:

$$z_E(t) = r_E e^{i\omega_E t} \tag{22.24}$$

where r_E is the radius of the orbit of the Earth and $\omega_E = 2\pi/P_E$, P_E being the tropical year.

For another planet P:

$$z_P(t) = r_P e^{i\omega_P t} \tag{22.25}$$

with corresponding definitions for r_P and ω_P.

The Tychonic system is merely obtained by a translation of $\tau(t) = -z_E(t)$. This translation varies with the time t. If we denote the positions of the Earth and the planets in the Tychonic system with z', we have

$$z'_E(t) = z_E(t) + \tau(t) = 0 \tag{22.26}$$

$$z'_P(t) = z_P(t) + \tau(t) = r_P e^{i\omega_P t} - r_E e^{i\omega_E t} \tag{22.27}$$

Consequently, the motion of each planet (including the Earth, as a matter of fact) is the sum of two circular motions, one around the Earth, and one around the Sun. Incidentally, we can choose the order of rotation, and a planet could be thought of moving on epicycles around a point moving around the Earth, with exactly the same positions.

In the Tychonic system, the Sun is located at $\tau(t)$ and we can look at the synodic motions of the planets by making the Sun still, that is by rotating

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all the motions by $-\omega_E t$, which corresponds to a multiplication by $e^{-i\omega_E t}$. Denoting the synodic positions with z'', we have

$$z''_P(t) = (z_P(t) + \tau(t))e^{-i\omega_E t} = r_P e^{i\omega_P t} e^{-i\omega_E t} + r_E e^{i\omega_E t} e^{-i\omega_E t} \tag{22.28}$$

$$= r_P e^{i(\omega_P - \omega_E)t} + r_E \tag{22.29}$$

This is a circular motion of period

$$S_P = \frac{2\pi}{\omega_P - \omega_E} = \frac{2\pi}{2\pi/P_P - 2\pi/P_E} = \frac{1}{1/P_P - 1/P_E} = \frac{P_P P_E}{P_E - P_P} \tag{22.30}$$

We thus have a simple formula for the computation of the synodic periods, and also of the tropical period of a planet from the synodic one, because

$$P_P = \frac{S_P P_E}{S_P + P_E} \tag{22.31}$$

and finally we can also see that in the synodic frame the motion of a planet is counterclockwise if $P_E > P_P$ (Mercury and Venus) and clockwise if $P_E < P_P$ (the superior planets).

We can now examine how this is implemented on Klein's clock.

22.4.3.2 The motions on the clock

The planets, as well as the lunar nodes, are all moving around the Sun (see figure 22.11). We first compute these motions in the moving frame 2. For Mercury, we have

$$\mathrm{V}_7^2 = \mathrm{V}_6^2 \times \left(-\frac{137}{33}\right) = -\frac{1}{365} \times \left(-\frac{137}{33}\right) = \frac{137}{12045} \tag{22.32}$$

$$\mathrm{P}_7^2 = \frac{12045}{137} = 87.9197\ldots \text{ days} \tag{22.33}$$

This in fact an approximation of the orbital period of Mercury, but one would have expected the synodic period (about 116 days). The direction of rotation (counterclockwise) is however correct, since $P_E > P_P$.

For Venus, we have

$$\mathrm{V}_8^2 = \mathrm{V}_6^2 \times \left(-\frac{39}{24}\right) = -\frac{1}{365} \times \left(-\frac{39}{24}\right) = \frac{13}{2920} \tag{22.34}$$

$$\mathrm{P}_8^2 = \frac{2920}{13} = 224.6153\ldots \text{ days} \tag{22.35}$$

This in fact an approximation of the orbital period of Venus, but one would have expected the synodic period (about 584 days). The direction of rotation (counterclockwise) is however correct, since $P_E > P_P$.

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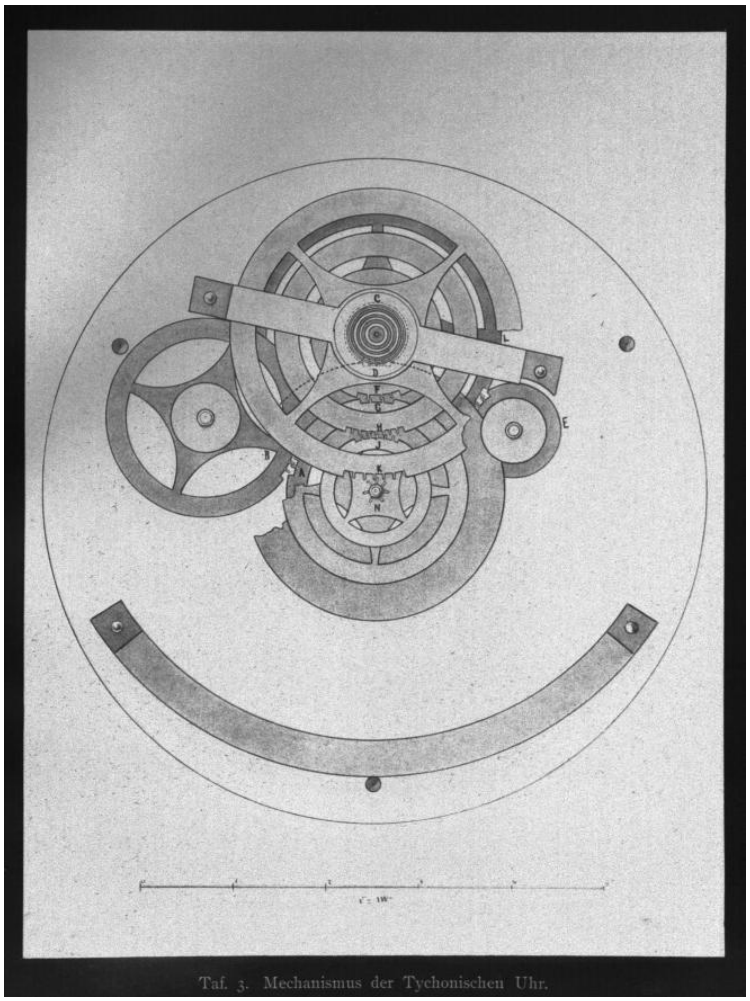


Figure 22.11: General view of the gears for the motions of the planets in Klein's clock. The axis of the Earth is at the center with the 7-leaves pinion on arbor 6 in the front. The group of wheels in the upper part are centered on the Sun. (source: [1])

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For Mars, we have

$$\mathrm{V}_9^2 = \mathrm{V}_6^2 \times \left(-\frac{42}{79}\right) = -\frac{1}{365} \times \left(-\frac{42}{79}\right) = \frac{42}{28835} \tag{22.36}$$

$$\mathrm{P}_9^2 = \frac{28835}{42} = 686.5476\ldots \text{ days} \tag{22.37}$$

This in fact an approximation of the orbital period of Mars, but one would have expected the synodic period (about 780 days). Moreover, the direction of rotation (counterclockwise) is wrong, since $P_E < P_P$.

For Jupiter, we have

$$\mathrm{V}_{10}^2 = \mathrm{V}_6^2 \times \left(-\frac{7}{83}\right) = -\frac{1}{365} \times \left(-\frac{7}{83}\right) = \frac{7}{30295} \tag{22.38}$$

$$\mathrm{P}_{10}^2 = \frac{30295}{7} = 4327.8571\ldots \text{ days} \tag{22.39}$$

This in fact an approximation of the orbital period of Jupiter, but one would have expected the synodic period (about 399 days). Moreover, the direction of rotation (counterclockwise) is wrong, since $P_E < P_P$.

For Saturn, we have

$$\mathrm{V}_{11}^2 = \mathrm{V}_6^2 \times \left(-\frac{7}{206}\right) = -\frac{1}{365} \times \left(-\frac{7}{206}\right) = \frac{7}{75190} \tag{22.40}$$

$$\mathrm{P}_{11}^2 = \frac{75190}{7} = 10741.4285\ldots \text{ days} \tag{22.41}$$

This in fact an approximation of the orbital period of Saturn, but one would have expected the synodic period (about 378 days). Moreover, the direction of rotation (counterclockwise) is wrong, since $P_E < P_P$.

For the line of nodes (see figure 22.12), we have

$$\mathrm{V}_{13}^2 = \mathrm{V}_6^2 \times \left(-\frac{72}{72}\right) \times \left(-\frac{60}{57}\right) = -\frac{1}{365} \times \frac{20}{19} = -\frac{4}{1387} \tag{22.42}$$

$$\mathrm{P}_{13}^2 = -\frac{1387}{4} = -346.75 \text{ days} \tag{22.43}$$

This is an approximation of the so-called eclipse year, the period of revolution of the nodes with respect to the Sun. This value is correct. Incidentally, Klein could also have made the nodes move around the Earth, both are possible.

Oechslin has given the same values, and he made the same observations about the incorrect revolution periods and rotation directions of the planets [10, p. 197, 237-238].

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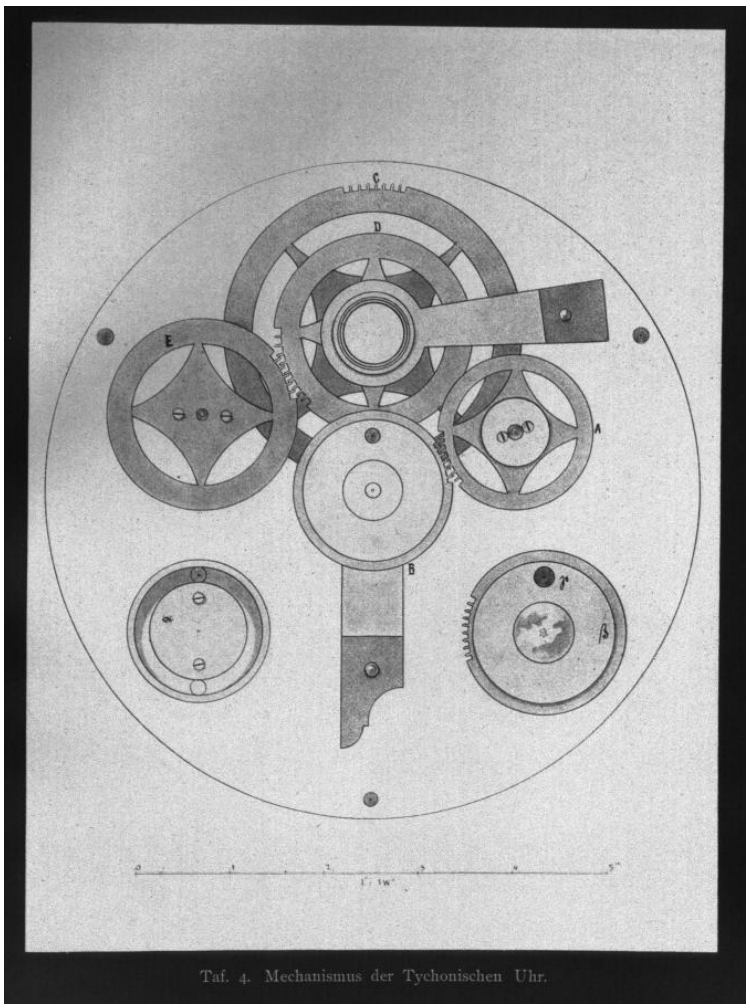


Figure 22.12: The motion of the lunar nodes in Klein's clock. (source: [1])

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22.4.4 The motion of the Moon

The Moon is carried by tube 15 and rotates around the Earth. The relative motion of tube 15 is derived from the motion of tube 7:

$$\mathrm{V}_{15}^2 = \mathrm{V}_7^2 \times \left(-\frac{78}{26}\right) \times \left(-\frac{60}{60}\right) = \mathrm{V}_7^2 \times 3 = \frac{137}{12045} \times 3 = \frac{137}{4015} \tag{22.44}$$

$$\mathrm{P}_{15}^2 = \frac{4015}{137} = 29.3065\ldots \text{ days} \tag{22.45}$$

This is an approximation of the synodic month, but a rather bad one. The same value is given by Oechslin.

22.4.5 The motion of the ecliptic

Between the 24-hour dial and the celestial disc, there is a ring 18 corresponding to the ecliptic. This ring carries a 365-teeth wheel which meshes with a 6-leaves pinion on the arbor 17 which is part of frame 2. This pinion advances by one tooth for every turn of frame 2, as a consequence of a finger on the fixed 24-hour dial (frame 16). Therefore, in the reference frame 2, we have

$$\mathrm{V}_{18}^2 = \mathrm{V}_{16}^2 \times \frac{1}{6} \times \left(-\frac{6}{365}\right) = -\mathrm{V}_{16}^2 \times \frac{1}{365} \tag{22.46}$$

$$= \mathrm{V}_2^{16} \times \frac{1}{365} = \mathrm{V}_2^0 \times \frac{1}{365} = -\frac{1}{365} \tag{22.47}$$

$$\mathrm{V}_{18}^0 = \mathrm{V}_{18}^2 + \mathrm{V}_2^0 = -\frac{1}{365} - 1 = -\frac{366}{365} \tag{22.48}$$

$$\mathrm{P}_{18}^0 = -\frac{365}{366} = -23 \text{ h } 56 \text{ m } 3.9344\ldots \text{ s} \tag{22.49}$$

The ecliptic ring 18 rotates clockwise in one sidereal day.

22.4.6 Summary of the motions

The Sun makes a turn clockwise in 24 hours and therefore goes through the midday line once a day. The ecliptic moves clockwise in one sidereal day, which corresponds to the apparent motion of the stars. The Moon does also rotate around the Earth in a synodic month with respect to the Sun. These motions are all correct.

However, the motions of all the planets appear to be incorrect, as Klein has given them tropical motions when synodic motions should have been used.

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Chapter 23

(Oechslin: 2.4)

Klein's Copernican clock in Prague (1752)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

23.1 Introduction

The clock described here was constructed in 1752 by Jan Klein (1684-1762) [1, p. 15].[1] This clock displays the Copernican system and belongs to a group of three similar clocks kept in the Clementinum in Prague.

This clock was first described in detail by Böhm in 1863 [1, 2]. It was again described by Oechslin in 1996, but Oechslin did not have the opportunity to disassemble the clock and based his description only on that published by Böhm.[2]

It is a table clock with two sides. One side of the clock shows the time on 12-hour and 24-hour dials. It also shows the sunrise, sunset, the Italic hours and the duration of day and night.

The other side shows the Copernican system, with the Sun at the center, and the planets Mercury, Venus, the Earth, Mars, Jupiter and Saturn revolving around it. Moreover, the Moon rotates around the Earth.

The clock is spring-driven (with two springs) and regulated by a pendulum, but the going work is not detailed by Oechslin. It is possibly that Oechslin only used Böhm's work, and did not have the opportunity to disassemble the clock.

I will describe each side of the clock separately.

[1] For biographical information on Klein, see the chapter on the geographic clock in Dresden.

[2] This clock was shown at the 1989 Hahn exhibition [19, p. 65]. See also Oechslin [9, p. 32-33, 49-50] and Michal [8].

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Figure 23.1: Left: Portrait of Jan Klein with his geographic clock from 1753/1754. (source: [1]) Right: portrait in Pelcl's biographical notice [12].

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23.2 Copernican system side

This side of the clock shows the Copernican system, with all the planets revolving around the Sun at the center, and the Moon revolving around the Earth. The various gears of this system are driven by the motion of arbor 1 which makes one turn clockwise in one day. All the motions are here expressed from the Copernican side.

$$\mathrm{V}_1^0 = -1 \tag{23.1}$$

$$\mathrm{P}_1^0 = -1 \tag{23.2}$$

This motion is first used to obtain that of the intermediate arbor 3:

$$\mathrm{V}_3^0 = \mathrm{V}_1^0 \times \frac{1}{5} \times \frac{1}{73} = \mathrm{V}_1^0 \times \frac{1}{365} = -\frac{1}{365} \tag{23.3}$$

$$\mathrm{P}_3^0 = -365 \text{ days} \tag{23.4}$$

Arbor 16 which is used in the other side of the clock gets the same motion:

$$\mathrm{V}_{16}^0 = \mathrm{V}_3^0 \times \left(-\frac{60}{60}\right) = -\mathrm{V}_3^0 \tag{23.5}$$

The arbor 3 thus makes a turn in 365 days.

23.2.1 The motion of the planets around the Sun

23.2.1.1 The motion of the Earth

The motion of arbor 3 is used to obtain the motion of the Earth on frame 9:

$$\mathrm{V}_9^0 = \mathrm{V}_3^0 \times \left(-\frac{60}{60}\right) = -\mathrm{V}_3^0 = \frac{1}{365} \tag{23.6}$$

$$\mathrm{P}_9^0 = 365 \text{ days} \tag{23.7}$$

The Earth/Moon system moves counterclockwise and makes a turn in 365 days. The same value is given by Oechslin. This is an approximation of the tropical year, but a very bad one, as the actual value is about 365.2422 days. Consequently, the Earth will quickly drift from its mean position.

The tube of frame 9 also carries a ring with the representation of stars, which does therefore also make a turn in one year [1, p. 20].

23.2.1.2 The motion of Mercury

The motion of arbor 3 is also used to obtain the motion of Mercury on tube 4:

$$\mathrm{V}_4^0 = \mathrm{V}_3^0 \times \left(-\frac{83}{20}\right) = \frac{1}{365} \times \frac{83}{20} = \frac{83}{7300} \tag{23.8}$$

$$\mathrm{P}_4^0 = \frac{7300}{83} = 87.9518\ldots \text{ days} \tag{23.9}$$

This is an approximation of the orbital period of Mercury and the same value is given by Oechslin.

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Figure 23.2: The Copernican system on Klein's clock. (source: [1])

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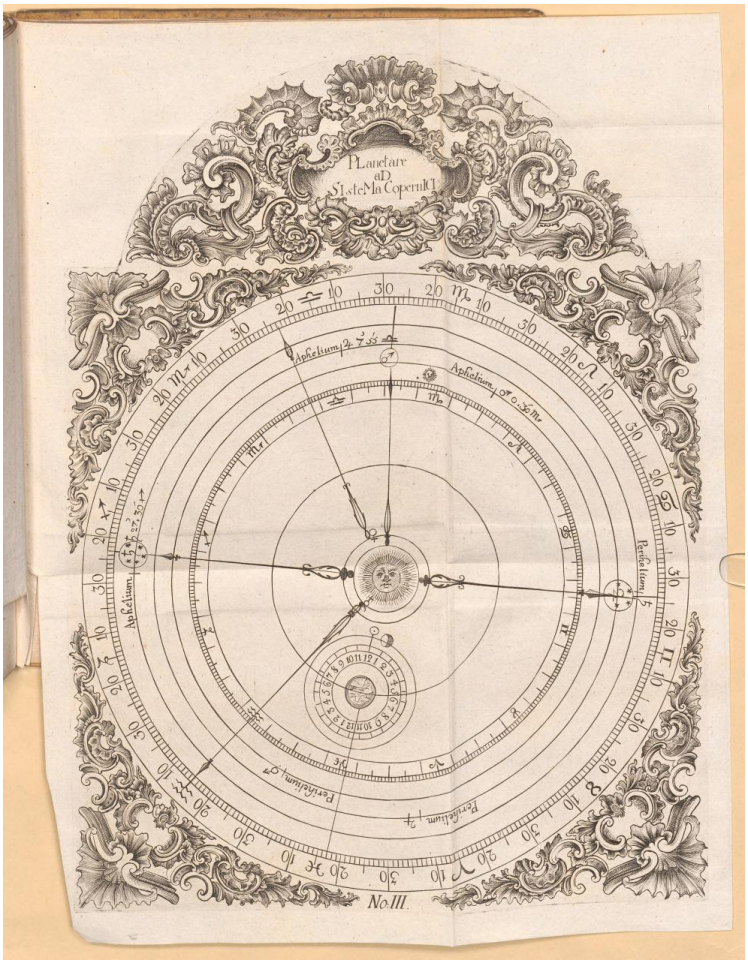


Figure 23.3: The Copernican system on Klein's clock. (source: [17])

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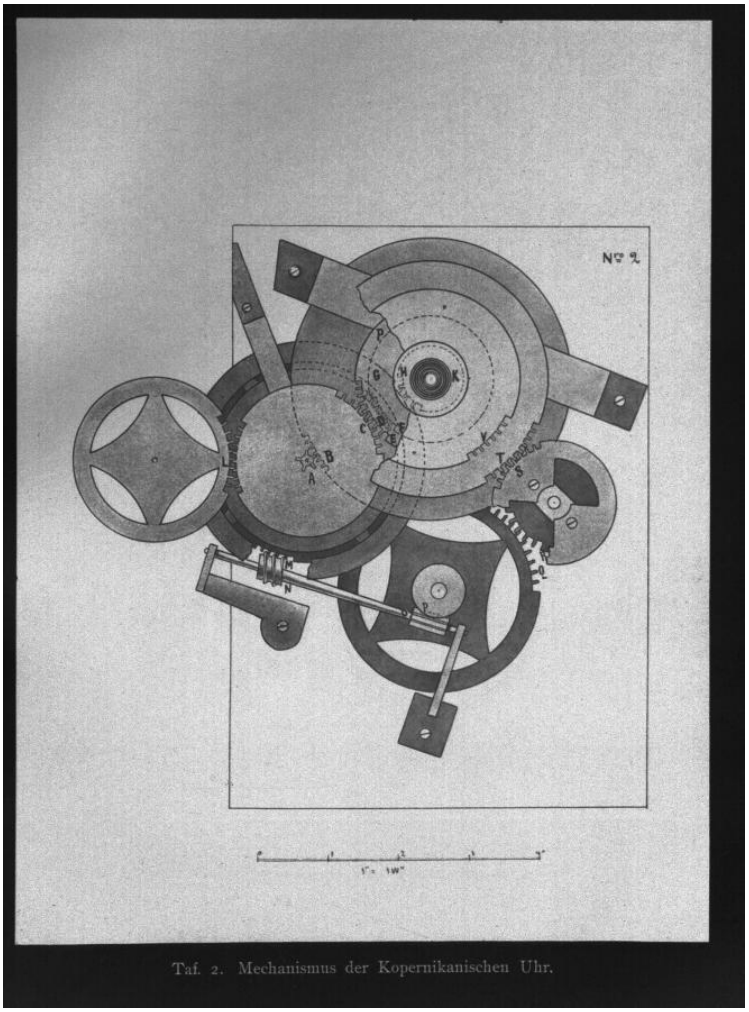


Figure 23.4: The gears of the Copernican system on Klein's clock. (source: [1])

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23.2.1.3 The motion of Venus

The motion of arbor 3 is also used to obtain the motion of Venus on tube 5:

$$V_5^0 = V_3^0 \times \left(-\frac{92}{57}\right) = \frac{1}{365} \times \frac{92}{57} = \frac{92}{20805} \tag{23.10}$$

$$P_5^0 = \frac{20805}{92} = 226.1413\ldots \text{ days} \tag{23.11}$$

This is an approximation of the orbital period of Venus and the same value is given by Oechslin. Note that Oechslin's drawing mistakenly gives 52 teeth to the 57-teeth wheel of the train.

23.2.1.4 The motion of Mars

The motion of arbor 3 is also used to obtain the motion of Mars on tube 6:

$$V_6^0 = V_3^0 \times \left(-\frac{51}{96}\right) = \frac{1}{365} \times \frac{51}{96} = \frac{17}{11680} \tag{23.12}$$

$$P_6^0 = \frac{11680}{17} = 687.0588\ldots \text{ days} \tag{23.13}$$

This is an approximation of the orbital period of Mars and the same value is given by Oechslin.

23.2.1.5 The motion of Jupiter

The motion of arbor 3 is also used to obtain the motion of Jupiter on tube 7:

$$V_7^0 = V_3^0 \times \left(-\frac{12}{142}\right) = \frac{1}{365} \times \frac{12}{142} = \frac{6}{25915} \tag{23.14}$$

$$P_7^0 = \frac{25915}{6} = 4319.1666\ldots \text{ days} \tag{23.15}$$

This is an approximation of the orbital period of Jupiter and the same value is given by Oechslin.

23.2.1.6 The motion of Saturn

The motion of arbor 3 is also used to obtain the motion of Saturn on tube 8:

$$V_8^0 = V_3^0 \times \left(-\frac{5}{146}\right) = \frac{1}{365} \times \frac{5}{146} = \frac{1}{10658} \tag{23.16}$$

$$P_8^0 = 10658 \text{ days} \tag{23.17}$$

This is an approximation of the orbital period of Saturn and the same value is given by Oechslin.

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23.2.2 The motions within the Earth-Moon system

At the position of the Earth, there is a planisphere on arbor 12, and the Moon on tube 15.

23.2.2.1 The rotation of the Earth

We can compute the motion of the Earth planisphere (arbor 12) with respect to frame 9 (direction of the Sun). This motion is based on the motion of frame 11, which replicates the motion of the central arbor 1:

$$\mathrm{V}_{11}^{0} = \mathrm{V}_{1}^{0} \times \left(-\frac{72}{36}\right) \times \left(-\frac{40}{80}\right) = \mathrm{V}_{1}^{0} \tag{23.18}$$

and then

$$\mathrm{V}_{12}^{9} = \mathrm{V}_{11}^{9} \times \left(-\frac{68}{68}\right) = -\mathrm{V}_{11}^{9} = \mathrm{V}_{9}^{11} = \mathrm{V}_{9}^{0} - \mathrm{V}_{11}^{0} = 1 + \mathrm{V}_{9}^{0} \tag{23.19}$$

However, the velocity of the Earth with respect to the Sun should be 1, and we should have a period of one day, which is not the case.

Oechslin doesn't obtain the same result and he finds $\mathrm{V}_{12}^{9} = 1$. His conclusion may be based on Böhm's analysis [1, p. 20] which is incorrect. Surprisingly, Böhm concludes that arbor 12 makes a turn in 24 hours, but this would only be true if the Earth were not rotating around the Sun.

23.2.2.2 The revolution of the Moon

The motion of the Moon (tube 15) is obtained using 2 pins on frame 11, and these pins move a 8-pointed starl whose arbor 13 is located on frame 9 (Oechslin doesn't show the construction clearly, but it is more explicit in Böhm's drawing, see figure 23.5). Another wheel on this arbor meshes with a wheel on arbor 14 which also seems to be located on frame 9. We thus have

$$\mathrm{V}_{15}^{9} = \mathrm{V}_{11}^{9} \times \left(-\frac{2}{8}\right) \times \left(-\frac{36}{36}\right) \times \left(-\frac{8}{59}\right) = \mathrm{V}_{11}^{9} \times \left(-\frac{2}{59}\right) \tag{23.20}$$

If $\mathrm{V}_{11}^{9} = -1$, then we would obtain a period of 29.5 days and the Moon would move couterclockwise on frame 9, which is what it should.

However, as we have seen above, we do not have $\mathrm{V}_{11}^{9} = -1$ and the synodic month is slightly different and worse than the value 29.5.

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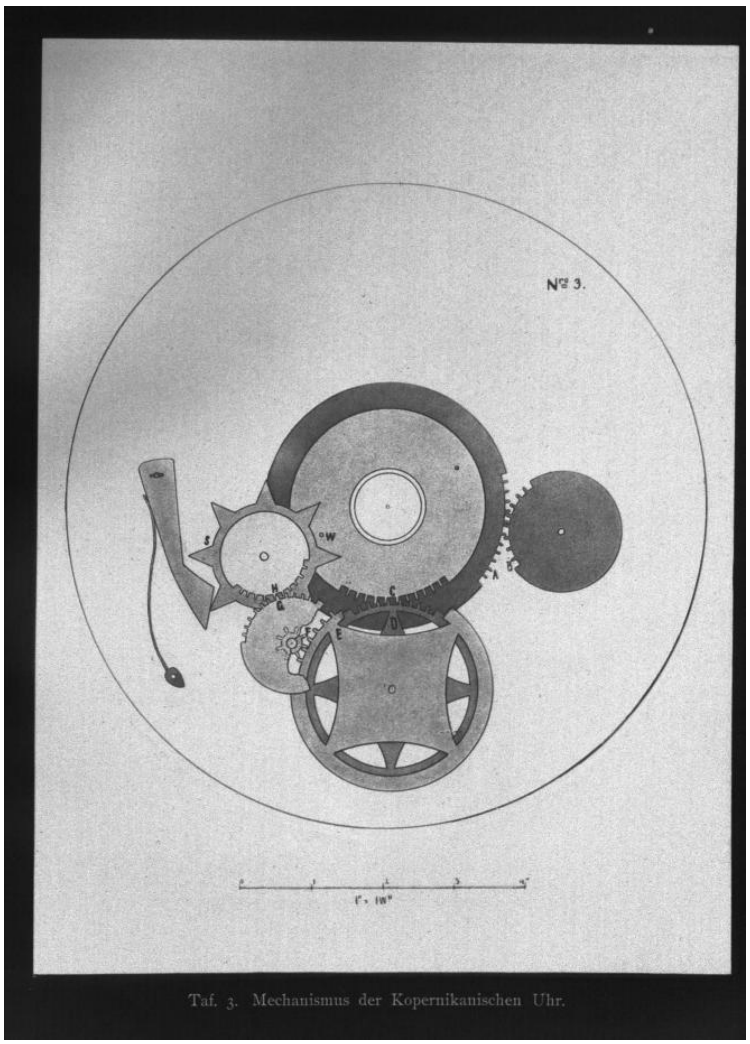


Figure 23.5: Detail of the motion of the Moon in Klein's clock. (source: [1])

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23.3 Time side

This side of the clock shows the time on 12-hour and 24-hour dials. It also shows the sunrise, sunset, the Italic hours and the duration of day and night.

The central arbor 18 of the dial carries the hand of the minutes and makes a turn clockwise in one hour. Seen from the front of the side, we have

$$\mathrm{V}_{18}^{0} = -24 \tag{23.21}$$

$$\mathrm{P}_{18}^{0} = -\frac{1}{24} = -1 \text{ h} \tag{23.22}$$

This arbor carries a 30-teeth wheel which meshes on the one hand with a similar wheel on arbor 19, and with a 60-teeth wheel on arbor 21. The former is used to derive the motion of the 12-hour hand (tube 20), the latter the motion of the 24-hour hand (tube 22):

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{18}^{0} \times \left(-\frac{30}{30}\right) \times \left(-\frac{6}{72}\right) = \mathrm{V}_{18}^{0} \times \frac{1}{12} = -2 \tag{23.23}$$

$$\mathrm{P}_{20}^{0} = -\frac{1}{2} = -12 \text{ h} \tag{23.24}$$

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{18}^{0} \times \left(-\frac{30}{60}\right) \times \left(-\frac{6}{72}\right) = \mathrm{V}_{18}^{0} \times \frac{1}{24} = -1 \tag{23.25}$$

$$\mathrm{P}_{22}^{0} = -1 \text{ day} \tag{23.26}$$

This side also obtains an input through arbor 16 which was seen earlier. This motion is transferred to that of arbor 17 (measured from this side):

$$\mathrm{V}_{17}^{0} = \mathrm{V}_{16}^{0} \times \frac{56}{56} = \mathrm{V}_{16}^{0} \tag{23.27}$$

The velocity of arbor 16 is measured from the Copernican side, and the velocity of arbor 17 from the time side, this explains why the sign is not changed.

Arbor 17 makes a turn in 365 days, this is an approximation of the tropical year, but not a very good one.

Arbor 17 carries a cam which is used to modify the Italic hour, the sunrise, the sunset, and the durations of day and night. This cam moves a moving 24-hour dial with four sector openings. This moving dial carries the 24 hours in Arabic numerals and is used for the Italic hours. The left and right openings are meant for the sunrise and sunset, an index on the moving dial being used to read the value which is on the fixed back (see figure 23.7). The moving 24-hour dial can also be used to read the times of sunrise and sunset, as these times correspond to the 12 and 24-hour marks of the moving dial.

The upper and lower openings are for the durations of the day and night. The values 12 are at the top and bottom, and when the moving dial moves, its index will show longer or shorter days and nights. Of course, these durations could also be deduced from the times of sunrise and sunset.

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Figure 23.6: The time side of Klein's clock. (source: [1])

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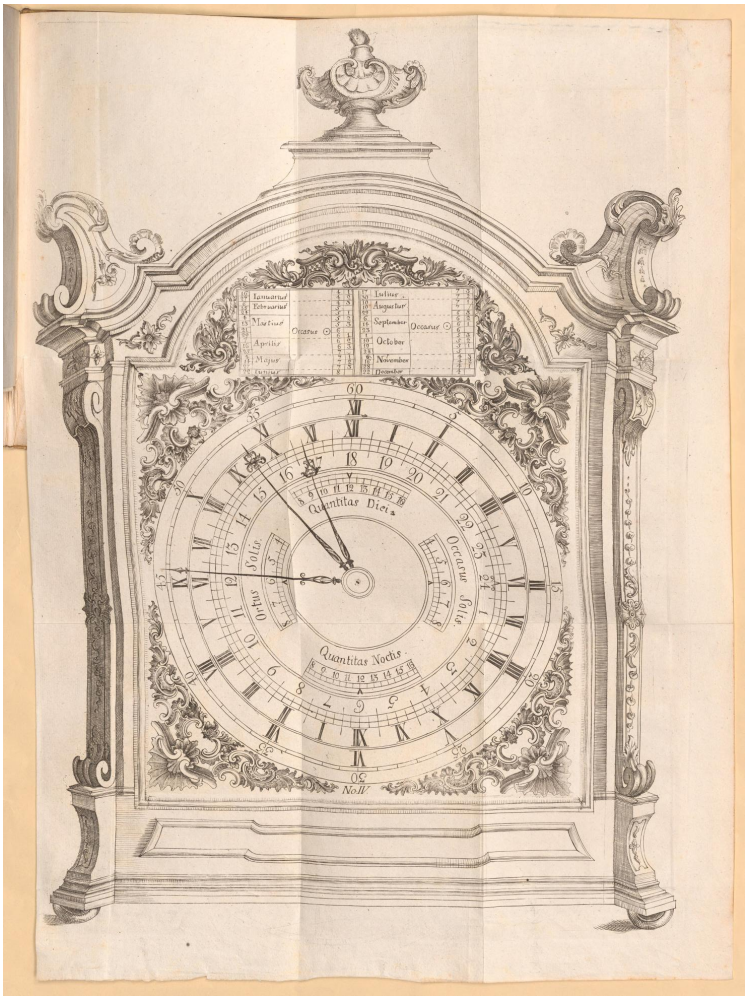


Figure 23.7: The time side on Klein's clock. (source: [17])

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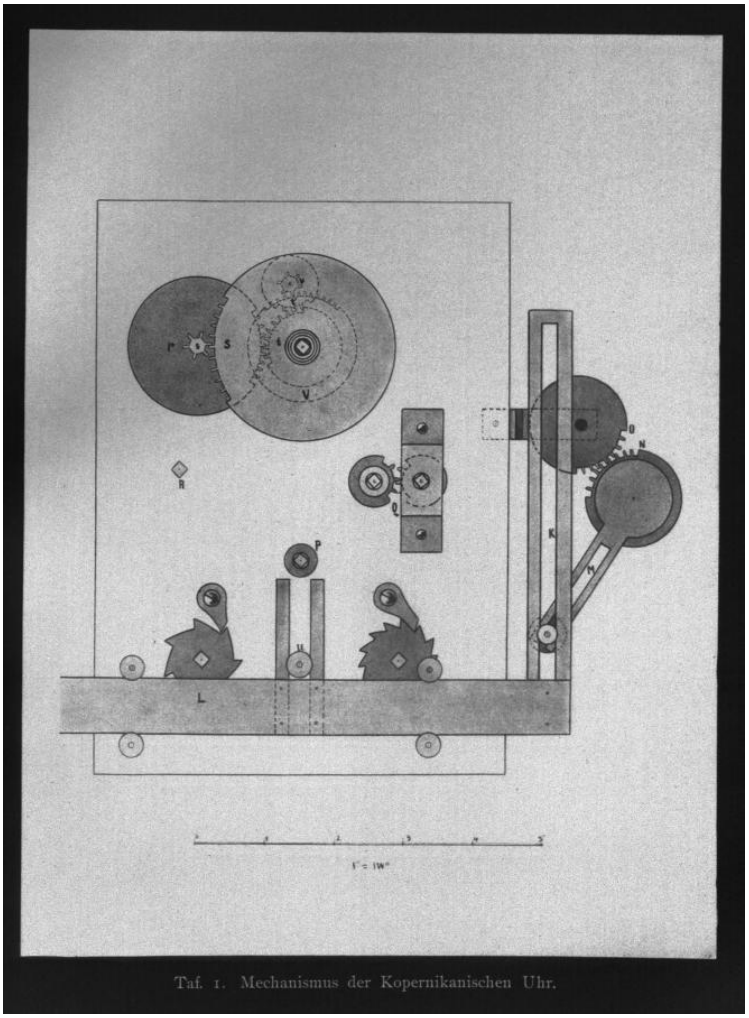


Figure 23.8: The gears of the time side of Klein's clock. (source: [1])

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23.4 Conclusion

The study of Klein's clock has been rather disappointing, with surprisingly bad values for the periods of the different motions, including those of the planets and the Earth around the Sun.

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Chapter 24

(Oechslin: 3.1)

Neßtfell's *Planetenmaschine* in Vienna (1745-1753)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

24.1 Introduction

The astronomical machine described here was constructed between 1745 and 1753 by Johann Georg Neßtfell (1694-1762).

Neßtfell was born in Alsfeld (Hessen) in a cabinetmaker family. He was trained as a cabinetmaker.[1] Because of a conflict with his corporation, he didn't become a master and offered his services to princes and the Church. In 1717, he was employed by the House of Schönborn and he made a number of works for Schloss Wiesentheid near Würzburg. He also made a number of book furnitures in monasteries, most of which do no longer exist. His specialty was marquetry in wood and precious materials such as tortoiseshell and mother-of-pearl.[2] Neßtfell was one of the most important ebenists at the service of the House of Schönborn.

Neßtfell's maternal grandfather was also working with watches or clocks,

[1]The biographical information given here is drawn from Hess [10], Henck [9], Sangl's summary [23] with additions from Stoehr [31] and Spenlé [29]. There are a number of other biographical mentions, such as by Stamminger [30, p. 249-250] and Maurice [15, v.1, p. 273-274]. The articles of Dotter [5] and Rübeling [22] have not been consulted. Weis's dissertation probably contains a lot of additional information, but I have not yet seen it [34]. A portrait of Neßtfell is shown by Hess and is reproduced here [10]. Spenlé [29, p. 145] and Dippold [4, p. 24] reproduce another portrait, which is also reproduced by Henck [9]. Some elements on Neßtfell are also given by Abeler [1] and by Mattl-Wurm [14, p. 39-42]. Bassermann-Jordan only illustrated the Vienna machine [2], [3].

[2]On Neßtfell's work in Schloss Wiesentheid, see Hess [10, p. 43-48]. See also Spenlé's article on a tortoiseshell casket made by Neßtfell in the 1720s [29].

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and organs, and Neßtfell was certainly familiar with metal work from an early age. In the 1720s, he was also assigned to surveying and map making tasks.

Then, in Banz Abbey, Neßtfell began to study astronomy. He could study books in the library and built hydraulic machines and scientific instruments.[3] It seems that two celestial globes made in 1692 by Coronelli in Venice and bought by Banz Abbey in the 1730s aroused particular interest in Neßtfell. In 1745, Neßtfell made a wooden orrery which came to the attention of Emperor Francis I (1708-1765) who was crowned in Frankfurt am Main in 1745. When the Emperor saw the orrery, he gave him 200 ducats.[4] Neßtfell was then called for an audience in Vienna and the Emperor asked him to make such a machine in metal. For the computations, he was helped by the student Johann Ludwig Fricker (1729-1766). The machine, described here, was delivered in 1753 in Vienna.

Neßtfell then received the title of court mechanician and a yearly salary.

Between 1755 and 1761, Neßtfell constructed a second orrery which is now kept in Munich. This machine was decorated by figures made by Johann Peter Wagner.

In 1761, Neßtfell published a description of his machines [18]. He died in 1762 in Würzburg.

[3]For Neßtfell's activity in Banz Abbey, see especially Hess [10, p. 49-60] and Dippold [4].

[4]See Hess [10, p. 22] and Oechslin [20, p. 221]. On possible persons who helped Neßtfell in the construction of his machines, see Oechslin [20, p. 219-220].

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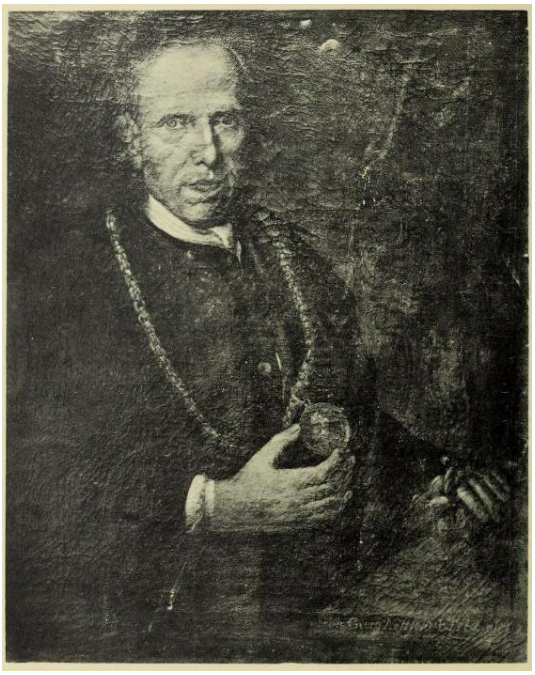


Figure 24.1: Johann Georg Neßtfell. (source: [10])

24.2 History and general description of the machine

Neßtfell's machine[5] was initially exhibited in the *k.-k. Hofbibliothek* in Vienna (now *Österreichische Nationalbibliothek*), then in 1768 in the *k.-k. Mechanisch-Physikalisches Kunstkabinett*,[6] then in 1797 in the *Thierkabinett*, then from 1806 to 1886 in the *k.-k. Physikalisch-Astronomisches Hofkabinett*. In 1868, Pater Mosheimer wrote a manuscript description of the machine [17]. Then, sometime after 1886 the machine was moved to the *Kunsthistorischesmuseum* in Vienna.[7] It is now exhibited in the *Naturhistorisches Museum* (museum of natural history) in Vienna.

After Neßtfell's death, Frater David a Sancto Cajetano restored his Vienna planetarium in the 1780s, and this may have brought him to develop his theory of epicyclic gears.[8]

[5]The best sources for Neßtfell's machines are Hess [10, p. 61-77] and Henck [9]. Besides Oechslin's work [20, p. 33, 36-37, 53-55, 199-203], see also King [11, p. 229-232] and Seelig's articles [24, 25, 28, 26, 27].

[6]See Weiskern's description [35, p. 58-64].

[7]See [13, p. 187]. Hauer [8, p. 3] seems to set the transfer in 1890. See also the less complete history given by Oechslin [20, p. 234]. This machine was part of the 1989 Hahn exhibition [32, p. 55, 57-59].

[8]See [6, p. 4] and [20, p. 227-228].

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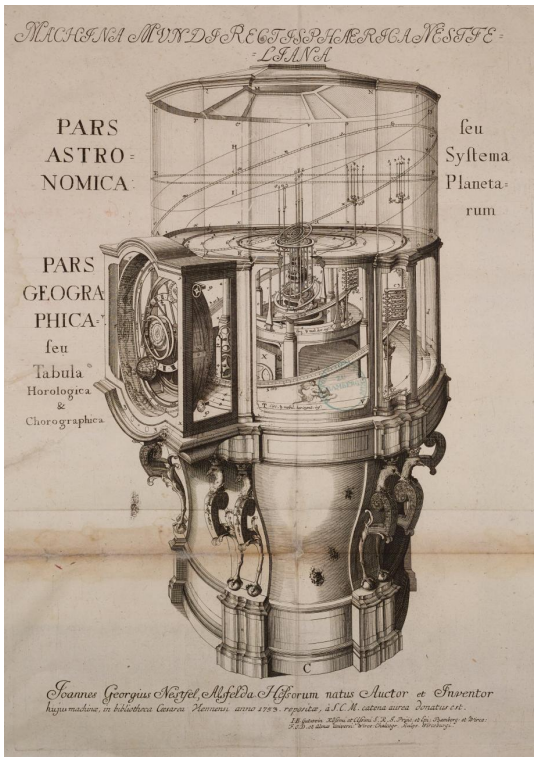


Figure 24.2: Engraving of Neßtfell's machine in Vienna, probably from 1754, given the year on the engraving showing the front dial. (Johann Balthasar Gutwein, source: [18])

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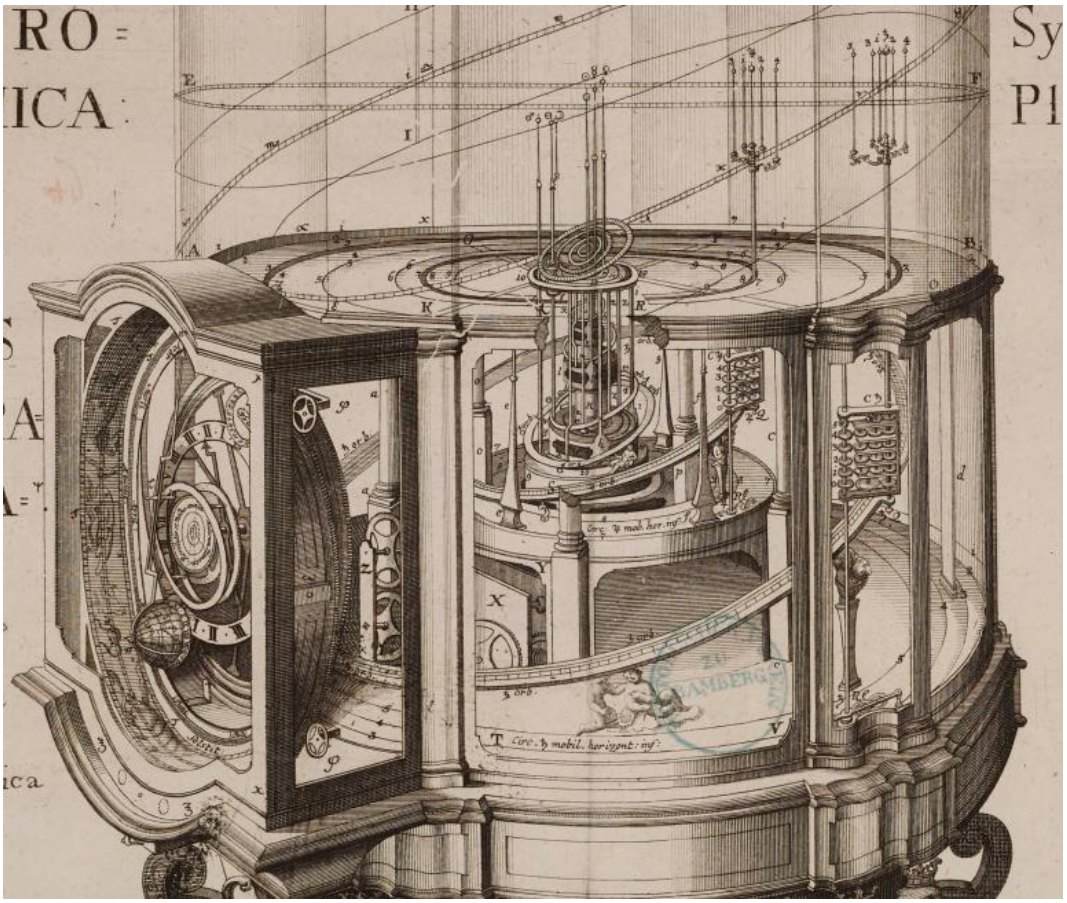


Figure 24.3: Detail of the previous engraving of Neßtfell's machine in Vienna, probably from 1754. (Johann Balthasar Gutwein, source: [18])

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Figure 24.4: General view of Neßtfell's machine in in the *Naturhistorisches-museum* in Vienna. (photograph by the author)

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Figure 24.5: Detail of Neßtfell's machine in Vienna. (photograph by the author)

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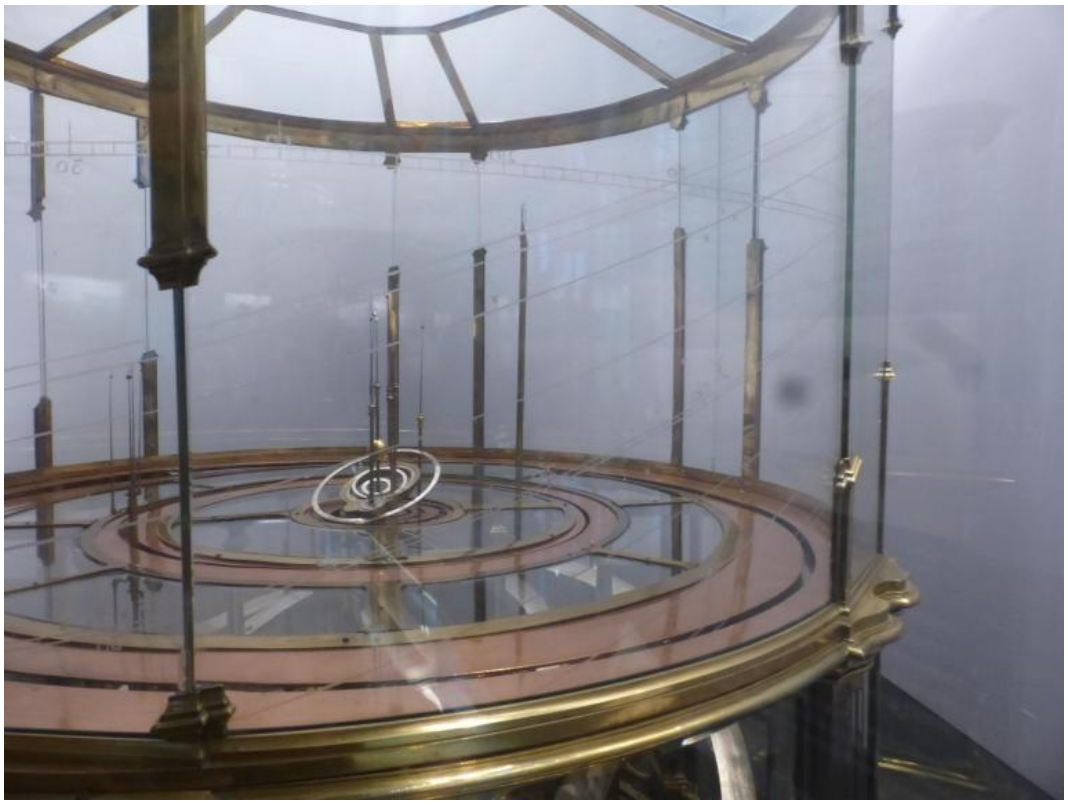


Figure 24.6: Detail of Neßtfell's machine in Vienna. (photograph by the author)

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Figure 24.7: Detail of Neßtfell's machine in Vienna. The system of Jupiter's satellites is visible at the upper right. (photograph by the author)

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It was also entirely restored in January-April 1980 for the exhibition *Maria Theresia und ihre Zeit* [12].

The machine is similar in appearance to Neßtfell's machine kept in Munich and which was constructed a few years later. The most obvious difference is that the base is decorated of simple columns in Munich, but of griffins in Vienna. There are also a number of sculpted figures in the Munich machine. However, there are also some differences in the gears. The descriptions of the two machines are very similar and overlap to a large extent, but that way they can be read in any order.

It should moreover be observed that Frater Fridericus's machine in Bamberg, made with the help of Johann Georg Fellwöck in 1772 (Oechslin 4.1), is really a simplified version of Neßtfell's machines. It has the same general cylindrical appearance, there is no orrery, and the entire front Earth-Mercury system is laid out horizontally.

Both of Neßtfell's machines are about 2 meters tall, they have a diameter of about 1 meter, and have a prismatic glass case containing the planetary system. Due to this construction, almost all the gears can be seen. In the front of the machine, there is a large dial representing the Earth-Mercury system.

These two machines are both very daunting, especially because it is rather difficult to understand them at first sight. I am going to split the machine in three parts: the going work, the orrery and the Earth-Mercury system.

24.3 The going work

The going work is set behind the Earth-Mercury system and it can be seen in Oechslin's figure 3.1.2 [20]. It is weight-driven and regulated by a pendulum. The going work contains two winding drums, that add up their strengths for a unique output on arbor 13 which makes one turn clockwise (seen from the front) in 24 hours. We have

$$\mathrm{T}^0_{13} = 1 \tag{24.1}$$

We can therefore deduce that the first drum (arbor 2) must have the velocity

$$\mathrm{T}^0_2 = \mathrm{T}^0_{13} \times \left(-\frac{54}{18}\right) \times \left(-\frac{18}{108}\right) = \mathrm{T}^0_{13} \times \frac{1}{2} = \frac{1}{2} \tag{24.2}$$

$$\mathrm{P}^0_2 = 2 \text{ days} \tag{24.3}$$

The second drum (arbor 8) must have the velocity

$$\mathrm{T}^0_8 = \mathrm{T}^0_{13} \times \left(-\frac{54}{86}\right) \times \left(-\frac{86}{18}\right) \times \left(-\frac{18}{108}\right) = \mathrm{T}^0_{13} \times \left(-\frac{1}{2}\right) \tag{24.4}$$

$$\mathrm{P}^0_8 = -2 \text{ days} \tag{24.5}$$

Each drum makes a turn in two days, but they turn in opposite directions.

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These two drums can be rewound using the winding arbors 1 (for the first drum) and 7 (for the second drum).

A gear train connects the drums to the escape wheel on arbor 6. We have

$$\mathrm{T}_6^0 = \mathrm{T}_2^0 \times \left(-\frac{108}{18}\right) \times \left(-\frac{88}{11}\right) \times \left(-\frac{72}{9}\right) \times \left(-\frac{63}{7}\right) \tag{24.6}$$

$$= \mathrm{T}_2^0 \times 3456 = 1728 \tag{24.7}$$

$$\mathrm{P}_6^0 = \frac{1}{1728} \text{ days} = 50 \text{ seconds} \tag{24.8}$$

The escape wheel having 27 teeth, we deduce that the pendulum should make a half swing in $\frac{50}{2\times 27} = \frac{25}{27}$ seconds. Its length should be about 86 cm.

The only output of the going work is arbor 13 and it immediately drives the Earth-Mercury system in the front. Moreover, on arbor 13 there is a 96-teeth wheel which drives a train towards the orrery, but which can be put out of mesh using arbor 12. For now, I will merely assume that the going work directly drives the orrery.

24.4 The orrery

The most unusual feature of this machine is that the planets do not move in a horizontal plane, or near a horizontal plane. In almost every orrery, the planets move either in or near a horizontal plane, or in or near a vertical plane. In Neßtfell's machine, the ecliptic is tilted by an angle of 23.5° and the planets move in or near that tilted plane.[9] Consequently, the plane of the equator is a horizontal plane, although it doesn't have any particular significance here. For instance, no horizontal plane is materialized through the Earth.

Moreover, the planets only move indirectly in the ecliptic. In fact, there are horizontal base motions, and the planets are on vertical arms that glide along tilted guides. This all gives a very unusual appearance to this machine.

As mentioned above, the input to the orrery is a gear train starting with the horizontal arbor 13 making one turn in a day. This arbor is used to drive the gears for Saturn. Then, the motion of arbor 13 is transferred to arbor 65 which drives the other planets:

$$\mathrm{V}_{65}^0 = \mathrm{V}_{13}^0 \times \left(-\frac{96}{88}\right) \times \left(-\frac{88}{96}\right) = \mathrm{V}_{13}^0 = -1 \tag{24.9}$$

Arbor 65 also makes one turn clockwise in a day.

All the planets are driven similarly, sitting on carriages which are stuck between moving wheels and wheels that are not moving. Neßtfell used large wheels which are about the size of the orbits, and Maurice and Oechslin assume that he took his inspiration from a British orrery by John Rowley which had been bought in 1723 in England by Prince Eugene Francis of Savoy-Carignano

[9] On this construction, see in particular [20, p. 146-147].

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(1663-1736). Neßtfell repaired this orrery, probably during his first trip to Vienna in 1745 [10, p. 23-24], [36, p. 42], [15, v.1, p. 273]. This orrery may have been similar to the one by Adams described in this book [20, p. 154, 210, 226].

I am first considering the mean motions of the planets, then the rotations of the planets and the satellites of Jupiter and the Moon. Finally, I am considering the motion of the Moon.

24.4.1 The mean motions and rotations of the planets

I am starting with Saturn and going backwards to Mercury. These systems are in fact all independent and could be described in any order, but I believe it is easier to look at them from the front to the back, and also from the outside to the inside. The descriptions below are very similar, because the structures are similar. However, I prefer to repeat the descriptions, so that the motions can be studied in any order.

All the planets are given a motion of rotation around their axis, but I will only analyze the rotation of Mercury, Venus, the Earth and Mars after each analysis of the mean motions. The rotations of Jupiter and Saturn will be examined when I describe their two satellite systems.

As mentioned above, Neßtfell first produces horizontal motions, and tilted slopes ensure that these motions are transferred near the ecliptic. I assume that each slope is adapted to its planet and that different planets therefore have different slopes. It is also possible that some of the orbits have been made eccentric, but this should be checked. In addition, one should keep in mind that the transfer from the horizontal plane to the tilted plane is a projection and that it does alter the motion.[10] But Neßtfell did not implement irregular motions, other than possibly those resulting from offset orbits.

The mean motions of the planets are obtained through epicyclical gear trains. In each case, there is an input motion and the rotation of a set of gears on a mobile carriage, this rotation being determined by a constraint on another wheel, in this case a fixed wheel. These constructions often lead to periods whose rational expression contains large primes:

- for Mercury, the period involves the prime number 317;
- for Venus, the period involves the prime number 2411;
- for the Earth, the period involves the prime number 296983;
- for Mars, the period involves the prime numbers 101 and 173;
- for Jupiter, the period involves the prime numbers 131 and 389;
- for Saturn, the period involves the prime number 51817.

[10] This was also observed by Oechslin, see [20, p. 201-202].

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Neßtfell's construction makes use of gears whose teeth numbers contain only small primes, the largest being 109. It is therefore tempting to view Neßtfell's construction as a way to avoid the use of large primes in gear trains.

However, I do not believe that Neßtfell chose to reach the exact ratios he finally produced. His objectif must have been much more to have the planets on moving carriages, and the complex ratios that these constructions entailed were rather side effects. This is therefore entirely different from Frater David's methods which aimed at obtaining specific ratios without resorting to too large primes. I am giving some details of Frater David's methods in the introduction of this book.

Neßtfell also gives the revolution periods of the planets in his description of the clock [18],[11] and the periods implemented on the clock are only approximations of these values:

Mercury	87d	23h	14m	40s		87.9685...
Venus	224d	17h	44m	11/12		224.7395...
Earth	365d	5h	48m	58s	1/4	365.2423...
Mars	686d	23h	31m	57s		686.9805...
Jupiter	4332d	14h	49m	31s	56‴	4332.6177...
Saturn	10759d	5h				10759.2083...

Neßtfell does not give the actual periods on his machine.

24.4.1.1 The motion of Saturn

On arbor 13, there is a 25-teeth wheel which meshes with a 897-teeth contrate wheel on frame 43. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}^0_{43} = \mathrm{V}^0_{13} \times \left(-\frac{25}{897}\right) = \frac{25}{897} \tag{24.10}$$

$$\mathrm{P}^0_{43} = \frac{897}{25} = 35.88 \text{ days} \tag{24.11}$$

Above the 897-teeth wheel, there is a 900-teeth wheel with interior gearing which meshes with a 25-teeth wheel which is part of a carriage 48 of which one pinion meshes with a fixed 690-teeth wheel.[12] This causes the entire carriage to move, and the carriage supports Saturn and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 44 on which the Saturn system moves up or down, as I will show later. The problem is to find the velocity of the carriage 48. I don't want to resort to some memorized formula. We need to be able to compute the motions without any cryptic magic. We can do so by setting ourself in the reference frame of the moving

[11] The number of days for Jupiter is mistakenly given as 4323, an obvious typo.

[12] There is a typo in the right part of Saturn's driving mechanism in Oechslin's drawing. 650 should be replaced by 690.

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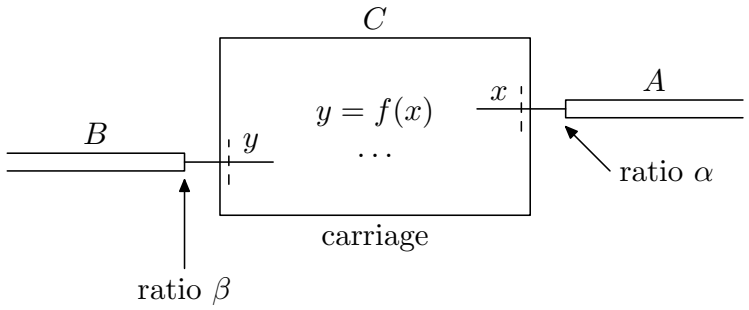


Figure 24.8: Principle of moving carriage C whose motion is determined by the motions of A and B.

carriage. We can merely compute the velocity of the 900-teeth wheel (frame 43) from that of the 690-teeth wheel (frame 47, fixed). We have

$$\mathrm{V}_{43}^{48} = \mathrm{V}_{47}^{48} \times \left(-\frac{690}{6}\right) \times \left(-\frac{109}{6}\right) \times \left(-\frac{103}{20}\right) \times \frac{25}{900} \tag{24.12}$$

$$= \mathrm{V}_{47}^{48} \times \left(-\frac{258221}{864}\right) = \mathrm{V}_{48}^{47} \times \frac{258221}{864} \tag{24.13}$$

It is important to note that the last ratio $\frac{25}{900}$ is positive, because the motion of arbor 44 is in the same direction as the frame 43, the 900-teeth wheel having an interior gearing.

Now, we have

$$\mathrm{V}_{43}^{48} = \mathrm{V}_{43}^{0} + \mathrm{V}_{0}^{48} = \mathrm{V}_{48}^{47} \times \frac{258221}{864} = \mathrm{V}_{48}^{0} \times \frac{258221}{864} \tag{24.14}$$

Therefore

$$\mathrm{V}_{43}^{0} = \mathrm{V}_{48}^{0} \times \frac{258221}{864} + \mathrm{V}_{48}^{0} = \mathrm{V}_{48}^{0} \times \left(\frac{258221}{864} + 1\right) = \mathrm{V}_{48}^{0} \times \frac{259085}{864} \tag{24.15}$$

And

$$\mathrm{V}_{48}^{0} = \mathrm{V}_{43}^{0} \times \frac{864}{259085} = \frac{25}{897} \times \frac{864}{259085} = \frac{1440}{15493283} \tag{24.16}$$

$$\mathrm{P}_{48}^{0} = \frac{15493283}{1440} = 10759.2243\ldots \text{ days} \tag{24.17}$$

This is an approximation of the sidereal orbital period of Saturn. The same value is given by Oechslin.

The motion of Saturn around its axis is described below in the section of Saturn's satellites.

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24.4.1.2 The motion of Jupiter

The mean motion of Jupiter is obtained like that of Saturn.

On arbor 65, there is a 30-teeth wheel which meshes with a 360-teeth contrate wheel on frame 66. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{66}^{0} = \mathrm{V}_{65}^{0} \times \left(-\frac{30}{360}\right) = \frac{30}{360} = \frac{1}{12} \tag{24.18}$$

$$\mathrm{P}_{66}^{0} = 12 \text{ days} \tag{24.19}$$

Above the 360-teeth wheel, there is a 361-teeth wheel which meshes with a 30-teeth wheel which is part of a carriage 73 of which one pinion meshes with a fixed 306-teeth wheel. This causes the entire carriage to move, and the carriage supports Jupiter and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 69 on which the Jupiter system moves up or down, as I will show later. The problem is to find the velocity of the carriage 73. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 361-teeth wheel (frame 66) from that of the 306-teeth wheel (frame 72, fixed). We have

$$\mathrm{V}_{66}^{73} = \mathrm{V}_{72}^{73} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \times \left(-\frac{14}{22}\right) \times \left(-\frac{22}{14}\right) \times \frac{30}{361} \tag{24.20}$$

$$= \mathrm{V}_{72}^{73} \times \left(-\frac{355725}{988}\right) = \mathrm{V}_{73}^{72} \times \frac{355725}{988} \tag{24.21}$$

The last ratio $\frac{30}{361}$ is positive for the same reason as in the case of Saturn.

Now, we have

$$\mathrm{V}_{66}^{73} = \mathrm{V}_{66}^{0} + \mathrm{V}_{0}^{73} = \mathrm{V}_{73}^{72} \times \frac{355725}{988} = \mathrm{V}_{73}^{0} \times \frac{355725}{988} \tag{24.22}$$

Therefore

$$\mathrm{V}_{66}^{0} = \mathrm{V}_{73}^{0} \times \frac{355725}{988} + \mathrm{V}_{73}^{0} = \mathrm{V}_{73}^{0} \times \left(\frac{355725}{988} + 1\right) = \mathrm{V}_{73}^{0} \times \frac{356713}{988} \tag{24.23}$$

And

$$\mathrm{V}_{73}^{0} = \mathrm{V}_{66}^{0} \times \frac{988}{356713} = \frac{1}{12} \times \frac{988}{356713} = \frac{247}{1070139} \tag{24.24}$$

$$\mathrm{P}_{73}^{0} = \frac{1070139}{247} = 4332.5465\ldots \text{ days} \tag{24.25}$$

This is an approximation of the sidereal orbital period of Jupiter. The same value is given by Oechslin.

The motion of Jupiter around its axis is described below in the section of Jupiter's satellites.

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24.4.1.3 The motion of Mars

The mean motion of Mars is obtained like those of Jupiter and Saturn.

On arbor 65, there is a 15-teeth wheel which meshes with a 101-teeth contrate wheel on frame 85. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}^0_{85} = \mathrm{V}^0_{65} \times \left(-\frac{15}{101}\right) = \frac{15}{101} \tag{24.26}$$

$$\mathrm{P}^0_{85} = \frac{101}{15} = 6.7333\ldots \text{ days} \tag{24.27}$$

Above the 101-teeth wheel, there is a 102-teeth wheel which meshes with a 15-teeth wheel which is part of a carriage 90 of which one pinion meshes with a fixed 97-teeth wheel. This causes the entire carriage to move, and the carriage supports Mars. All these motions take place in a horizontal plane, but the carriage itself has an arbor 92 on which Mars moves up or down, as I will show later. The problem is to find the velocity of the carriage 90. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 102-teeth wheel (frame 85) from that of the 97-teeth wheel (frame 89, fixed). We have

$$\mathrm{V}^{90}_{85} = \mathrm{V}^{90}_{89} \times \left(-\frac{97}{4}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \frac{15}{102} \tag{24.28}$$

$$= \mathrm{V}^{90}_{89} \times \left(-\frac{879305}{8704}\right) = \mathrm{V}^{89}_{90} \times \frac{879305}{8704} \tag{24.29}$$

The last ratio $\frac{15}{102}$ is positive for the same reason as in the cases of Jupiter and Saturn.

Now, we have

$$\mathrm{V}^{90}_{85} = \mathrm{V}^0_{85} + \mathrm{V}^{90}_0 = \mathrm{V}^{89}_{90} \times \frac{879305}{8704} = \mathrm{V}^0_{90} \times \frac{879305}{8704} \tag{24.30}$$

Therefore

$$\mathrm{V}^0_{85} = \mathrm{V}^0_{90} \times \frac{879305}{8704} + \mathrm{V}^0_{90} = \mathrm{V}^0_{90} \times \left(\frac{879305}{8704} + 1\right) = \mathrm{V}^0_{90} \times \frac{888009}{8704} \tag{24.31}$$

And

$$\mathrm{V}^0_{90} = \mathrm{V}^0_{85} \times \frac{8704}{888009} = \frac{15}{101} \times \frac{8704}{888009} = \frac{43520}{29896303} \tag{24.32}$$

$$\mathrm{P}^0_{90} = \frac{29896303}{43520} = 686.9554\ldots \text{ days} \tag{24.33}$$

This is an approximation of the sidereal orbital period of Mars. The same value is given by Oechslin.

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Mars also rotates around its axis. In the (synodic) reference frame 90, we have

$$\mathrm{V}_{92}^{90} = \mathrm{V}_{89}^{90} \times \left(-\frac{97}{4}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \left(-\frac{13}{21}\right) \times \left(-\frac{22}{14}\right) \tag{24.34}$$

$$= \mathrm{V}_{89}^{90} \times \left(-\frac{513227}{768}\right) = \mathrm{V}_{90}^{0} \times \frac{513227}{768} \tag{24.35}$$

$$= \frac{43520}{29896303} \times \frac{513227}{768} = \frac{87248590}{89688909} \tag{24.36}$$

And in the absolute frame:

$$\mathrm{V}_{92}^{0} = \mathrm{V}_{92}^{90} + \mathrm{V}_{90}^{0} = \mathrm{V}_{90}^{0} \times \left(\frac{513227}{768} + 1\right) = \mathrm{V}_{90}^{0} \times \frac{513995}{768} \tag{24.37}$$

$$= \frac{43520}{29896303} \times \frac{513995}{768} = \frac{87379150}{89688909} \tag{24.38}$$

$$\mathrm{P}_{92}^{0} = \frac{89688909}{87379150} = 1.0264\ldots \text{ days} = 24 \text{ h } 38 \text{ mn } 3.87\ldots \text{ s} \tag{24.39}$$

The same value is given by Oechslin. This period is close to the actual sidereal rotation of Mars.

24.4.1.4 The motion of the Earth

The mean motion of the Earth is obtained like those of Mars, Jupiter and Saturn.

On arbor 65, there is a 13-teeth wheel which meshes with a 170-teeth contrate wheel on frame 93. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{93}^{0} = \mathrm{V}_{65}^{0} \times \left(-\frac{13}{170}\right) = \frac{13}{170} \tag{24.40}$$

$$\mathrm{P}_{93}^{0} = \frac{170}{13} = 13.0769\ldots \text{ days} \tag{24.41}$$

Above the 170-teeth wheel, there is a 217-teeth wheel which meshes with a 16-teeth wheel which is part of a carriage 98 of which one pinion meshes with a fixed 83-teeth wheel. This causes the entire carriage to move, and the carriage supports the Earth. All these motions take place in a horizontal plane, but the carriage itself has an arbor 100 on which the Earth moves up or down, as I will show later. The problem is to find the velocity of the carriage 98. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 217-teeth wheel (frame 93) from that of the 83-teeth wheel (frame 97, fixed). We have

$$\mathrm{V}_{93}^{98} = \mathrm{V}_{97}^{98} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \frac{16}{217} \tag{24.42}$$

$$= \mathrm{V}_{97}^{98} \times \left(-\frac{286350}{10633}\right) = \mathrm{V}_{98}^{97} \times \frac{286350}{10633} \tag{24.43}$$

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The last ratio $\frac{16}{217}$ is positive for the same reason as in the cases of Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{93}^{98} = \mathrm{V}_{93}^{0} + \mathrm{V}_{0}^{98} = \mathrm{V}_{98}^{97} \times \frac{286350}{10633} = \mathrm{V}_{98}^{0} \times \frac{286350}{10633} \tag{24.44}$$

Therefore

$$\mathrm{V}_{93}^{0} = \mathrm{V}_{98}^{0} \times \frac{286350}{10633} + \mathrm{V}_{98}^{0} = \mathrm{V}_{98}^{0} \times \left(\frac{286350}{10633} + 1\right) = \mathrm{V}_{98}^{0} \times \frac{296983}{10633} \tag{24.45}$$

And

$$\mathrm{V}_{98}^{0} = \mathrm{V}_{93}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{24.46}$$

$$\mathrm{P}_{98}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{24.47}$$

This is an approximation of the sidereal year. The same value is given by Oechslin.

The Earth also rotates around its axis. In the (synodic) reference frame 98, we have

$$\mathrm{V}_{100}^{98} = \mathrm{V}_{97}^{98} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \left(-\frac{14}{56}\right) \times \left(-\frac{56}{14}\right) \tag{24.48}$$

$$= \mathrm{V}_{97}^{98} \times \left(-\frac{143175}{392}\right) = \mathrm{V}_{98}^{0} \times \frac{143175}{392} \tag{24.49}$$

$$= \frac{138229}{50487110} \times \frac{143175}{392} = \frac{80779335}{80779376} \tag{24.50}$$

$$\mathrm{P}_{100}^{98} = \frac{80779376}{80779335} = 1.0000005\ldots \text{ days} = 86400.0438\ldots \text{ seconds} \tag{24.51}$$

The Earth practically makes one turn around its axis in 24 hours with respect to the Sun. It should do so in exactly 24 hours, but it doesn't. In fact, Oechslin writes that the Earth makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}_{100}^{0} = \mathrm{V}_{100}^{98} + \mathrm{V}_{98}^{0} = \mathrm{V}_{98}^{0} \times \left(\frac{143175}{392} + 1\right) = \mathrm{V}_{98}^{0} \times \frac{143567}{392} \tag{24.52}$$

$$= \frac{138229}{50487110} \times \frac{143567}{392} = \frac{405002507}{403896880} \tag{24.53}$$

$$\mathrm{P}_{100}^{0} = \frac{403896880}{405002507} = 23 \text{ h } 56 \text{ mn } 4.1343\ldots \text{ s} \tag{24.54}$$

This period is close to the actual sidereal rotation of the Earth.

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24.4.1.5 The motion of Venus

The mean motion of Venus is obtained like those of the Earth, Mars, Jupiter and Saturn.

On arbor 65, there is a 20-teeth wheel which meshes with a 96-teeth contrate wheel on frame 113. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}^0_{113} = \mathrm{V}^0_{65} \times \left(-\frac{20}{96}\right) = \frac{5}{24} \tag{24.55}$$

$$\mathrm{P}^0_{113} = \frac{24}{5} = 4.8 \text{ days} \tag{24.56}$$

Above the 96-teeth wheel, there is a 103-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 118 of which one pinion meshes with a fixed 65-teeth wheel.[13] This causes the entire carriage to move, and the carriage supports Venus. All these motions take place in a horizontal plane, but the carriage itself has an arbor 120 on which Venus moves up or down, as I will show later. The problem is to find the velocity of the carriage 118. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 103-teeth wheel (frame 113) from that of the 65-teeth wheel (frame 117, fixed). We have

$$\mathrm{V}^{118}_{113} = \mathrm{V}^{118}_{117} \times \left(-\frac{65}{4}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{22}{10}\right) \times \frac{21}{103} \tag{24.57}$$

$$= \mathrm{V}^{118}_{117} \times \left(-\frac{4719}{103}\right) = \mathrm{V}^{117}_{118} \times \frac{4719}{103} \tag{24.58}$$

The last ratio $\frac{21}{103}$ is positive for the same reason as in the cases of the Earth, Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}^{118}_{113} = \mathrm{V}^0_{113} + \mathrm{V}^{118}_0 = \mathrm{V}^{117}_{118} \times \frac{4719}{103} = \mathrm{V}^0_{118} \times \frac{4719}{103} \tag{24.59}$$

Therefore

$$\mathrm{V}^0_{113} = \mathrm{V}^0_{118} \times \frac{4719}{103} + \mathrm{V}^0_{118} = \mathrm{V}^0_{118} \times \left(\frac{4719}{103} + 1\right) = \mathrm{V}^0_{118} \times \frac{4822}{103} \tag{24.60}$$

And

$$\mathrm{V}^0_{118} = \mathrm{V}^0_{113} \times \frac{103}{4822} = \frac{5}{24} \times \frac{103}{4822} = \frac{515}{115728} \tag{24.61}$$

$$\mathrm{P}^0_{118} = \frac{115728}{515} = 224.7145\ldots \text{ days} \tag{24.62}$$

[13] However, Oechslin also writes "198/65" for the number of teeth, and it isn't clear what he meant. Perhaps there is an unused wheel of 198 teeth.

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This is an approximation of the sidereal orbital period of Venus. The same value is given by Oechslin.

Venus also rotates around its axis. In the (synodic) reference frame 118, we have

$$\mathrm{V}_{120}^{118} = \mathrm{V}_{117}^{118} \times \left(-\frac{65}{4}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{22}{10}\right) \times \left(-\frac{10}{20}\right) \times \left(-\frac{20}{10}\right) \tag{24.63}$$

$$= \mathrm{V}_{117}^{118} \times \left(-\frac{1573}{7}\right) = \mathrm{V}_{118}^{0} \times \frac{1573}{7} \tag{24.64}$$

$$= \frac{515}{115728} \times \frac{1573}{7} = \frac{810095}{810096} \tag{24.65}$$

$$\mathrm{P}_{120}^{118} = \frac{810096}{810095} = 1.000001\ldots \text{ days} = 86400.1066\ldots \text{ seconds} \tag{24.66}$$

It would seem that Neßtfell attempted here too to have Venus make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. Like in the case of the Earth, Oechslin writes that Venus makes one turn with respect to the Sun in one day, but this is not true.

Incidentally, we can see that Nessfell uses a ratio $\frac{n+1}{n}$, and such ratios appear in several places, although there does not seem to be any good reason for it.

Now, in the absolute frame, we have:

$$\mathrm{V}_{120}^{0} = \mathrm{V}_{120}^{118} + \mathrm{V}_{118}^{0} = \mathrm{V}_{118}^{0} \times \left(\frac{1573}{7} + 1\right) = \mathrm{V}_{118}^{0} \times \frac{1580}{7} \tag{24.67}$$

$$= \frac{515}{115728} \times \frac{1580}{7} = \frac{203425}{202524} \tag{24.68}$$

$$\mathrm{P}_{120}^{0} = \frac{202524}{203425} = 23 \text{ h } 53 \text{ mn } 37.3213\ldots \text{ s} \tag{24.69}$$

The same value is given by Oechslin. This period is very inaccurate, because Venus rotates much more slowly. It makes one turn in 243 days. But in the 18th century, Venus's motion around its axis was still a mystery, and some thought it was rotating in 23 hours, others in 25 days.

24.4.1.6 The motion of Mercury

The mean motion of Mercury is obtained like those of Venus, the Earth, Mars, Jupiter and Saturn.

On arbor 65, there is a 21-teeth wheel which meshes with a 84-teeth contrate wheel on frame 121. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{121}^{0} = \mathrm{V}_{65}^{0} \times \left(-\frac{21}{84}\right) = \frac{1}{4} \tag{24.70}$$

$$\mathrm{P}_{121}^{0} = 4 \text{ days} \tag{24.71}$$

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Above the 84-teeth wheel, there is a 88-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 126 of which one pinion meshes with a fixed 60-teeth wheel. This causes the entire carriage to move, and the carriage supports Mercury. All these motions take place in a horizontal plane, but the carriage itself has an arbor 128 on which Mercury moves up or down, as I will show later. The problem is to find the velocity of the carriage 126. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 88-teeth wheel (frame 121) from that of the 60-teeth wheel (frame 125, fixed). We have

$$\mathrm{V}_{121}^{126} = \mathrm{V}_{125}^{126} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{19}\right) \times \left(-\frac{30}{7}\right) \times \frac{21}{88} \tag{24.72}$$

$$= \mathrm{V}_{125}^{126} \times \left(-\frac{8775}{418}\right) = \mathrm{V}_{126}^{125} \times \frac{8775}{418} \tag{24.73}$$

The last ratio $\frac{21}{88}$ is positive for the same reason as in the cases of Venus, the Earth, Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{121}^{126} = \mathrm{V}_{121}^{0} + \mathrm{V}_{0}^{126} = \mathrm{V}_{126}^{125} \times \frac{8775}{418} = \mathrm{V}_{126}^{0} \times \frac{8775}{418} \tag{24.74}$$

Therefore

$$\mathrm{V}_{121}^{0} = \mathrm{V}_{126}^{0} \times \frac{8775}{418} + \mathrm{V}_{126}^{0} = \mathrm{V}_{126}^{0} \times \left(\frac{8775}{418} + 1\right) = \mathrm{V}_{126}^{0} \times \frac{9193}{418} \tag{24.75}$$

And

$$\mathrm{V}_{126}^{0} = \mathrm{V}_{121}^{0} \times \frac{418}{9193} = \frac{1}{4} \times \frac{418}{9193} = \frac{209}{18386} \tag{24.76}$$

$$\mathrm{P}_{126}^{0} = \frac{18386}{209} = 87.9712\ldots \text{ days} \tag{24.77}$$

This is an approximation of the sidereal orbital period of Mercury. The same value is given by Oechslin.

Mercury also rotates around its axis. In the (synodic) reference frame 126, we have

$$\mathrm{V}_{128}^{126} = \mathrm{V}_{125}^{126} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{19}\right) \times \left(-\frac{30}{7}\right) \times \left(-\frac{12}{18}\right) \times \left(-\frac{18}{12}\right) \tag{24.78}$$

$$= \mathrm{V}_{125}^{126} \times \left(-\frac{11700}{133}\right) = \mathrm{V}_{126}^{0} \times \frac{11700}{133} \tag{24.79}$$

$$= \frac{209}{18386} \times \frac{11700}{133} = \frac{64350}{64351} \tag{24.80}$$

$$\mathrm{P}_{128}^{126} = \frac{64351}{64350} = 1.00001\ldots \text{ days} = 86401.3426\ldots \text{ seconds} \tag{24.81}$$

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Again, it seems that Neßtfell attempted here too to have Mercury make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. And like in the case of the Earth and Venus, Oechslin writes that Mercury makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}^0_{128} = \mathrm{V}^{126}_{128} + \mathrm{V}^0_{126} = \mathrm{V}^0_{126} \times \left(\frac{11700}{133} + 1\right) = \mathrm{V}^0_{126} \times \frac{11833}{133} \tag{24.82}$$

$$= \frac{209}{18386} \times \frac{11833}{133} = \frac{130163}{128702} \tag{24.83}$$

$$\mathrm{P}^0_{128} = \frac{128702}{130163} = 23 \text{ h } 43 \text{ mn } 50.2128\ldots \text{ s} \tag{24.84}$$

Oechslin gives the same value, although there is a typo in his numerical evaluation.

This period is of course very inaccurate, because Mercury rotates much more slowly. It makes one turn in about 59 days. But in the 18th century, Mercury's motion around its axis was still a mystery.

Finally, there also seems to be a dial for the orbital rotation of Mercury. This dial is located on frame 131 which merely replicates the (fixed) frame 125:

$$\mathrm{V}^{126}_{131} = \mathrm{V}^{126}_{125} \times \left(-\frac{60}{6}\right) \times \left(-\frac{6}{60}\right) = \mathrm{V}^{126}_{125} \tag{24.85}$$

$$\mathrm{V}^0_{131} = \mathrm{V}^{126}_{131} + \mathrm{V}^0_{126} = \mathrm{V}^{126}_{125} + \mathrm{V}^0_{126} = \mathrm{V}^0_{125} \tag{24.86}$$

24.4.2 The motion of the Sun

The motion of the Sun on tube 130 is derived from that of Mercury:

$$\mathrm{V}^{126}_{130} = \mathrm{V}^{126}_{125} \times \left(-\frac{60}{6}\right) \times \left(-\frac{6}{10}\right) \times \left(-\frac{9}{22}\right) \tag{24.87}$$

$$= \mathrm{V}^{126}_{125} \times \left(-\frac{27}{11}\right) = \mathrm{V}^{125}_{126} \times \frac{27}{11} = \mathrm{V}^0_{126} \times \frac{27}{11} \tag{24.88}$$

$$\mathrm{V}^0_{130} = \mathrm{V}^{126}_{130} + \mathrm{V}^0_{126} = \mathrm{V}^0_{126} \times \left(\frac{27}{11} + 1\right) \tag{24.89}$$

$$= \mathrm{V}^0_{126} \times \frac{38}{11} = \frac{209}{18386} \times \frac{38}{11} = \frac{361}{9193} \tag{24.90}$$

$$\mathrm{P}^0_{130} = \frac{9193}{361} = 25.4653\ldots \text{ days} \tag{24.91}$$

The same value is given by Oechslin. This is a good approximation of the period of rotation of the Sun which is about 25 days at the equator. The motion is counterclockwise, as it should be. However, on the Munich machine, Neßtfell used a slightly different period, and had the Sun rotate clockwise (from above), which is wrong.[14]

[14] See also [20, p. 203].

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24.4.3 The satellites of Jupiter and Saturn

24.4.3.1 The system of Jupiter

The gears for the rotation of Jupiter and its satellites are contained in frame 74 which is gliding along a tilted slope. This frame therefore does have a fixed orientation with respect to the Sun. The motion of frame 74 is therefore exactly that of the carriage 73 seen earlier:

$$\mathrm{V}^0_{74} = \mathrm{V}^0_{73} \tag{24.92}$$

The input to frame 74 is arbor 69 whose motion is

$$\mathrm{V}^{73}_{69} = \mathrm{V}^{73}_{72} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \tag{24.93}$$

$$= \mathrm{V}^{73}_{72} \times \left(-\frac{450585}{104}\right) = \mathrm{V}^{72}_{73} \times \frac{450585}{104} = \mathrm{V}^0_{73} \times \frac{450585}{104} \tag{24.94}$$

$$= \frac{247}{1070139} \times \frac{450585}{104} = \frac{2853705}{2853704} \tag{24.95}$$

We can then easily compute the velocities of Jupiter and its four satellites in the rotating frame. All these motions are directly obtained from arbor 69, and not one from the other.[15]

$$\mathrm{V}^{74}_{76} = \mathrm{V}^{74}_{69} \times \left(-\frac{29}{37}\right) \times \left(-\frac{37}{12}\right) = \mathrm{V}^{74}_{69} \times \frac{29}{12} \text{ (Jupiter)} \tag{24.96}$$

$$\mathrm{V}^{74}_{78} = \mathrm{V}^{74}_{69} \times \left(-\frac{31}{32}\right) \times \left(-\frac{28}{48}\right) = \mathrm{V}^{74}_{69} \times \frac{217}{384} \text{ (Io)} \tag{24.97}$$

$$\mathrm{V}^{74}_{80} = \mathrm{V}^{74}_{69} \times \left(-\frac{26}{42}\right) \times \left(-\frac{15}{33}\right) = \mathrm{V}^{74}_{69} \times \frac{65}{231} \text{ (Europa)} \tag{24.98}$$

$$\mathrm{V}^{74}_{82} = \mathrm{V}^{74}_{69} \times \left(-\frac{20}{43}\right) \times \left(-\frac{12}{40}\right) = \mathrm{V}^{74}_{69} \times \frac{6}{43} \text{ (Ganymede)} \tag{24.99}$$

$$\mathrm{V}^{74}_{84} = \mathrm{V}^{74}_{69} \times \left(-\frac{22}{43}\right) \times \left(-\frac{7}{60}\right) = \mathrm{V}^{74}_{69} \times \frac{77}{1290} \text{ (Callisto)} \tag{24.100}$$

And in the absolute frame:

$$\mathrm{V}^0_{76} = \mathrm{V}^{74}_{76} + \mathrm{V}^0_{74} = \mathrm{V}^{74}_{69} \times \frac{29}{12} + \mathrm{V}^0_{74} = \frac{2853705}{2853704} \times \frac{29}{12} + \frac{247}{1070139} \tag{24.101}$$

$$= \frac{82765349}{34244448} \tag{24.102}$$

$$\mathrm{P}^0_{76} = \frac{34244448}{82765349} = 9.9300\ldots \text{ hours} = 9 \text{ h } 55 \text{ mn } 48.2973 \text{ s} \tag{24.103}$$

[15] In the gear train for Io, Oechslin mistakenly wrote 28 instead of 48.

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The same value is given by Oechslin. This is an approximation of the rotation of Jupiter around its axis which is completed in about 9.8 hours.

$$\mathrm{V}_{78}^{0} = \mathrm{V}_{78}^{74} + \mathrm{V}_{74}^{0} = \mathrm{V}_{69}^{74} \times \frac{217}{384} + \mathrm{V}_{74}^{0} = \frac{2853705}{2853704} \times \frac{217}{384} + \frac{247}{1070139} \tag{24.104}$$

$$= \frac{619506913}{1095822336} \tag{24.105}$$

$$\mathrm{P}_{78}^{0} = \frac{1095822336}{619506913} = 1.7688\ldots \text{ days (Io)} \tag{24.106}$$

$$\mathrm{V}_{80}^{0} = \mathrm{V}_{80}^{74} + \mathrm{V}_{74}^{0} = \mathrm{V}_{69}^{74} \times \frac{65}{231} + \mathrm{V}_{74}^{0} = \frac{2853705}{2853704} \times \frac{65}{231} + \frac{247}{1070139} \tag{24.107}$$

$$= \frac{185642977}{659205624} \tag{24.108}$$

$$\mathrm{P}_{80}^{0} = \frac{659205624}{185642977} = 3.5509\ldots \text{ days (Europa)} \tag{24.109}$$

$$\mathrm{V}_{82}^{0} = \mathrm{V}_{82}^{74} + \mathrm{V}_{74}^{0} = \mathrm{V}_{69}^{74} \times \frac{6}{43} + \mathrm{V}_{74}^{0} = \frac{2853705}{2853704} \times \frac{6}{43} + \frac{247}{1070139} \tag{24.110}$$

$$= \frac{25725829}{184063908} \tag{24.111}$$

$$\mathrm{P}_{82}^{0} = \frac{184063908}{25725829} = 7.1548\ldots \text{ days (Ganymede)} \tag{24.112}$$

$$\mathrm{V}_{84}^{0} = \mathrm{V}_{84}^{74} + \mathrm{V}_{74}^{0} = \mathrm{V}_{69}^{74} \times \frac{77}{1290} + \mathrm{V}_{74}^{0} = \frac{2853705}{2853704} \times \frac{77}{1290} + \frac{247}{1070139} \tag{24.113}$$

$$= \frac{44116993}{736255632} \tag{24.114}$$

$$\mathrm{P}_{84}^{0} = \frac{736255632}{44116993} = 16.6887\ldots \text{ days (Callisto)} \tag{24.115}$$

The same values are given by Oechslin. The periods of the satellites are all good approximations of the actual ones.

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24.4.3.2 The system of Saturn

The structure of the Saturn system is similar to that of Jupiter, except that it contains two worms for the slow motion of Saturn's fifth satellite, Iapetus.



Figure 24.9: Detail of Neßtfell's machine in Vienna. The gears for Saturn's satellites are visible in the back. The largest ring is for Saturn, the other one for Jupiter. (photograph by the author)

The gears for the rotation of Saturn and its satellites are contained in frame 49 which is gliding along a tilted slope. This frame therefore does have a fixed orientation with respect to the Sun. The motion of frame 49 is therefore exactly that of the carriage 48 seen earlier:

$$\mathrm{V}_{49}^{0} = \mathrm{V}_{48}^{0} \tag{24.116}$$

The input to frame 49 is arbor 44 whose motion is

$$\mathrm{V}_{44}^{48} = \mathrm{V}_{47}^{48} \times \left(-\frac{690}{6}\right) \times \left(-\frac{109}{6}\right) \times \left(-\frac{103}{20}\right) \tag{24.117}$$

$$= \mathrm{V}_{47}^{48} \times \left(-\frac{258221}{24}\right) = \mathrm{V}_{48}^{47} \times \frac{258221}{24} = \mathrm{V}_{48}^{0} \times \frac{258221}{24} \tag{24.118}$$

$$= \frac{1440}{15493283} \times \frac{258221}{24} = \frac{673620}{673621} \tag{24.119}$$

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Figure 24.10: Detail of Neßtfell's machine in Vienna. (photograph by the author)

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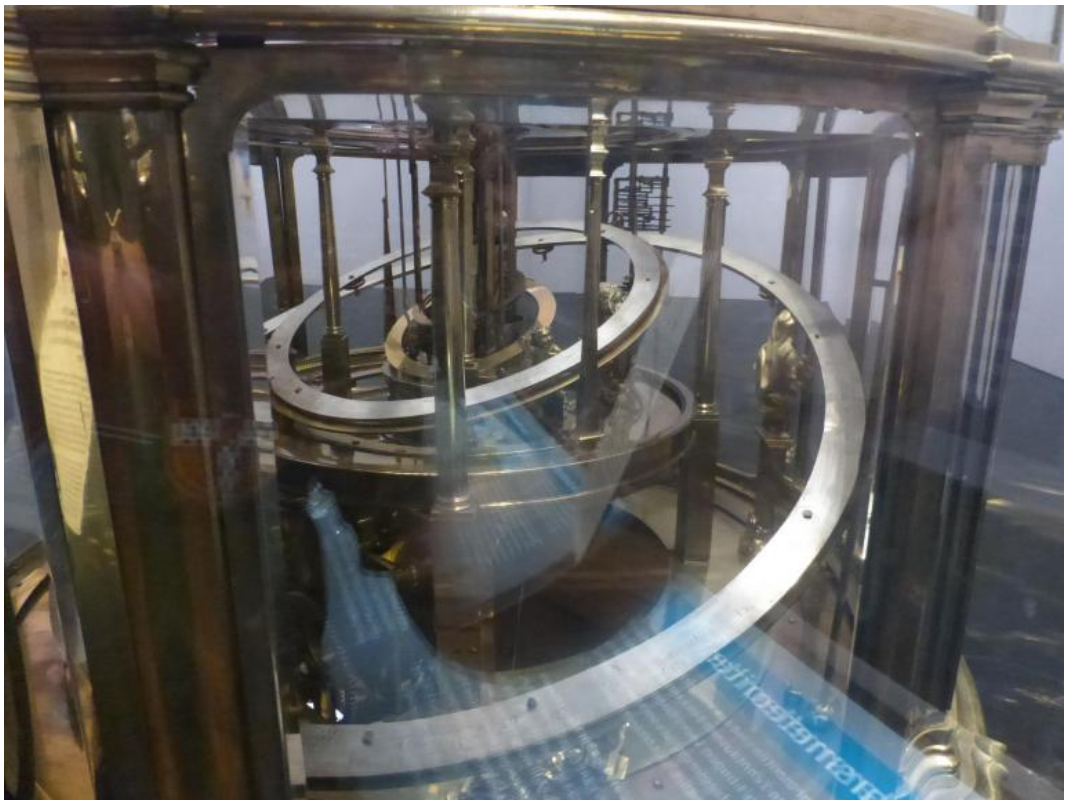


Figure 24.11: Detail of Neßtfell's machine in Vienna. (photograph by the author)

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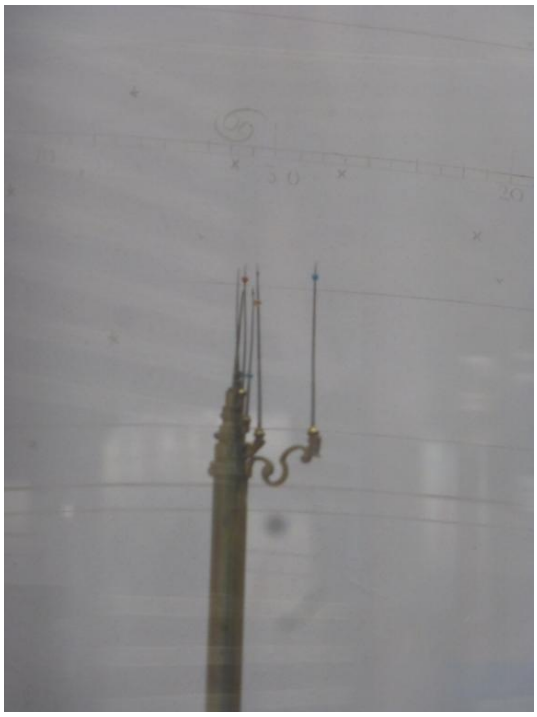


Figure 24.12: The satellites of Saturn on Neßtfell's machine in Vienna. (photograph by the author)

We can then easily compute the velocities of Saturn and its five satellites in the rotating frame. All these motions are directly obtained from arbor 44, and not one from the other.[16]

$$\mathrm{V}_{51}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{30}{40}\right) \times \left(-\frac{40}{10}\right) = \mathrm{V}_{44}^{49} \times 3 \text{ (Saturn)} \tag{24.120}$$

$$\mathrm{V}_{53}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{24}{19}\right) \times \left(-\frac{13}{31}\right) = \mathrm{V}_{44}^{49} \times \frac{312}{589} \text{ (Tethys)} \tag{24.121}$$

$$\mathrm{V}_{55}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{19}{29}\right) \times \left(-\frac{29}{52}\right) = \mathrm{V}_{44}^{49} \times \frac{19}{52} \text{ (Dione)} \tag{24.122}$$

$$\mathrm{V}_{57}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{47}{30}\right) \times \left(-\frac{6}{43}\right) = \mathrm{V}_{44}^{49} \times \frac{47}{215} \text{ (Rhea)} \tag{24.123}$$

$$\mathrm{V}_{59}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{29}{34}\right) \times \left(-\frac{5}{68}\right) = \mathrm{V}_{44}^{49} \times \frac{145}{2312} \text{ (Titan)} \tag{24.124}$$

$$\mathrm{V}_{63}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{2}{8}\right) \times \left(-\frac{1}{20}\right) \times \left(-\frac{53}{11}\right) \times \left(-\frac{9}{43}\right) \tag{24.125}$$

$$= \mathrm{V}_{44}^{49} \times \frac{477}{37840} \text{ (Iapetus)} \tag{24.126}$$

[16] In the case of Iapetus, Oechslin's drawing forgot to give the number of teeth 43 for the wheel on tube 63.

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And in the absolute frame:

$$\mathrm{V}_{51}^{0} = \mathrm{V}_{51}^{49} + \mathrm{V}_{49}^{0} = \mathrm{V}_{44}^{49} \times 3 + \mathrm{V}_{49}^{0} = \frac{673620}{673621} \times 3 + \frac{1440}{15493283} \tag{24.127}$$

$$= \frac{46481220}{15493283} \tag{24.128}$$

$$\mathrm{P}_{51}^{0} = \frac{15493283}{46481220} = 7.9997\ldots \text{ hours (Saturn)} \tag{24.129}$$

$$7 \text{ h } 59 \text{ m } 59.1505\ldots \text{ s} \tag{24.130}$$

The same value is given by Oechslin. The actual period of rotation of Saturn is about 10.65 hours.

$$\mathrm{V}_{53}^{0} = \mathrm{V}_{53}^{49} + \mathrm{V}_{49}^{0} = \mathrm{V}_{44}^{49} \times \frac{312}{589} + \mathrm{V}_{49}^{0} = \frac{673620}{673621} \times \frac{312}{589} + \frac{1440}{15493283} \tag{24.131}$$

$$= \frac{4834745280}{9125543687} \tag{24.132}$$

$$\mathrm{P}_{53}^{0} = \frac{9125543687}{4834745280} = 1.8874\ldots \text{ days (Tethys)} \tag{24.133}$$

$$\mathrm{V}_{55}^{0} = \mathrm{V}_{55}^{49} + \mathrm{V}_{49}^{0} = \mathrm{V}_{44}^{49} \times \frac{19}{52} + \mathrm{V}_{49}^{0} = \frac{673620}{673621} \times \frac{19}{52} + \frac{1440}{15493283} \tag{24.134}$$

$$= \frac{73611705}{201412679} \tag{24.135}$$

$$\mathrm{P}_{55}^{0} = \frac{201412679}{73611705} = 2.7361\ldots \text{ days (Dione)} \tag{24.136}$$

$$\mathrm{V}_{57}^{0} = \mathrm{V}_{57}^{49} + \mathrm{V}_{49}^{0} = \mathrm{V}_{44}^{49} \times \frac{47}{215} + \mathrm{V}_{49}^{0} = \frac{673620}{673621} \times \frac{47}{215} + \frac{1440}{15493283} \tag{24.137}$$

$$= \frac{145698564}{666211169} \tag{24.138}$$

$$\mathrm{P}_{57}^{0} = \frac{666211169}{145698564} = 4.5725\ldots \text{ days (Rhea)} \tag{24.139}$$

$$\mathrm{V}_{59}^{0} = \mathrm{V}_{59}^{49} + \mathrm{V}_{49}^{0} = \mathrm{V}_{44}^{49} \times \frac{145}{2312} + \mathrm{V}_{49}^{0} = \frac{673620}{673621} \times \frac{145}{2312} + \frac{1440}{15493283} \tag{24.140}$$

$$= \frac{562462995}{8955117574} \tag{24.141}$$

$$\mathrm{P}_{59}^{0} = \frac{8955117574}{562462995} = 15.9212\ldots \text{ days (Titan)} \tag{24.142}$$

$$\mathrm{V}_{63}^{0} = \mathrm{V}_{63}^{49} + \mathrm{V}_{49}^{0} = \mathrm{V}_{44}^{49} \times \frac{477}{37840} + \mathrm{V}_{49}^{0} \tag{24.143}$$

$$= \frac{673620}{673621} \times \frac{477}{37840} + \frac{1440}{15493283} \tag{24.144}$$

$$= \frac{372238731}{29313291436} \tag{24.145}$$

$$\mathrm{P}_{63}^{0} = \frac{29313291436}{372238731} = 78.7486\ldots \text{ days (Iapetus)} \tag{24.146}$$

The same values are given by Oechslin. These period are good approximations of the actual sidereal periods of the satellites of Saturn.

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24.4.4 The motion of the Moon

24.4.4.1 The mean motion

The main motion of the Moon is that of frame 104 which rotates around the axis of the Earth. Its motion is obtained through arbor 102 which is located on the Earth carriage 98. This arbor carries a 33-teeth wheel which meshes with a 20-teeth wheel at right angle, and whose arbor 103 carries a worm meshing with a 15-teeth wheel located on frame 104. We can compute the motion of frame 104 in frame 98:

$$\mathrm{V}_{104}^{98} = \mathrm{V}_{102}^{98} \times \left(-\frac{33}{20}\right) \times \left(-\frac{1}{15}\right) = \mathrm{V}_{102}^{98} \times \frac{11}{100} \tag{24.147}$$

and

$$\mathrm{V}_{102}^{98} = \mathrm{V}_{97}^{98} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \left(-\frac{17}{21}\right) \times \left(-\frac{27}{71}\right) \tag{24.148}$$

$$= \mathrm{V}_{97}^{98} \times \left(-\frac{21905775}{194824}\right) \tag{24.149}$$

and therefore

$$\mathrm{V}_{104}^{98} = \mathrm{V}_{97}^{98} \times \left(-\frac{21905775}{194824}\right) \times \frac{11}{100} = \mathrm{V}_{97}^{98} \times \left(-\frac{9638541}{779296}\right) \tag{24.150}$$

$$= \mathrm{V}_{98}^{97} \times \frac{9638541}{779296} = \mathrm{V}_{98}^{0} \times \frac{9638541}{779296} \tag{24.151}$$

$$= \frac{138229}{50487110} \times \frac{9638541}{779296} = \frac{228490119}{6747453760} \tag{24.152}$$

$$\mathrm{P}_{104}^{98} = \frac{6747453760}{228490119} = 29.5306\ldots \text{ days} \tag{24.153}$$

The same value is given by Oechslin. This is the motion of the Moon with respect to the Sun, and an excellent approximation of the synodic month.

We can also compute the motion of the Moon in an absolute (sidereal) frame:

$$\mathrm{V}_{104}^{0} = \mathrm{V}_{104}^{98} + \mathrm{V}_{98}^{0} = \frac{228490119}{6747453760} + \frac{138229}{50487110} = \frac{4198388311}{114706713920} \tag{24.154}$$

$$\mathrm{P}_{104}^{0} = \frac{114706713920}{4198388311} = 27.3216\ldots \text{ days} \tag{24.155}$$

The same value is also given by Oechslin. This is an approximation of the sidereal month.

24.4.4.2 The nodes and the apsides

Neßtfell's construction for the nodes and apsides is at first sight very opaque (figure 24.13) and Oechslin's drawing (figure 24.14) does not help much to understand it. However, I believe that I have managed to understand it correctly. Here it goes.

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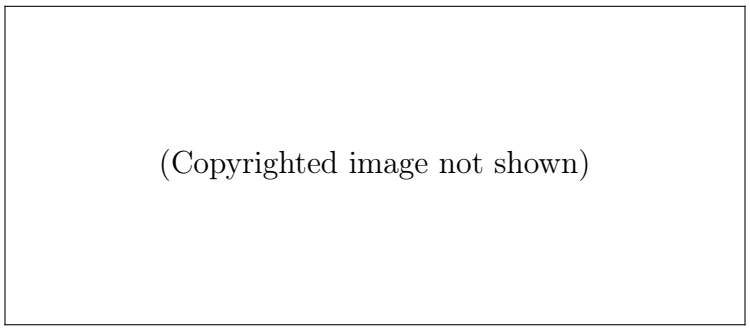


Figure 24.13: Neßtfell's construction for the motion of the nodes and apsides of the Moon.

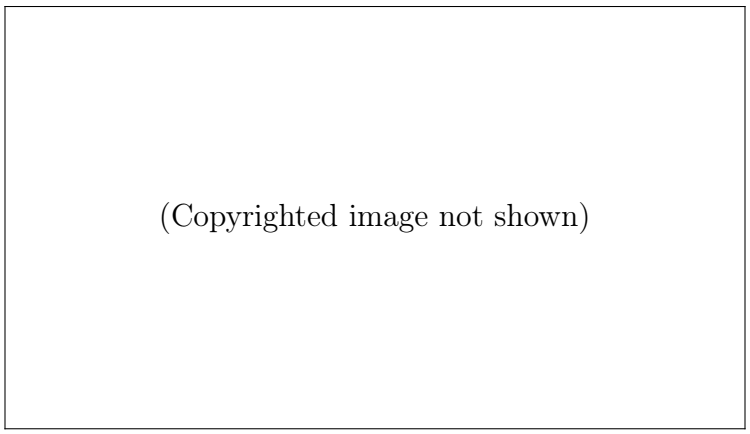


Figure 24.14: Oechslin's drawing of Neßtfell's construction.

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There is a rotating frame 106 carrying a number of gears for the motions of the lunar nodes and apsides. This frame rotates with carriage 98, but remains in the plane of the ecliptic.

There is also a fixed frame 105 which carries a 138-teeth wheel. This wheel meshes with two different wheels. First it meshes with a 29-teeth wheel for the revolution of the nodes. The arbor of this wheel carries a 31-teeth wheel which meshes with a 7-teeth wheel whose arbor carries a worm. This worm meshes with a 20-teeth wheel which moves the nodes on arbor 109.

We can compute the motion of arbor 109 with respect to frame 106:

$$\mathrm{V}_{109}^{106} = \mathrm{V}_{105}^{106} \times \left(-\frac{138}{29}\right) \times \left(-\frac{31}{7}\right) \times \frac{1}{20} = \mathrm{V}_{105}^{106} \times \frac{2139}{2030} \tag{24.156}$$

$$= \mathrm{V}_{106}^{105} \times \left(-\frac{2139}{2030}\right) = \mathrm{V}_{98}^{0} \times \left(-\frac{2139}{2030}\right) \tag{24.157}$$

$$= \frac{138229}{50487110} \times \left(-\frac{2139}{2030}\right) = -\frac{42238833}{14641261900} \tag{24.158}$$

$$\mathrm{P}_{109}^{106} = -\frac{14641261900}{42238833} = -346.6303\ldots \text{ days} \tag{24.159}$$

This is the period of revolution of the nodes with respect to the Sun, the so-called eclipse year. I have assumed that the worm makes the 20-teeth wheel move in the correct direction, but I cannot check that this is actually the case.

We can then compute the motion of the nodes in the absolute frame:

$$\mathrm{V}_{109}^{0} = \mathrm{V}_{109}^{106} + \mathrm{V}_{106}^{0} = \mathrm{V}_{109}^{106} + \mathrm{V}_{98}^{0} \tag{24.160}$$

$$= -\frac{42238833}{14641261900} + \frac{138229}{50487110} = -\frac{2152423}{14641261900} \tag{24.161}$$

$$\mathrm{P}_{109}^{0} = -\frac{14641261900}{2152423} = -6802.2233\ldots \text{ days} \tag{24.162}$$

which is a good approximation of the actual value of 6798 days. This value is negative, because the lunar nodes are retrograding. The same value is given by Oechslin.

Second, the 138-teeth wheel of frame 105 meshes with a 11-teeth wheel for the revolution of the apsides. The arbor of this wheel carries a 2-threaded worm which meshes with a 6-teeth wheel. And the arbor of this wheel carries another worm which meshes with a 37-teeth wheel which moves the apsides on arbor 112.

We can compute the motion of arbor 112 with respect to frame 106:

$$\mathrm{V}_{112}^{106} = \mathrm{V}_{105}^{106} \times \left(-\frac{138}{11}\right) \times \left(-\frac{2}{6}\right) \times \left(-\frac{1}{37}\right) = \mathrm{V}_{105}^{106} \times \left(-\frac{46}{407}\right) \tag{24.163}$$

$$= \mathrm{V}_{106}^{105} \times \frac{46}{407} = \mathrm{V}_{98}^{0} \times \frac{46}{407} \tag{24.164}$$

$$= \frac{138229}{50487110} \times \frac{46}{407} = \frac{3179267}{10274126885} \tag{24.165}$$

$$\mathrm{P}_{112}^{106} = \frac{10274126885}{3179267} = 3231.6024\ldots \text{ days} \tag{24.166}$$

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This is a good approximation of the revolution of the apsides, *but with respect to the absolute frame*. And therefore it is false.[17]

However, we should have found a period of about 411 days, which is the period of revolution of the apsides with respect to the Sun. It would then seem that Neßtfell mixed up his computations, and used a gear train meant for the absolute motion of the apsides in the context of a synodic motion of the apsides. The problem is not a mere typo or one wheel that could be easily fixed.

If we now compute the motion of the apsides in the absolute frame, we of course do not obtain anything meaningful:

$$\mathrm{V}^0_{112} = \mathrm{V}^{106}_{112} + \mathrm{V}^0_{106} = \mathrm{V}^{106}_{112} + \mathrm{V}^0_{98} \tag{24.167}$$

$$= \frac{3179267}{10274126885} + \frac{138229}{50487110} = \frac{62617737}{20548253770} \tag{24.168}$$

$$\mathrm{P}^0_{112} = \frac{20548253770}{62617737} = 328.1538\ldots \text{ days} \tag{24.169}$$

Interestingly, Oechslin has found a period of 411 days in the absolute frame using an incorrect computation, but dismissed it as unmeaningful.

[17] About the error in the motion of the apsides, see also [20, p. 203].

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24.5 The Earth-Mercury system

The front part of the machine is the Earth-Mercury system and is focused on displaying the apparent motion of Mercury. Both the Earth and Mercury rotate around the central axis which represents the Sun. Such a main display with Mercury on an astronomical clock is unusual, but the choice of that display is certainly a consequence of the interest for the transits of Mercury in the 18th century. Neßtfell certainly did not choose to display the Earth-Venus system, because transits of Venus are much less frequent than those of Mercury. This representation also has a pedogogical function, it serves to highlight a particular part of the solar system, as stressed by Meier.[18] Finally, it is also a way to illustrate the retrogradations of the planets, as Neßtfell explained himself, and as Oechslin observed [20, p. 36].

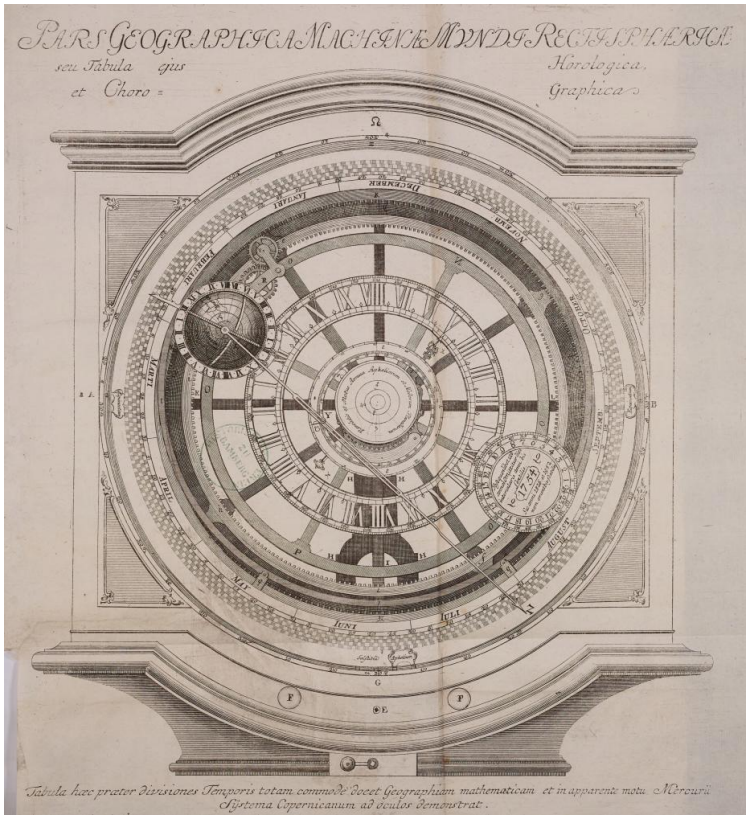


Figure 24.15: Engraving of the Earth-Mercury system on Neßtfell's machine in Vienna, probably from 1754, given the year on the engraving. (Johann Balthasar Gutwein, source: [18])

The central axis of the panel represents the position of the Sun and the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves around the Sun as I will describe

[18] Cf. [16, p. 44-45].

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Figure 24.16: The front side of Neßtfell's machine in Vienna. (photograph by the author)

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Figure 24.17: The 24-hour dial on the front side of Neßtfell's machine in Vienna. This dial gives also the year, but there is a mention that the year 1828 should not be a leap year. On the engraving of figure 24.15, it is 1826 that should not be a leap year. (photograph by the author)

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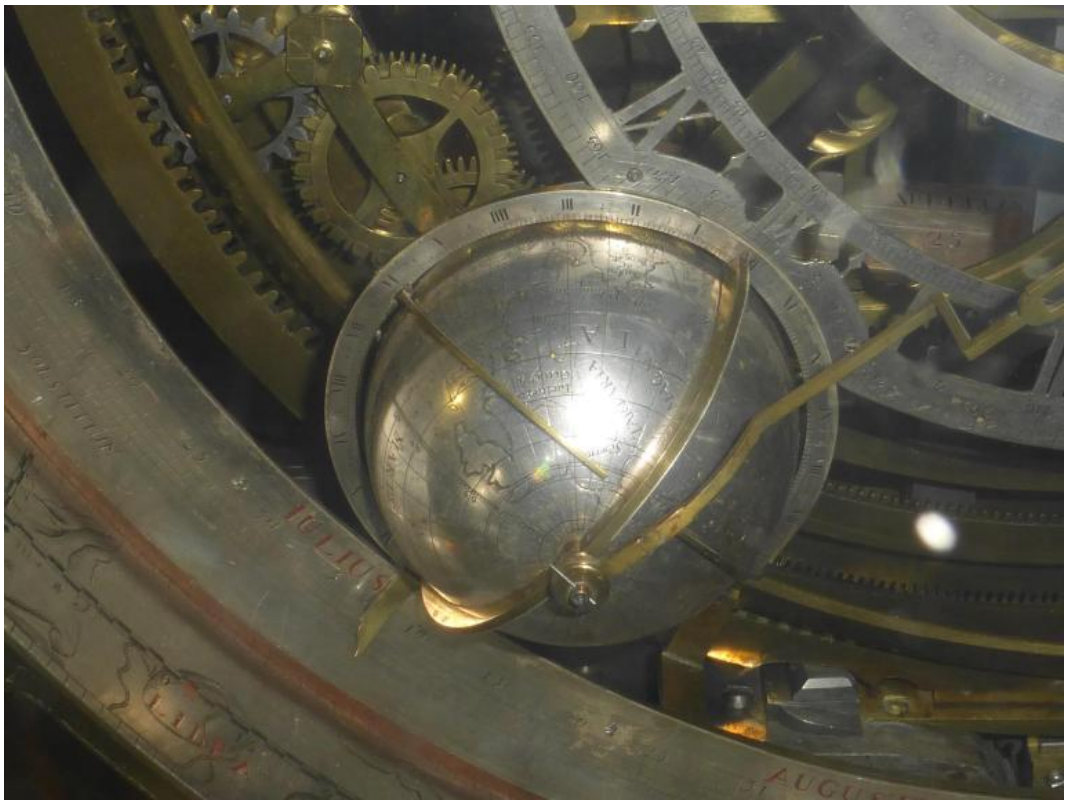


Figure 24.18: The Earth sphere on Neßtfell's machine in Vienna. (photograph by the author)

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Figure 24.19: The central front dial of Neßtfell's machine in Vienna. Mercury is at the lower right. (photograph by the author)

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Figure 24.20: The mention of the fall equinox on the calendar dial. (photograph by the author)

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below.

The input to the Earth-Mercury system is arbor 13 which comes from the going work. This arbor makes one turn clockwise in one day:

$$\mathrm{V}_{13}^{0} = -\mathrm{T}_{13}^{0} = -1 \tag{24.170}$$

This arbor is also used to derive several other auxiliary motions which will be described later.

24.5.1 The mean orbital motion of the Earth

Arbor 13 carries a 26-teeth wheel which meshes with a 340-teeth wheel on frame 14. The axis of this frame is the central axis of the front part. The velocity of frame 14 (seen from the front) is

$$\mathrm{V}_{14}^{0} = \mathrm{V}_{13}^{0} \times \left(-\frac{26}{340}\right) = \frac{13}{170} \tag{24.171}$$

As I note below, it is possible that the distance between the axes of the 26-teeth wheel and the 340-teeth wheel varies, because of the motion of the line of apsides.

Frame 14 also carries an interior wheel of 434 teeth, which meshes with a 32-teeth wheel which is part of a train located on frame 20 and ending with a 32-teeth wheel on tube 22. This tube is not fixed, although one might expect it to be fixed. The frame 20 also carries the Earth's meridian as well as an arbor for the rotation of the Earth. We might therefore expect frame 20 to rotate in a tropical year.

The motion of frame 20 is derived from a train laid between the meridian's axis and a fixed 249-teeth wheel on frame 19. We can compute the motion of frame 20 by first considering the relative motion of frame 19 with respect to that of frame 14:

$$\mathrm{V}_{19}^{20} = \mathrm{V}_{14}^{20} \times \frac{434}{32} \times \left(-\frac{14}{50}\right) \times \left(-\frac{7}{69}\right) \times \left(-\frac{24}{19}\right) \times \frac{19}{249} \tag{24.172}$$

$$= \mathrm{V}_{14}^{20} \times \left(-\frac{10633}{286350}\right) \tag{24.173}$$

$$= \left(\mathrm{V}_{20}^{0} + \mathrm{V}_{0}^{14}\right) \times \frac{10633}{286350} \tag{24.174}$$

Hence

$$\mathrm{V}_{14}^{0} \times \frac{10633}{286350} = \mathrm{V}_{20}^{0} \times \left(1 + \frac{10633}{286350}\right) = \mathrm{V}_{20}^{0} \times \frac{296983}{286350} \tag{24.175}$$

And

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{14}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{24.176}$$

$$\mathrm{P}_{20}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{24.177}$$

This is an approximation of the tropical year. The same value is given by Oechslin.

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24.5.2 The rotation of the Earth around its axis

We can now also compute the velocity of the central tube 22:

$$\mathrm{V}_{22}^{20} = \mathrm{V}_{15}^{20} = \mathrm{V}_{14}^{20} \times \frac{434}{32} = \left(\mathrm{V}_{14}^{0} - \mathrm{V}_{20}^{0}\right) \times \frac{217}{16} \tag{24.178}$$

$$= \left(\frac{13}{170} - \frac{138229}{50487110}\right) \times \frac{217}{16} = \frac{372255}{5048711} \times \frac{217}{16} = \frac{80779335}{80779376} \tag{24.179}$$

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{22}^{20} + \mathrm{V}_{20}^{0} = \frac{80779335}{80779376} + \frac{138229}{50487110} = \frac{405002507}{403896880} \tag{24.180}$$

$$\mathrm{P}_{22}^{0} = \frac{403896880}{405002507} = 0.9972\ldots \text{ days} = 86164.1343\ldots \text{ seconds} \tag{24.181}$$

The central tube 22 actually rotates with the velocity of the sidereal day.

Moreover, arbor 15 which carries the 32-teeth wheel meshing with the interior gearing also carries the frame for the Earth's meridian. Because of the two 32-teeth wheels, the one on the central fixed tube 22, and the one on tube 15, arbor 15 replicates the motion of the central tube. The Earth therefore rotates around a vertical axis (and not a tilted one) and makes one turn in one sidereal day in the absolute frame.

24.5.3 The line of apsides

As mentioned above, the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves like the line of the apsides. However, since the motion of the Earth is obtained through the 26-teeth wheel on arbor 13 meshing with the 340-teeth wheel on frame 14, this implies that there is a variable distance between the two axes. If this is the case, then the teeth are probably such that the meshing is maintained, even though the distance varies.[19]

The motion of the line of apsides is that of tube 33. The slow motion of tube 33 is obtained from that of arbor 13, with four intermediate worms:

$$\mathrm{V}_{33}^{0} = \mathrm{V}_{13}^{0} \times \frac{1}{34} \times \frac{1}{30} \times \frac{1}{15} \times \left(-\frac{1}{499}\right) = \mathrm{V}_{13}^{0} \times \left(-\frac{1}{7634700}\right) \tag{24.182}$$

$$= \frac{1}{7634700} \tag{24.183}$$

$$\mathrm{P}_{33}^{0} = 7634700 \text{ days} \approx 20900 \text{ years} \tag{24.184}$$

The same value is given by Oechslin. This is a good approximation of the period of tropical apsidal precession which is about 21000 years.

[19] See Oechslin's observations on this problem [20, p. 154-155, 199-200].

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24.5.4 Two more front gears

The front panel carries two large rings, one with 499 teeth (frame 37) and the other with 650 teeth (frame 35). The velocities of these frames are:

$$\mathrm{V}^0_{35} = \mathrm{V}^0_{13} \times \frac{1}{34} \times \frac{1}{30} \times \left(-\frac{1}{14}\right) \times \left(-\frac{1}{650}\right) = \mathrm{V}^0_{13} \times \frac{1}{9282000} \tag{24.185}$$

$$= -\frac{1}{9282000} \tag{24.186}$$

$$\mathrm{P}^0_{35} = -9282000 \text{ days} \approx -25413 \text{ years} \tag{24.187}$$

The same value is given by Oechslin.

$$\mathrm{V}^0_{37} = \mathrm{V}^0_{13} \times \frac{1}{34} \times \frac{1}{30} \times \left(-\frac{1}{15}\right) \times \frac{1}{499} = \mathrm{V}^0_{33} \tag{24.188}$$

$$\mathrm{P}^0_{37} = \mathrm{P}^0_{33} \approx 20900 \text{ years} \tag{24.189}$$

Frame 35 corresponds to the period of precession of the equinoxes. The Munich machine uses a slightly different ratio for that period. The correct value should however have been closer to 26000 years.

However, given that the front panel has the Earth rotate in a tropical year, which is confirmed by the motion of the apsides (which otherwise would not take place in about 21000 years), the zodiac is actually fixed. The actual constellations should then move *counterclockwise* and this is presumably what Neßtfell attempted to display, in order to show where in the sky Mercury is located. But on Neßtfell's machine, frame 35 moves clockwise, as if he wanted to show a precession.

In any case, frame 35 is only used to position the actual axis of the Earth, if I understand it correctly.

Frame 37 is again the period of precession of the apsides with respect to the equinoxes. Frame 37 actually moves like frame 33 and seems to be used as a dial showing the position of the apsides.

24.5.5 The motion of Mercury

The main front reference frame for Mercury is frame 33. We have seen above that this frame moves with the motion of the line of apsides of the orbit of the Earth. The Mercury wheel is a wheel on arbor 24 which pivots eccentrically on frame 33. This will account for the elliptical orbit of Mercury, but it does assume that the line of apsides of Mercury coincides with that of the Earth, which is not the case.

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24.5.5.1 The mean motion of Mercury

The mean motion of Mercury is given by the Mercury wheel on arbor 24. This motion is derived from that of tube 22. We have

$$\mathrm{V}_{24}^{33} = \mathrm{V}_{22}^{33} \times \left(-\frac{18}{68}\right) \times \left(-\frac{7}{163}\right) = \mathrm{V}_{22}^{33} \times \frac{63}{5542} \tag{24.190}$$

$$\mathrm{V}_{24}^{0} = \mathrm{V}_{24}^{33} + \mathrm{V}_{33}^{0} = \mathrm{V}_{22}^{33} \times \frac{63}{5542} + \mathrm{V}_{33}^{0} = \left(\mathrm{V}_{22}^{0} - \mathrm{V}_{33}^{0}\right) \times \frac{63}{5542} + \mathrm{V}_{33}^{0} \tag{24.191}$$

$$= \mathrm{V}_{22}^{0} \times \frac{63}{5542} + \mathrm{V}_{33}^{0} \times \left(1 - \frac{63}{5542}\right) = \mathrm{V}_{22}^{0} \times \frac{63}{5542} + \mathrm{V}_{33}^{0} \times \frac{5479}{5542} \tag{24.192}$$

$$= \frac{405002507}{403896880} \times \frac{63}{5542} + \frac{1}{7634700} \times \frac{5479}{5542} \tag{24.193}$$

$$= \frac{572949380244583}{50263193608696800} \tag{24.194}$$

$$\mathrm{P}_{24}^{0} = \frac{50263193608696800}{572949380244583} = 87.7271\ldots \text{ days} \tag{24.195}$$

The same value is given by Oechslin. This is a good approximation of the orbital period of Mercury. The same value is used in the Munich machine.

Of course, we could have ignored the term $\frac{1}{7634700}$, and this would only have slightly altered the orbital period of Mercury. But the exact ratio resulting from Neßtfell's construction is the one given above.

24.5.5.2 The inclination of Mercury's orbit

Neßtfell has Mercury move radially depending on its distance from its line of nodes. The orbit of Mercury is tilted by about 7 degrees, but on Neßtfell's panel, this angle is about 5 degrees, according to Oechslin. This tilted orbital plane is represented by frame 29 which carries a 259-teeth wheel. This is rotated using gears located on frame 33. We first compute the velocity of

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frame 29 with respect to frame 33:

$$\mathrm{V}_{29}^{33} = \mathrm{V}_{22}^{33} \times \left(-\frac{26}{96}\right) \times \left(-\frac{1}{14}\right) \times \left(-\frac{1}{26}\right) \times \frac{1}{16} \times \left(-\frac{1}{259}\right) \tag{24.196}$$

$$= \mathrm{V}_{22}^{33} \times \frac{1}{5569536} \tag{24.197}$$

$$\mathrm{V}_{29}^{0} = \mathrm{V}_{29}^{33} + \mathrm{V}_{33}^{0} = \mathrm{V}_{22}^{33} \times \frac{1}{5569536} + \mathrm{V}_{33}^{0} \tag{24.198}$$

$$= \left(\mathrm{V}_{22}^{0} - \mathrm{V}_{33}^{0}\right) \times \frac{1}{5569536} + \mathrm{V}_{33}^{0} \tag{24.199}$$

$$= \mathrm{V}_{22}^{0} \times \frac{1}{5569536} + \mathrm{V}_{33}^{0} \times \left(1 - \frac{1}{5569536}\right) \tag{24.200}$$

$$= \mathrm{V}_{22}^{0} \times \frac{1}{5569536} + \mathrm{V}_{33}^{0} \times \frac{5569535}{5569536} \tag{24.201}$$

$$= \frac{405002507}{403896880} \times \frac{1}{5569536} + \frac{1}{7634700} \times \frac{5569535}{5569536} \tag{24.202}$$

$$= \frac{3142112029261}{10102586296593530880} \tag{24.203}$$

$$\mathrm{P}_{29}^{0} = \frac{10102586296593530880}{3142112029261} = 3215221.5460\ldots \text{ days} \approx 8800 \text{ years} \tag{24.204}$$

The same value is given by Oechslin. This is the period of the revolution of the nodes resulting from Neßtfell's construction, but it is possible that Neßtfell aimed at having the nodes fixed and that his gears merely provide a very long period. The period of the revolution of the nodes of Mercury was actually not well known in the 18th century.

Incidentally, even though Oechslin didn't work out the exact fractions, he still provided the period 3215221.546060 days, which he could only have obtained by taking accurate values of each term. However, the exact evaluation of the above ratio is 3215221.546053…, and this shows that even with computations to many decimal places, it may still not yield an entirely correct result.

24.5.5.3 The viewpoint from the Earth

Finally, there is a guide linking the Earth to Mercury, and showing the backdrop of the sky where Mercury is located. Neßtfell did however not attempt to show the transits of Mercury.

24.5.6 The calendar part

The front panel also has a calendar display, which I am not describing here. It shows the year in an opening and it has a mention about the year 1828 not being a leap year. However, Gutwein's engraving (figure 24.15) mentions the

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year 1826. I don't know if a similar mention appears on the Munich machine.[20]

Neßtfell's description of the clock also mentions that a day should be removed every 126 years [18, p. 54]. After that time, the excess of the Julian year to the tropical year accumulates to about a day, but it is not totally clear how Neßtfell arrived at that value. In any case, on Gutwein's engraving from 1754, this is indicated by the two years 1826 and 1952.

24.6 Conclusion

Neßtfell's clock follows a very original construction, and one can only be in awe of his imagination, and also in admiration of Oechslin's efforts to render the structure of the machine in a plan, which is certainly not easy.

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Chapter 25

(Oechslin: 3.2)

Neßtfell's *Planetenmaschine* in Munich (c1755-1761)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

25.1 Introduction

The astronomical machine described here was constructed around 1755-1761 by Johann Georg Neßtfell (1694-1762).[1] The machine was constructed for Adam Friedrich Graf von Seinsheim (1708-1779) who was the Prince-Bishop of Würzburg from 1755 to 1779 and Prince-Bishop of Bamberg from 1757 to 1779 and Neßtfell was given 4000 gulden for it.[2]

The clock described here was located in the *physikalische Kabinet* of the University of Würzburg. It entered the collections of the *Bayerisches Nationalmuseum* in Munich in 1877 where it is now exhibited (Inv.-Nr. Phys 254).[4]

This machine is very similar in appearance to Neßtfell's machine kept in Vienna and which was constructed in 1753. The Munich machine was badly damaged during WWII and was considered beyond repair, but thanks to the remarkable work of Ludwig Oechslin and others, it could be restored at the

[1] For a biographical notice of Neßtfell, see the description of the Vienna machine.

[2] For a summary of the history of this machine, see Hess [4, p. 73-77] and Oechslin [9, p. 223]. On possible persons who helped Neßtfell in the construction of his machines, see Oechslin [9, p. 219-220]. One of them was the mechanician Johann Georg Fellwöck (sometimes spelled Felbäch or Felbeck) (1728-1810).[3] The best sources for Neßtfell's machines are Hess [4] and Henck [3]. Besides Oechslin's work [9, p. 33, 36-37, 53-55, 199-203], see also King [5, p. 229-232] and Seelig's articles [13, 14, 17, 15, 16].

[4] Cf. [1, p. 19, 23] and [16, p. 441]. Röntgen confirms the year 1877 [11, p. 12], but in 1882 Schlereth still locates the machine in Würzburg [12, p. 44]. Seidl shows the location of Neßtfell's machine in Munich in 1902 [18, pl .XLVI]. This machine was exhibited at the 1989 Hahn exhibition [19, p. 60-61]. See also [9, p. 234].

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end of the 1980s.[5] This was to a great extent possible because of the existence of the Vienna machine.

The most obvious difference between the two machines is that the base is decorated of simple columns in Munich, but of griffins in Vienna. There are also a number of sculpted figures in the Munich machine.[6] However, there are also some differences in the gears. The descriptions of the two machines are very similar and overlap to a large extent, but that way they can be read in any order.

It should moreover be observed that Frater Fridericus's machine in Bamberg, made with the help of Johann Georg Fellwöck in 1772 (Oechslin 4.1), is really a simplified version of Neßtfell's machines. It has the same general cylindrical appearance, there is no orrery, and the entire front Earth-Mercury system is laid out horizontally.

Both of Neßtfell's machines are about 2 meters tall, they have a diameter of about 1 meter, and have a prismatic glass case containing the planetary system. Due to this construction, almost all the gears can be seen. In the front of the machine, there is a large dial representing the Earth-Mercury system.

These two machines are both very daunting, especially because it is rather difficult to understand them at first sight. I am going to split the machine in three parts: the going work, the orrery and the Earth-Mercury system.

[5] On the restoration of the machine for the 1988-1989 exhibitions, see [13, 10, 8, 17].

[6] On these figures by Johann Peter Wagner (1730-1809), see Seelig [14, 15].

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Figure 25.1: The location of Neßtfell's machine in 1902. (source: [18, pl .XLVI])

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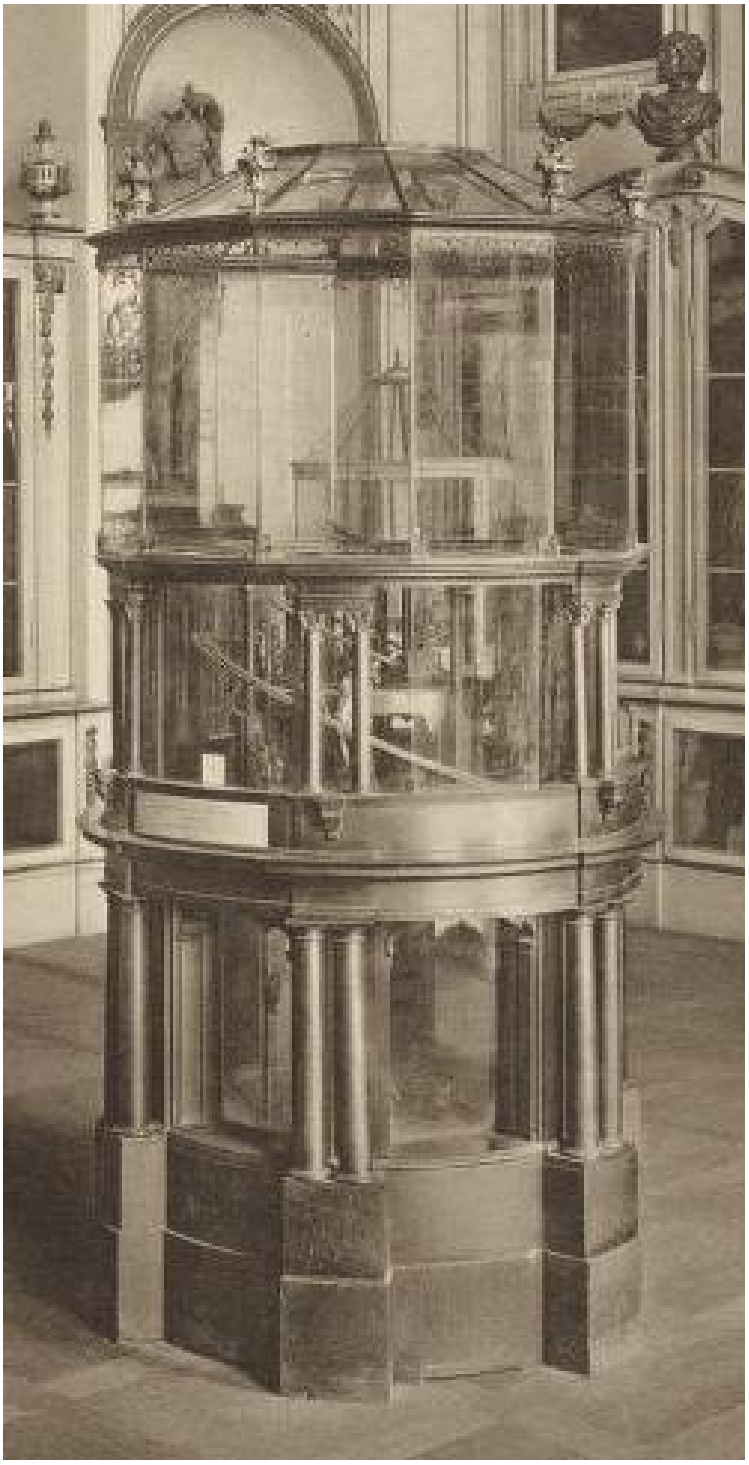


Figure 25.2: Neßtfell's machine in 1902 (detail). (source: [18, pl .XLVI])

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Figure 25.3: Neßtfell's machine as it appears today. (photograph by the author)

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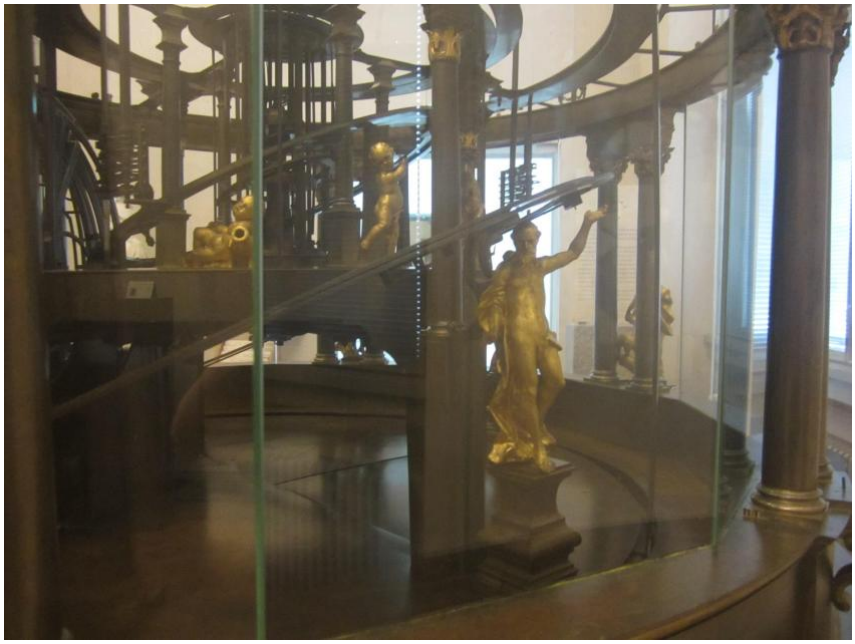


Figure 25.4: Detail of some of the figures inside Neßtfell's machine. (photograph by the author)

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Figure 25.5: The gears from the front dial of Neßtfell's machine seen from the side. (photograph by the author)

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Figure 25.6: The top of the case of Neßtfell's machine. (photograph by the author)

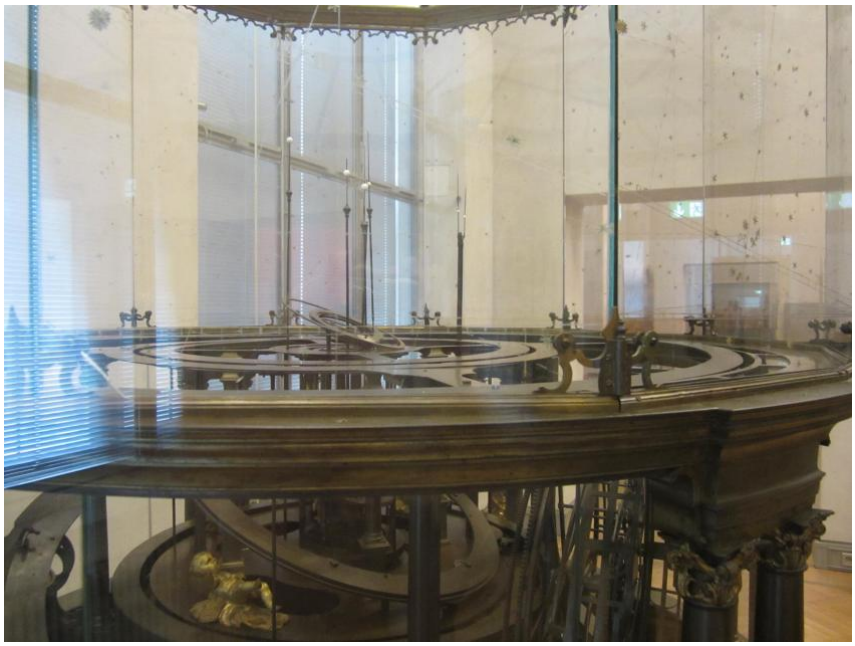


Figure 25.7: The upper level of Neßtfell's machine, which all the planets moving in a tilted plane. (photograph by the author)

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25.2 The going work

The going work is set behind the Earth-Mercury system. It is weight-driven and regulated by a pendulum. The going work contains two winding drums, that add up their strengths for a unique output on arbor 9 which makes one turn clockwise (seen from the front) in 24 hours. We have

$$\mathrm{T}_9^0 = 1 \tag{25.1}$$

We can therefore deduce that the first drum (arbor 2) must have the velocity

$$\mathrm{T}_2^0 = \mathrm{T}_9^0 \times \left(-\frac{60}{15}\right) \times \left(-\frac{15}{90}\right) = \mathrm{T}_9^0 \times \frac{2}{3} = \frac{2}{3} \tag{25.2}$$

$$\mathrm{P}_2^0 = 1.5 \text{ days} \tag{25.3}$$

The second drum (arbor 8) has the same velocity:

$$\mathrm{T}_8^0 = \mathrm{T}_2^0 \times \left(-\frac{90}{15}\right) \times \left(-\frac{15}{90}\right) = \mathrm{T}_2^0 \tag{25.4}$$

$$\mathrm{P}_8^0 = 1.5 \text{ days} \tag{25.5}$$

Each drum makes a turn in 36 hours and they turn in the same directions. In the Vienna machine, the two drums make a turn in two days.

These two drums can be rewound using the winding arbors 1 (for the first drum) and 7 (for the second drum).

A gear train connects the drums to the escape wheel on arbor 6. We have

$$\mathrm{T}_6^0 = \mathrm{T}_2^0 \times \left(-\frac{90}{15}\right) \times \left(-\frac{60}{10}\right) \times \left(-\frac{60}{8}\right) \times \left(-\frac{56}{7}\right) \tag{25.6}$$

$$= \mathrm{T}_2^0 \times 2160 = 1440 \tag{25.7}$$

$$\mathrm{P}_6^0 = \frac{1}{1440} \text{ days} = 60 \text{ seconds} \tag{25.8}$$

The escape wheel having 30 teeth, we deduce that the pendulum should make a half swing in $\frac{60}{2\times 30} = 1$ seconds. Its length should be about 1 m.

The only output of the going work is arbor 9 which makes one turn clockwise in one day. This arbor is prolonged by arbor 11 through a coupling. These two arbors can be desynchronized using a shiftable 70-teeth wheel on an arbor also named 11 by Oechslin, but which I will call 11′. The arbor 11 immediately drives the Earth-Mercury system in the front. On arbor 11, there is also a 30-teeth wheel which drives a train towards the orrery. For now, I will merely assume that the going work directly drives the orrery.

The motion of arbor 11 is also transferred to arbor 46 which drives Saturn.

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25.3 The Ptolemaic allegories

At the bottom of the machine, there are several allegories of the planets rotating around the Earth. This is a very simplified representation of the Ptolemaic system. The Earth is located is the middle and pictured as a wooden sculpture by Johann Peter Wagner (1730-1809).[7] It is surrounded by the Sun, Mars, Jupiter and Saturn. The planets Mercury and Venus do not seem to be there, and are perhaps missing.

The allegories move on a carrousel, all at the same speed. This motion is obtained from arbor 11′ whose motion is transferred to the arbor 140, then to the arbor 141. All these arbors make one turn in 24 hours. The arbor 141 carries a 180-teeth wheel which meshes wish a similar wheel on tube 142. This apparently causes the motion of the allegories on arbor 145. This arbor has the following velocity:

$$\mathrm{V}^0_{142} = 1 \tag{25.9}$$

$$\mathrm{V}^0_{145} = \mathrm{V}^0_{142} \times \frac{56}{14} \times \left(-\frac{14}{28}\right) \times \left(-\frac{14}{29}\right) = \frac{28}{29} \tag{25.10}$$

$$\mathrm{P}^0_{145} = \frac{29}{28} = 24.8571\ldots \text{ hours} \tag{25.11}$$

which may be an approximation of the lunar day.

25.4 The orrery

The most unusual feature of this machine is that the planets do not move in a horizontal plane, or near a horizontal plane. In almost every orrery, the planets move either in or near a horizontal plane, or in or near a vertical plane. In Neßtfell's machine, the ecliptic is tilted by an angle of 23.5° and the planets move in or near that tilted plane.[8] Consequently, the plane of the equator is a horizontal plane, although it doesn't have any particular significance here. For instance, no horizontal plane is materialized through the Earth.

Moreover, the planets only move indirectly in the ecliptic. In fact, there are horizontal base motions, and the planets are on vertical arms that glide along tilted guides. This all gives a very unusual appearance to this machine.

As mentioned above, the input to the orrery is a gear train starting with the horizontal arbor 11 making one turn clockwise in a day. The motion of this arbor is transferred to arbor 46 which is used to drive the gears for Saturn. Then, the motion of arbor 11 is transferred to arbor 71 which drives the other planets:

$$\mathrm{V}^0_{71} = \mathrm{V}^0_{11} \times \left(-\frac{30}{78}\right) \times \left(-\frac{78}{30}\right) = \mathrm{V}^0_{11} = -1 \tag{25.12}$$

[7] For more on these allegories and Johann Peter Wagner, see Seelig [14, 15].

[8] On this construction, see in particular [9, p. 146-147].

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Arbor 71 also makes one turn clockwise in a day.

All the planets are driven similarly, sitting on carriages which are stuck between moving wheels and wheels that are not moving. Neßtfell used large wheels which are about the size of the orbits, and Oechslin assumes that he took his inspiration from a British orrery by John Rowley he had been working on and which had been bought in 1723 in England by Prince Eugene Francis of Savoy-Carignano (1663-1736) [21, p. 42]. This orrery may have been similar to the one by Adams described in this book [9, p. 154, 210].

I am first considering the mean motions of the planets, then the rotations of the planets and the satellites of Jupiter and the Moon. Finally, I am considering the motion of the Moon.

25.4.1 The mean motions and rotations of the planets

I am starting with Saturn and going backwards to Mercury. These systems are in fact all independent and could be described in any order, but I believe it is easier to look at them from the front to the back, and also from the outside to the inside. The descriptions below are very similar, because the structures are similar. However, I prefer to repeat the descriptions, so that the motions can be studied in any order.

All the planets are given a motion of rotation around their axis, but I will only analyze the rotation of Mercury, Venus, the Earth and Mars after each analysis of the mean motions. The rotations of Jupiter and Saturn will be examined when I describe their two satellite systems.

As mentioned above, Neßtfell first produces horizontal motions, and tilted slopes ensure that these motions are transferred near the ecliptic. I assume that each slope is adapted to its planet and that different planets therefore have different slopes. It is also possible that some of the orbits have been made eccentric, but this should be checked. In addition, one should keep in mind that the transfer from the horizontal plane to the tilted plane is a projection and that it does alter the motion.[9] But Neßtfell did not implement irregular motions, other than possibly those resulting from offset orbits.

The mean motions of the planets are obtained through epicyclical gear trains. In each case, there is an input motion and the rotation of a set of gears on a mobile carriage, this rotation being determined by a constraint on another wheel, in this case a fixed wheel. These constructions often lead to periods whose rational expression contains large primes:

- for Mercury, the period involves the prime number 317;
- for Venus, the period involves the prime number 2411;
- for the Earth, the period involves the prime number 296983;
- for Mars, the period involves the prime numbers 101 and 173;

[9] This was also observed by Oechslin, see [9, p. 201-202].

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- for Jupiter, the period involves the prime numbers 131 and 389;
- for Saturn, the period involves the prime number 51817.

Neßtfell's construction makes use of gears whose teeth numbers contain only small primes, the largest being 109. It is therefore tempting to view Neßtfell's construction as a way to avoid the use of large primes in gear trains.

However, I do not believe that Neßtfell chose to reach the exact ratios he finally produced. His objectif must have been much more to have the planets on moving carriages, and the complex ratios that these constructions entailed were rather side effects. This is therefore entirely different from Frater David's methods which aimed at obtaining specific ratios without resorting to too large primes. I am giving some details of Frater David's methods in the introduction of this book.

Neßtfell also gives the revolution periods of the planets in his description of the clock [7],[10] and the periods implemented on the clock are only approximations of these values:

Mercury	87d	23h	14m	40s		87.9685...
Venus	224d	17h	44m	11/12		224.7395...
Earth	365d	5h	48m	58s	1/4	365.2423...
Mars	686d	23h	31m	57s		686.9805...
Jupiter	4332d	14h	49m	31s	56‴	4332.6177...
Saturn	10759d	5h				10759.2083...

Neßtfell does not give the actual periods on his machine.

25.4.1.1 The motion of Saturn

On arbor 46, there is a 25-teeth wheel which meshes with a 897-teeth contrate wheel on frame 47. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{47}^0 = \mathrm{V}_{46}^0 \times \left(-\frac{25}{897}\right) = \frac{25}{897} \tag{25.13}$$

$$\mathrm{P}_{47}^0 = \frac{897}{25} = 35.88 \text{ days} \tag{25.14}$$

Above the 897-teeth wheel, there is a 900-teeth wheel with interior gearing which meshes with a 25-teeth wheel which is part of a carriage 54 of which one pinion meshes with a fixed 690-teeth wheel. This causes the entire carriage to move, and the carriage supports Saturn and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 48 on which the Saturn system moves up or down, as I will show later. The problem is to find the velocity of the carriage 54. I don't want to resort to some memorized formula. We need to be able to compute the motions without any cryptic

[10]The number of days for Jupiter is mistakenly given as 4323, an obvious typo.

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magic. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 900-teeth wheel (frame 47) from that of the 690-teeth wheel (frame 53, fixed). We have

$$\mathrm{V}_{47}^{54} = \mathrm{V}_{53}^{54} \times \left(-\frac{690}{25}\right) \times \left(-\frac{25}{8}\right) \times \left(-\frac{109}{9}\right) \times \left(-\frac{103}{10}\right) \times \left(-\frac{25}{25}\right) \times \frac{25}{900} \tag{25.15}$$

$$= \mathrm{V}_{53}^{54} \times \left(-\frac{258221}{864}\right) = \mathrm{V}_{54}^{53} \times \frac{258221}{864} \tag{25.16}$$

It is important to note that the last ratio $\frac{25}{900}$ is positive, because the motion of arbor 48 is in the same direction as the frame 47, the 900-teeth wheel having an interior gearing.

Now, we have

$$\mathrm{V}_{47}^{54} = \mathrm{V}_{47}^{0} + \mathrm{V}_{0}^{54} = \mathrm{V}_{54}^{53} \times \frac{258221}{864} = \mathrm{V}_{54}^{0} \times \frac{258221}{864} \tag{25.17}$$

Therefore

$$\mathrm{V}_{47}^{0} = \mathrm{V}_{54}^{0} \times \frac{258221}{864} + \mathrm{V}_{54}^{0} = \mathrm{V}_{54}^{0} \times \left(\frac{258221}{864} + 1\right) = \mathrm{V}_{54}^{0} \times \frac{259085}{864} \tag{25.18}$$

And

$$\mathrm{V}_{54}^{0} = \mathrm{V}_{47}^{0} \times \frac{864}{259085} = \frac{25}{897} \times \frac{864}{259085} = \frac{1440}{15493283} \tag{25.19}$$

$$\mathrm{P}_{54}^{0} = \frac{15493283}{1440} = 10759.2243\ldots \text{ days} \tag{25.20}$$

This is an approximation of the sidereal orbital period of Saturn. The same value is given by Oechslin.

The motion of Saturn around its axis is described below in the section of Saturn's satellites.

25.4.1.2 The motion of Jupiter

The mean motion of Jupiter is obtained like that of Saturn.

On arbor 71, there is a 30-teeth wheel which meshes with a 360-teeth contrate wheel on frame 72. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{72}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{30}{360}\right) = \frac{30}{360} = \frac{1}{12} \tag{25.21}$$

$$\mathrm{P}_{72}^{0} = 12 \text{ days} \tag{25.22}$$

Above the 360-teeth wheel, there is a 361-teeth wheel which meshes with a 30-teeth wheel which is part of a carriage 77 of which one pinion meshes

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with a fixed 306-teeth wheel. This causes the entire carriage to move, and the carriage supports Jupiter and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 73 on which the Jupiter system moves up or down, as I will show later. The problem is to find the velocity of the carriage 77. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 361-teeth wheel (frame 72) from that of the 306-teeth wheel (frame 76, fixed). We have

$$\mathrm{V}_{72}^{77} = \mathrm{V}_{76}^{77} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \times \frac{30}{361} \tag{25.23}$$

$$= \mathrm{V}_{76}^{77} \times \left(-\frac{355725}{988}\right) = \mathrm{V}_{77}^{76} \times \frac{355725}{988} \tag{25.24}$$

The last ratio $\frac{30}{361}$ is positive for the same reason as in the case of Saturn. Now, we have

$$\mathrm{V}_{72}^{77} = \mathrm{V}_{72}^{0} + \mathrm{V}_{0}^{77} = \mathrm{V}_{77}^{76} \times \frac{355725}{988} = \mathrm{V}_{77}^{0} \times \frac{355725}{988} \tag{25.25}$$

Therefore

$$\mathrm{V}_{72}^{0} = \mathrm{V}_{77}^{0} \times \frac{355725}{988} + \mathrm{V}_{77}^{0} = \mathrm{V}_{77}^{0} \times \left(\frac{355725}{988} + 1\right) = \mathrm{V}_{77}^{0} \times \frac{356713}{988} \tag{25.26}$$

And

$$\mathrm{V}_{77}^{0} = \mathrm{V}_{72}^{0} \times \frac{988}{356713} = \frac{1}{12} \times \frac{988}{356713} = \frac{247}{1070139} \tag{25.27}$$

$$\mathrm{P}_{77}^{0} = \frac{1070139}{247} = 4332.5465\ldots \text{ days} \tag{25.28}$$

This is an approximation of the sidereal orbital period of Jupiter. The same value is given by Oechslin.

The motion of Jupiter around its axis is described below in the section of Jupiter's satellites.

25.4.1.3 The motion of Mars

The mean motion of Mars is obtained like those of Jupiter and Saturn.

On arbor 71, there is a 30-teeth wheel which meshes with a 202-teeth contrate wheel on frame 89. (In Vienna, we have the equivalent ratio 15/101.) This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{89}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{30}{202}\right) = \frac{15}{101} \tag{25.29}$$

$$\mathrm{P}_{89}^{0} = \frac{101}{15} = 6.7333\ldots \text{ days} \tag{25.30}$$

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Above the 202-teeth wheel, there is a 204-teeth wheel which meshes with a 30-teeth wheel (102/15 in Vienna) which is part of a carriage 94 of which one pinion meshes with a fixed 194-teeth wheel (97 in Vienna). This causes the entire carriage to move, and the carriage supports Mars. All these motions take place in a horizontal plane, but the carriage itself has an arbor 97 on which Mars moves up or down, as I will show later. The problem is to find the velocity of the carriage 94. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 204-teeth wheel (frame 89) from that of the 194-teeth wheel (frame 93, fixed). We have

$$\mathrm{V}_{89}^{94} = \mathrm{V}_{93}^{94} \times \left(-\frac{194}{8}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \frac{30}{204} \tag{25.31}$$

$$= \mathrm{V}_{93}^{94} \times \left(-\frac{879305}{8704}\right) = \mathrm{V}_{94}^{93} \times \frac{879305}{8704} \tag{25.32}$$

The last ratio $\frac{30}{204}$ is positive for the same reason as in the cases of Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{89}^{94} = \mathrm{V}_{89}^{0} + \mathrm{V}_{0}^{94} = \mathrm{V}_{94}^{93} \times \frac{879305}{8704} = \mathrm{V}_{94}^{0} \times \frac{879305}{8704} \tag{25.33}$$

Therefore

$$\mathrm{V}_{89}^{0} = \mathrm{V}_{94}^{0} \times \frac{879305}{8704} + \mathrm{V}_{94}^{0} = \mathrm{V}_{94}^{0} \times \left(\frac{879305}{8704} + 1\right) = \mathrm{V}_{94}^{0} \times \frac{888009}{8704} \tag{25.34}$$

And

$$\mathrm{V}_{94}^{0} = \mathrm{V}_{89}^{0} \times \frac{8704}{888009} = \frac{15}{101} \times \frac{8704}{888009} = \frac{43520}{29896303} \tag{25.35}$$

$$\mathrm{P}_{94}^{0} = \frac{29896303}{43520} = 686.9554\ldots \text{ days} \tag{25.36}$$

This is an approximation of the sidereal orbital period of Mars. The same value is given by Oechslin.

Mars also rotates around its axis. In the (synodic) reference frame 94, we have

$$\mathrm{V}_{97}^{94} = \mathrm{V}_{93}^{94} \times \left(-\frac{194}{8}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \left(-\frac{13}{21}\right) \times \left(-\frac{22}{14}\right) \tag{25.37}$$

$$= \mathrm{V}_{93}^{94} \times \left(-\frac{513227}{768}\right) = \mathrm{V}_{94}^{0} \times \frac{513227}{768} \tag{25.38}$$

$$= \frac{43520}{29896303} \times \frac{513227}{768} = \frac{87248590}{89688909} \tag{25.39}$$

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And in the absolute frame:

$$\mathrm{V}_{97}^{0} = \mathrm{V}_{97}^{94} + \mathrm{V}_{94}^{0} = \mathrm{V}_{94}^{0} \times \left(\frac{513227}{768} + 1 \right) = \mathrm{V}_{94}^{0} \times \frac{513995}{768} \tag{25.40}$$

$$= \frac{43520}{29896303} \times \frac{513995}{768} = \frac{87379150}{89688909} \tag{25.41}$$

$$\mathrm{P}_{97}^{0} = \frac{89688909}{87379150} = 1.0264\ldots \text{ days} = 24 \text{ h } 38 \text{ mn } 3.87\ldots \text{ s} \tag{25.42}$$

The same value is given by Oechslin. This period is close to the actual sidereal rotation of Mars.

25.4.1.4 The motion of the Earth

The mean motion of the Earth is obtained like those of Mars, Jupiter and Saturn.

On arbor 71, there is a 13-teeth wheel which meshes with a 170-teeth contrate wheel on frame 98. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{98}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{13}{170} \right) = \frac{13}{170} \tag{25.43}$$

$$\mathrm{P}_{98}^{0} = \frac{170}{13} = 13.0769\ldots \text{ days} \tag{25.44}$$

Above the 170-teeth wheel, there is a 217-teeth wheel which meshes with a 16-teeth wheel which is part of a carriage 103 of which one pinion meshes with a fixed 83-teeth wheel. This causes the entire carriage to move, and the carriage supports the Earth. All these motions take place in a horizontal plane, but the carriage itself has an arbor 107 on which the Earth moves up or down, as I will show later. The problem is to find the velocity of the carriage 103. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 217-teeth wheel (frame 98) from that of the 83-teeth wheel (frame 102, fixed). We have

$$\mathrm{V}_{98}^{103} = \mathrm{V}_{102}^{103} \times \left(-\frac{83}{8} \right) \times \left(-\frac{69}{7} \right) \times \left(-\frac{50}{14} \right) \times \frac{16}{217} \tag{25.45}$$

$$= \mathrm{V}_{102}^{103} \times \left(-\frac{286350}{10633} \right) = \mathrm{V}_{103}^{102} \times \frac{286350}{10633} \tag{25.46}$$

The last ratio $\frac{16}{217}$ is positive for the same reason as in the cases of Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{98}^{103} = \mathrm{V}_{98}^{0} + \mathrm{V}_{0}^{103} = \mathrm{V}_{103}^{102} \times \frac{286350}{10633} = \mathrm{V}_{103}^{0} \times \frac{286350}{10633} \tag{25.47}$$

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Therefore

$$\mathrm{V}^0_{98} = \mathrm{V}^0_{103} \times \frac{286350}{10633} + \mathrm{V}^0_{103} = \mathrm{V}^0_{103} \times \left(\frac{286350}{10633} + 1\right) = \mathrm{V}^0_{103} \times \frac{296983}{10633} \tag{25.48}$$

And

$$\mathrm{V}^0_{103} = \mathrm{V}^0_{98} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{25.49}$$

$$\mathrm{P}^0_{103} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{25.50}$$

This is an approximation of the sidereal year. The same value is given by Oechslin.

The Earth also rotates around its axis. In the (synodic) reference frame 103, we have

$$\mathrm{V}^{103}_{107} = \mathrm{V}^{103}_{102} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \tag{25.51}$$

$$\times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{30}{30}\right) \tag{25.52}$$

$$= \mathrm{V}^{103}_{102} \times \left(-\frac{143175}{392}\right) = \mathrm{V}^0_{103} \times \frac{143175}{392} \tag{25.53}$$

$$= \frac{138229}{50487110} \times \frac{143175}{392} = \frac{80779335}{80779376} \tag{25.54}$$

$$\mathrm{P}^{103}_{107} = \frac{80779376}{80779335} = 1.0000005\ldots \text{ days} = 86400.0438\ldots \text{ seconds} \tag{25.55}$$

The Earth practically makes one turn around its axis in 24 hours with respect to the Sun. It should do so in exactly 24 hours, but it doesn't. In fact, Oechslin writes that the Earth makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}^0_{107} = \mathrm{V}^{103}_{107} + \mathrm{V}^0_{103} = \mathrm{V}^0_{103} \times \left(\frac{143175}{392} + 1\right) = \mathrm{V}^0_{103} \times \frac{143567}{392} \tag{25.56}$$

$$= \frac{138229}{50487110} \times \frac{143567}{392} = \frac{405002507}{403896880} \tag{25.57}$$

$$\mathrm{P}^0_{107} = \frac{403896880}{405002507} = 23 \text{ h } 56 \text{ mn } 4.1343\ldots \text{ s} \tag{25.58}$$

This period is close to the actual sidereal rotation of the Earth.

25.4.1.5 The motion of Venus

The mean motion of Venus is obtained like those of the Earth, Mars, Jupiter and Saturn.

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On arbor 71, there is a 20-teeth wheel which meshes with a 96-teeth contrate wheel on frame 122. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}^0_{122} = \mathrm{V}^0_{71} \times \left(-\frac{20}{96}\right) = \frac{5}{24} \tag{25.59}$$

$$\mathrm{P}^0_{122} = \frac{24}{5} = 4.8 \text{ days} \tag{25.60}$$

Above the 96-teeth wheel, there is a 103-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 127 of which one pinion meshes with a fixed 65-teeth wheel. This causes the entire carriage to move, and the carriage supports Venus. All these motions take place in a horizontal plane, but the carriage itself has an arbor 130 on which Venus moves up or down, as I will show later. The problem is to find the velocity of the carriage 127. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 103-teeth wheel (frame 122) from that of the 65-teeth wheel (frame 126, fixed). We have

$$\mathrm{V}^{127}_{122} = \mathrm{V}^{127}_{126} \times \left(-\frac{65}{8}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{44}{10}\right) \times \frac{21}{103} \tag{25.61}$$

$$= \mathrm{V}^{127}_{126} \times \left(-\frac{4719}{103}\right) = \mathrm{V}^{126}_{127} \times \frac{4719}{103} \tag{25.62}$$

The last ratio $\frac{21}{103}$ is positive for the same reason as in the cases of the Earth, Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}^{127}_{122} = \mathrm{V}^0_{122} + \mathrm{V}^{127}_0 = \mathrm{V}^{126}_{127} \times \frac{4719}{103} = \mathrm{V}^0_{127} \times \frac{4719}{103} \tag{25.63}$$

Therefore

$$\mathrm{V}^0_{122} = \mathrm{V}^0_{127} \times \frac{4719}{103} + \mathrm{V}^0_{127} = \mathrm{V}^0_{127} \times \left(\frac{4719}{103} + 1\right) = \mathrm{V}^0_{127} \times \frac{4822}{103} \tag{25.64}$$

And

$$\mathrm{V}^0_{127} = \mathrm{V}^0_{122} \times \frac{103}{4822} = \frac{5}{24} \times \frac{103}{4822} = \frac{515}{115728} \tag{25.65}$$

$$\mathrm{P}^0_{127} = \frac{115728}{515} = 224.7145\ldots \text{ days} \tag{25.66}$$

This is an approximation of the sidereal orbital period of Venus. The same value is given by Oechslin.

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Venus also rotates around its axis. In the (synodic) reference frame 127, we have

$$\mathrm{V}_{130}^{127} = \mathrm{V}_{126}^{127} \times \left(-\frac{65}{8}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{44}{10}\right) \times \left(-\frac{20}{25}\right) \times \left(-\frac{25}{20}\right) \tag{25.67}$$

$$= \mathrm{V}_{126}^{127} \times \left(-\frac{1573}{7}\right) = \mathrm{V}_{127}^{0} \times \frac{1573}{7} \tag{25.68}$$

$$= \frac{515}{115728} \times \frac{1573}{7} = \frac{810095}{810096} \tag{25.69}$$

$$\mathrm{P}_{130}^{127} = \frac{810096}{810095} = 1.000001\ldots \text{ days} = 86400.1066\ldots \text{ seconds} \tag{25.70}$$

It would seem that Neßtfell attempted here too to have Venus make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. Like in the case of the Earth, Oechslin writes that Venus makes one turn with respect to the Sun in one day, but this is not true.

Incidentally, we can see that Nessfell uses a ratio $\frac{n+1}{n}$, and such ratios appear in several places, although there does not seem to be any good reason for it.

Now, in the absolute frame, we have:

$$\mathrm{V}_{130}^{0} = \mathrm{V}_{130}^{127} + \mathrm{V}_{127}^{0} = \mathrm{V}_{127}^{0} \times \left(\frac{1573}{7} + 1\right) = \mathrm{V}_{127}^{0} \times \frac{1580}{7} \tag{25.71}$$

$$= \frac{515}{115728} \times \frac{1580}{7} = \frac{203425}{202524} \tag{25.72}$$

$$\mathrm{P}_{130}^{0} = \frac{202524}{203425} = 23 \text{ h } 53 \text{ mn } 37.3213\ldots \text{ s} \tag{25.73}$$

The same value is given by Oechslin.

This period is very inaccurate, because Venus rotates much more slowly. It makes one turn in 243 days. But in the 18th century, Venus's motion around its axis was still a mystery, and some thought it was rotating in 23 hours, others in 25 days. The same problem occurs on the Vienna orrery.

25.4.1.6 The motion of Mercury

The mean motion of Mercury is obtained like those of Venus, the Earth, Mars, Jupiter and Saturn.

On arbor 71, there is a 21-teeth wheel which meshes with a 84-teeth contrate wheel on frame 131. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}_{131}^{0} = \mathrm{V}_{71}^{0} \times \left(-\frac{21}{84}\right) = \frac{1}{4} \tag{25.74}$$

$$\mathrm{P}_{131}^{0} = 4 \text{ days} \tag{25.75}$$

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Above the 84-teeth wheel, there is a 88-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 136 of which one pinion meshes with a fixed 60-teeth wheel. This causes the entire carriage to move, and the carriage supports Mercury. All these motions take place in a horizontal plane, but the carriage itself has an arbor 138 on which Mercury moves up or down, as I will show later. The problem is to find the velocity of the carriage 136. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 88-teeth wheel (frame 131) from that of the 60-teeth wheel (frame 135, fixed). We have

$$\mathrm{V}_{131}^{136} = \mathrm{V}_{135}^{136} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{7}\right) \times \left(-\frac{30}{19}\right) \times \frac{21}{88} \tag{25.76}$$

$$= \mathrm{V}_{135}^{136} \times \left(-\frac{8775}{418}\right) = \mathrm{V}_{136}^{135} \times \frac{8775}{418} \tag{25.77}$$

The last ratio $\frac{21}{88}$ is positive for the same reason as in the cases of Venus, the Earth, Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{131}^{136} = \mathrm{V}_{131}^{0} + \mathrm{V}_{0}^{136} = \mathrm{V}_{136}^{135} \times \frac{8775}{418} = \mathrm{V}_{136}^{0} \times \frac{8775}{418} \tag{25.78}$$

Therefore

$$\mathrm{V}_{131}^{0} = \mathrm{V}_{136}^{0} \times \frac{8775}{418} + \mathrm{V}_{136}^{0} = \mathrm{V}_{136}^{0} \times \left(\frac{8775}{418} + 1\right) = \mathrm{V}_{136}^{0} \times \frac{9193}{418} \tag{25.79}$$

And

$$\mathrm{V}_{136}^{0} = \mathrm{V}_{131}^{0} \times \frac{418}{9193} = \frac{1}{4} \times \frac{418}{9193} = \frac{209}{18386} \tag{25.80}$$

$$\mathrm{P}_{136}^{0} = \frac{18386}{209} = 87.9712\ldots \text{ days} \tag{25.81}$$

This is an approximation of the sidereal orbital period of Mercury. The same value is given by Oechslin.

Mercury also rotates around its axis. In the (synodic) reference frame 136, we have

$$\mathrm{V}_{138}^{136} = \mathrm{V}_{135}^{136} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{7}\right) \times \left(-\frac{30}{19}\right) \times \left(-\frac{20}{12}\right) \times \left(-\frac{12}{20}\right) \tag{25.82}$$

$$= \mathrm{V}_{135}^{136} \times \left(-\frac{11700}{133}\right) = \mathrm{V}_{136}^{0} \times \frac{11700}{133} \tag{25.83}$$

$$= \frac{209}{18386} \times \frac{11700}{133} = \frac{64350}{64351} \tag{25.84}$$

$$\mathrm{P}_{138}^{136} = \frac{64351}{64350} = 1.00001\ldots \text{ days} = 86401.3426\ldots \text{ seconds} \tag{25.85}$$

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Again, it seems that Neßtfell attempted here too to have Mercury make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. And like in the case of the Earth and Venus, Oechslin writes that Mercury makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}^0_{138} = \mathrm{V}^{136}_{138} + \mathrm{V}^0_{136} = \mathrm{V}^0_{136} \times \left(\frac{11700}{133} + 1\right) = \mathrm{V}^0_{136} \times \frac{11833}{133} \tag{25.86}$$

$$= \frac{209}{18386} \times \frac{11833}{133} = \frac{130163}{128702} \tag{25.87}$$

$$\mathrm{P}^0_{138} = \frac{128702}{130163} = 23 \text{ h } 43 \text{ mn } 50.2128\ldots \text{ s} \tag{25.88}$$

Oechslin gives the same value, although there is a typo in his numerical evaluation.

This period is of course very inaccurate, because Mercury rotates much more slowly. It makes one turn in about 59 days. But in the 18th century, Mercury's motion around its axis was still a mystery. The same problem occurs in the Vienna machine.

25.4.2 The motion of the Sun

The motion of the Sun on tube 139 is derived from that of Mercury:

$$\mathrm{V}^{136}_{139} = \mathrm{V}^{136}_{135} \times \left(-\frac{60}{6}\right) \times \left(-\frac{20}{45}\right) \tag{25.89}$$

$$= \mathrm{V}^{136}_{135} \times \frac{40}{9} = \mathrm{V}^{135}_{136} \times \left(-\frac{40}{9}\right) = \mathrm{V}^0_{136} \times \left(-\frac{40}{9}\right) \tag{25.90}$$

$$\mathrm{V}^0_{139} = \mathrm{V}^{136}_{139} + \mathrm{V}^0_{136} = \mathrm{V}^0_{136} \times \left(1 - \frac{40}{9}\right) \tag{25.91}$$

$$= \mathrm{V}^0_{136} \times \left(-\frac{31}{9}\right) = \frac{209}{18386} \times \left(-\frac{31}{9}\right) = -\frac{6479}{165474} \tag{25.92}$$

$$\mathrm{P}^0_{139} = -\frac{165474}{6479} = -25.5400\ldots \text{ days} \tag{25.93}$$

The same value is given by Oechslin. This is a good approximation of the period of rotation of the Sun which is about 25 days at the equator, but the direction of rotation is wrong. The rotation should have been counterclockwise (from above), and in fact Neßtfell did it correctly in the Vienna machine.[11]

[11] See also [9, p. 203].

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25.4.3 The satellites of Jupiter and Saturn

25.4.3.1 The system of Jupiter

The gears for the rotation of Jupiter and its satellites are contained in a unnamed frame which is gliding along a tilted slope. I will call this frame J. The gears use exactly the same ratios as in the Vienna machine, although one wheel has a different teeth-count. The frame J therefore does have a fixed orientation with respect to the Sun. The motion of this frame is therefore exactly that of the carriage 77 seen earlier.

$$\mathrm{V}^0_J = \mathrm{V}^0_{77} \tag{25.94}$$

The input to frame J is arbor 73 whose motion is

$$\mathrm{V}^{77}_{73} = \mathrm{V}^{77}_{76} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \tag{25.95}$$

$$= \mathrm{V}^{77}_{76} \times \left(-\frac{450585}{104}\right) = \mathrm{V}^{76}_{77} \times \frac{450585}{104} = \mathrm{V}^0_{77} \times \frac{450585}{104} \tag{25.96}$$

$$= \frac{247}{1070139} \times \frac{450585}{104} = \frac{2853705}{2853704} \tag{25.97}$$

We can then easily compute the velocities of Jupiter and its four satellites in the rotating frame. All these motions are directly obtained from arbor 73, and not one from the other.

$$\mathrm{V}^J_{80} = \mathrm{V}^J_{73} \times \left(-\frac{29}{38}\right) \times \left(-\frac{38}{12}\right) = \mathrm{V}^J_{73} \times \frac{29}{12} \text{ (Jupiter)} \tag{25.98}$$

$$\mathrm{V}^J_{82} = \mathrm{V}^J_{73} \times \left(-\frac{31}{32}\right) \times \left(-\frac{28}{48}\right) = \mathrm{V}^J_{73} \times \frac{217}{384} \text{ (Io)} \tag{25.99}$$

$$\mathrm{V}^J_{84} = \mathrm{V}^J_{73} \times \left(-\frac{26}{42}\right) \times \left(-\frac{15}{33}\right) = \mathrm{V}^J_{73} \times \frac{65}{231} \text{ (Europa)} \tag{25.100}$$

$$\mathrm{V}^J_{86} = \mathrm{V}^J_{73} \times \left(-\frac{20}{43}\right) \times \left(-\frac{12}{40}\right) = \mathrm{V}^J_{73} \times \frac{6}{43} \text{ (Ganymede)} \tag{25.101}$$

$$\mathrm{V}^J_{88} = \mathrm{V}^J_{73} \times \left(-\frac{22}{43}\right) \times \left(-\frac{7}{60}\right) = \mathrm{V}^J_{73} \times \frac{77}{1290} \text{ (Callisto)} \tag{25.102}$$

And in the absolute frame:

$$\mathrm{V}^0_{80} = \mathrm{V}^J_{80} + \mathrm{V}^0_J = \mathrm{V}^J_{73} \times \frac{29}{12} + \mathrm{V}^0_J = \frac{2853705}{2853704} \times \frac{29}{12} + \frac{247}{1070139} \tag{25.103}$$

$$= \frac{82765349}{34244448} \tag{25.104}$$

$$\mathrm{P}^0_{80} = \frac{34244448}{82765349} = 9.9300\ldots \text{ hours} = 9 \text{ h } 55 \text{ mn } 48.2973 \text{ s} \tag{25.105}$$

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This is an approximation of the rotation of Jupiter around its axis which is completed in about 9.8 hours. The same value is given by Oechslin.

$$\mathrm{V}^0_{82} = \mathrm{V}^J_{82} + \mathrm{V}^0_J = \mathrm{V}^J_{73} \times \frac{217}{384} + \mathrm{V}^0_J = \frac{2853705}{2853704} \times \frac{217}{384} + \frac{247}{1070139} \tag{25.106}$$

$$= \frac{619506913}{1095822336} \tag{25.107}$$

$$\mathrm{P}^0_{82} = \frac{1095822336}{619506913} = 1.7688\ldots \text{ days (Io)} \tag{25.108}$$

$$\mathrm{V}^0_{84} = \mathrm{V}^J_{84} + \mathrm{V}^0_J = \mathrm{V}^J_{73} \times \frac{65}{231} + \mathrm{V}^0_J = \frac{2853705}{2853704} \times \frac{65}{231} + \frac{247}{1070139} \tag{25.109}$$

$$= \frac{185642977}{659205624} \tag{25.110}$$

$$\mathrm{P}^0_{84} = \frac{659205624}{185642977} = 3.5509\ldots \text{ days (Europa)} \tag{25.111}$$

$$\mathrm{V}^0_{86} = \mathrm{V}^J_{86} + \mathrm{V}^0_J = \mathrm{V}^J_{73} \times \frac{6}{43} + \mathrm{V}^0_J = \frac{2853705}{2853704} \times \frac{6}{43} + \frac{247}{1070139} \tag{25.112}$$

$$= \frac{25725829}{184063908} \tag{25.113}$$

$$\mathrm{P}^0_{86} = \frac{184063908}{25725829} = 7.1548\ldots \text{ days (Ganymede)} \tag{25.114}$$

$$\mathrm{V}^0_{88} = \mathrm{V}^J_{88} + \mathrm{V}^0_J = \mathrm{V}^J_{73} \times \frac{77}{1290} + \mathrm{V}^0_J = \frac{2853705}{2853704} \times \frac{77}{1290} + \frac{247}{1070139} \tag{25.115}$$

$$= \frac{44116993}{736255632} \tag{25.116}$$

$$\mathrm{P}^0_{88} = \frac{736255632}{44116993} = 16.6887\ldots \text{ days (Callisto)} \tag{25.117}$$

The same values are given by Oechslin. The periods of the satellites are all good approximations of the actual ones.

25.4.3.2 The system of Saturn

The structure of the Saturn system is similar to that of Jupiter, except that it contains two worms for the slow motion of Saturn's fifth satellite, Iapetus. The ratios are the same as in the Vienna machine, except for Titan. There are also a few minor teeth-count differences, but which do not alter the ratios.

The gears for the rotation of Saturn and its satellites are contained in a frame that I call S which is gliding along a tilted slope. This frame therefore does have a fixed orientation with respect to the Sun. The motion of frame S is therefore exactly that of the carriage 54 seen earlier:

$$\mathrm{V}^0_S = \mathrm{V}^0_{54} \tag{25.118}$$

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The input to frame S is arbor 48 whose motion is

$$\mathrm{V}_{48}^{54} = \mathrm{V}_{53}^{54} \times \left(-\frac{690}{25}\right) \times \left(-\frac{25}{8}\right) \times \left(-\frac{109}{9}\right) \times \left(-\frac{103}{10}\right) \times \left(-\frac{25}{25}\right) \tag{25.119}$$

$$= \mathrm{V}_{53}^{54} \times \left(-\frac{258221}{24}\right) = \mathrm{V}_{54}^{53} \times \frac{258221}{24} = \mathrm{V}_{54}^{0} \times \frac{258221}{24} \tag{25.120}$$

$$= \frac{1440}{15493283} \times \frac{258221}{24} = \frac{673620}{673621} \tag{25.121}$$

We can then easily compute the velocities of Saturn and its five satellites in the rotating frame. All these motions are directly obtained from arbor 48, and not one from the other.

$$\mathrm{V}_{57}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{30}{52}\right) \times \left(-\frac{52}{10}\right) = \mathrm{V}_{48}^{S} \times 3 \text{ (Saturn)} \tag{25.122}$$

$$\mathrm{V}_{59}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{24}{19}\right) \times \left(-\frac{26}{62}\right) = \mathrm{V}_{48}^{S} \times \frac{312}{589} \text{ (Tethys)} \tag{25.123}$$

$$\mathrm{V}_{61}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{19}{31}\right) \times \left(-\frac{31}{52}\right) = \mathrm{V}_{48}^{S} \times \frac{19}{52} \text{ (Dione)} \tag{25.124}$$

$$\mathrm{V}_{63}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{47}{30}\right) \times \left(-\frac{12}{86}\right) = \mathrm{V}_{48}^{S} \times \frac{47}{215} \text{ (Rhea)} \tag{25.125}$$

$$\mathrm{V}_{65}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{27}{41}\right) \times \left(-\frac{8}{84}\right) = \mathrm{V}_{48}^{S} \times \frac{18}{287} \text{ (Titan)} \tag{25.126}$$

$$\mathrm{V}_{69}^{S} = \mathrm{V}_{48}^{S} \times \left(-\frac{2}{8}\right) \times \left(-\frac{2}{40}\right) \times \left(-\frac{53}{11}\right) \times \left(-\frac{9}{43}\right) \tag{25.127}$$

$$= \mathrm{V}_{48}^{S} \times \frac{477}{37840} \text{ (Iapetus)} \tag{25.128}$$

And in the absolute frame:

$$\mathrm{V}_{57}^{0} = \mathrm{V}_{57}^{S} + \mathrm{V}_{S}^{0} = \mathrm{V}_{48}^{S} \times 3 + \mathrm{V}_{S}^{0} = \frac{673620}{673621} \times 3 + \frac{1440}{15493283} \tag{25.129}$$

$$= \frac{46481220}{15493283} \tag{25.130}$$

$$\mathrm{P}_{57}^{0} = \frac{15493283}{46481220} = 7.9997\ldots \text{ hours (Saturn)} \tag{25.131}$$

$$= 7 \text{ h } 59 \text{ m } 59.1505\ldots \text{ s} \tag{25.132}$$

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The same value is given by Oechslin. The actual period of rotation of Saturn is about 10.65 hours.

$$\mathrm{V}^0_{59} = \mathrm{V}^S_{59} + \mathrm{V}^0_S = \mathrm{V}^S_{48} \times \frac{312}{589} + \mathrm{V}^0_S = \frac{673620}{673621} \times \frac{312}{589} + \frac{1440}{15493283} \tag{25.133}$$

$$= \frac{4834745280}{9125543687} \tag{25.134}$$

$$\mathrm{P}^0_{59} = \frac{9125543687}{4834745280} = 1.8874\ldots \text{ days (Tethys)} \tag{25.135}$$

$$\mathrm{V}^0_{61} = \mathrm{V}^S_{61} + \mathrm{V}^0_S = \mathrm{V}^S_{48} \times \frac{19}{52} + \mathrm{V}^0_S = \frac{673620}{673621} \times \frac{19}{52} + \frac{1440}{15493283} \tag{25.136}$$

$$= \frac{73611705}{201412679} \tag{25.137}$$

$$\mathrm{P}^0_{61} = \frac{201412679}{73611705} = 2.7361\ldots \text{ days (Dione)} \tag{25.138}$$

$$\mathrm{V}^0_{63} = \mathrm{V}^S_{63} + \mathrm{V}^0_S = \mathrm{V}^S_{48} \times \frac{47}{215} + \mathrm{V}^0_S = \frac{673620}{673621} \times \frac{47}{215} + \frac{1440}{15493283} \tag{25.139}$$

$$= \frac{145698564}{666211169} \tag{25.140}$$

$$\mathrm{P}^0_{63} = \frac{666211169}{145698564} = 4.5725\ldots \text{ days (Rhea)} \tag{25.141}$$

$$\mathrm{V}^0_{65} = \mathrm{V}^S_{65} + \mathrm{V}^0_S = \mathrm{V}^S_{48} \times \frac{18}{287} + \mathrm{V}^0_S = \frac{673620}{673621} \times \frac{18}{287} + \frac{1440}{15493283} \tag{25.142}$$

$$= \frac{279291960}{4446572221} \tag{25.143}$$

$$\mathrm{P}^0_{65} = \frac{4446572221}{279291960} = 15.9208\ldots \text{ days (Titan)} \tag{25.144}$$

$$\mathrm{V}^0_{69} = \mathrm{V}^S_{69} + \mathrm{V}^0_S = \mathrm{V}^S_{48} \times \frac{477}{37840} + \mathrm{V}^0_S \tag{25.145}$$

$$= \frac{673620}{673621} \times \frac{477}{37840} + \frac{1440}{15493283} \tag{25.146}$$

$$= \frac{372238731}{29313291436} \tag{25.147}$$

$$\mathrm{P}^0_{69} = \frac{29313291436}{372238731} = 78.7486\ldots \text{ days (Iapetus)} \tag{25.148}$$

The same values are given by Oechslin. These period are good approximations of the actual sidereal periods of the satellites of Saturn. Only the period of Titan slightly differs from that in the Vienna machine.

25.4.4 The motion of the Moon

25.4.4.1 The mean motion

The main motion of the Moon is that of frame 111 which rotates around the axis of the Earth. Its motion is obtained through arbor 99 which is located on the Earth carriage 103. We can compute the motion of frame 111 in frame

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103:

$$\mathrm{V}_{111}^{103} = \mathrm{V}_{99}^{103} \times \left(-\frac{37}{49}\right) \times \left(-\frac{12}{39}\right) \times \left(-\frac{12}{26}\right) \times \left(-\frac{6}{19}\right) \tag{25.149}$$

$$= \mathrm{V}_{99}^{103} \times \frac{5328}{157339} \tag{25.150}$$

and

$$\mathrm{V}_{99}^{103} = \mathrm{V}_{102}^{103} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) = \mathrm{V}_{102}^{103} \times \left(-\frac{143175}{392}\right) \tag{25.151}$$

and therefore

$$\mathrm{V}_{111}^{103} = \mathrm{V}_{102}^{103} \times \left(-\frac{143175}{392}\right) \times \frac{5328}{157339} = \mathrm{V}_{102}^{103} \times \left(-\frac{95354550}{7709611}\right) \tag{25.152}$$

$$= \mathrm{V}_{103}^{102} \times \frac{95354550}{7709611} = \mathrm{V}_{103}^{0} \times \frac{95354550}{7709611} \tag{25.153}$$

$$= \frac{138229}{50487110} \times \frac{95354550}{7709611} = \frac{295599105}{8729221319} \tag{25.154}$$

$$\mathrm{P}_{111}^{103} = \frac{8729221319}{295599105} = 29.5306\ldots \text{ days} \tag{25.155}$$

The same value is given by Oechslin. This is the motion of the Moon with respect to the Sun, and an excellent approximation of the synodic month. It is however a slightly different ratio than the one used in the Vienna machine.

We can also compute the motion of the Moon in an absolute (sidereal) frame:

$$\mathrm{V}_{111}^{0} = \mathrm{V}_{111}^{103} + \mathrm{V}_{103}^{0} = \frac{295599105}{8729221319} + \frac{138229}{50487110} = \frac{3194988991}{87292213190} \tag{25.156}$$

$$\mathrm{P}_{111}^{0} = \frac{87292213190}{3194988991} = 27.3216\ldots \text{ days} \tag{25.157}$$

The same value is given by Oechslin. This is an approximation of the sidereal month. It is also a slightly different ratio than the one used in the Vienna machine.

25.4.4.2 The nodes and the apsides

Neßtfell's construction for the nodes and apsides in the Munich machine is much simpler than in the Vienna machine. Here, we have a frame 113 which rotates with the Earth. This frame carries a number of gears, two of which mesh with a 138-teeth wheel on a fixed frame 112. One of these wheels drives the train to the nodes tube 117, and the other one the train to the apsides tube 121.

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We can compute the motion of tube 117 with respect to frame 113:

$$\mathrm{V}_{117}^{113} = \mathrm{V}_{112}^{113} \times \left(-\frac{138}{29}\right) \times \left(-\frac{31}{35}\right) \times \left(-\frac{10}{30}\right) \times \left(-\frac{30}{40}\right) \tag{25.158}$$

$$= \mathrm{V}_{112}^{113} \times \frac{2139}{2030} = \mathrm{V}_{113}^{112} \times \left(-\frac{2139}{2030}\right) = \mathrm{V}_{103}^{0} \times \left(-\frac{2139}{2030}\right) \tag{25.159}$$

$$= \frac{138229}{50487110} \times \left(-\frac{2139}{2030}\right) = -\frac{42238833}{14641261900} \tag{25.160}$$

$$\mathrm{P}_{117}^{113} = -\frac{14641261900}{42238833} = -346.6303\ldots \text{ days} \tag{25.161}$$

This is is the period of revolution of the nodes with respect to the Sun, the so-called eclipse year. The same ratio was used in the Vienna machine.

We can then compute the motion of the nodes in the absolute frame:

$$\mathrm{V}_{117}^{0} = \mathrm{V}_{117}^{113} + \mathrm{V}_{113}^{0} = \mathrm{V}_{117}^{113} + \mathrm{V}_{103}^{0} \tag{25.162}$$

$$= -\frac{42238833}{14641261900} + \frac{138229}{50487110} = -\frac{2152423}{14641261900} \tag{25.163}$$

$$\mathrm{P}_{117}^{0} = -\frac{14641261900}{2152423} = -6802.2233\ldots \text{ days} \tag{25.164}$$

which is a good approximation of the actual value of 6798 days. The same value is given by Oechslin. This value is negative, because the lunar nodes are retrograding.

Second, the 138-teeth wheel of frame 112 meshes with a 16-teeth wheel for the revolution of the apsides. This wheel meshes in turn with a 12-teeth wheel, whose arbor carries a 2-threaded worm which meshes with a 11-teeth wheel. And the arbor of this wheel carries another 2-threaded worm which meshes with a 37-teeth wheel which moves the apsides on tube 121.

We can compute the motion of tube 121 with respect to frame 113:

$$\mathrm{V}_{121}^{113} = \mathrm{V}_{112}^{113} \times \left(-\frac{138}{16}\right) \times \left(-\frac{16}{12}\right) \times \frac{2}{11} \times \frac{2}{37} = \mathrm{V}_{112}^{113} \times \frac{46}{407} \tag{25.165}$$

$$= \mathrm{V}_{113}^{112} \times \left(-\frac{46}{407}\right) = \mathrm{V}_{103}^{0} \times \left(-\frac{46}{407}\right) \tag{25.166}$$

$$= \frac{138229}{50487110} \times \left(-\frac{46}{407}\right) = -\frac{3179267}{10274126885} \tag{25.167}$$

$$\mathrm{P}_{121}^{113} = -\frac{10274126885}{3179267} = -3231.6024\ldots \text{ days} \tag{25.168}$$

This is a good approximation of the revolution of the apsides, *but with respect to the absolute frame*. And therefore it is false.[12] Moreover, the value is negative, and it should have been positive, which it is in the Vienna machine.

We should actually have found a period of about 411 days, which is the period of revolution of the apsides with respect to the Sun. It would then

[12] About the error in the motion of the apsides, see also [9, p. 203].

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seem that Neßtfell mixed up his computations, and used a gear train meant for the absolute motion of the apsides in the context of a synodic motion of the apsides. The problem is not a mere typo or one wheel that could be easily fixed.

If we now compute the motion of the apsides in the absolute frame, we obtain

$$\mathrm{V}^0_{121} = \mathrm{V}^{113}_{121} + \mathrm{V}^0_{113} = \mathrm{V}^{113}_{121} + \mathrm{V}^0_{103} \tag{25.169}$$

$$= -\frac{3179267}{10274126885} + \frac{138229}{50487110} = \frac{49900669}{20548253770} \tag{25.170}$$

$$\mathrm{P}^0_{121} = \frac{20548253770}{49900669} = 411.7831\ldots \text{ days} \tag{25.171}$$

but this is the value that should have been obtained with respect to the Sun.

25.5 The Earth-Mercury system

The front part of the machine is the Earth-Mercury system and is focused on displaying the apparent motion of Mercury. It is similar to the Earth-Mercury system in Vienna's machine. Both the Earth and Mercury rotate around the central axis which represents the Sun. Such a main display with Mercury on an astronomical clock is unusual, but the choice of that display is certainly a consequence of the interest for the transits of Mercury in the 18th century. Neßtfell certainly did not choose to display the Earth-Venus system, because transits of Venus are much less frequent than those of Mercury. This representation also has a pedogogical function, it serves to highlight a particular part of the solar system, as stressed by Meier.[13] Finally, it is also a way to illustrate the retrogradations of the planets, as Neßtfell explained himself, and as Oechslin observed [9, p. 36].

The central axis of the panel represents the position of the Sun and the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves around the Sun as I will describe below.

The input to the Earth-Mercury system is arbor 11 which comes from the going work. This arbor makes one turn clockwise in one day:

$$\mathrm{V}^0_{11} = -\mathrm{T}^0_{11} = -1 \tag{25.172}$$

This arbor is also used to derive several other auxiliary motions which will be described later.

25.5.1 The mean orbital motion of the Earth

Arbor 11 carries a 26-teeth wheel which meshes with a 340-teeth wheel on frame 12. The axis of this frame is the central axis of the front part. The

[13] Cf. [6, p. 44-45].

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velocity of frame 12 (seen from the front) is

$$\mathrm{V}_{12}^{0} = \mathrm{V}_{11}^{0} \times \left(-\frac{26}{340}\right) = \frac{13}{170} \tag{25.173}$$

As I note below, it is possible that the distance between the axes of the 26-teeth wheel and the 340-teeth wheel varies, because of the motion of the line of apsides.

Frame 12 also carries an interior wheel of 434 teeth, which meshes with a 32-teeth wheel which is part of a train located on frame 18 and ending with a 32-teeth wheel on tube 20. This tube is not fixed, although one might expect it to be fixed. The frame 18 also carries the Earth's meridian as well as an arbor for the rotation of the Earth. We might therefore expect frame 18 to rotate in a tropical year.

The motion of frame 18 is derived from a train laid between the meridian's axis and a fixed 249-teeth wheel on frame 17. We can compute the motion of frame 18 by first considering the relative motion of frame 17 with respect to that of frame 12:

$$\mathrm{V}_{17}^{18} = \mathrm{V}_{12}^{18} \times \frac{434}{32} \times \left(-\frac{14}{50}\right) \times \left(-\frac{7}{30}\right) \times \left(-\frac{30}{69}\right) \times \left(-\frac{24}{249}\right) \tag{25.174}$$

$$= \mathrm{V}_{12}^{18} \times \left(-\frac{10633}{286350}\right) = \left(\mathrm{V}_{18}^{0} + \mathrm{V}_{0}^{12}\right) \times \frac{10633}{286350} \tag{25.175}$$

Hence

$$\mathrm{V}_{12}^{0} \times \frac{10633}{286350} = \mathrm{V}_{18}^{0} \times \left(1 + \frac{10633}{286350}\right) = \mathrm{V}_{18}^{0} \times \frac{296983}{286350} \tag{25.176}$$

And

$$\mathrm{V}_{18}^{0} = \mathrm{V}_{12}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{25.177}$$

$$\mathrm{P}_{18}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{25.178}$$

This is an approximation of the tropical year. The same value is given by Oechslin.

25.5.2 The rotation of the Earth around its axis

We can now also compute the velocity of the central tube 20:

$$\mathrm{V}_{20}^{18} = \mathrm{V}_{13}^{18} = \mathrm{V}_{12}^{18} \times \frac{434}{32} = \left(\mathrm{V}_{12}^{0} - \mathrm{V}_{18}^{0}\right) \times \frac{217}{16} \tag{25.179}$$

$$= \left(\frac{13}{170} - \frac{138229}{50487110}\right) \times \frac{217}{16} = \frac{372255}{5048711} \times \frac{217}{16} = \frac{80779335}{80779376} \tag{25.180}$$

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{20}^{18} + \mathrm{V}_{18}^{0} = \frac{80779335}{80779376} + \frac{138229}{50487110} = \frac{405002507}{403896880} \tag{25.181}$$

$$\mathrm{P}_{20}^{0} = \frac{403896880}{405002507} = 0.9972\ldots \text{ days} = 86164.1343\ldots \text{ seconds} \tag{25.182}$$

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The central tube 20 actually rotates with the velocity of the sidereal day.

Moreover, arbor 13 which carries the 32-teeth wheel meshing with the interior gearing also carries the frame for the Earth's meridian. Because of the two 32-teeth wheels, the one on the central fixed tube 20, and the one on tube 13, arbor 13 replicates the motion of the central tube. The Earth therefore rotates around a vertical axis (and not a tilted one) and makes one turn in one sidereal day in the absolute frame.

25.5.3 The line of apsides

As mentioned above, the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves like the line of the apsides. However, since the motion of the Earth is obtained through the 26-teeth wheel on arbor 11 meshing with the 340-teeth wheel on frame 12, this implies that there is a variable distance between the two axes. If this is the case, then the teeth are probably such that the meshing is maintained, even though the distance varies.[14]

The motion of the line of apsides is that of tube 34. The slow motion of tube 34 is obtained from that of arbor 11, with four intermediate worms:

$$\mathrm{V}_{34}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{1}{61}\right) \times \left(-\frac{1}{26}\right) \times \left(-\frac{1}{11}\right) \times \frac{1}{447} \tag{25.183}$$

$$= \mathrm{V}_{11}^0 \times \left(-\frac{1}{7798362}\right) = \frac{1}{7798362} \tag{25.184}$$

$$\mathrm{P}_{34}^0 = 7798362 \text{ days} \approx 21351 \text{ years} \tag{25.185}$$

The same value is given by Oechslin. This is a good approximation of the period of tropical apsidal precession which is about 21000 years. The ratios used here are slightly different from those used in the Vienna machine, resulting in a slightly less accurate period.

25.5.4 A front gear

The front panel carries one large ring with 352 teeth (frame 38). The velocity of this frame is:

$$\mathrm{V}_{38}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{1}{103}\right) \times \left(-\frac{1}{16}\right) \times \left(-\frac{1}{16}\right) \times \left(-\frac{1}{352}\right) \tag{25.186}$$

$$= \mathrm{V}_{11}^0 \times \frac{1}{9281536} = -\frac{1}{9281536} \tag{25.187}$$

$$\mathrm{P}_{38}^0 = -9281536 \text{ days} \approx -25412 \text{ years} \tag{25.188}$$

The same value is given by Oechslin. This is the period of precession of the equinoxes. Oechslin adds a question mark to the value 352, and perhaps he

[14] See Oechslin's observations on this problem [9, p. 154-155, 199-200].

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meant that the wheel is missing. It appears in any case that with a 352-teeth wheel, we obtain almost the same period as in the Vienna machine, and perhaps this is what Oechslin has been trying to achieve, if this is a reconstitution. Yet, the period obtained is somewhat off the more accurate value of 26000 years. If this wheel has been missing, then 352 is perhaps not the value that Neßtfell had used. Perhaps Neßtfell had actually used a 360-teeth wheel, in which case he would have obtained a period of about 25990 years, much closer to what was then thought to be the period of precession of equinoxes.

However, given that the front panel has the Earth rotate in a tropical year, which is confirmed by the motion of the apsides (which otherwise would not take place in about 21000 years), the zodiac is actually fixed. The actual constellations should then move *counterclockwise* and this is presumably what Neßtfell attempted to display, in order to show where in the sky Mercury is located. But on Neßtfell's machine, frame 38 moves clockwise, as if he wanted to show a precession.

In any case, frame 38 is only used to position the actual axis of the Earth, if I understand it correctly.

25.5.5 The motion of Mercury

The main front reference frame for Mercury is frame 34. We have seen above that this frame moves with the motion of the line of apsides of the orbit of the Earth. The Mercury wheel is a wheel on arbor 24 which pivots eccentrically on frame 34. This will account for the elliptical orbit of Mercury, but it does assume that the line of apsides of Mercury coincides with that of the Earth, which is not the case.

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25.5.5.1 The mean motion of Mercury

The mean motion of Mercury is given by the Mercury wheel on arbor 24. This motion is derived from that of tube 20. We have

$$\mathrm{V}_{24}^{34} = \mathrm{V}_{20}^{34} \times \left(-\frac{36}{18}\right) \times \left(-\frac{18}{36}\right) \times \left(-\frac{18}{68}\right) \times \left(-\frac{7}{163}\right) \tag{25.189}$$

$$= \mathrm{V}_{20}^{34} \times \frac{63}{5542} \tag{25.190}$$

$$\mathrm{V}_{24}^{0} = \mathrm{V}_{24}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{20}^{34} \times \frac{63}{5542} + \mathrm{V}_{34}^{0} = \left(\mathrm{V}_{20}^{0} - \mathrm{V}_{34}^{0}\right) \times \frac{63}{5542} + \mathrm{V}_{34}^{0} \tag{25.191}$$

$$= \mathrm{V}_{20}^{0} \times \frac{63}{5542} + \mathrm{V}_{34}^{0} \times \left(1 - \frac{63}{5542}\right) = \mathrm{V}_{20}^{0} \times \frac{63}{5542} + \mathrm{V}_{34}^{0} \times \frac{5479}{5542} \tag{25.192}$$

$$= \frac{405002507}{403896880} \times \frac{63}{5542} + \frac{1}{7634700} \times \frac{5479}{5542} \tag{25.193}$$

$$= \frac{572949380244583}{50263193608696800} \tag{25.194}$$

$$\mathrm{P}_{24}^{0} = \frac{50263193608696800}{572949380244583} = 87.7271\ldots \text{ days} \tag{25.195}$$

This is a good approximation of the orbital period of Mercury. The same value was used in the Vienna machine.

Of course, we could have ignored the term $\frac{1}{7634700}$, and this would only have slightly altered the orbital period of Mercury. This is actually what Oechslin did, and he gave the period as 87.728107 days. But the exact ratio resulting from Neßtfell's construction is the one given above, not the one given by Oechslin.

25.5.5.2 The inclination of Mercury's orbit

Neßtfell has Mercury move radially depending on its distance from its line of nodes. The orbit of Mercury is tilted by about 7 degrees, but on Neßtfell's panel for the Vienna machine, this angle is about 5 degrees, according to Oechslin. This tilted orbital plane is represented by frame 30 which carries a 259-teeth wheel. This is rotated using gears located on frame 34. We first

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compute the velocity of frame 30 with respect to frame 34:

$$\mathrm{V}_{30}^{34} = \mathrm{V}_{20}^{34} \times \left(-\frac{52}{96}\right) \times \left(-\frac{1}{14}\right) \times \left(-\frac{1}{26}\right) \tag{25.196}$$

$$\times \left(-\frac{1}{16}\right) \times \frac{1}{16} \times \left(-\frac{8}{259}\right) \tag{25.197}$$

$$= \mathrm{V}_{20}^{34} \times \frac{1}{5569536} \tag{25.198}$$

$$\mathrm{V}_{30}^{0} = \mathrm{V}_{30}^{34} + \mathrm{V}_{34}^{0} = \mathrm{V}_{20}^{34} \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \tag{25.199}$$

$$= \left(\mathrm{V}_{20}^{0} - \mathrm{V}_{34}^{0}\right) \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \tag{25.200}$$

$$= \mathrm{V}_{20}^{0} \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \times \left(1 - \frac{1}{5569536}\right) \tag{25.201}$$

$$= \mathrm{V}_{20}^{0} \times \frac{1}{5569536} + \mathrm{V}_{34}^{0} \times \frac{5569535}{5569536} \tag{25.202}$$

$$= \frac{405002507}{403896880} \times \frac{1}{5569536} + \frac{1}{7634700} \times \frac{5569535}{5569536} \tag{25.203}$$

$$= \frac{3142112029261}{10102586296593530880} \tag{25.204}$$

$$\mathrm{P}_{30}^{0} = \frac{10102586296593530880}{3142112029261} = 3215221.5460\ldots \text{ days} \approx 8800 \text{ years} \tag{25.205}$$

This is the period of the revolution of the nodes resulting from Neßtfell's construction, but it is possible that Neßtfell aimed at having the nodes fixed and that his gears merely provide a very long period. The exact same period was used in the Vienna machine, although with a slightly different train. The period of the revolution of the nodes of Mercury was actually not well known in the 18th century.

Oechslin, instead, found a wrong period of about 15207 years.

25.5.5.3 The viewpoint from the Earth

Finally, there is a guide linking the Earth to Mercury, and showing the backdrop of the sky where Mercury is located. Neßtfell did however not attempt to show the transits of Mercury.

25.5.6 The calendar part

The front panel also has a calendar display, which I am not describing here. It shows the year in an opening like the Vienna machine. The Vienna machine has a mention about the year 1828 not being a leap year. However, Gutwein's engraving of the front of the Vienna machine (figure 24.15) mentions the year 1826. I don't know if a similar mention appears here.[15]

[15] See in particular Fowler's brief description [2, p. 26-27].

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Neßtfell's description of the clock also mentions that a day should be removed every 126 years [7, p. 54]. After that time, the excess of the Julian year to the tropical year accumulates to about a day, but it is not totally clear how Neßtfell arrived at that value. In any case, on Gutwein's engraving from 1754, this is indicated by the two years 1826 and 1952.

25.6 Conclusion

Neßtfell's clock follows a very original construction, and one can only be in awe of his imagination, and also in admiration of Oechslin's efforts to render the structure of the machine in a plan, which is certainly not easy.

25.7 References

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Chapter 26

(Oechslin: 7.1)

Rinderle's clock in Furtwangen (1787)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

26.1 Introduction

The clock described here was constructed in 1787 by Thaddäus Rinderle, born Matthias Rinderle (1748-1824). The case was made by Matthias Faller (1707-1791).[1]

Rinderle became a Benedictine and priest in 1772. He studied mathematics in Salzburg and invented optical and mechanical instruments.[2]

The history of the ownership of Rinderle's geographical-astronomical clock is very well known[3] and in 1859 it entered the collection if the Furtwangen horology school, and later the *Furtwangen Uhrenmuseum* where it is now held (figure 26.1). There is also a 16-pages manuscript of the clock, presumably by Rinderle.[4]

Rinderle's clock has two dials on top of each other.[5] The largest dial

[1] See [18, p. 78] and [3].

[2] For a comprehensive biography of Rinderle, see Schmidt's work [18]. The geographical-astronomical clock is covered especially on pages 73-81. In his catalogue of the Furtwangen museum published in 1925, Kistner devoted several pages to Rinderle [8, p. 44-54]. A more recent biographical notice is that of Schäffner, but I haven't seen it [17]. There are numerous other mentions of Rinderle, of which I am giving a few in the references section. Lübke mentioned Rinderle's clock, but with a wrong illustration [11, p. 272], [12, p. 240-241]. In his article on the Johann brothers, Abeler has suggested that Nicolaus Alexius Johann may have been in touch with Rinderle, before or after 1781 [1, p. 203]. See also Abeler's dictionary [2].

[3] [18, p. 80]

[4] [18, p. 75] The manuscript was reproduced and transcribed in 2007 [22].

[5] The fundamental descriptions of the clock are those of Wenzel [22] and Oechslin [15].

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Figure 26.1: Thaddäus Rinderle's clock (1787). (source: Wikipedia)

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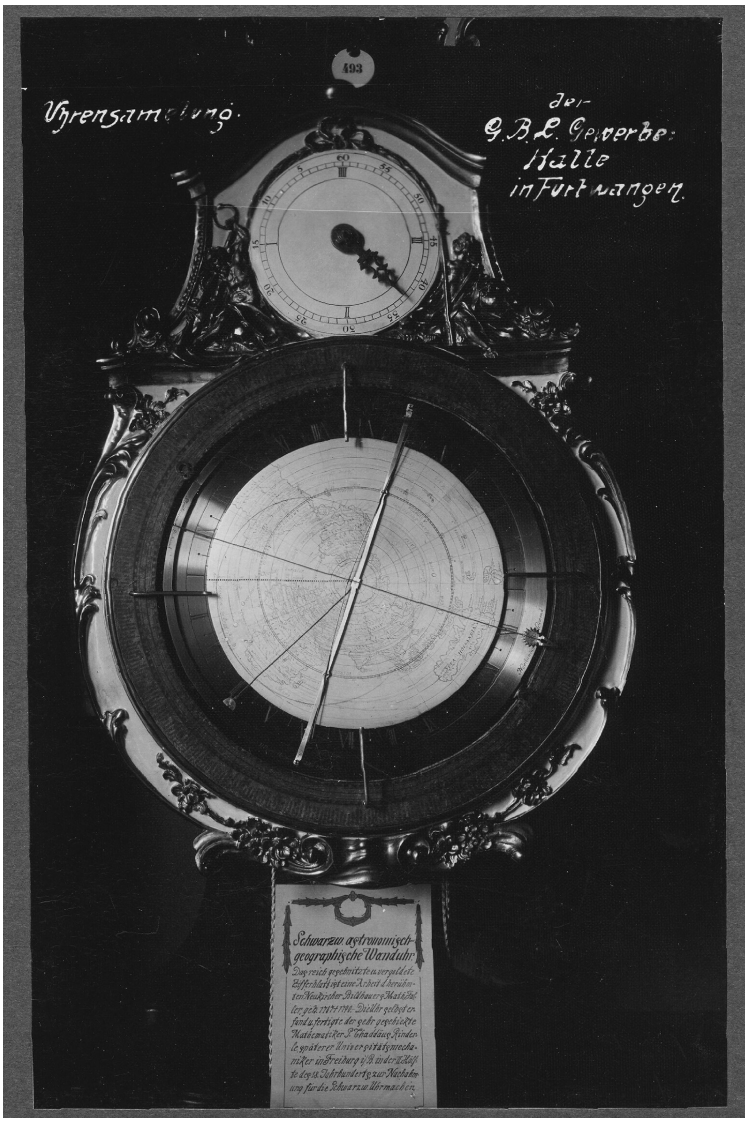


Figure 26.2: Thaddäus Rinderle's clock (1787). (source: `https://global.museum-digital.org`, licence CC BY-SA)

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displays a fixed calendar at its rim, a Northern planisphere, the motion of the Sun (which also serves to indicate the time) and of the Moon, the lunar nodes, and the limit between day and night. The small dial at the top shows the minutes which move counterclockwise.

The clock was studied in the 1980s by Johann Wenzel, but his study was left unpublished. Also at the beginning of the 1980s, Rinderle's clock was faithfully reproduced by Wilfried Dorer.[6] Meanwhile, in 1996, Oechslin described Rinderle's clock [15], not knowing about Wenzel's work. Oechslin did not have the opportunity to disassemble the clock [15, p. 5] and used details on the clock provided by Richard Mühe and Beatrice Techen from the Furtwangen museum.

And when Wenzel's work was published in 2007 [22], it didn't mention Oechslin's work at all and it is unfortunate that the *Furtwangen Uhrenmuseum* did not attempt a consolidate work. The 2007 book also gives a facsimile and transcription of Rinderle's handwritten description of the clock, presumably from 1787. This description does however not enter into the details of the gears. Finally, Hahn also published a summary of the clock in 2008 and suggested a number of improvements in the gears [7].

Now, there are basically two independent descriptions of the clock, that of Wenzel and that of Oechslin, and they do not provide exactly the same teeth-counts and conclusions. I will therefore split this analysis in two parts, one for Oechslin's description, and one for Wenzel's description. I have chosen this order, because it is the order of publication, but also because some errors found in Oechslin's description are corrected in Wenzel's description.

26.2 Oechslin's description

As mentioned above, Rinderle's clock was not directly examined by Oechslin, and this certainly explains some of the problems mentioned in this section.[7]

26.2.1 The base motions and assumptions

When working through Oechslin's plan of the clock, things are somewhat confusing, because one is tempted to assume that the central arbor makes a turn in 24 hours on the main dial and that the hand of the small dial shows the

One of the first descriptions of Rinderle's clock is in volume 3 or Ruef's *Freyburger Beyträge* in 1790 [16, p. 494-495]. There we also learn that Rinderle had built a calculating machine working on 13 places and for the four operations (see also Schmidt [18, p. 64] and Wenzel [22, p. 37]). Another early description of the clock is that of Steyrer in 1796 [21, p. 23-29]. After his death, Schneller also devoted a few pages to the clock [19]. See also Kistner's catalogue of the Furtwangen museum published in 1925 [8, p. 44-54]. Several other shorter mentions were published later.

[6] [22, p. 38]

[7] For Oechslin's description, see particularly [15, p. 37, 50-51].

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minutes and thus makes a turn in one hour. But these assumptions reveal inconsistencies. For instance, assuming that the arbor of the small dial makes a turn in 60 minutes, Oechslin deduced that the central arbor 7 of the main dial makes a turn in 26.1333... hours. Moreover, even if we assume that something is wrong in the train from the small dial to the main dial, and that the central arbor makes a turn in 24 hours (as suggested by Oechslin's drawing), we still find very bad values for the motion of the Sun (366.2468... days) and the Moon (27.3964... days).

After spending some time with Oechslin's plan, and before I even read Wenzel's analysis, I came to the conclusion that the central arbor 7 of the main dial is not meant to make a turn in 26.1333... hours, and not even in 24 hours, but in one sidereal day. This arbor carries the planisphere and in that case the motion of the planisphere should be thought of with respect to the stars.

This assumption is supported by the periods which are then obtained for the Moon and the Sun, although the period of precession of the nodes remains somewhat inaccurate.

However, even with this assumption, there seems to be a problem with the small dial. The minute hand does not make a turn in one hour, and it isn't clear what would have to be corrected so that the minute hand turns in one hour. We will see below that there is a solution to this problem, but meanwhile I was looking for an even simpler one, which is that since the hours of the main dial are sidereal hours, a possibility would be that the small dial also shows the sidereal time. And indeed, if we assume that the small minute hand makes a turn in 60 sidereal minutes, then the ratio between the velocity of the minute arbor and that of the central arbor 7 must be 24. This isn't the case with the values given by Oechslin, but it can easily be fixed if the 98-teeth wheel on arbor 7 is given instead 90 teeth, and this is what I will assume below. However, again, we will see below a better solution, but which entails more changes to the gears.

26.2.2 The going work

The clock is weight-driven and regulated by a pendulum. Assuming that the clock is based on sidereal time, I will first express the velocities of the wheels in turns per sidereal days (t/sd), and not mean days. We start with

$$\mathrm{V}_5^0 = 24 \text{ t/sd (counterclockwise)} \tag{26.1}$$

then

$$\mathrm{V}_3^0 = \mathrm{V}_5^0 \times \frac{50}{12} \times \frac{9}{100} = \mathrm{V}_5^0 \times \frac{3}{8} = 9 \text{ t/sd} \tag{26.2}$$

The velocity of the escape wheel then is

$$\mathrm{V}_1^0 = \mathrm{V}_3^0 \times \left(-\frac{100}{9}\right) \times \left(-\frac{72}{8}\right) = \mathrm{V}_3^0 \times 100 = 900 \text{ t/sd} \tag{26.3}$$

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This gives a period of

$$\mathrm{P}_1^0 = \frac{1}{900}\ \mathrm{sd} = 96 \text{ sidereal seconds} \tag{26.4}$$

and since the escape wheel has 36 teeth, this gives a half-oscillation of the pendulum in 4/3 sidereal seconds. This may seem strange, but it is manageable.

However, in order to express these values in seconds we need to choose a value for the sidereal day, because this value is not contained in the clock. So, this also means that the ratios I will give will not have any particular significance with respect to Rinderle. I will assume that a sidereal day is $\sigma = 86164/86400$ mean days, but the reader can experiment other ratios and will obtain different values for the astronomical motions.

So, I now assume that

$$\mathrm{V}_5^0 = 24 \times \frac{1}{\sigma} \tag{26.5}$$

$$\mathrm{V}_3^0 = 9 \times \frac{1}{\sigma} \tag{26.6}$$

26.2.3 The motion of the planisphere

The central arbor 7 of the main dial moves a Northern planisphere. We first compute the velocity of the intermediate arbor 6:

$$\mathrm{V}_6^0 = \mathrm{V}_3^0 \times \left(-\frac{61}{61}\right) = -\mathrm{V}_3^0 = -9 \times \frac{1}{\sigma} \tag{26.7}$$

Then, the velocity of arbor 7 is (with the correction $98 \to 90$)

$$\mathrm{V}_7^0 = \mathrm{V}_6^0 \times \left(-\frac{10}{90}\right) = \frac{1}{\sigma} \tag{26.8}$$

$$\mathrm{P}_7^0 = \sigma \tag{26.9}$$

that is one turn in one sidereal day.

26.2.4 The motion of the Moon

The motion of the Moon on tube 9 is derived from that of the central arbor 7:

$$\mathrm{V}_9^0 = \mathrm{V}_7^0 \times \left(-\frac{23}{103}\right) \times \left(-\frac{17}{104}\right) = \mathrm{V}_7^0 \times \frac{391}{10712} \tag{26.10}$$

$$\mathrm{P}_9^0 = \frac{10712}{391}\sigma \tag{26.11}$$

If we take $\sigma = 86164/86400$, we find $\mathrm{P}_9^0 = 27.3215\ldots$ days. This is an approximation of the tropical month.

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26.2.5 The motion of the Sun

The motion of the Sun on tube 11 is derived from that of the Moon:

$$\mathrm{V}_{11}^0 = \mathrm{V}_9^0 \times \left(-\frac{36}{72}\right) \times \left(-\frac{19}{127}\right) = \mathrm{V}_9^0 \times \frac{19}{254} \tag{26.12}$$

$$= \mathrm{V}_7^0 \times \frac{391}{10712} \times \frac{19}{254} = \mathrm{V}_7^0 \times \frac{7429}{2720848} \tag{26.13}$$

$$\mathrm{P}_{11}^0 = \frac{2720848}{7429}\sigma \tag{26.14}$$

If we take $\sigma = 86164/86400$, we find $\mathrm{P}_{11}^0 = 365.2464\ldots$ days. This is an approximation of the tropical year.

26.2.6 The motion of the lunar nodes

The lunar nodes are shown by a hand carried by tube 14. This hand ends with the symbols ☊ (ascending node) and ☋ (descending node). The motion of this tube is derived from that of the Sun:

$$\mathrm{V}_{14}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{40}{94}\right) \times \left(-\frac{20}{24}\right) \times \left(-\frac{10}{76}\right) = \mathrm{V}_{11}^0 \times \left(-\frac{125}{2679}\right) \tag{26.15}$$

$$= \mathrm{V}_7^0 \times \frac{7429}{2720848} \times \left(-\frac{125}{2679}\right) = \mathrm{V}_7^0 \times \left(-\frac{48875}{383639568}\right) \tag{26.16}$$

$$\mathrm{P}_{14}^0 = -\frac{383639568}{48875}\sigma \tag{26.17}$$

If we take $\sigma = 86164/86400$, we find $\mathrm{P}_{14}^0 = -7827.9624\ldots$ days. This is a very bad approximation of the period of precession of the lunar nodes which is about 6798 days. This bad value may be due to wrong gear ratios. For instance, with a ratio 10/66 instead of 10/76 in the train, we would have found the almost perfect value of $-6797.9673\ldots$ days. The value is negative, because there is a retrogradation of the lunar nodes. See [15, p. 51] on that motion.

26.2.7 The day/night separation

On the front dial, we can also see a (usually) bent line. This is actually a spring which is bent either towards the Sun or against it, and it shows which part of the Earth is lit or not. At the equinoxes, this spring is a straight line. This bending is caused by a fixed cam which is located behind the planisphere. This cam is shown in Wenzel's analysis [22, p. 21].[8] It is the motion of the Sun in one year which causes the spring to bend in one direction or the other. Similar constructions had been used in earlier clocks.[9]

[8] See also [15, p. 141].

[9] See for instance Fowler [4].

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26.3 Wenzel's analysis

Wenzel's analysis published in 2007 [22] assumes that arbor 3, which is the rope arbor, makes a turn in two hours, but there are several differences in the gears with respect to Oechslin's drawing. This is certainly explained by the fact that Oechslin didn't use first-hand data for the clock. First, arbor 6 carries a 73-teeth wheel and not a 61-teeth wheel, which may have been in typo in Oechslin's notes. Second, the central arbor 7 carries a 100-teeth wheel, and not a 98-teeth wheel as in Oechslin's notes. The train between the small dial and the main dial is also slightly different. Oechslin has an arbor 4 with two pinions of 12 and 9 teeth, but Wenzel describes two pinions of 12 teeth [22, p. 27]. If Wenzel's description is correct, and if we assume the small dial to show the minutes,

$$\mathrm{V}_5^0 = 24 \tag{26.18}$$

then the velocity of arbor 3 is

$$\mathrm{V}_3^0 = \mathrm{V}_5^0 \times \frac{50}{12} \times \frac{12}{100} = \mathrm{V}_5^0 \times \frac{1}{2} = 12 \tag{26.19}$$

$$\mathrm{P}_3^0 = \frac{1}{12} = 2 \text{ hours} \tag{26.20}$$

From this it also follows that the velocity of the escape wheel is

$$\mathrm{V}_1^0 = \mathrm{V}_3^0 \times \left(-\frac{100}{9}\right) \times \left(-\frac{72}{8}\right) = \mathrm{V}_3^0 \times 100 = 1200 \tag{26.21}$$

$$\mathrm{P}_1^0 = \frac{1}{1200} = 72 \text{ s} \tag{26.22}$$

and given that the escape wheel has 36 teeth, we obtain a half-oscillation in one second, that is that of a seconds pendulum. We have therefore a much more natural configuration than that implied by Oechslin.

The central arbor 7 then has the following velocity:

$$\mathrm{V}_7^0 = \mathrm{V}_3^0 \times \left(-\frac{61}{73}\right) \times \left(-\frac{10}{100}\right) = \mathrm{V}_3^0 \times \frac{61}{730} = 12 \times \frac{61}{730} = \frac{366}{365} \tag{26.23}$$

$$\mathrm{P}_7^0 = \frac{365}{366} = 23 \text{ h } 56 \text{ m } 3.9344 \text{ s} \tag{26.24}$$

This also agrees with my first conclusions (after having analyzed Oechslin's drawing and before having read Wenzel's analysis).

The remaining periods can be obtained like in the case of Oechslin, by taking $\sigma = 365/366$. We thus find

26.3.1 The motion of the Moon

$$\mathrm{P}_9^0 = \frac{10712}{391}\sigma = \frac{10712}{391} \times \frac{365}{366} = \frac{1954940}{71553} = 27.3215\ldots \text{ days} \tag{26.25}$$

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26.3.2 The motion of the Sun

$$\mathrm{P}_{11}^0 = \frac{2720848}{7429}\sigma = \frac{2720848}{7429} \times \frac{365}{366} = \frac{496554760}{1359507} = 365.2461\ldots \text{ days} \tag{26.26}$$

26.3.3 The motion of the lunar nodes

In the case of the nodes, Wenzel gives the teeth-count 66 instead of Oechslin's 76, as I had suggested above (before having read Wenzel's analysis). We have

$$\mathrm{V}_{14}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{40}{94}\right) \times \left(-\frac{20}{24}\right) \times \left(-\frac{10}{66}\right) = \mathrm{V}_{11}^0 \times \left(-\frac{250}{4653}\right) \tag{26.27}$$

$$= \mathrm{V}_7^0 \times \frac{7429}{2720848} \times \left(-\frac{250}{4653}\right) = \mathrm{V}_7^0 \times \left(-\frac{928625}{6330052872}\right) \tag{26.28}$$

$$\mathrm{P}_{14}^0 = -\frac{6330052872}{928625}\sigma = -\frac{6330052872}{928625} \times \frac{365}{366} \tag{26.29}$$

$$= -\frac{77015643276}{11329225} = -6797.9621\ldots \text{ days} \tag{26.30}$$

26.4 Summary on the errors and interpretation of the clock

It appears that there are notable differences between the descriptions of Oechslin and Wenzel. These differences are certainly due to Oechslin using some other unreliable source of data for the clock.

In a first interpretation, I assumed that the small dial did not show the (mean) minutes, but the sidereal minutes, but this assumption rested on my trust into most of the train given by Oechslin between the main dial and the small dial. I expected some figures to be wrong, but I intended to minimize the number of errors, and I found that changing the number of teeth of the 98-teeth wheel on arbor 7 to 90 solved the problem. It was perhaps not entirely satisfying, but it was a possible solution.

However, Wenzel's analysis made it possible to go back on the right track, with a small dial really showing the minutes, a rope arbor with a simple period (two hours), and a central arbor 7 making a turn in a sidereal day. But this entailed three corrections in the train, instead of the only one I had tried to find. I am now siding with Wenzel's analysis, which seems to be the best one.

This does not explain why Oechslin's drawing and calculations contain four teeth-count errors, and why these errors were not corrected by him, given that he noticed that some of the periods he obtained were incorrect.

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Chapter 27

(Oechslin: 10.3)

Seige's metal orrery fragment in Prague (c1781?)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

27.1 Introduction

The orrery metal fragment described here was constructed by Engelbert Seige (1737-1811), probably around 1781. Before entering into the details of this fragment, a few words on Seige are in order.[1]

Engelbert Wenzel Seige was born on April 25, 1737 in Chlumec, near Ústí nad Labem, at about 20 km from the Osek monastery, in the Czech Republic. Seige entered this monastery at the age of 18 and became priest in 1764. He was also a professor of Greek between 1775 and 1783. He constructed many telescopes and astronomical clocks.

Seige worked on his most important clock for about ten years, between 1781 and 1791. This clock is now exhibited in the technical museum in Prague. The museum also keeps two fragments of astronomical clocks, the fragment described here, and a fragment in wood.

Seige was prior of the Osek monastery until 1808 and he died in 1811.

The present metal fragment features Uranus which was discovered in 1781, but with a very gross approximation of its period. It may well be the first machine showing the motion of Uranus. Hahn's *Weltmaschine* in Nuremberg (Oechslin 8.2) also shows Uranus, but with a more accurate period, and it must have been constructed a few years later.[2]

[1] Very little is known about the life of Seige, and I am drawing these few lines from Hrůšová's thesis [4] who seems to have borrowed it from [1].

[2] Faber seems to suggest that one of Seige's fragments is that of an unfinished clock at the time of his death [3, p. 9].

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Seige's machine is made of a cubic base which contains the going work, as well as the gears for the four sides of the base. It is topped by an orrery in the middle, and four satellite systems at the corner, which is a somewhat unusual arrangement. The general structure is however very simple, only with many long gear trains. Many of the trains are independent from each other. The only more advanced construction is that of the Sun-Moon system at the base of the machine. I will first describe the base, then the upper parts.[3]

Seige's *Weltmaschine* from 1791 (Oechslin 10.1) seems to be the finalization of this fragment.

27.2 The base

27.2.1 The going work

The machine is weight driven and regulated by a pendulum and an anchor escapement. The drum is on arbor 1. If one computes the period of this drum from the period of arbor 8 which is indicated as 24 hours by Oechslin, then we reach a period of 64 hours. And eventually, we arrive at the escape wheel making a turn in 160 seconds and the pendulum being more than seven meters long. This is obviously wrong.

If Oechslin's drawing is correct, arbor 1 can't make a turn in 64 hours. The pendulum which is visible on some pictures seems to be about one meter long. We should therefore have the escape wheel making a turn in about a minute and arbor 1 making a turn in 24 hours. This also entails that arbor 8 should make a turn in nine hours and it does not fit with Oechslin writing that arbor 8 shows the motion of the Earth in 24 hours.

I will therefore assume that arbor 1 makes a turn in 24 hours:

$$\mathrm{V}_1^0 = 1 \tag{27.1}$$

I am measuring the velocities in the going work from the right in Oechslin's plate.

We can then deduce the velocities of arbors 2, 3, 4 and 5, the latter being that of the escape wheel:

$$\mathrm{V}_2^0 = \mathrm{V}_1^0 \times \left(-\frac{80}{10}\right) = -8 \tag{27.2}$$

[3]This fragment was also described by Oechslin, see [8, p. 34, 198].

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This arbor makes a turn clockwise in three hours.

$$V_3^0 = V_2^0 \times \left(-\frac{60}{10}\right) = 48 \tag{27.3}$$

$$P_3^0 = \frac{1}{48} \text{ days} = 30 \text{ minutes} \tag{27.4}$$

$$V_4^0 = V_3^0 \times \left(-\frac{60}{16}\right) = -180 \tag{27.5}$$

$$P_4^0 = -\frac{1}{180} \text{ days} = -8 \text{ minutes} \tag{27.6}$$

$$V_5^0 = V_4^0 \times \frac{56}{7} = -1440 \tag{27.7}$$

$$P_5^0 = -\frac{1}{1440} \text{ days} = -60 \text{ seconds} \tag{27.8}$$

Note that on Oechslin's drawing the escapement arbor is shown vertically, but it is actually horizontal. My choices for the orientations rely on Oechslin's drawing.

If my assumption is correct, the escape wheel should make a turn in exactly one minute and the pendulum should be about one meter long.

However, as we will see below, the period of arbor 3 leads to a problem in all the upper mechanisms of the machine. The orrery and corner subsystems only turn as they should if arbor 3 makes a turn in one hour. Oechslin also observed some of these inconsistencies [8, p. 198].

27.2.2 The outputs of the going work

The going work drives a number of arbors that were meant to display some motions. On one side, there is the arbor 8, supporting a wheel which according to the above calculations makes one turn in nine hours. However, Oechslin writes that it makes one turn in 24 hours.

Another side shows the motion of the Sun and the Moon. Its input is arbor 10 whose motion is derived from that of arbor 2:

$$V_{10}^0 = V_2^0 \times \left(-\frac{30}{60}\right) \times \left(-\frac{8}{32}\right) = -1 \tag{27.9}$$

(All directions are viewed from the right or the bottom on Oechslin's plate.)

So, arbor 10 makes one turn clockwise in one day.

On another side, we have the three arbors 110, 111 and 112. Their velocities

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(now measured from the left) are:

$$V^0_{110} = V^0_1 \times \frac{11}{56} \times \left(-\frac{10}{76}\right) = -\frac{55}{2128} \tag{27.10}$$

$$P^0_{110} = -\frac{2128}{55} = -38.6909\ldots \text{ days} \tag{27.11}$$

$$V^0_{111} = V^0_2 \times \frac{40}{30} = -\frac{32}{3} \tag{27.12}$$

$$P^0_{111} = -\frac{3}{32} = -2 \text{ hours } 15 \text{ minutes} \tag{27.13}$$

$$V^0_{112} = V^0_3 \times \frac{15}{60} = 12 \tag{27.14}$$

$$P^0_{112} = \frac{1}{12} = 2 \text{ hours} \tag{27.15}$$

It isn't clear what these three arbors were meant to represent.

On the fourth side, we have the three arbors 113, 114, and 115 whose periods also remain clouded in mystery:

$$V^0_{113} = V^0_1 \times \left(-\frac{11}{56}\right) = -\frac{11}{56} \tag{27.16}$$

$$P^0_{113} = -\frac{56}{11} = -5.0909\ldots \text{ days} \tag{27.17}$$

$$V^0_{114} = V^0_2 \times \left(-\frac{20}{40}\right) = 4 \tag{27.18}$$

$$P^0_{114} = \frac{1}{4} = 6 \text{ hours} \tag{27.19}$$

$$V^0_{115} = V^0_2 \times \left(-\frac{45}{89}\right) = \frac{360}{89} \tag{27.20}$$

$$P^0_{115} = \frac{89}{360} = 5.9333\ldots \text{ hours} \tag{27.21}$$

And finally, there are two outputs to the upper part of the machine, here measured as seen from above:

$$V^0_{27} = V^0_3 \times \left(-\frac{10}{60}\right) \times \left(-\frac{10}{40}\right) = 2 \tag{27.22}$$

$$P^0_{27} = \frac{1}{2} = 12 \text{ hours} \tag{27.23}$$

$$V^0_{83} = V^0_3 \times \left(-\frac{20}{40}\right) \times \left(-\frac{12}{36}\right) = -8 \tag{27.24}$$

$$P^0_{83} = -\frac{1}{8} = -3 \text{ hours} \tag{27.25}$$

However, if arbor 27 makes one turn in 12 hours, the periods in the upper central orrery, as well as in the two corner systems driven by that arbor are too

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small by a factor two. Obviously, these systems assume that arbor 27 makes one turn in 24 hours. I will therefore assume below that it is so.

The same is actually true for arbor 83 which should actually be making a turn in 6 hours and not 3 hours. Both of these problems go back to the velocity of arbor 3. I have computed above that this arbor makes a turn in 30 minutes. But if it did a turn in one hour, this would solve our problem. Perhaps it would make a turn in one hour and perhaps some teeth counts are incorrect. Perhaps the ratio between arbors 2 and 3 is 3 and not 6.

27.2.3 The base system of the Sun and the Moon

As mentioned above, the input to this system is arbor 10 which makes one turn clockwise (from the front) in one day:

$$\mathrm{V}_{10}^{0} = -1 \tag{27.26}$$

27.2.3.1 The motion of the Sun

The motion of the Sun corresponds to tube 18. We can easily compute its velocity following the train from arbor 10 to tube 18:

$$\mathrm{V}_{11}^{0} = \mathrm{V}_{10}^{0} \times \left(-\frac{30}{30}\right) = 1 \tag{27.27}$$

$$\mathrm{V}_{12}^{0} = \mathrm{V}_{11}^{0} \times \left(-\frac{10}{53}\right) = -\frac{10}{53} \tag{27.28}$$

$$\mathrm{V}_{13}^{0} = \mathrm{V}_{12}^{0} \times \left(-\frac{14}{64}\right) = \left(-\frac{10}{53}\right) \times \left(-\frac{7}{32}\right) = \frac{35}{848} \tag{27.29}$$

$$\mathrm{V}_{16}^{0} = \mathrm{V}_{13}^{0} \times \left(-\frac{20}{62}\right) = \frac{35}{848} \times \left(-\frac{10}{31}\right) = -\frac{175}{13144} \tag{27.30}$$

$$\mathrm{V}_{17}^{0} = \mathrm{V}_{16}^{0} \times \left(-\frac{25}{37}\right) = \left(-\frac{175}{13144}\right) \times \left(-\frac{25}{37}\right) = \frac{4375}{486328} \tag{27.31}$$

$$\mathrm{V}_{18}^{0} = \mathrm{V}_{17}^{0} \times \left(-\frac{21}{69}\right) = \frac{4375}{486328} \times \left(-\frac{7}{23}\right) = -\frac{30625}{11185544} \tag{27.32}$$

$$\mathrm{P}_{18}^{0} = -\frac{11185544}{30625} = -365.2422\ldots \text{ days} \tag{27.33}$$

The same value is given by Oechslin.

Note that the Sun moves clockwise here, but should move counterclockwise. In fact, all the motions are in the wrong directions, and it is possible that the input motion was meant to turn in the opposite direction. This would give correct motions of the Sun, of the Moon, and of the nodes and apsides.

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27.2.3.2 The mean motion of the Moon

The mean motion of the Moon is given by frame 14 whose motion is derived from that of arbor 13:

$$\mathrm{V}^0_{14} = \mathrm{V}^0_{13} \times \left(-\frac{47}{53}\right) = \frac{35}{848} \times \left(-\frac{47}{53}\right) = -\frac{1645}{44944} \tag{27.34}$$

$$\mathrm{P}^0_{14} = -\frac{44944}{1645} = -27.3215\ldots \text{ days} \tag{27.35}$$

The same value is given by Oechslin. It is an approximation of the tropical month.

27.2.3.3 The lunar apsides

The motion of the lunar apsides is given by tube 21, obtained through a derivation from the motion of arbor 17. We have:

$$\mathrm{V}^0_{19} = \mathrm{V}^0_{17} \times \left(-\frac{19}{60}\right) = \frac{4375}{486328} \times \left(-\frac{19}{60}\right) = -\frac{16625}{5835936} \tag{27.36}$$

$$\mathrm{V}^0_{20} = \mathrm{V}^0_{19} \times \left(-\frac{19}{53}\right) = \left(-\frac{16625}{5835936}\right) \times \left(-\frac{19}{53}\right) = \frac{315875}{309304608} \tag{27.37}$$

$$\mathrm{V}^0_{21} = \mathrm{V}^0_{20} \times \left(-\frac{20}{66}\right) = \frac{315875}{309304608} \times \left(-\frac{10}{33}\right) = -\frac{1579375}{5103526032} \tag{27.38}$$

$$\mathrm{P}^0_{21} = -\frac{5103526032}{1579375} = -3231.3579\ldots \text{ days} \tag{27.39}$$

which is about 8.85 years. The same value is given by Oechslin.

27.2.3.4 The lunar nodes

The motion of the lunar nodes is given by tube 25, obtained through a long derivation from the motion of arbor 20. We have:

$$\mathrm{V}^0_{22} = \mathrm{V}^0_{20} \times \left(-\frac{23}{22}\right) = \frac{315875}{309304608} \times \left(-\frac{23}{22}\right) = -\frac{7265125}{6804701376} \tag{27.40}$$

$$\mathrm{V}^0_{23} = \mathrm{V}^0_{22} \times \left(-\frac{24}{38}\right) = \left(-\frac{7265125}{6804701376}\right) \times \left(-\frac{12}{19}\right) = \frac{382375}{567058448} \tag{27.41}$$

$$\mathrm{V}^0_{24} = \mathrm{V}^0_{23} \times \left(-\frac{29}{41}\right) = \frac{382375}{567058448} \times \left(-\frac{29}{41}\right) = -\frac{11088875}{23249396368} \tag{27.42}$$

$$\mathrm{V}^0_{25} = \mathrm{V}^0_{24} \times \left(-\frac{33}{107}\right) = \left(-\frac{11088875}{23249396368}\right) \times \left(-\frac{33}{107}\right) \tag{27.43}$$

$$= \frac{33266625}{226153219216} \tag{27.44}$$

$$\mathrm{P}^0_{25} = \frac{226153219216}{33266625} = 6798.2014\ldots \text{ days} \tag{27.45}$$

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which is an excellent approximation of the period of precession of the lunar nodes. The same value is given by Oechslin.

There are currently no hands and the Moon seems to have an eccentric motion based on the motion of the lunar apsides.

27.3 The four corner systems

The upper part of the machine has a central orrery and four corner systems. Two of these systems are driven with the input from arbor 27 which should be making one turn in 24 hours. The other two systems are driven with the input from arbor 83 which should be making one turn in six hours.

$$\mathrm{V(h)}^0_{27} = 1 \tag{27.46}$$
$$\mathrm{P(h)}^0_{27} = 1 \text{ day} \tag{27.47}$$
$$\mathrm{V(h)}^0_{83} = -4 \tag{27.48}$$
$$\mathrm{P(h)}^0_{83} = -6 \text{ hours} \tag{27.49}$$

The motion of arbor 83 is first split into those of arbor 98 (daily Sun/Moon system) and 85 (satellites of Jupiter). And the motion of arbor 27 is split into those of arbor 52 (yearly Sun/Moon system) and 67 (satellites of Saturn). We therefore first compute the velocities of these four arbors:

$$\mathrm{V}^0_{98} = \mathrm{V}^0_{83} \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{80}\right) = \mathrm{V}^0_{83} \times \left(-\frac{1}{2}\right) = 2 \tag{27.50}$$
$$\mathrm{P}^0_{98} = \frac{1}{2} \text{ days} = 12 \text{ hours} \tag{27.51}$$
$$\mathrm{V}^0_{85} = \mathrm{V}^0_{83} \times \left(-\frac{45}{83}\right) \times \left(-\frac{27}{89}\right) = \mathrm{V}^0_{83} \times \frac{1215}{7387} = -\frac{4860}{7387} \tag{27.52}$$
$$\mathrm{P}^0_{85} = -\frac{7387}{4860} = -1.5199\ldots \text{ days} \tag{27.53}$$
$$\mathrm{V}^0_{52} = \mathrm{V(h)}^0_{27} \times \left(-\frac{30}{60}\right) \times \left(-\frac{19}{85}\right) \times \left(-\frac{27}{65}\right) \tag{27.54}$$
$$= \mathrm{V(h)}^0_{27} \times \left(-\frac{513}{11050}\right) = -\frac{513}{11050} \tag{27.55}$$
$$\mathrm{P}^0_{52} = -\frac{11050}{513} = -21.5399\ldots \text{ days} \tag{27.56}$$
$$\mathrm{V(h)}^0_{67} = \mathrm{V(h)}^0_{27} \times \left(-\frac{36}{58}\right) \times \left(-\frac{76}{72}\right) = \mathrm{V(h)}^0_{27} \times \frac{19}{29} = \frac{19}{29} \tag{27.57}$$
$$\mathrm{P(h)}^0_{67} = \frac{29}{19} = 1.5263\ldots \text{ days} \tag{27.58}$$

On the arbor 27, Oechslin's plate has a 50-teeth wheel, but this leads to incorrect periods for the satellites. Oechslin suggested in his calculations that

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the 50-teeth wheel be replaced by a 36-teeth wheel. However, if one wished to correct to periods, the 58-teeth wheel with which the 50-teeth wheel meshes should of course also be adapted, be it only for its size. Moreover, this is only one of several possible ways to fix the problem. For instance, the 76-teeth wheel could also have been replaced with a 55-teeth wheel for similar results.

27.3.1 The Sun/Moon daily system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the Earth. The Sun therefore makes one turn *clockwise* around the Earth in one day. There are four separate trains and they are all driven by the input on arbor 98.

For the motion of the Sun on tube 105, we have

$$\mathrm{V}^0_{105} = \mathrm{V}^0_{98} \times \left(-\frac{30}{60}\right) = -1 \tag{27.59}$$

$$\mathrm{P}^0_{105} = -1 \text{ day} \tag{27.60}$$

The motion of the Moon is given by the central arbor 101:

$$\mathrm{V}^0_{101} = \mathrm{V}^0_{98} \times \left(-\frac{21}{23}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{42}{74}\right) \tag{27.61}$$

$$= \mathrm{V}^0_{98} \times \left(-\frac{2058}{4255}\right) = -\frac{4116}{4255} \tag{27.62}$$

$$\mathrm{P}^0_{101} = -\frac{4255}{4116} = -24 \text{ h } 48 \text{ m } 37.7842\ldots \text{ s} \tag{27.63}$$

The same value is given by Oechslin. It is an approximation of the apparent diurnal motion of the Moon, which is also clockwise.

The motion of the sky is given by tube 108:

$$\mathrm{V}^0_{108} = \mathrm{V}^0_{98} \times \left(-\frac{27}{34}\right) \times \left(-\frac{40}{42}\right) \times \left(-\frac{59}{89}\right) \tag{27.64}$$

$$= \mathrm{V}^0_{98} \times \left(-\frac{5310}{10591}\right) = -\frac{10620}{10591} \tag{27.65}$$

$$\mathrm{P}^0_{108} = -\frac{10591}{10620} = -0.9972\ldots \text{ days} = -23 \text{ h } 56 \text{ m } 4.0677\ldots \text{ s} \tag{27.66}$$

This is an approximation of the sidereal day. The same value is given by Oechslin.

Finally, the motion of the lunar nodes is given on tube 104:

$$\mathrm{V}^0_{104} = \mathrm{V}^0_{98} \times \left(-\frac{28}{27}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{43}{83}\right) \tag{27.67}$$

$$= \mathrm{V}^0_{98} \times \left(-\frac{16856}{33615}\right) = -\frac{33712}{33615} \tag{27.68}$$

$$\mathrm{P}^0_{104} = -\frac{33615}{33712} = -0.9971\ldots \text{ days} = -23 \text{ h } 55 \text{ m } 51.4000\ldots \text{ s} \tag{27.69}$$

The same value is given by Oechslin.

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27.3.2 The satellites of Jupiter

This system is driven by the input on arbor 85.

The velocities of the four satellites are:

$$\mathrm{V}^0_{86} = \mathrm{V}^0_{85} \times \left(-\frac{61}{71}\right) = \left(-\frac{4860}{7387}\right) \times \left(-\frac{61}{71}\right) = \frac{296460}{524477} \tag{27.70}$$

$$\mathrm{P}^0_{86} = \frac{524477}{296460} = 1.7691\ldots \text{ days (Io)} \tag{27.71}$$

$$\mathrm{V}^0_{89} = \mathrm{V}^0_{85} \times \left(-\frac{42}{49}\right) \times \left(-\frac{27}{46}\right) \times \left(-\frac{57}{67}\right) = \mathrm{V}^0_{85} \times \left(-\frac{4617}{10787}\right) \tag{27.72}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{4617}{10787}\right) = \frac{22438620}{79683569} \tag{27.73}$$

$$\mathrm{P}^0_{89} = \frac{79683569}{22438620} = 3.5511\ldots \text{ days (Europa)} \tag{27.74}$$

$$\mathrm{V}^0_{92} = \mathrm{V}^0_{85} \times \left(-\frac{44}{53}\right) \times \left(-\frac{22}{37}\right) \times \left(-\frac{34}{79}\right) = \mathrm{V}^0_{85} \times \left(-\frac{32912}{154919}\right) \tag{27.75}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{32912}{154919}\right) = \frac{159952320}{1144386653} \tag{27.76}$$

$$\mathrm{P}^0_{92} = \frac{1144386653}{159952320} = 7.1545\ldots \text{ days (Ganymede)} \tag{27.77}$$

$$\mathrm{V}^0_{95} = \mathrm{V}^0_{85} \times \left(-\frac{26}{53}\right) \times \left(-\frac{22}{48}\right) \times \left(-\frac{32}{79}\right) = \mathrm{V}^0_{85} \times \left(-\frac{1144}{12561}\right) \tag{27.78}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{1144}{12561}\right) = \frac{1853280}{30929369} \tag{27.79}$$

$$\mathrm{P}^0_{95} = \frac{30929369}{1853280} = 16.6889\ldots \text{ days (Callisto)} \tag{27.80}$$

The same values are given by Oechslin. These are good approximations of the actual periods of the satellites. The satellites all move counterclockwise as they should. Note however that the 26-teeth wheel used in Callisto's train is mistakenly marked as having 72 teeth in Oechslin's drawing, but his calculations are correct.

The 1791 machine (Oechslin 10.1) uses the same ratios.

27.3.3 The satellites of Saturn

This system is driven by the input on arbor 67. I am here assuming that the motion of arbor 67 is the one taking into account the correction suggested by Oechslin, but other corrections are possible.

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With this caveat, the velocities of the five satellites are then:

$$\mathrm{V}^0_{68} = \mathrm{V(h)}^0_{67} \times \left(-\frac{38}{47}\right) = \frac{19}{29} \times \left(-\frac{38}{47}\right) = -\frac{722}{1363} \tag{27.81}$$

$$\mathrm{P}^0_{68} = -\frac{1363}{722} = -1.8878\ldots \text{ days (Tethys)} \tag{27.82}$$

$$\mathrm{V}^0_{71} = \mathrm{V(h)}^0_{67} \times \left(-\frac{21}{26}\right) \times \left(-\frac{18}{21}\right) \times \left(-\frac{33}{41}\right) = \mathrm{V(h)}^0_{67} \times \left(-\frac{297}{533}\right) \tag{27.83}$$

$$= \frac{19}{29} \times \left(-\frac{297}{533}\right) = -\frac{5643}{15457} \tag{27.84}$$

$$\mathrm{P}^0_{71} = -\frac{15457}{5643} = -2.7391\ldots \text{ days (Dione)} \tag{27.85}$$

$$\mathrm{V}^0_{74} = \mathrm{V(h)}^0_{67} \times \left(-\frac{38}{62}\right) \times \left(-\frac{38}{44}\right) \times \left(-\frac{30}{47}\right) = \mathrm{V(h)}^0_{67} \times \left(-\frac{5415}{16027}\right) \tag{27.86}$$

$$= \frac{19}{29} \times \left(-\frac{5415}{16027}\right) = -\frac{102885}{464783} \tag{27.87}$$

$$\mathrm{P}^0_{74} = -\frac{464783}{102885} = -4.5175\ldots \text{ days (Rhea)} \tag{27.88}$$

$$\mathrm{V}^0_{77} = \mathrm{V(h)}^0_{67} \times \left(-\frac{37}{39}\right) \times \left(-\frac{12}{29}\right) \times \left(-\frac{10}{41}\right) = \mathrm{V(h)}^0_{67} \times \left(-\frac{1480}{15457}\right) \tag{27.89}$$

$$= \frac{19}{29} \times \left(-\frac{1480}{15457}\right) = -\frac{28120}{448253} \tag{27.90}$$

$$\mathrm{P}^0_{77} = -\frac{448253}{28120} = -15.9407\ldots \text{ days (Titan)} \tag{27.91}$$

$$\mathrm{V}^0_{81} = \mathrm{V(h)}^0_{67} \times \left(-\frac{13}{30}\right) \times \left(-\frac{8}{46}\right) \times \left(-\frac{12}{47}\right) = \mathrm{V(h)}^0_{67} \times \left(-\frac{104}{5405}\right) \tag{27.92}$$

$$= \frac{19}{29} \times \left(-\frac{104}{5405}\right) = -\frac{1976}{156745} \tag{27.93}$$

$$\mathrm{P}^0_{81} = -\frac{156745}{1976} = -79.3243\ldots \text{ days (Iapetus)} \tag{27.94}$$

The same values are given by Oechslin.

In this system, the satellites move clockwise, but they should actually move counterclockwise. The 1791 machine (Oechslin 10.1) uses the same ratios, but the satellites move correctly.

27.3.4 The Sun/Moon annual system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the zodiac. There are four separate trains and they are all driven by the input on arbor 52.

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The motion of the Sun is that of tube 65. We have

$$V^0_{65} = V^0_{52} \times \left(-\frac{20}{74}\right) \times \left(-\frac{67}{69}\right) \times \left(-\frac{20}{89}\right) \tag{27.95}$$

$$= V^0_{52} \times \left(-\frac{13400}{227217}\right) = \left(-\frac{513}{11050}\right) \times \left(-\frac{13400}{227217}\right) \tag{27.96}$$

$$= \frac{45828}{16738319} \tag{27.97}$$

$$P^0_{65} = \frac{16738319}{45828} = 365.2421\ldots \text{ days} \tag{27.98}$$

The same value is given by Oechslin.

The motion of the Moon is that of the central arbor 55. We have

$$V^0_{55} = V^0_{52} \times \left(-\frac{22}{23}\right) \times \left(-\frac{28}{29}\right) \times \left(-\frac{35}{41}\right) = V^0_{52} \times \left(-\frac{21560}{27347}\right) \tag{27.99}$$

$$= \left(-\frac{513}{11050}\right) \times \left(-\frac{21560}{27347}\right) = \frac{1106028}{30218435} \tag{27.100}$$

$$P^0_{55} = \frac{30218435}{1106028} = 27.3215\ldots \text{ days} \tag{27.101}$$

The same value is given by Oechslin.

The motion of the lunar nodes is that of tube 59. We have

$$V^0_{59} = V^0_{52} \times \left(-\frac{43}{57}\right) \times \left(-\frac{10}{72}\right) \times \left(-\frac{10}{75}\right) \times \left(-\frac{22}{97}\right) \tag{27.102}$$

$$= V^0_{52} \times \frac{473}{149283} \tag{27.103}$$

$$= \left(-\frac{513}{11050}\right) \times \frac{473}{149283} = -\frac{473}{3215550} \tag{27.104}$$

$$P^0_{59} = -\frac{3215550}{473} = -6798.2029\ldots \text{ days} \tag{27.105}$$

This value is negative because the lunar nodes are retrograding. The same value is given by Oechslin.

Finally, the motion of the lunar apsides is that of tube 62. We have

$$V^0_{62} = V^0_{52} \times \left(-\frac{19}{82}\right) \times \left(-\frac{10}{79}\right) \times \left(-\frac{20}{88}\right) = V^0_{52} \times \left(-\frac{475}{71258}\right) \tag{27.106}$$

$$= \left(-\frac{513}{11050}\right) \times \left(-\frac{475}{71258}\right) = \frac{9747}{31496036} \tag{27.107}$$

$$P^0_{62} = \frac{31496036}{9747} = 3231.3569\ldots \text{ days} \tag{27.108}$$

or about 8.85 years. This is the period of precession of the lunar apsides. The same value is given by Oechslin.

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27.4 The central orrery

This system is driven by the input on arbor 28 whose motion is derived from the motion of arbor 27. I am assuming that arbor 27 makes one turn in a day:

$$\mathrm{V}^0_{28} = \mathrm{V(h)}^0_{27} \times \left(-\frac{20}{40}\right) = -\frac{1}{2} \tag{27.109}$$

This arbor makes one turn in two days. The motion of this arbor is used to obtain the motion of arbor 30:

$$\mathrm{V}^0_{30} = \mathrm{V}^0_{28} \times \left(-\frac{10}{51}\right) \times \left(-\frac{25}{87}\right) = \mathrm{V}^0_{28} \times \frac{250}{4437} = -\frac{125}{4437} \tag{27.110}$$

This motion is then used to derive the motions of all the planets.

First, the motion of Mercury is given by the central arbor 31 whose velocity is

$$\mathrm{V}^0_{31} = \mathrm{V}^0_{30} \times \left(-\frac{46}{114}\right) \tag{27.111}$$

$$= \left(-\frac{125}{4437}\right) \times \left(-\frac{23}{57}\right) = \frac{2875}{252909} \tag{27.112}$$

$$\mathrm{P}^0_{31} = \frac{252909}{2875} = 87.9683\ldots \text{ days} \tag{27.113}$$

The same value is given by Oechslin. It is a good approximation of the orbit period of Mercury, and it turns counterclockwise as it should.

The motions of the other planets are computed similarly. Venus is on tube 34, but we first compute the motion of arbor 33, which is also useful for Mars. We have:

$$\mathrm{V}^0_{33} = \mathrm{V}^0_{30} \times \left(-\frac{59}{67}\right) \times \left(-\frac{37}{75}\right) \tag{27.114}$$

$$= \mathrm{V}^0_{30} \times \frac{2183}{5025} = \left(-\frac{125}{4437}\right) \times \frac{2183}{5025} = -\frac{10915}{891837} \tag{27.115}$$

$$\mathrm{V}^0_{34} = \mathrm{V}^0_{33} \times \left(-\frac{32}{88}\right) = \left(-\frac{10915}{891837}\right) \times \left(-\frac{32}{88}\right) = \frac{43660}{9810207} \tag{27.116}$$

$$\mathrm{P}^0_{34} = \frac{9810207}{43660} = 224.6955\ldots \text{ days} \tag{27.117}$$

The same value is given by Oechslin.

For the Earth which is on tube 46, we first compute the motion of arbor 45

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which is useful later for Jupiter:

$$V^0_{45} = V^0_{30} \times \left(-\frac{33}{59}\right) \times \left(-\frac{29}{76}\right) = V^0_{30} \times \frac{957}{4484} \tag{27.118}$$

$$= \left(-\frac{125}{4437}\right) \times \frac{957}{4484} = -\frac{1375}{228684} \tag{27.119}$$

$$V^0_{46} = V^0_{45} \times \left(-\frac{51}{112}\right) = \left(-\frac{1375}{228684}\right) \times \left(-\frac{51}{112}\right) = \frac{1375}{502208} \tag{27.120}$$

$$P^0_{46} = \frac{502208}{1375} = 365.2421\ldots \text{ days} \tag{27.121}$$

The same value is given by Oechslin.

For Mars which is on tube 37, we first compute the motion of arbor 36 which is reused later in the Saturn train. We have:

$$V^0_{36} = V^0_{33} \times \left(-\frac{42}{62}\right) \times \left(-\frac{22}{71}\right) = V^0_{33} \times \frac{462}{2201} \tag{27.122}$$

$$= \left(-\frac{10915}{891837}\right) \times \frac{462}{2201} = -\frac{1680910}{654311079} \tag{27.123}$$

$$V^0_{37} = V^0_{36} \times \left(-\frac{51}{90}\right) = \left(-\frac{1680910}{654311079}\right) \times \left(-\frac{51}{90}\right) = \frac{168091}{115466661} \tag{27.124}$$

$$P^0_{37} = \frac{115466661}{168091} = 686.9294\ldots \text{ days} \tag{27.125}$$

The same value is given by Oechslin.

For Jupiter, we have:

$$V^0_{49} = V^0_{45} \times \left(-\frac{19}{68}\right) \times \left(-\frac{18}{59}\right) \times \left(-\frac{41}{91}\right) = V^0_{45} \times \left(-\frac{7011}{182546}\right) \tag{27.126}$$

$$= \left(-\frac{1375}{228684}\right) \times \left(-\frac{7011}{182546}\right) = \frac{169125}{732374552} \tag{27.127}$$

$$P^0_{49} = \frac{732374552}{169125} = 4330.3742\ldots \text{ days} \tag{27.128}$$

The same value is given by Oechslin.

For Saturn which is on tube 40, we first compute the motion of arbor 39 which is also used in the train for Uranus:

$$V^0_{39} = V^0_{36} \times \left(-\frac{25}{42}\right) \times \left(-\frac{25}{67}\right) = V^0_{36} \times \frac{625}{2814} \tag{27.129}$$

$$= \left(-\frac{1680910}{654311079}\right) \times \frac{625}{2814} = -\frac{75040625}{131516526879} \tag{27.130}$$

$$V^0_{40} = V^0_{39} \times \left(-\frac{15}{92}\right) = \left(-\frac{75040625}{131516526879}\right) \times \left(-\frac{15}{92}\right) \tag{27.131}$$

$$= \frac{375203125}{4033173490956} \tag{27.132}$$

$$P^0_{40} = \frac{4033173490956}{375203125} = 10749.3067\ldots \text{ days} \tag{27.133}$$

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The same value is given by Oechslin.

Finally, Uranus is on tube 43 and its motion is computed as follows:

$$V^0_{43} = V^0_{39} \times \left(-\frac{12}{73}\right) \times \left(-\frac{47}{78}\right) \times \left(-\frac{59}{82}\right) = V^0_{39} \times \left(-\frac{2773}{38909}\right) \quad (27.134)$$

$$= \left(-\frac{75040625}{131516526879}\right) \times \left(-\frac{2773}{38909}\right) = \frac{208087653125}{5117176544335011} \quad (27.135)$$

$$P^0_{43} = \frac{5117176544335011}{208087653125} = 24591.4472\ldots \text{ days} \quad (27.136)$$

This is a period of about 67.3 years. Oechslin obtains the same value. The actual orbital period of Uranus is about 84 years. Given that the period of Uranus was found to be about 83 years already in 1782, it is possible that Seige's construction is a very early one and based on an early account of the discovery of Uranus (then not yet called Uranus), and this warrants attributing the present machine to about 1781. It is possibly the first machine showing the motion of Uranus. Hahn's *Weltmaschine* in Nuremberg (Oechslin 8.2) also shows Uranus, but with a more accurate period, and it must have been constructed a few years later.

27.5 References

[1] *Album Ossecense, oder Verzeichnis der Mitglieder des Cistercienser-Stiftes Ossegg vom Jahre 1645-1896*. Verlag des Cistercienser-Stiftes Ossegg, 1896.

[2] Friedrich Böttcher. Die Cistercienser und ihre in Böhmen und Sachsen noch bestehenden Stifter. *Zeitschrift für die historische Theologie*, 17:269–335, 1847. [see p. 331 for Seige].

[3] T. E. Faber. Ein Blick nach Böhmen. *Ueberlieferungen zur Geschichte, Literatur und Kunst der Vor- und Mitwelt*, 2:1–24, 1827.

[4] Lucie Hrůšová. *Astronomické funkce a uměleckohistorická analýza planetária P. Engelberta Seige (1791) ve sbírkách Národního technického muzea v Praze*. Master's thesis, Astronomical Institute of Charles University, Prague, 2020. [Astronomical functions and art historical analysis of the planetarium of P. Engelbert Seige (1791) in the collections of the National Technical Museum in Prague].

[5] Radko Kynčl. *Hodiny a hodinky*. Prag: Aventinum, 2001.

[6] Radko Kynčl. *Measuring of Time — Catalogue of the Exhibition*. Prague: National Technical Museum, 2018. [not seen].

[7] Jaroslav Nový. Restaurátorská zpráva k opravě a rekonstrukci Seigeho planetária, 2011. [not seen].

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[8] Ludwig Oechslin. *Astronomische Uhren und Welt-Modelle der Priestermechaniker im 18. Jahrhundert*. Neuchâtel: Antoine Simonin, 1996. [2 volumes and portfolio of plates].

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CH. 27. SEIGE'S METAL ORRERY FRAGMENT IN PRAGUE (C1781?) [O:10.3]

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Chapter 28

(Oechslin: 10.2)

Seige's wood orrery fragment in Prague (c1785?)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

28.1 Introduction

The orrery fragment described here was constructed by Engelbert Seige (1737-1811),[1] probably around 1785. It features Uranus which was discovered in 1781. This fragment is in wood.[2]

Unfortunately, many parts are incomplete, and it is not totally clear how they all fitted together. We can however get a general idea of this machine. The left part seems to have been the going work and provided a base motion. The upper part was to be an orrery showing the motions of the planets Mercury to Uranus. The right part seems to be meant to display a partial orrery, perhaps only for Mercury to Mars, but it is incomplete. Oechslin made a complete drawing, but he only figured out the motions of the upper orrery. We are not doing better, but it is probably possible to find out more about the functions of this fragment.

28.2 The base period

We can see that the motion of arbor 6 is directly transferred to that of the Earth. It must therefore have been meant to make a turn in one tropical year. Assuming this to be about 365.2422 days, we can move backwards to arbor 18

[1] For some biographical information on Seige, see the chapter devoted to Seige's metal fragment.

[2] A description of this fragment was given by Oechslin, see [8, p. 34].

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which should make one turn in two days. Assuming this to be the case

$$\mathrm{V}_{18}^0 = -\frac{1}{2} \tag{28.1}$$

we obtain

$$\mathrm{V}_6^0 = \mathrm{V}_{18}^0 \times \left(-\frac{12}{60}\right) \times \left(-\frac{10}{60}\right) \times \frac{12}{73} = \mathrm{V}_{18}^0 \times \frac{2}{365} = -\frac{1}{365} \tag{28.2}$$

It seems therefore that arbor 6 was meant to make one turn clockwise (seen from above) in 365 days. This is also the period given by Oechslin. The Earth itself is on tube 9 and its velocity is

$$\mathrm{V}_9^0 = \mathrm{V}_6^0 \times \left(-\frac{48}{48}\right) = \frac{1}{365} \tag{28.3}$$

$$\mathrm{P}_9^0 = 365 \text{ days} \tag{28.4}$$

28.3 The planets of the upper orrery

For the other planets, we then have

$$\mathrm{V}_7^0 = \mathrm{V}_6^0 \times \left(-\frac{87}{21}\right) = \left(-\frac{1}{365}\right) \times \left(-\frac{87}{21}\right) = \frac{29}{2555} \tag{28.5}$$

$$\mathrm{P}_7^0 = \frac{2555}{29} = 88.1034\ldots \text{ days (Mercury)} \tag{28.6}$$

$$\mathrm{V}_8^0 = \mathrm{V}_6^0 \times \left(-\frac{60}{37}\right) = \left(-\frac{1}{365}\right) \times \left(-\frac{60}{37}\right) = \frac{12}{2701} \tag{28.7}$$

$$\mathrm{P}_8^0 = \frac{2701}{12} = 225.0833\ldots \text{ days (Venus)} \tag{28.8}$$

$$\mathrm{V}_{10}^0 = \mathrm{V}_6^0 \times \left(-\frac{34}{64}\right) = \left(-\frac{1}{365}\right) \times \left(-\frac{34}{64}\right) = \frac{17}{11680} \tag{28.9}$$

$$\mathrm{P}_{10}^0 = \frac{11680}{17} = 687.0588\ldots \text{ days (Mars)} \tag{28.10}$$

$$\mathrm{V}_{11}^0 = \mathrm{V}_6^0 \times \left(-\frac{7}{83}\right) = \left(-\frac{1}{365}\right) \times \left(-\frac{7}{83}\right) = \frac{7}{30295} \tag{28.11}$$

$$\mathrm{P}_{11}^0 = \frac{30295}{7} = 4327.8571\ldots \text{ days (Jupiter)} \tag{28.12}$$

$$\mathrm{V}_{14}^0 = \mathrm{V}_6^0 \times \left(-\frac{36}{36}\right) \times \left(-\frac{8}{40}\right) \times \left(-\frac{10}{59}\right) = \left(-\frac{1}{365}\right) \times \left(-\frac{2}{59}\right) \tag{28.13}$$

$$= \frac{2}{21535} \tag{28.14}$$

$$\mathrm{P}_{14}^0 = \frac{21535}{2} = 10767.5 \text{ days (Saturn)} \tag{28.15}$$

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$$V_{17}^0 = V_6^0 \times \left(-\frac{20}{40}\right) \times \left(-\frac{10}{50}\right) \times \left(-\frac{10}{83}\right) = \left(-\frac{1}{365}\right) \times \left(-\frac{1}{83}\right) \quad (28.16)$$

$$= \frac{1}{30295} \quad (28.17)$$

$$P_{17}^0 = 30295 \text{ days (about 83 years) (Uranus)} \quad (28.18)$$

Oechslin has put a question mark after the 83-teeth wheel of Uranus, and it isn't clear why, because this wheel does not seem missing. The same periods are given by Oechslin, except for Uranus, where Oechslin used the teeth count 48 instead of 40 in his computations and found therefore the wrong period of 36354 days. I am assuming that the correct value is the one given in Oechslin's drawing.

Given that caveat, the periods obtained by Seige are acceptable, but they are not very accurate.

28.4 References

[1] *Album Ossecense, oder Verzeichnis der Mitglieder des Cistercienser-Stiftes Ossegg vom Jahre 1645-1896*. Verlag des Cistercienser-Stiftes Ossegg, 1896.

[2] Friedrich Böttcher. Die Cistercienser und ihre in Böhmen und Sachsen noch bestehenden Stifter. *Zeitschrift für die historische Theologie*, 17:269–335, 1847. [see p. 331 for Seige].

[3] T. E. Faber. Ein Blick nach Böhmen. *Ueberlieferungen zur Geschichte, Literatur und Kunst der Vor- und Mitwelt*, 2:1–24, 1827.

[4] Lucie Hrůšová. *Astronomické funkce a uměleckohistorická analýza planetária P. Engelberta Seige (1791) ve sbírkách Národního technického muzea v Praze*. Master's thesis, Astronomical Institute of Charles University, Prague, 2020. [Astronomical functions and art historical analysis of the planetarium of P. Engelbert Seige (1791) in the collections of the National Technical Museum in Prague].

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[6] Radko Kynčl. *Measuring of Time — Catalogue of the Exhibition*. Prague: National Technical Museum, 2018. [not seen].

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CH. 28. SEIGE'S WOOD ORRERY FRAGMENT IN PRAGUE (C1785?) [O:10.2]

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Chapter 29

(Oechslin: 10.1)

Seige's *Weltmaschine* in Prague (1791)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

29.1 Introduction

The machine described here was constructed by Engelbert Seige (1737-1811)[1] and is dated 1791.

According to Hrůšová, this machine was first located in the library of the Osek monastery, Czech Republic (at least around 1827), and transferred in 1847 to the room of natural sciences [4, 2].

It was restored in 1885 by Richard Swoboda and Hrůšová believes that Swoboda added the globes on each side of the clock. I find this statement doubtful.

Seige's machine was studied by Ludwig Oechslin around 1990 and later restored by Jaroslav Nový and included in the permanent exhibition of the technical museum in Prague [8].[2]

It is the most complex machine we know of Seige. It is however not that complex. It only happens that there are many dials, multiple celestial spheres, of which two are on sides. But what also adds to the difficulty of understanding this machine is that Oechslin's plan takes some time to assimilate, because there are many gear trains, and that the gears concerning some dials are not located on the plan as they are in reality. This is therefore perhaps the machine where this companion analysis will be the most needed.

[1] For some biographical information on Seige, see the chapter devoted to Seige's metal fragment.

[2] For a description of this machine, see [9, p. 34]. It was also briefly described by King [5, p. 243-244].

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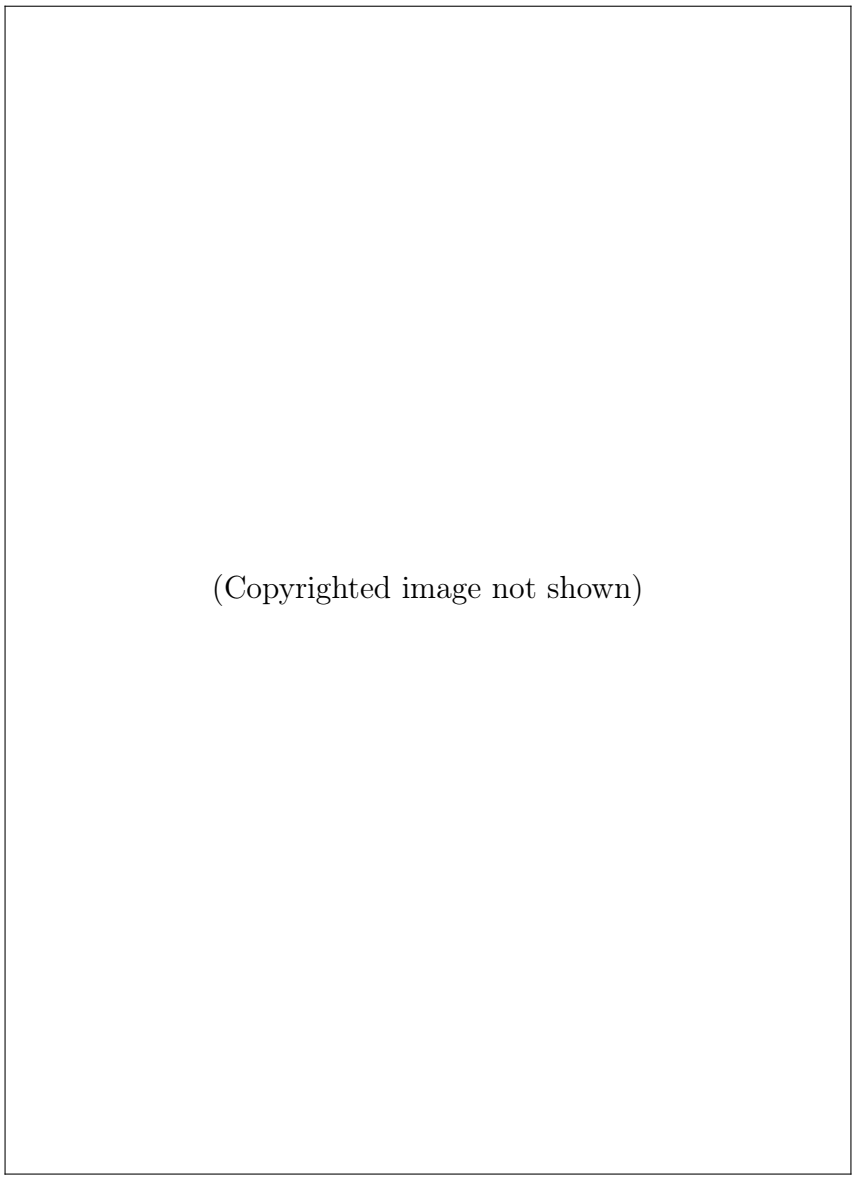


Figure 29.1: Seige's machine (from Hrůšová [4]).

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The machine is made of a cubic base which is surmounted by a central orrery encased in a kind of armillary sphere, and this orrery is surrounded by four smaller systems, like in the metal fragment (Oechslin 10.3). These smaller systems are also encased in armillary spheres. The remaining metal fragment is probably a prototype of this machine.

Two of the four faces of the base are sided by spheres within armillary spheres. One of these spheres shows the shadow of the Earth on a yearly basis. This side also contains seven smaller dials and an inscription plate.

The other sphere shows the Sun/Moon/Earth daily system. This side also contains seven smaller dials and an inscription plate. The small dials of these two opposite sides are clearly inspired by those of Frater David's 1769 clock (Oechslin 6.1). Seige must have seen this clock, and decided to copy its dials. This is not a mere hypothesis, when one considers the dial for the epacts.

The two other faces are simpler dials: the annual dial and the shadow of the Earth daily dial.

In my analysis below, I normally take the vantage point of the outside of the dials, but on the plate this can mean above, or below, or sideways. For the upper parts, the top of the plate coincides with the top of the systems.

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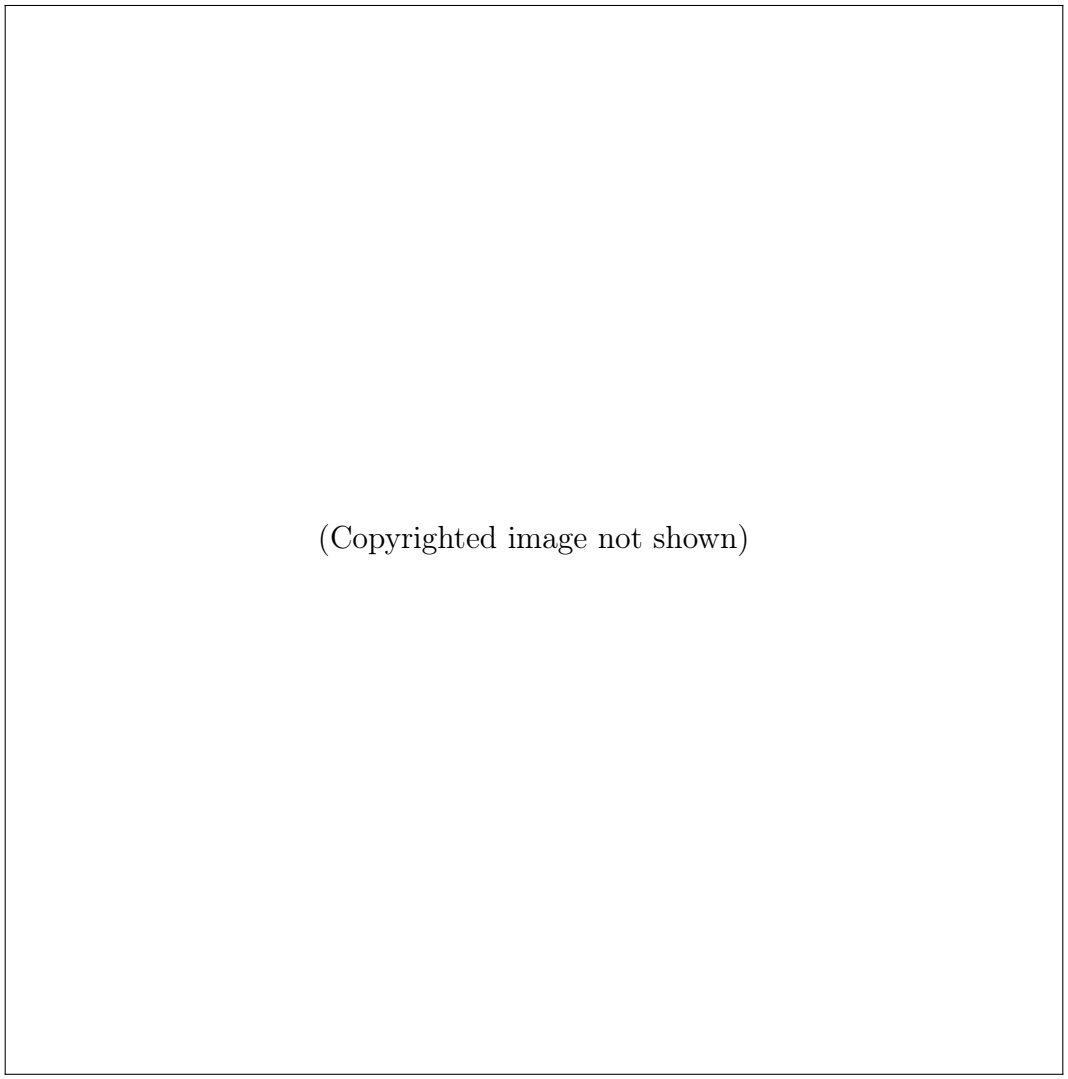


Figure 29.2: The inside of the machine seen from above. We can clearly see the two vertical arbors leading to the upper systems. (from Hrůšová [4]).

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29.2 The going work

The clock is weight-driven and regulated by a pendulum. The drum on arbor 1 makes one turn on two days. Its velocity, measured from the right on Oechslin's plate, is:

$$\mathrm{V}_1^0 = \frac{1}{2} \tag{29.1}$$

$$\mathrm{P}_1^0 = 2 \text{ days} \tag{29.2}$$

The train to the escape wheel goes through arbors 2 and 3, as well as two unnumbered arbors which I am naming x and y. We have:

$$\mathrm{V}_2^0 = \mathrm{V}_1^0 \times \left(-\frac{80}{10}\right) = -4 \tag{29.3}$$

$$\mathrm{P}_2^0 = -\frac{1}{4} = -6 \text{ hours} \tag{29.4}$$

$$\mathrm{V}_3^0 = \mathrm{V}_2^0 \times \left(-\frac{60}{10}\right) = 24 \tag{29.5}$$

$$\mathrm{P}_3^0 = \frac{1}{24} = 1 \text{ hour} \tag{29.6}$$

$$\mathrm{V}_x^0 = \mathrm{V}_3^0 \times \left(-\frac{60}{6}\right) = -240 \tag{29.7}$$

$$\mathrm{P}_x^0 = -6 \text{ minutes} \tag{29.8}$$

$$\mathrm{V}_y^0 = \mathrm{V}_x^0 \times \frac{48}{8} = -1440 \tag{29.9}$$

$$\mathrm{P}_y^0 = -\frac{1}{1440} = -1 \text{ minute} \tag{29.10}$$

The 30-teeth wheel on arbor y is the escape wheel and its velocity is measured from above. It makes one turn in a minute and the pendulum makes a half-oscillation in one second.

Arbors 1 and 3 drive the Earth shadow system. Arbors 2 and 3 drive the Sun/Earth/Moon system on the opposite side.

Arbors 97 and 146 drive the upper part of the machine, both motions being derived from that of arbor 3.

29.3 The four sides of the base

29.3.1 The annual dial

This dial shows the yearly motion of the Sun, the Moon, the lunar nodes, and the lunar apsides The dial pictures a calendar, with the days and months running clockwise. Consequently, Seige chose to have the hands for the Sun, the Moon, the lunar nodes and the lunar apsides move clockwise. The long

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double hand is that of the lunar nodes and pictures the head and tail of a dragon.

The calendar is actually shown on four concentric rings. Each ring contains 365.25 days and the alternating black and white cells shift from one ring to the next. The first ring is for the first year in a 4-year cycle, the second ring is for the second year, and so on. There are a total of 1461 cells.

Within the months, a number of saints are shown. And the inner ring shows the various sundays and the moving feasts. This ring can be set each year so that Easter falls on the right date.

This dial is driven by arbor 80. The motion originates from arbor 2. If we measure the motion of arbor 80 from the left and arbor 17 from the above (on Oechslin's plan), we have:

$$\mathrm{V}_{17}^0 = \mathrm{V}_2^0 \times \frac{30}{48} = -\frac{5}{2} \tag{29.11}$$

$$\mathrm{V}_{80}^0 = \mathrm{V}_{17}^0 \times \frac{16}{40} = \left(-\frac{5}{2}\right) \times \frac{2}{5} = -1 \tag{29.12}$$

$$\mathrm{P}_{80}^0 = -1 \text{ day} \tag{29.13}$$

All the remaining motions are measured from the front of the dial, that is from the left on Oechslin's drawing.

The motion of the Sun is that of tube 88. We first compute the velocity of arbor 83.

$$\mathrm{V}_{83}^0 = \mathrm{V}_{80}^0 \times \left(-\frac{30}{30}\right) \times \left(-\frac{10}{53}\right) \times \left(-\frac{14}{64}\right) = \mathrm{V}_{80}^0 \times \left(-\frac{35}{848}\right) \tag{29.14}$$

$$= \frac{35}{848} \tag{29.15}$$

$$\mathrm{V}_{85}^0 = \mathrm{V}_{83}^0 \times \left(-\frac{20}{62}\right) \times \left(-\frac{25}{37}\right) = \mathrm{V}_{83}^0 \times \frac{250}{1147} = \frac{35}{848} \times \frac{250}{1147} \tag{29.16}$$

$$= \frac{4375}{486328} \tag{29.17}$$

$$\mathrm{V}_{88}^0 = \mathrm{V}_{85}^0 \times \left(-\frac{21}{69}\right) = \frac{4375}{486328} \times \left(-\frac{7}{23}\right) = -\frac{30625}{11185544} \tag{29.18}$$

$$\mathrm{P}_{88}^0 = -\frac{11185544}{30625} = -365.2422 \text{ days} \tag{29.19}$$

This is an approximation of the tropical year.[3] The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

The motion of the lunar apsides is that of tube 91. We first compute the motion of arbor 90 which is also the basis of the train for the lunar nodes. We

[3]Incidentally, Oechslin observed that Seige used five different values of the tropical year [9, p. 177]. Another one was used in the metal fragment (Oechslin 10.3).

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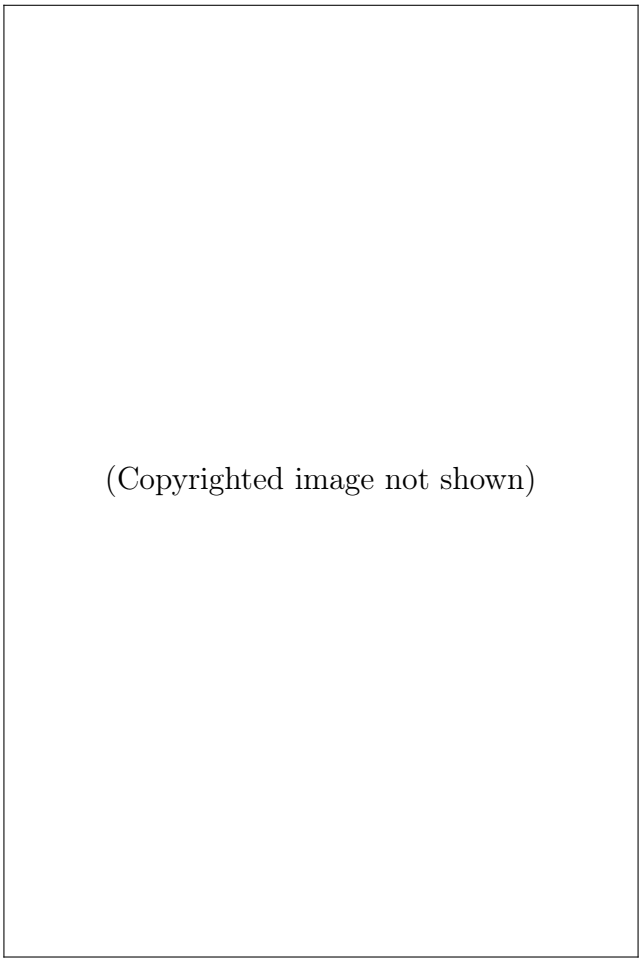


Figure 29.3: The annual dial (from Hrůšová [4]).

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have

$$V^0_{90} = V^0_{85} \times \left(-\frac{19}{60}\right) \times \left(-\frac{19}{53}\right) = V^0_{85} \times \frac{361}{3180} = \frac{4375}{486328} \times \frac{361}{3180} \tag{29.20}$$

$$= \frac{315875}{309304608} \tag{29.21}$$

Then

$$V^0_{91} = V^0_{90} \times \left(-\frac{20}{66}\right) = \frac{315875}{309304608} \times \left(-\frac{20}{66}\right) = -\frac{1579375}{5103526032} \tag{29.22}$$

$$P^0_{91} = -\frac{5103526032}{1579375} = -3231.3579\ldots \text{ days} \tag{29.23}$$

which is about 8.85 years. The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

The motion of the lunar nodes is that of tube 95. We have

$$V^0_{95} = V^0_{90} \times \left(-\frac{23}{22}\right) \times \left(-\frac{24}{38}\right) \times \left(-\frac{29}{41}\right) \times \left(-\frac{33}{107}\right) \tag{29.24}$$

$$= V^0_{90} \times \frac{12006}{83353} \tag{29.25}$$

$$= \frac{315875}{309304608} \times \frac{12006}{83353} = \frac{33266625}{226153219216} \tag{29.26}$$

$$P^0_{95} = \frac{226153219216}{33266625} = 6798.2014\ldots \text{ days} \tag{29.27}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

Finally, the mean motion of the Moon is that of frame 84. We have

$$V^0_{84} = V^0_{83} \times \left(-\frac{47}{53}\right) = \frac{35}{848} \times \left(-\frac{47}{53}\right) = -\frac{1645}{44944} \tag{29.28}$$

$$P^0_{84} = -27.3215\ldots \text{ days} \tag{29.29}$$

This is an approximation of the tropical month. The same value is given by Oechslin. The same ratio was used in the metal fragment (Oechslin 10.3).

The motion of the Moon is corrected using a cam which is part of tube 91 and moves with the apsides. A feeler on this cam then moves the corrected Moon back and forth, depending on the position of the Moon with respect to the line of apsides.

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29.3.2 The daily Earth shadow system

This dial is opposite to the one for the yearly system described in the previous section. It shows the Sun and the shadow of the Earth moving around a fixed Earth.

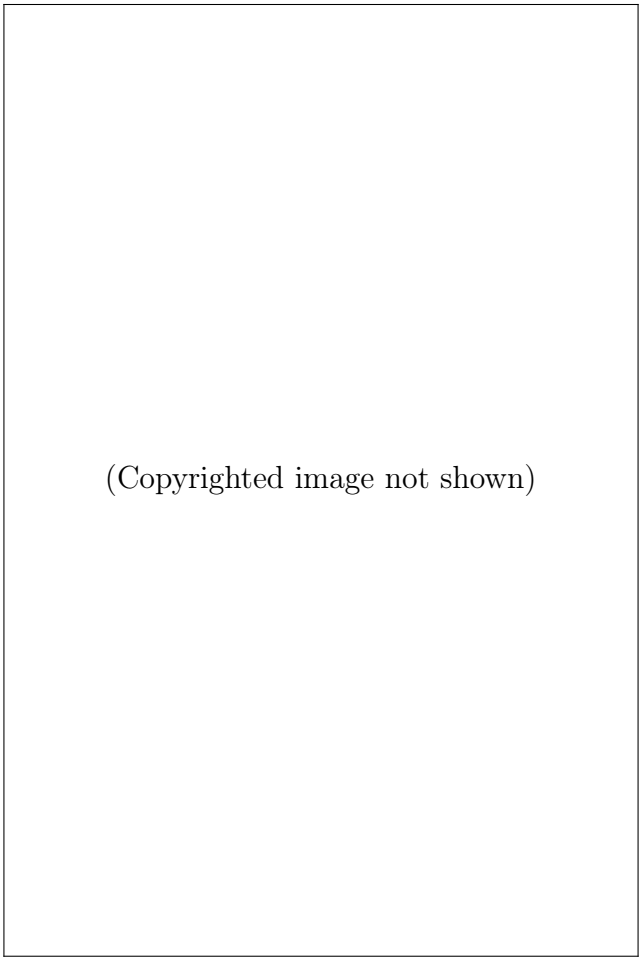


Figure 29.4: The daily Earth shadow system (from Hrůšová [4]).

This system is driven by arbor 78. This motion also originates from arbor 2, with a similar train as in the case of arbor 80. We measure the motion from the side of the dial:

$$\mathrm{V}_{78}^{0} = \mathrm{V}_{2}^{0} \times \left(-\frac{30}{48}\right) \times \frac{16}{40} = \mathrm{V}_{2}^{0} \times \left(-\frac{1}{4}\right) = -4 \times \left(-\frac{1}{4}\right) = 1 \tag{29.30}$$

$$\mathrm{P}_{78}^{0} = 1 \text{ day} \tag{29.31}$$

The only difference with arbor 80 is that the motion is in the opposite direction.

The Sun and the shadow of the Earth move clockwise with frame 79 and

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makes a turn in one day. We have

$$\mathrm{V}^0_{79} = \mathrm{V}^0_{78} \times \left(-\frac{96}{96}\right) = -1 \tag{29.32}$$

The shadow always goes through the North pole and the inclination of the Earth was not taken into account in this dial.[4]

[4]See also [9, p. 149] on this construction.

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29.3.3 The side of the yearly Earth shadow system

This side has seven dials around a central horizontal arbor leading to an armillary sphere containing the Earth shadow system.

29.3.3.1 The yearly Earth shadow system

In this system, the horizontal axis going through the armillary sphere represents the rotation axis of the Earth. The Earth has its North pole towards the center of the machine, and the South pole towards the outside. It makes one turn in a day. It seems that the shadow of the Earth as well as the Sun are pivoting as a consequence of a cam that rotates in one year. The Sun does not rotate around the Earth, but it moves either to the Northern latitudes or to the Southern latitudes, and the shadow of the Earth is always opposite to the Sun.

The motion of this system is derived from those of arbors 1 and 3. There are two tubes, tube 12 and tube 15. The motion of tube 12 is obtained from arbor 3. First, we have the motion of arbor 11, here measured from the outside:

$$\mathrm{V}_{11}^0 = \mathrm{V}_3^0 \times \frac{12}{48} = 24 \times \frac{1}{4} = 6 \tag{29.33}$$

Then

$$\mathrm{V}_{12}^0 = \mathrm{V}_{11}^0 \times \left(-\frac{16}{96}\right) = 6 \times \left(-\frac{1}{6}\right) = -1 \tag{29.34}$$

$$\mathrm{P}_{12}^0 = -1 \tag{29.35}$$

Tube 12 makes a turn clockwise (seen from the outside) in one day. This corresponds to a counterclockwise motion seen from the North pole, which is indeed the motion of the Earth with respect to the Sun.

Similarly, we consider the motion of tube 13, also measured from the outside:

$$\mathrm{V}_{13}^0 = \mathrm{V}_1^0 \times \frac{11}{56} = \frac{1}{2} \times \frac{11}{56} = \frac{11}{112} \tag{29.36}$$

And

$$\mathrm{V}_{15}^0 = \mathrm{V}_{13}^0 \times \left(-\frac{10}{76}\right) \times \left(-\frac{25}{118}\right) = \mathrm{V}_{13}^0 \times \frac{125}{4484} = \frac{11}{112} \times \frac{125}{4484} \tag{29.37}$$

$$= \frac{1375}{502208} \tag{29.38}$$

$$\mathrm{P}_{15}^0 = \frac{502208}{1375} = 365.2421\ldots \text{ days} \tag{29.39}$$

This is an approximation of the tropical year. The same value is given by Oechslin.

Tube 15 also carries the equation of time cam which is used to show the true time in the time dial below.

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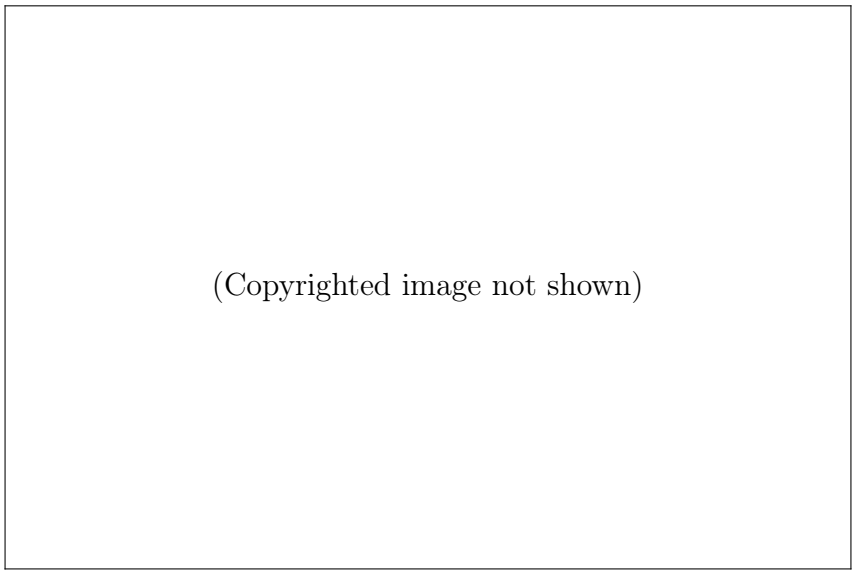


Figure 29.5: The yearly Earth shadow system (from Hrůšová [4]).

29.3.3.2 The other dials

There are seven dials and an inscription plate. This inscription plate recalls the restoration by Richard Swoboda in 1885.

From the bottom and going clockwise, we have 1) the inscription plate, 2) the day of the week, 3) the age of the Moon, 4) the golden number, the epacts and the indiction, 5) the time, 6) the solar cycle and the dominical letter, 7) a dial for the anomalistic month, and 8) a dial for the draconic month. However, it seems that some dials have been mixed up, as they do not all correspond to the gears which are underneath. I will however consider the dials in the apparent order, not in the order of the gears underneath.

Except for the time dial, all of the motions of the dials are derived from that of arbor 18 which is obtained from the motion of arbor 2. The motion of the time dial is instead derived from the motion of arbor 3.

We have

$$\mathrm{V}_{18}^{0} = \mathrm{V}_{17}^{0} \times \left(-\frac{16}{40}\right) = \left(-\frac{5}{2}\right) \times \left(-\frac{2}{5}\right) = 1 \tag{29.40}$$

This arbor makes one turn counterclockwise (from the outside) in one day.

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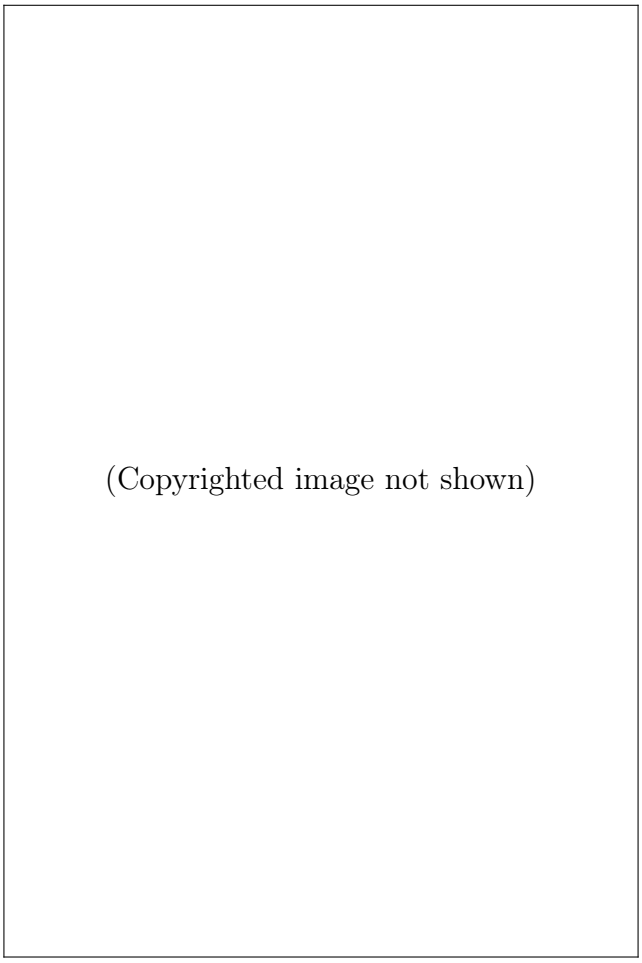


Figure 29.6: The dials of the yearly Earth shadow system (from Hrůšová [4]).

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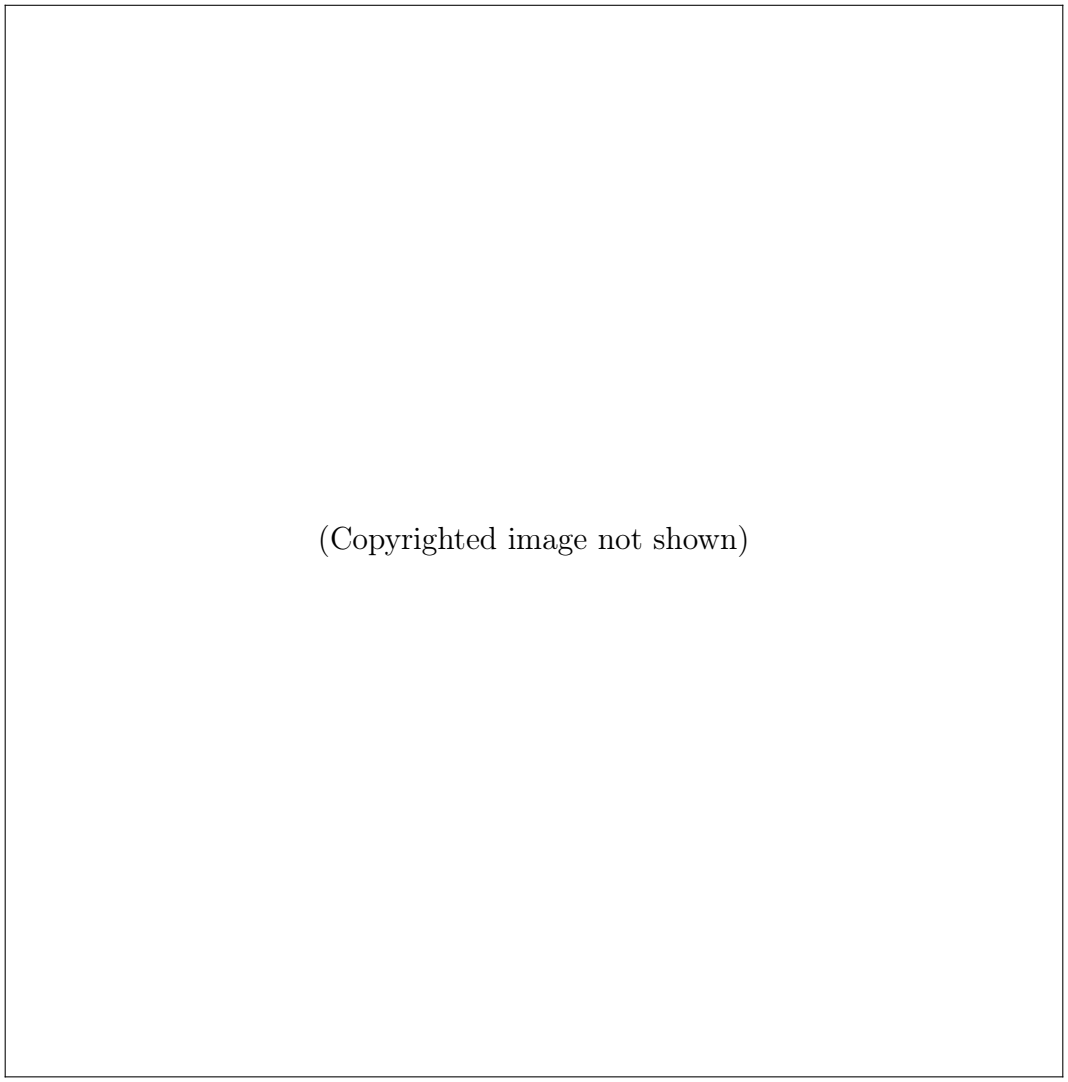


Figure 29.7: The gears for the dials of the yearly Earth shadow system. Note however that the dials do not perfectly correspond to the gears. For instance, the gears at the bottom left correspond to the day of the week, but the gears above correspond to the anomalistic month, not to the synodic month, as would be expected. Some of the dials have been misplaced. (from Hrůšová [4]).

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29.3.3.2.1 The inscription plate

The inscription plate records the 1885 restoration of the machine by Richard Swoboda who was teacher in Hrob (Klostergrab in German).

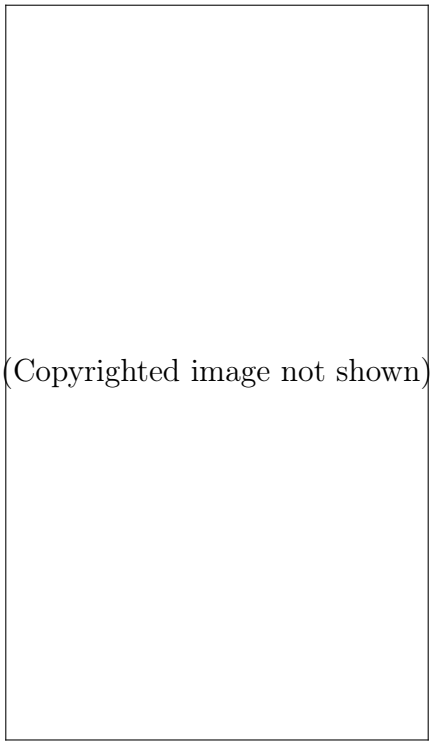


Figure 29.8: The inscription plate (from Hrůšová [4]).

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29.3.3.2.2 The day of the week

The hand of this dial is on arbor 19 and it makes one turn clockwise (from the outside) in seven days:

$$\mathrm{V}_{19}^{0} = \mathrm{V}_{18}^{0} \times \left(-\frac{12}{84}\right) = \mathrm{V}_{18}^{0} \times \left(-\frac{1}{7}\right) = -\frac{1}{7} \tag{29.41}$$

$$\mathrm{P}_{19}^{0} = -7 \text{ days} \tag{29.42}$$

The dial also shows the hours, but only every six hours.

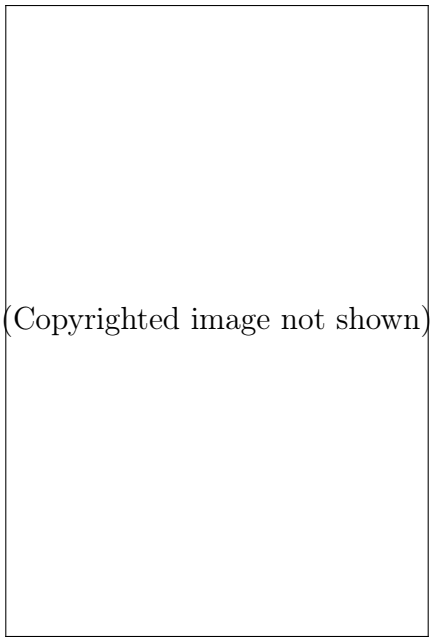


Figure 29.9: The day of the week (from Hrůšová [4]).

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29.3.3.2.3 The age of the Moon

This dial shows the age of the Moon. The hand is carried by arbor 25.

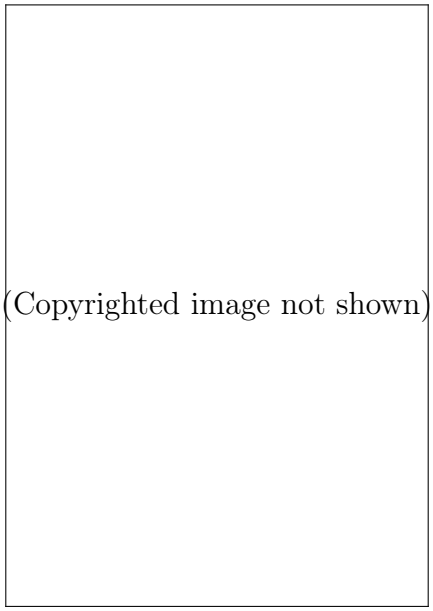


Figure 29.10: The age of the Moon (from Hrůšová [4]).

In order to compute the velocity of this arbor, we first compute the velocity of arbor 21, as it is also used for the anomalistic month:

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{18}^{0} \times \left(-\frac{41}{54}\right) \times \left(-\frac{8}{53}\right) = \mathrm{V}_{18}^{0} \times \frac{164}{1431} = \frac{164}{1431} \tag{29.43}$$

Then

$$\mathrm{V}_{25}^{0} = \mathrm{V}_{21}^{0} \times \left(-\frac{34}{83}\right) \times \left(-\frac{44}{61}\right) \times \left(-\frac{40}{40}\right) = \mathrm{V}_{21}^{0} \times \left(-\frac{1496}{5063}\right) \tag{29.44}$$

$$= \frac{164}{1431} \times \left(-\frac{1496}{5063}\right) = -\frac{245344}{7245153} \tag{29.45}$$

$$\mathrm{P}_{25}^{0} = -\frac{7245153}{245344} = -29.5305\ldots \text{ days} \tag{29.46}$$

The hand makes a turn clockwise in one synodic month. The same value is given by Oechslin.

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29.3.3.2.4 The golden number, the epacts and the indiction

This dial shows the values of the golden number, the epacts and the indiction. There are two series of epacts, the series for the 18th and 19th centuries on the outer circle, and the series for the 20th, 21st and 22nd centuries on the inner circle. This dial was obviously copied from the almost identical dial found in Frater David's clock from 1769 (Oechslin 6.1).

Like for the solar cycle dial, the two hands are driven by the arbor 31 which makes one turn in a year. So, we first consider the motion of arbor 31. This motion is itself derived from the motion of arbor 27 which we also compute, as it is used for the draconic month:

$$\mathrm{V}_{27}^0 = \mathrm{V}_{18}^0 \times \left(-\frac{12}{66}\right) \times \left(-\frac{43}{74}\right) = \mathrm{V}_{18}^0 \times \frac{43}{407} = \frac{43}{407} \tag{29.47}$$

$$\mathrm{V}_{31}^0 = \mathrm{V}_{27}^0 \times \left(-\frac{17}{48}\right) \times \left(-\frac{12}{64}\right) \times \left(-\frac{16}{41}\right) = \mathrm{V}_{27}^0 \times \left(-\frac{17}{656}\right) \tag{29.48}$$

$$= \frac{43}{407} \times \left(-\frac{17}{656}\right) = -\frac{731}{266992} \tag{29.49}$$

$$\mathrm{P}_{31}^0 = -\frac{266992}{731} = -365.2421\ldots \text{ days} \tag{29.50}$$

The same value is given by Oechslin.

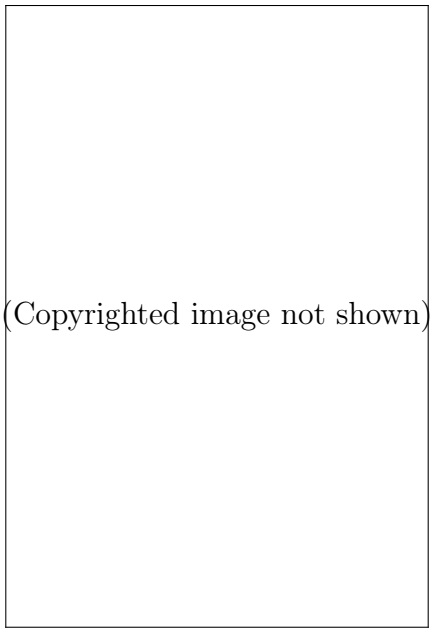


Figure 29.11: The golden number, the epacts and the indiction (from Hrůšová [4]).

An arm on arbor 31 then makes a 19-teeth wheel on arbor 32 advance intermittently by one tooth. This wheel then moves two levers on arbor 35

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which advance by one tooth the 19-teeth wheel on arbor 36 and the 15-teeth wheel on tube 37. The former carries the hand of the golden number and epacts, and the latter carries the hand of the indiction. Both hands move clockwise.

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29.3.3.2.5 The time

The central arbor of this dial is arbor 8. We first obtain the velocity of arbor 5 (measured from the outside):

$$\mathrm{V}_5^0 = \mathrm{V}_3^0 \times \frac{24}{24} = 24 \tag{29.51}$$

Then

$$\mathrm{V}_8^0 = \mathrm{V}_5^0 \times \left(-\frac{24}{24}\right) \times \left(-\frac{36}{36}\right) \times \left(-\frac{40}{40}\right) = -24 \tag{29.52}$$

So, this arbor makes one turn clockwise (from above) in one hour and holds the minute hand.

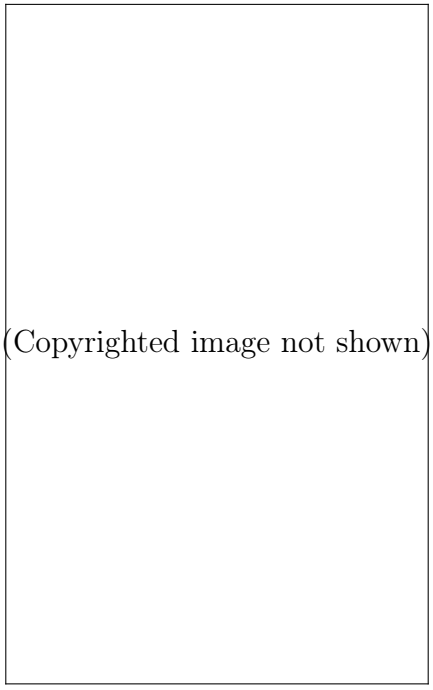


Figure 29.12: The time dial (from Hrůšová [4]).

The motion of tube 10 is derived from it and it makes one turn clockwise in 12 hours:

$$\mathrm{V}_{10}^0 = \mathrm{V}_8^0 \times \left(-\frac{15}{60}\right) \times \left(-\frac{20}{60}\right) = \mathrm{V}_8^0 \times \frac{1}{12} = -2 \tag{29.53}$$

Finally, there is a moving hour minute dial which moves back and forth using a rack with a feeler on the equation cam on the tube 15 described earlier. This rack meshes with a 24-teeth wheel on the tube carrying the moving dial. When the minute hand is read on the moving dial, we get the minutes of true time. The hour may need to be corrected. For instance, if the mean time is 1:55 and the minutes of true time show 01 minute, then the true time is actually 2:01. In other cases, the hours may need to be decreased.

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29.3.3.2.6 The solar cycle and the dominical letter

This dial shows the values of the solar cycle and the dominical letter. Its hand makes a turn clockwise in 28 years.

We have seen above that the 19-teeth wheel on arbor 32 advances intermittently by one tooth every year. Another lever on arbor 33 transmits this intermittent motion to the 28-teeth wheel on arbor 34. This wheel therefore advances by one tooth every year, and its arbor carries the hand for the solar cycle. The dial also shows the values of the corresponding dominical letters during the 18th century. This correspondence was no longer valid after 1800.

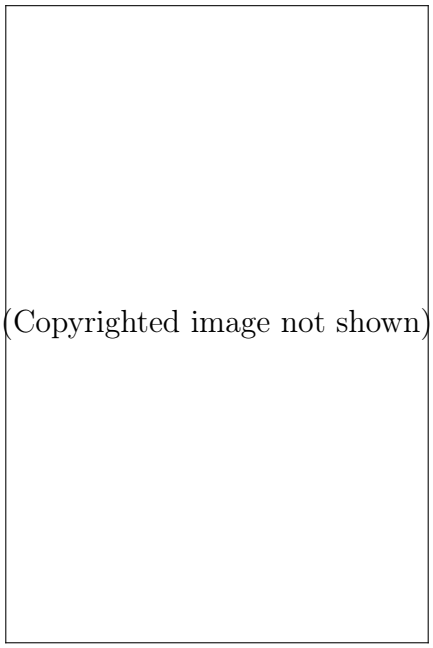


Figure 29.13: The solar cycle and the dominical letter (from Hrůšová [4]).

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29.3.3.2.7 The anomalistic month

This dial shows the number of days since the last mean perigee of the Moon, assuming it was set on 0 on passage of the mean Moon to a mean perigee.

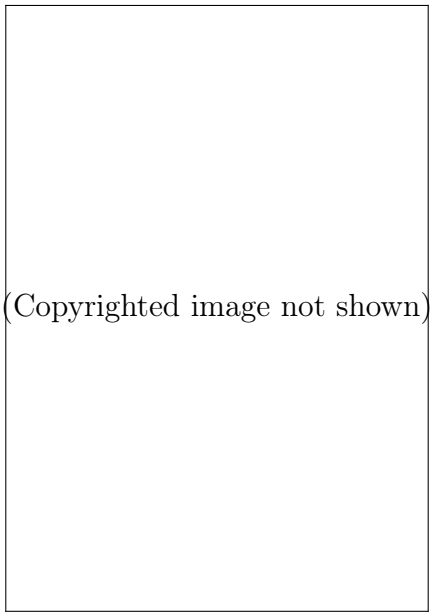


Figure 29.14: The anomalistic month (from Hrůšová [4]).

The hand of this dial is carried by arbor 22. Its motion is derived from the arbor 21 whose velocity was computed above. We have

$$\mathrm{V}^0_{22} = \mathrm{V}^0_{21} \times \left(-\frac{19}{60}\right) = \frac{164}{1431} \times \left(-\frac{19}{60}\right) = -\frac{779}{21465} \tag{29.54}$$

$$\mathrm{P}^0_{22} = -\frac{21465}{779} = -27.5545\ldots \text{ days} \tag{29.55}$$

This is the period of the anomalistic month. The same value is given by Oechslin. The hand makes one turn clockwise in an anomalistic month.

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29.3.3.2.8 The draconic month

This dial seems to show the angle since the last ascending node of the Moon, and therefore its hands on arbor 28 rotates in a draconic month.

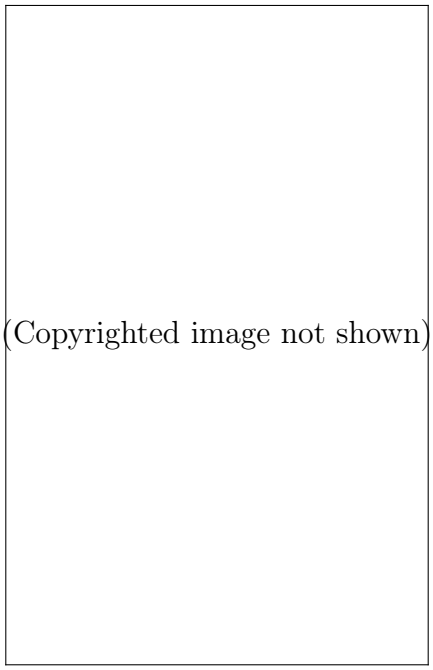


Figure 29.15: The draconic month (from Hrůšová [4]).

We have:

$$\mathrm{V}_{28}^{0} = \mathrm{V}_{27}^{0} \times \left(-\frac{24}{69}\right) = \frac{43}{407} \times \left(-\frac{24}{69}\right) = -\frac{344}{9361} \tag{29.56}$$

$$\mathrm{P}_{28}^{0} = -\frac{9361}{344} = -27.2122\ldots \text{ days} \tag{29.57}$$

The same value is given by Oechslin.

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29.3.4 The side of the Sun/Earth/Moon daily system

This side has seven dials around a central horizontal arbor leading to an armillary sphere containing the Sun/Earth/Moon daily system.

29.3.4.1 The Sun/Earth/Moon daily system

The motion of this system is derived from that of tube 68 which makes one turn in a sidereal day. This motion is itself derived from that of arbor 2. We have

$$\mathrm{V}_{68}^0 = \mathrm{V}_2^0 \times \left(-\frac{45}{89}\right) \times \left(-\frac{59}{119}\right) = \mathrm{V}_2^0 \times \frac{2655}{10591} \tag{29.58}$$

$$= -4 \times \frac{2655}{10591} = -\frac{10620}{10591} \tag{29.59}$$

$$\mathrm{P}_{68}^0 = -\frac{10591}{10620} = -23 \text{ h } 56 \text{ m } 4.0677\ldots \text{ s} \tag{29.60}$$

The same value is given by Oechslin.

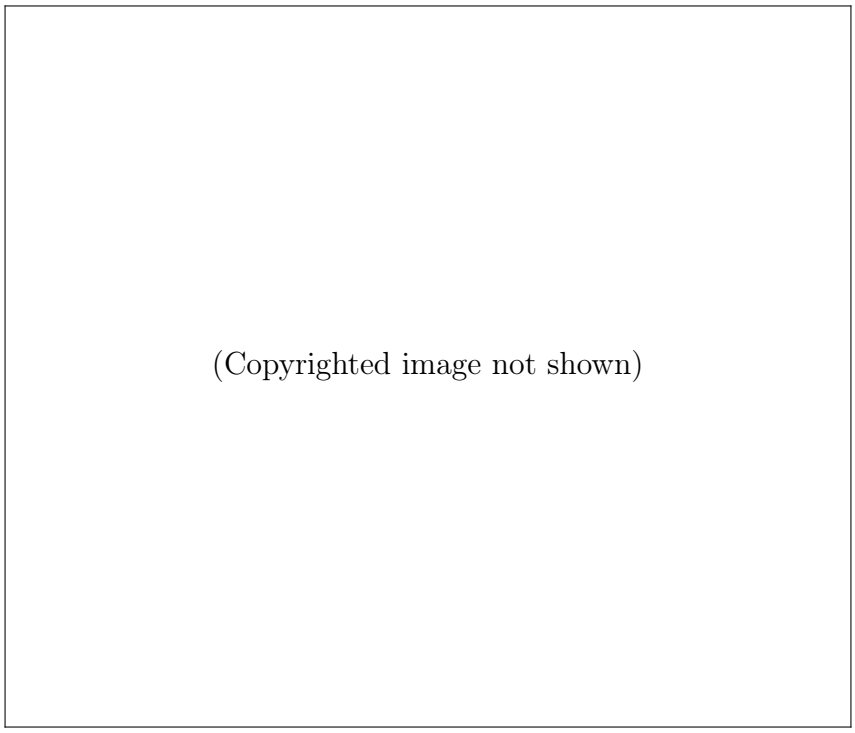


Figure 29.16: The Sun/Earth/Moon system (from Hrůšová [4]).

29.3.4.2 The other dials

There are seven dials and an inscription plate. This inscription plate recalls the construction of the machine in 1791.

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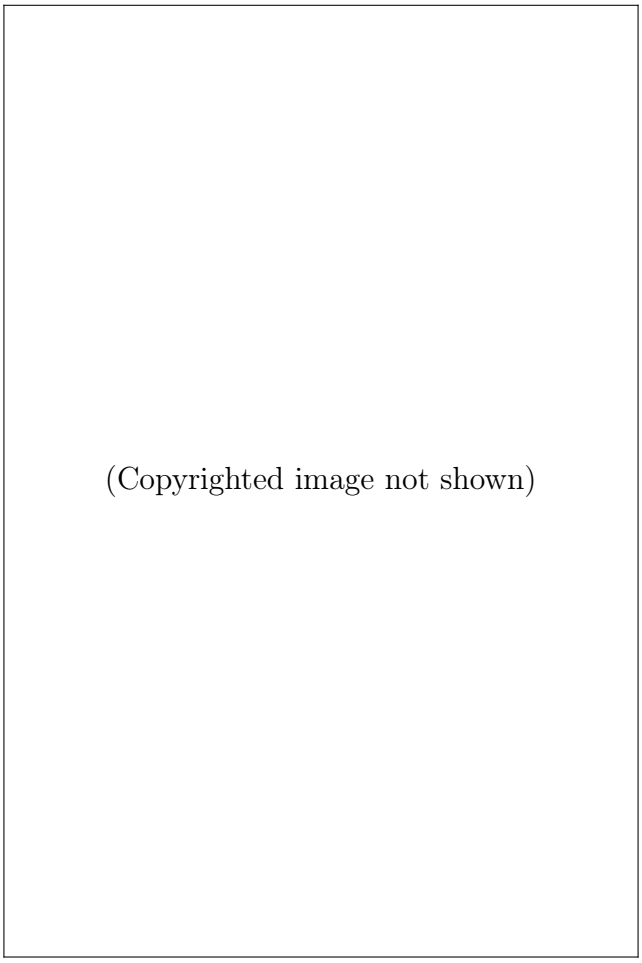


Figure 29.17: The dials of the Sun/Earth/Moon system (from Hrůšová [4]).

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From the bottom and going clockwise, we have 1) the inscription plate, 2) the dial of Venus, 3) the dial of Saturn, 4) the dial of Mars, 5) the mean and true time, 6) the dial of Uranus, 7) the dial of Jupiter, and 8) the dial of Mercury.

The motion of the hours is derived from that of arbor 3. The motions of the other dials are derived from that of arbor 43, which is itself derived from the motion of arbor 2. We have (measured from the side of dials)

$$\mathrm{V}_{43}^0 = \mathrm{V}_2^0 \times \left(-\frac{20}{40}\right) \times \left(-\frac{24}{48}\right) \times \left(-\frac{21}{42}\right) = \mathrm{V}_2^0 \times \left(-\frac{1}{8}\right) \tag{29.61}$$

$$(-4) \times \left(-\frac{1}{8}\right) = \frac{1}{2} \tag{29.62}$$

The central arbor for the hours is arbor 40. Its velocity (also measured from the side of the dial) is

$$\mathrm{V}_{40}^0 = \mathrm{V}_3^0 \times \left(-\frac{24}{48}\right) \times \left(-\frac{16}{48}\right) \times \left(-\frac{20}{80}\right) = \mathrm{V}_3^0 \times \left(-\frac{1}{24}\right) \tag{29.63}$$

$$= 24 \times \left(-\frac{1}{24}\right) = -1 \tag{29.64}$$

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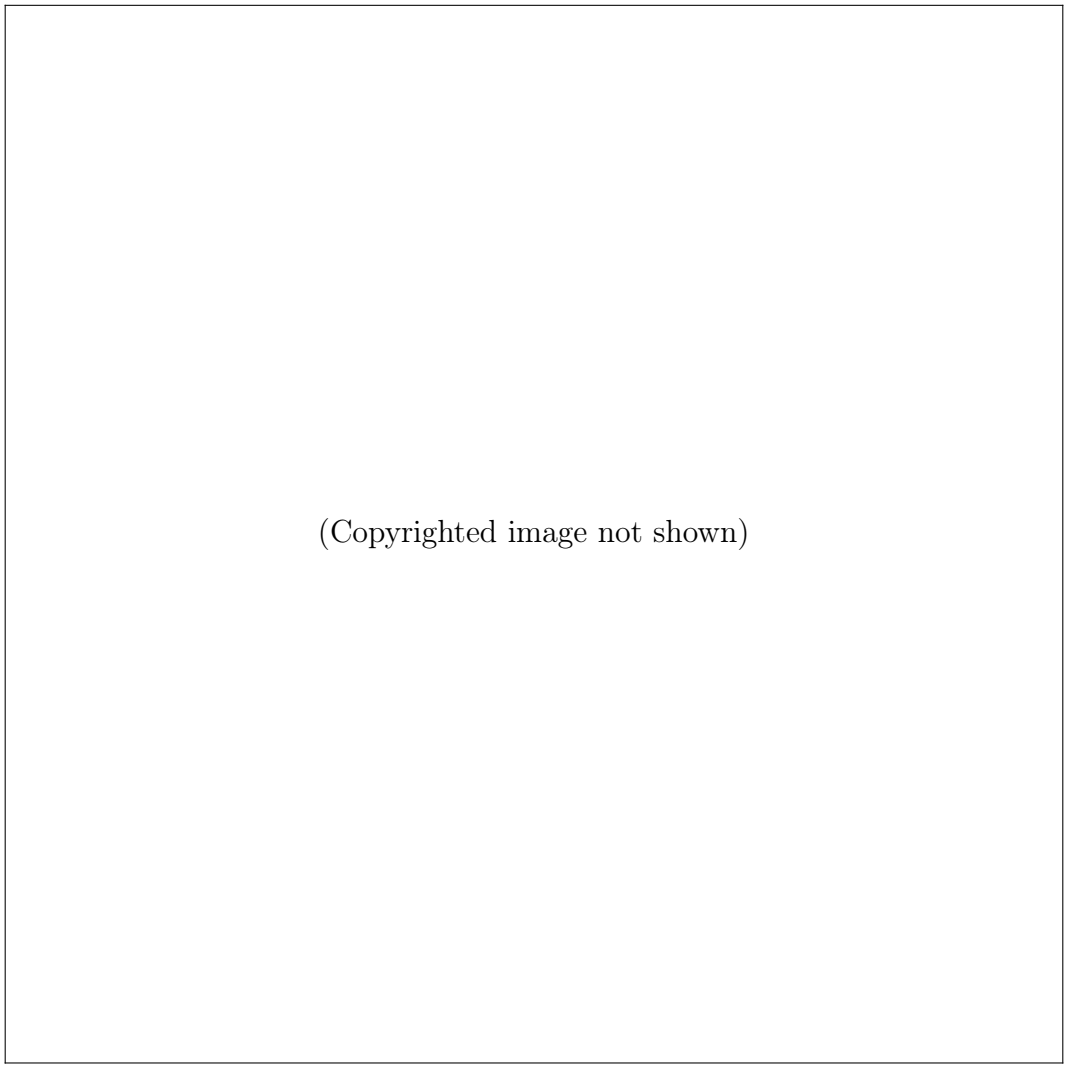


Figure 29.18: The gears for the planet dials (from Hrůšová [4]).

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29.3.4.2.1 The inscription plate

The inscription plate records the construction of the machine in 1791.

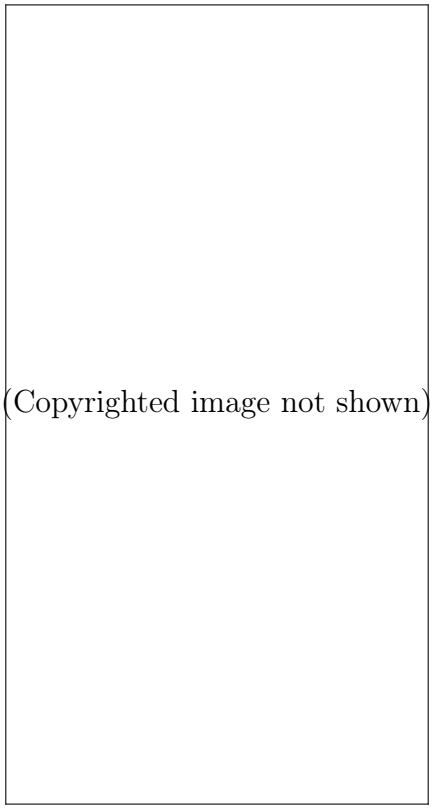


Figure 29.19: The inscription plate (from Hrůšová [4]).

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29.3.4.2.2 The Venus dial

This dial shows the longitude of Venus. The central arbor 51 carries the hand of Venus. We first compute the velocity of arbor 50 which is also used in the trains for Mars and Saturn. We have (measured from the front of the dial) is

$$\mathrm{V}^0_{50} = \mathrm{V}^0_{43} \times \left(-\frac{23}{76}\right) \times \left(-\frac{10}{60}\right) = \mathrm{V}^0_{43} \times \frac{23}{456} = \frac{1}{2} \times \frac{23}{456} = \frac{23}{912} \tag{29.65}$$

$$\mathrm{V}^0_{51} = \mathrm{V}^0_{50} \times \left(-\frac{12}{68}\right) = \frac{23}{912} \times \left(-\frac{12}{68}\right) = -\frac{23}{5168} \tag{29.66}$$

$$\mathrm{P}^0_{51} = -\frac{5168}{23} = -224.6956\ldots \text{ days} \tag{29.67}$$

The same value is given by Oechslin. The hand moves clockwise and makes one turn in the orbital period of Venus.

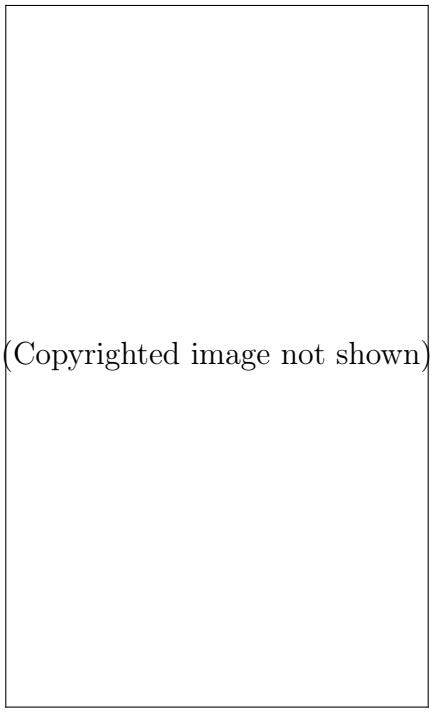


Figure 29.20: The Venus dial (from Hrůšová [4]).

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29.3.4.2.3 The Saturn dial

The trains of Mars and Saturn are both derived from arbor 53, and we first compute the motion of arbor 53. Then, the hand of Saturn is carried by arbor 57.

We have

$$\mathrm{V}^0_{53} = \mathrm{V}^0_{50} \times \left(-\frac{20}{72}\right) \times \left(-\frac{67}{73}\right) = \mathrm{V}^0_{50} \times \frac{335}{1314} \tag{29.68}$$

$$= \frac{23}{912} \times \frac{335}{1314} = \frac{7705}{1198368} \tag{29.69}$$

$$\mathrm{V}^0_{57} = \mathrm{V}^0_{53} \times \left(-\frac{12}{46}\right) \times \left(-\frac{15}{77}\right) \times \left(-\frac{20}{68}\right) = \mathrm{V}^0_{53} \times \left(-\frac{450}{30107}\right) \tag{29.70}$$

$$= \frac{7705}{1198368} \times \left(-\frac{450}{30107}\right) = -\frac{8375}{87147984} \tag{29.71}$$

$$\mathrm{P}^0_{57} = -\frac{87147984}{8375} = -10405.7294\ldots \text{ days} \tag{29.72}$$

This seems a bad approximation of the orbital period of Saturn, as the expected value is about 10747 days, but the same value was obtained by Oechslin.

Now, since the periods of Venus and Mars are very accurate, it is likely that the Saturn train contains an error, but the error is not merely that of a single ratio. If the three ratios from arbor 53 to arbor 57 were replaced by 15/41, 15/72 and 15/79, we would for instance have obtained:

$$\mathrm{V(h)}^0_{57} = \mathrm{V}^0_{53} \times \left(-\frac{15}{41}\right) \times \left(-\frac{15}{72}\right) \times \left(-\frac{15}{79}\right) \tag{29.73}$$

$$\mathrm{P(h)}^0_{57} = -10746.9994\ldots \text{ days} \tag{29.74}$$

which would have been much better.

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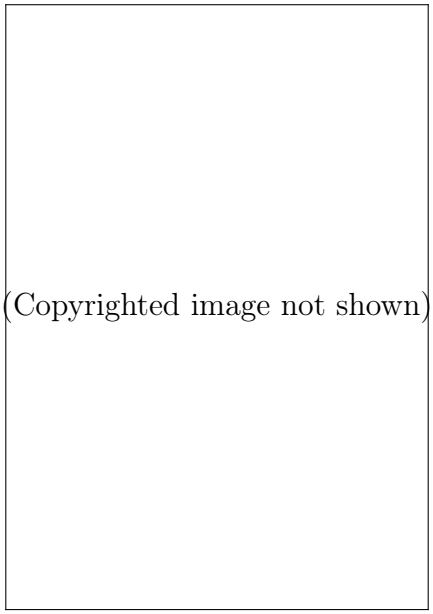


Figure 29.21: The Saturn dial (from Hrůšová [4]).

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29.3.4.2.4 The Mars dial

The train for Mars is derived from the motion of arbor 53. The hand for Mars is carried by arbor 54. We have

$$\mathrm{V}^0_{54} = \mathrm{V}^0_{53} \times \left(-\frac{12}{53}\right) = \frac{7705}{1198368} \times \left(-\frac{12}{53}\right) = -\frac{7705}{5292792} \tag{29.75}$$

$$\mathrm{P}^0_{54} = -\frac{5292792}{7705} = -686.9295\ldots \text{ days} \tag{29.76}$$

The same value is given by Oechslin. The Mars hand moves clockwise and makes a turn in the orbital period of Mars. The period found here is a good approximation of the actual orbital period.

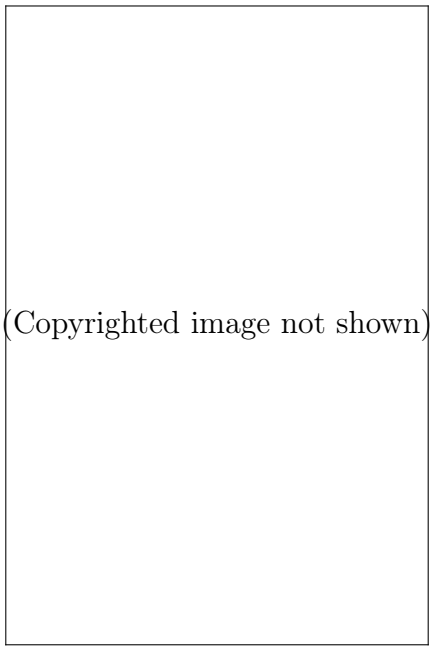


Figure 29.22: The Mars dial (from Hrůšová [4]).

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29.3.4.2.5 The mean and true time

This dial actually has a fixed outer 24-hours dial for the display of the mean time, and a moving inner 24-hours dial for the display of the true time. The latter is moved by a equation cam on tube 47, which is itself located around tube 68. We therefore first need to compute the motion of this cam. We have

$$\mathrm{V}_{47}^{0} = \mathrm{V}_{43}^{0} \times \left(-\frac{12}{25}\right) \times \left(-\frac{14}{42}\right) \times \left(-\frac{14}{69}\right) \times \left(-\frac{14}{83}\right) \tag{29.77}$$

$$= \mathrm{V}_{43}^{0} \times \frac{784}{143175} = \frac{1}{2} \times \frac{784}{143175} = \frac{392}{143175} \tag{29.78}$$

$$\mathrm{P}_{47}^{0} = \frac{143175}{392} = 365.2423\ldots \text{ days} \tag{29.79}$$

The same value is given by Oechslin. The equation cam therefore makes a turn in a tropical year. This cam has only been located around the center of this side of the machine for convenience and there is no relationship with the daily Sun/Earth/Moon system.

The cam moves a rack which moves a 30-teeth wheel on the tube 48 carrying the moving inner dial.

The central arbor 40 carries the hour hand and makes a turn in one day, as described above.

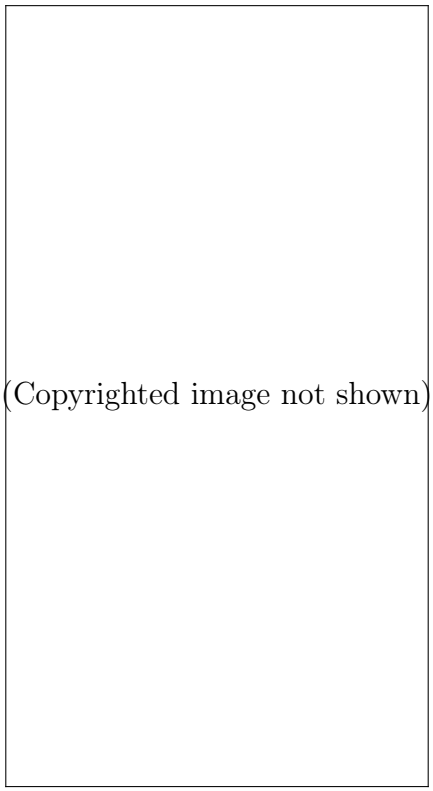


Figure 29.23: The mean and true time (from Hrůšová [4]).

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29.3.4.2.6 The Uranus dial

For the dials of Mercury, Jupiter and Uranus, we first compute the velocities of arbors 59 and 62:

$$\mathrm{V}^0_{59} = \mathrm{V}^0_{43} \times \left(-\frac{10}{51}\right) \times \left(-\frac{25}{87}\right) = \mathrm{V}^0_{43} \times \frac{250}{4437} = \frac{1}{2} \times \frac{250}{4437} = \frac{125}{4437} \tag{29.80}$$

$$\mathrm{V}^0_{62} = \mathrm{V}^0_{59} \times \left(-\frac{14}{78}\right) \times \left(-\frac{8}{79}\right) = \mathrm{V}^0_{59} \times \frac{56}{3081} \tag{29.81}$$

$$= \frac{125}{4437} \times \frac{56}{3081} = \frac{7000}{13670397} \tag{29.82}$$

We can then compute the motion of arbor 66 which carries the hand of Uranus:

$$\mathrm{V}^0_{66} = \mathrm{V}^0_{62} \times \left(-\frac{16}{109}\right) \times \left(-\frac{31}{53}\right) \times \left(-\frac{29}{39}\right) = \mathrm{V}^0_{62} \times \left(-\frac{14384}{225303}\right) \tag{29.83}$$

$$= \frac{7000}{13670397} \times \left(-\frac{14384}{225303}\right) = -\frac{3472000}{106206257079} \tag{29.84}$$

$$\mathrm{P}^0_{66} = -\frac{106206257079}{3472000} = -30589.3597\ldots \text{ days} \approx -83.75 \text{ years} \tag{29.85}$$

The same value is given by Oechslin. This is again an excellent approximation of Uranus' orbital period which is about 30588.74 days.

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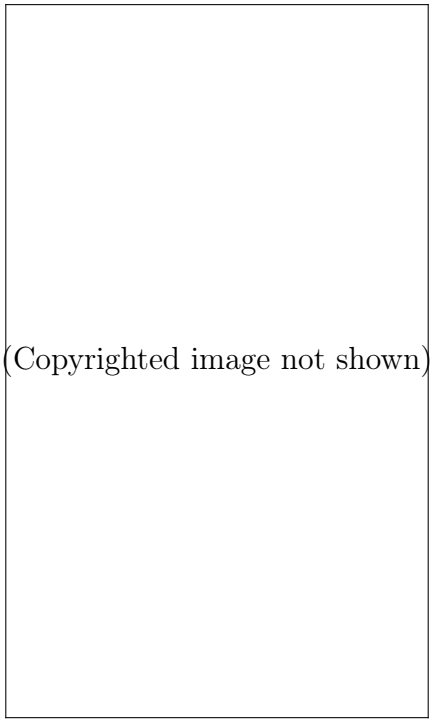


Figure 29.24: The Uranus dial (from Hrůšová [4]).

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29.3.4.2.7 The Jupiter dial

The hand of Jupiter is carried by the arbor 63 whose velocity is

$$\mathrm{V}_{63}^0 = \mathrm{V}_{62}^0 \times \left(-\frac{23}{51}\right) = \frac{7000}{13670397} \times \left(-\frac{23}{51}\right) = -\frac{161000}{697190247} \tag{29.86}$$

$$\mathrm{P}_{63}^0 = -\frac{697190247}{161000} = -4330.3742\ldots \text{ days} \tag{29.87}$$

The same value is given by Oechslin. This is an excellent approximation of Jupiter's orbital period.

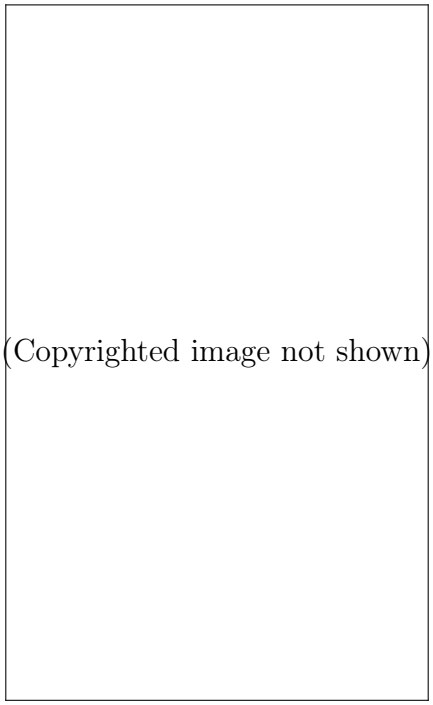


Figure 29.25: The Jupiter dial (from Hrůšová [4]).

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29.3.4.2.8 The Mercury dial

The hand of Mercury is carried by the arbor 60 whose velocity is

$$\mathrm{V}_{60}^0 = \mathrm{V}_{59}^0 \times \left(-\frac{23}{57}\right) = \frac{125}{4437} \times \left(-\frac{23}{57}\right) = -\frac{2875}{252909} \tag{29.88}$$

$$\mathrm{P}_{60}^0 = -\frac{252909}{2875} = -87.9683\ldots \text{ days} \tag{29.89}$$

The same value is given by Oechslin. This is an excellent approximation of Mercury's orbital period.

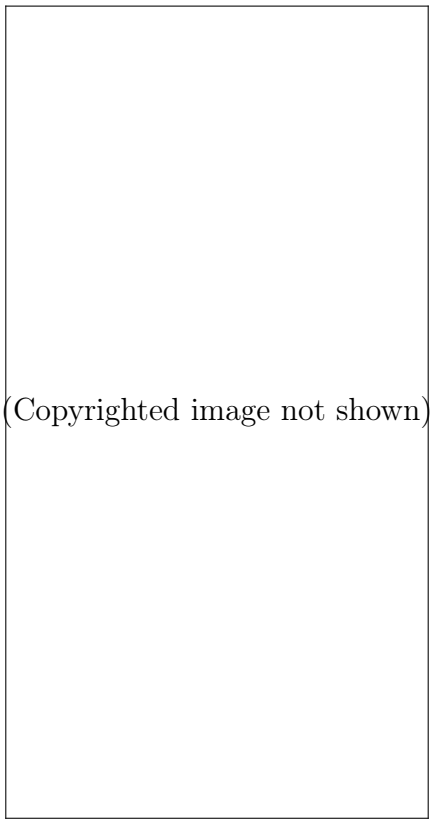


Figure 29.26: The Mercury dial (from Hrůšová [4]).

29.4 The upper part of the machine

The upper part of the machine derives its motions from the two arbors 97 and 146. The motion of arbor 97 is derived from that of arbor 3:

$$\mathrm{V}_{97}^0 = \mathrm{V}_3^0 \times \left(-\frac{12}{48}\right) \times \left(-\frac{12}{48}\right) \times \left(-\frac{12}{36}\right) \tag{29.90}$$

$$= \mathrm{V}_3^0 \times \left(-\frac{1}{48}\right) = 24 \times \left(-\frac{1}{48}\right) = -\frac{1}{2} \tag{29.91}$$

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The motion of arbor 146 is also derived from that of arbor 3:

$$\mathrm{V}^0_{146} = \mathrm{V}^0_3 \times \left(-\frac{24}{48}\right) \times \frac{12}{36} = \mathrm{V}^0_3 \times \left(-\frac{1}{6}\right) = 24 \times \left(-\frac{1}{6}\right) = -4 \qquad (29.92)$$

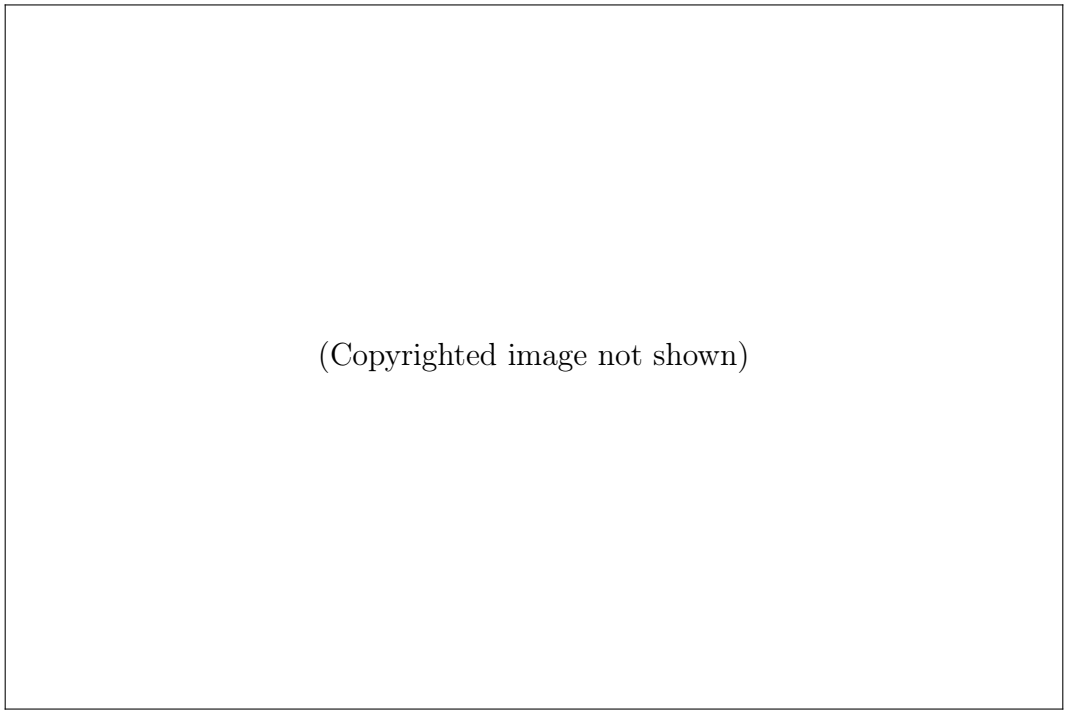


Figure 29.27: The upper part of the machine (from Hrůšová [4]).

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29.4.1 The Sun/Moon annual system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the zodiac. There are four separate trains and they are all driven by the arbor 118. This arbor takes its motion from arbor 97 which makes one turn in two days. We have

$$\mathrm{V}^0_{118} = \mathrm{V}^0_{97} \times \left(-\frac{30}{106}\right) \times \left(-\frac{28}{96}\right) = \mathrm{V}^0_{97} \times \frac{35}{424} \tag{29.93}$$

$$= \left(-\frac{1}{2}\right) \times \frac{35}{424} = -\frac{35}{848} \tag{29.94}$$

The motion of the Sun is that of tube 129. We have

$$\mathrm{V}^0_{129} = \mathrm{V}^0_{118} \times \left(-\frac{21}{31}\right) \times \left(-\frac{10}{37}\right) \times \left(-\frac{25}{69}\right) \tag{29.95}$$

$$= \mathrm{V}^0_{118} \times \left(-\frac{1750}{26381}\right) = \left(-\frac{35}{848}\right) \times \left(-\frac{1750}{26381}\right) \tag{29.96}$$

$$= \frac{30625}{11185544} \tag{29.97}$$

$$\mathrm{P}^0_{129} = \frac{11185544}{30625} = 365.2422\ldots \text{ days} \tag{29.98}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment, but not in the Sun/Moon annual system.

The motion of the Moon is that of the central arbor 119. We have

$$\mathrm{V}^0_{119} = \mathrm{V}^0_{118} \times \left(-\frac{47}{53}\right) = \left(-\frac{35}{848}\right) \times \left(-\frac{47}{53}\right) = \frac{1645}{44944} \tag{29.99}$$

$$\mathrm{P}^0_{119} = \frac{44944}{1645} = 27.3215\ldots \text{ days} \tag{29.100}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment, but not in the Sun/Moon annual system.

The motion of the lunar nodes is that of tube 123. We have

$$\mathrm{V}^0_{123} = \mathrm{V}^0_{118} \times \left(-\frac{19}{44}\right) \times \left(-\frac{8}{50}\right) \times \left(-\frac{9}{51}\right) \times \left(-\frac{19}{65}\right) \tag{29.101}$$

$$= \mathrm{V}^0_{118} \times \frac{1083}{303875} \tag{29.102}$$

$$= \left(-\frac{35}{848}\right) \times \frac{1083}{303875} = -\frac{7581}{51537200} \tag{29.103}$$

$$\mathrm{P}^0_{123} = -\frac{51537200}{7581} = -6798.2060\ldots \text{ days} \tag{29.104}$$

The same value is given by Oechslin. This value is negative because the lunar nodes are retrograding. A slightly different ratio was used in the metal fragment.

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Finally, the motion of the lunar apsides is that of tube 126. We have

$$\mathrm{V}^0_{126} = \mathrm{V}^0_{118} \times \left(-\frac{16}{75}\right) \times \left(-\frac{8}{71}\right) \times \left(-\frac{17}{109}\right) \tag{29.105}$$

$$= \mathrm{V}^0_{118} \times \left(-\frac{2176}{580425}\right) \tag{29.106}$$

$$= \left(-\frac{35}{848}\right) \times \left(-\frac{2176}{580425}\right) = \frac{952}{6152505} \tag{29.107}$$

$$\mathrm{P}^0_{126} = \frac{6152505}{952} = 6462.7153\ldots \text{ days} \tag{29.108}$$

This should be the period of precession of the lunar apsides. Oechslin obtained the same value, but it is of course incorrect.[5] There is obviously a missing factor 2, as the correct value is about 3231.36 days, or about 8.85 years.

[5] See [9, p. 205] on this error.

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29.4.2 The Sun/Moon daily system

This system shows the motion of the Sun, the Moon, the lunar nodes and the lunar apsides with respect to the Earth. The Sun therefore makes one turn *clockwise* around the Earth in one day. There are four separate trains and they are all driven by the input on arbor 161. The motion of this arbor is in turn derived from that of arbor 146 which makes one turn in 6 hours. We have

$$\mathrm{V}^0_{161} = \mathrm{V}^0_{146} \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{40}\right) \times \left(-\frac{40}{80}\right) \tag{29.109}$$

$$= \mathrm{V}^0_{146} \times \left(-\frac{1}{2}\right) = (-4) \times \left(-\frac{1}{2}\right) = 2 \tag{29.110}$$

Arbor 161 therefore makes one turn counterclockwise (from above) in 12 hours.

For the motion of the Sun on tube 168, we have

$$\mathrm{V}^0_{168} = \mathrm{V}^0_{161} \times \left(-\frac{30}{60}\right) = -1 \tag{29.111}$$

$$\mathrm{P}^0_{168} = -1 \text{ day} \tag{29.112}$$

The same ratio was used in the metal fragment.

The motion of the Moon is given by the central arbor 164:

$$\mathrm{V}^0_{164} = \mathrm{V}^0_{161} \times \left(-\frac{21}{23}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{42}{74}\right) \tag{29.113}$$

$$= \mathrm{V}^0_{161} \times \left(-\frac{2058}{4255}\right) = -\frac{4116}{4255} \tag{29.114}$$

$$\mathrm{P}^0_{164} = -\frac{4255}{4116} = -24 \text{ h } 48 \text{ m } 37.7842\ldots \text{ s} \tag{29.115}$$

The same value is given by Oechslin.

This is an approximation of the apparent diurnal motion of the Moon, which is also clockwise. The same ratio was used in the metal fragment.

The motion of the sky is given by tube 171:

$$\mathrm{V}^0_{171} = \mathrm{V}^0_{161} \times \left(-\frac{27}{34}\right) \times \left(-\frac{40}{42}\right) \times \left(-\frac{59}{89}\right) \tag{29.116}$$

$$= \mathrm{V}^0_{161} \times \left(-\frac{5310}{10591}\right) = -\frac{10620}{10591} \tag{29.117}$$

$$\mathrm{P}^0_{171} = -\frac{10591}{10620} = -0.9972\ldots \text{ days} = -23 \text{ h } 56 \text{ m } 4.0677\ldots \text{ s} \tag{29.118}$$

The same value is given by Oechslin. This is an approximation of the sidereal day. The same ratio was used in the metal fragment.

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Finally, the motion of the lunar nodes is given on tube 167:

$$\mathrm{V}^0_{167} = \mathrm{V}^0_{161} \times \left(-\frac{28}{27}\right) \times \left(-\frac{28}{30}\right) \times \left(-\frac{43}{83}\right) \tag{29.119}$$

$$= \mathrm{V}^0_{161} \times \left(-\frac{16856}{33615}\right) = -\frac{33712}{33615} \tag{29.120}$$

$$\mathrm{P}^0_{167} = -\frac{33615}{33712} = -0.9971\ldots \text{ days} = -23 \text{ h } 55 \text{ m } 51.4000\ldots \text{ s} \tag{29.121}$$

The same value is given by Oechslin. The same ratio was used in the metal fragment.

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29.4.3 The satellites of Jupiter

This system takes its input from arbor 148 whose motion is itself derived from that of arbor 146 which makes one turn in 6 hours. We have

$$\mathrm{V}^0_{148} = \mathrm{V}^0_{146} \times \left(-\frac{45}{83}\right) \times \left(-\frac{27}{89}\right) \tag{29.122}$$

$$= \mathrm{V}^0_{146} \times \frac{1215}{7387} = (-4) \times \frac{1215}{7387} = -\frac{4860}{7387} \tag{29.123}$$

The same ratio was used in the metal fragment.

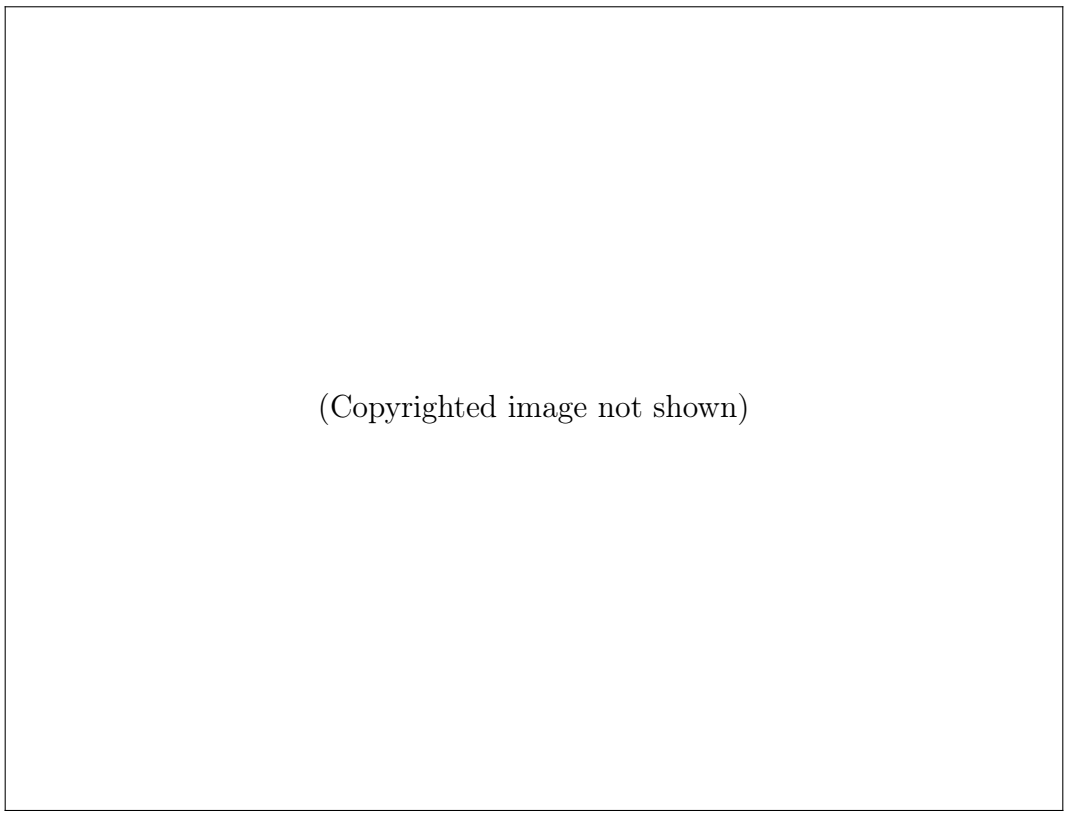


Figure 29.28: The satellites of Jupiter (from Hrůšová [4]).

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The velocities of the four satellites are:

$$\mathrm{V}^0_{149} = \mathrm{V}^0_{148} \times \left(-\frac{61}{71}\right) = \left(-\frac{4860}{7387}\right) \times \left(-\frac{61}{71}\right) = \frac{296460}{524477} \tag{29.124}$$

$$\mathrm{P}^0_{149} = \frac{524477}{296460} = 1.7691\ldots \text{ days (Io)} \tag{29.125}$$

$$\mathrm{V}^0_{152} = \mathrm{V}^0_{148} \times \left(-\frac{42}{49}\right) \times \left(-\frac{27}{46}\right) \times \left(-\frac{57}{67}\right) = \mathrm{V}^0_{148} \times \left(-\frac{4617}{10787}\right) \tag{29.126}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{4617}{10787}\right) = \frac{22438620}{79683569} \tag{29.127}$$

$$\mathrm{P}^0_{152} = \frac{79683569}{22438620} = 3.5511\ldots \text{ days (Europa)} \tag{29.128}$$

$$\mathrm{V}^0_{155} = \mathrm{V}^0_{148} \times \left(-\frac{44}{53}\right) \times \left(-\frac{22}{37}\right) \times \left(-\frac{34}{79}\right) = \mathrm{V}^0_{148} \times \left(-\frac{32912}{154919}\right) \tag{29.129}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{32912}{154919}\right) = \frac{159952320}{1144386653} \tag{29.130}$$

$$\mathrm{P}^0_{155} = \frac{1144386653}{159952320} = 7.1545\ldots \text{ days (Ganymede)} \tag{29.131}$$

$$\mathrm{V}^0_{158} = \mathrm{V}^0_{148} \times \left(-\frac{26}{53}\right) \times \left(-\frac{22}{48}\right) \times \left(-\frac{32}{79}\right) = \mathrm{V}^0_{148} \times \left(-\frac{1144}{12561}\right) \tag{29.132}$$

$$= \left(-\frac{4860}{7387}\right) \times \left(-\frac{1144}{12561}\right) = \frac{1853280}{30929369} \tag{29.133}$$

$$\mathrm{P}^0_{158} = \frac{30929369}{1853280} = 16.6889\ldots \text{ days (Callisto)} \tag{29.134}$$

The same values are given by Oechslin. These are good approximations of the actual periods of the satellites. The satellites all move counterclockwise as they should. The exact same trains were used in the metal fragment.

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29.4.4 The satellites of Saturn

This system takes its input from arbor 131 whose motion is itself derived from that of arbor 97 which makes one turn in two days. We have

$$\mathrm{V}^0_{131} = \mathrm{V}^0_{97} \times \left(-\frac{72}{58}\right) \times \left(-\frac{76}{72}\right) \tag{29.135}$$

$$= \mathrm{V}^0_{97} \times \frac{38}{29} = \left(-\frac{1}{2}\right) \times \frac{38}{29} = -\frac{19}{29} \tag{29.136}$$

This is the same ratio as the one used in the metal fragment, except that it moves in the opposite direction.

(Copyrighted image not shown)

Figure 29.29: The satellites of Saturn (from Hrůšová [4]).

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The velocities of the five satellites are then:

$$V^0_{132} = V^0_{131} \times \left(-\frac{38}{47}\right) = \frac{19}{29} \times \left(-\frac{38}{47}\right) = -\frac{722}{1363} \tag{29.137}$$

$$P^0_{132} = -\frac{1363}{722} = -1.8878\ldots \text{ days (Tethys)} \tag{29.138}$$

$$V^0_{135} = V^0_{131} \times \left(-\frac{21}{26}\right) \times \left(-\frac{18}{21}\right) \times \left(-\frac{33}{41}\right) = V^0_{131} \times \left(-\frac{297}{533}\right) \tag{29.139}$$

$$= \frac{19}{29} \times \left(-\frac{297}{533}\right) = -\frac{5643}{15457} \tag{29.140}$$

$$P^0_{135} = -\frac{15457}{5643} = -2.7391\ldots \text{ days (Dione)} \tag{29.141}$$

$$V^0_{138} = V^0_{131} \times \left(-\frac{38}{62}\right) \times \left(-\frac{38}{44}\right) \times \left(-\frac{30}{47}\right) = V^0_{131} \times \left(-\frac{5415}{16027}\right) \tag{29.142}$$

$$= \frac{19}{29} \times \left(-\frac{5415}{16027}\right) = -\frac{102885}{464783} \tag{29.143}$$

$$P^0_{138} = -\frac{464783}{102885} = -4.5175\ldots \text{ days (Rhea)} \tag{29.144}$$

$$V^0_{141} = V^0_{131} \times \left(-\frac{37}{39}\right) \times \left(-\frac{12}{29}\right) \times \left(-\frac{10}{41}\right) = V^0_{131} \times \left(-\frac{1480}{15457}\right) \tag{29.145}$$

$$= \frac{19}{29} \times \left(-\frac{1480}{15457}\right) = -\frac{28120}{448253} \tag{29.146}$$

$$P^0_{141} = -\frac{448253}{28120} = -15.9407\ldots \text{ days (Titan)} \tag{29.147}$$

$$V^0_{144} = V^0_{131} \times \left(-\frac{13}{30}\right) \times \left(-\frac{8}{46}\right) \times \left(-\frac{12}{47}\right) = V^0_{131} \times \left(-\frac{104}{5405}\right) \tag{29.148}$$

$$= \frac{19}{29} \times \left(-\frac{104}{5405}\right) = -\frac{1976}{156745} \tag{29.149}$$

$$P^0_{144} = -\frac{156745}{1976} = -79.3243\ldots \text{ days (Iapetus)} \tag{29.150}$$

The same values are given by Oechslin. We have here the exact same ratios as in the metal fragment, and the satellites move counterclockwise, as they should. The error in the metal fragment, where the satellites would have moved clockwise, has been corrected.

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29.4.5 The central orrery

At the center of the upper part of the machine, we have an orrery showing the mean motions of the planets from Mercury to Uranus. This orrery is mounted inside an armillary sphere.

This orrery takes its motion from arbor 98. This motion is itself derived from arbor 97 which also drives the Sun/Moon annual system and the satellites of Saturn.

We have

$$\mathrm{V}^0_{98} = \mathrm{V}^0_{97} \times \left(-\frac{23}{57}\right) = \left(-\frac{1}{2}\right) \times \left(-\frac{23}{57}\right) = \frac{23}{114} \tag{29.151}$$

We next compute the motion of arbor 99 which is used in several trains:

$$\mathrm{V}^0_{99} = \mathrm{V}^0_{98} \times \left(-\frac{10}{51}\right) = \frac{23}{114} \times \left(-\frac{10}{51}\right) = -\frac{115}{2907} \tag{29.152}$$

The motion of Mercury is that of arbor 100:

$$\mathrm{V}^0_{100} = \mathrm{V}^0_{99} \times \left(-\frac{25}{87}\right) = \left(-\frac{115}{2907}\right) \times \left(-\frac{25}{87}\right) = \frac{2875}{252909} \tag{29.153}$$

$$\mathrm{P}^0_{100} = \frac{252909}{2875} = 87.9683\ldots \text{ days} \tag{29.154}$$

The same value is given by Oechslin. The exact same ratio was used in the metal fragment.

The motion of Venus is that of tube 101:

$$\mathrm{V}^0_{101} = \mathrm{V}^0_{99} \times \left(-\frac{9}{80}\right) = \left(-\frac{115}{2907}\right) \times \left(-\frac{9}{80}\right) = \frac{23}{5168} \tag{29.155}$$

$$\mathrm{P}^0_{101} = \frac{5168}{23} = 224.6956\ldots \text{ days} \tag{29.156}$$

The same value is given by Oechslin.

This ratio was not used in the metal fragment.

The motion of the Earth is that of tube 110, but we first compute the motion of arbor 109 which is useful for Jupiter and Uranus. We have

$$\mathrm{V}^0_{109} = \mathrm{V}^0_{99} \times \left(-\frac{21}{51}\right) \times \left(-\frac{13}{33}\right) = \mathrm{V}^0_{99} \times \frac{91}{561} \tag{29.157}$$

$$= \left(-\frac{115}{2907}\right) \times \frac{91}{561} = -\frac{10465}{1630827} \tag{29.158}$$

$$\mathrm{V}^0_{110} = \mathrm{V}^0_{109} \times \left(-\frac{32}{75}\right) = \left(-\frac{10465}{1630827}\right) \times \left(-\frac{32}{75}\right) = \frac{66976}{24462405} \tag{29.159}$$

$$\mathrm{P}^0_{110} = \frac{24462405}{66976} = 365.2413\ldots \text{ days} \tag{29.160}$$

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This is a somewhat less accurate value of the tropical year, but Oechslin obtained it too.

The motion of Mars is that of tube 104, but we first compute the motion of arbor 103 which is also useful for Saturn. We have

$$\mathrm{V}^0_{103} = \mathrm{V}^0_{99} \times \left(-\frac{12}{59}\right) \times \left(-\frac{34}{58}\right) = \mathrm{V}^0_{99} \times \frac{204}{1711} \tag{29.161}$$

$$= \left(-\frac{115}{2907}\right) \times \frac{204}{1711} = -\frac{460}{97527} \tag{29.162}$$

And then

$$\mathrm{V}^0_{104} = \mathrm{V}^0_{103} \times \left(-\frac{25}{81}\right) = \left(-\frac{460}{97527}\right) \times \left(-\frac{25}{81}\right) = \frac{11500}{7899687} \tag{29.163}$$

$$\mathrm{P}^0_{104} = \frac{7899687}{11500} = 686.9293\ldots \text{ days} \tag{29.164}$$

The same value is given by Oechslin.

The motion of Jupiter is that of tube 113, but we first compute the motion of arbor 112 which is also useful for Uranus. We have

$$\mathrm{V}^0_{112} = \mathrm{V}^0_{109} \times \left(-\frac{12}{61}\right) \times \left(-\frac{17}{54}\right) = \mathrm{V}^0_{109} \times \frac{34}{549} \tag{29.165}$$

$$= \left(-\frac{10465}{1630827}\right) \times \frac{34}{549} = -\frac{20930}{52666119} \tag{29.166}$$

$$\mathrm{V}^0_{113} = \mathrm{V}^0_{112} \times \left(-\frac{43}{74}\right) = \left(-\frac{20930}{52666119}\right) \times \left(-\frac{43}{74}\right) = \frac{449995}{1948646403} \tag{29.167}$$

$$\mathrm{P}^0_{113} = \frac{1948646403}{449995} = 4330.3734\ldots \text{ days} \tag{29.168}$$

The same value is given by Oechslin.

The motion of Saturn is that of tube 107. We have

$$\mathrm{V}^0_{107} = \mathrm{V}^0_{103} \times \left(-\frac{17}{59}\right) \times \left(-\frac{14}{56}\right) \times \left(-\frac{23}{84}\right) = \mathrm{V}^0_{103} \times \left(-\frac{391}{19824}\right) \tag{29.169}$$

$$= \left(-\frac{460}{97527}\right) \times \left(-\frac{391}{19824}\right) = \frac{44965}{483343812} \tag{29.170}$$

$$\mathrm{P}^0_{107} = \frac{483343812}{44965} = 10749.3341\ldots \text{ days} \tag{29.171}$$

The same value is given by Oechslin.

Finally, the motion of Uranus is that of tube 116. We have

$$\mathrm{V}^0_{116} = \mathrm{V}^0_{112} \times \left(-\frac{17}{41}\right) \times \left(-\frac{19}{30}\right) \times \left(-\frac{26}{83}\right) = \mathrm{V}^0_{112} \times \left(-\frac{4199}{51045}\right) \tag{29.172}$$

$$= \left(-\frac{20930}{52666119}\right) \times \left(-\frac{4199}{51045}\right) = \frac{54418}{1664608077} \tag{29.173}$$

$$\mathrm{P}^0_{116} = \frac{1664608077}{54418} = 30589.2917\ldots \text{ days} \approx 83.75 \text{ years} \tag{29.174}$$

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The same value is given by Oechslin.

All these motions are counterclockwise as seen from above.

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Chapter 30

(Oechslin: 1.1)

Stuart's clock in Salzburg (1735)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

30.1 Introduction

The clock described here was designed in 1735 by Bernard Stuart (1706-1755) and constructed by the clockmaker Jakob Pentele (or Bentele) (1701?-1773) and the cabinetmaker Thomas Ableuthner. It was made for Leopold Anton von Firmian (1679-1744), who was Prince-Archbishop of Salzburg from 1727 until his death. The Prince-Archbishop had a special interest in clocks.[1]

Stuart was born Alexander Stuart in 1706 (on March 31 according to Zauner,[2] or on November 30 according to Gärtner[3]) in Scotland to a noble family.[4]

When he had the required age, he was sent to a new seminary in Germany by his uncle Maurus Stuart who was himself a missionary. In 1725 he entered the Scots benedictine monastery in Regensburg which had been founded in the 11th century, where after having made the vows in 1727 he studied philosophy and theology. He became a priest in 1730 and was then (or in 1728?) sent to

[1] See Husty's account on Leopold's clocks and sundials [12].

[2] [30] This was then for instance copied by Wurzbach [29], although Wurzbach also used Gärtner's account.

[3] [9]

[4] The biographical information given here is drawn from Paricius [21], Stetten [24], Hübner [10], Lipowsky [16], Zauner [30], Pillwein [22], Gärtner [9], Esterl [4], Mayr [18], Wurzbach [29], Lindner [15], Ilg [13], Schulte [23], Fischer [5], Maurice [17, v.1, p. 249, 267], Abeler [1] and Husty [12]. A number of authors are merely copying what was written before them, and there are sometimes simplifications. There is in particular a doubt concerning Stuart's date of birth, as Zauner (1813) and Gärtner (1821) give different dates. I am not sure which one is right. I have tried to provide a consolidated biographical notice, and have added precise sources where needed. The most complete informations on Stuart are probably found in Husty's thesis [11], but I haven't seen it.

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the Salzburg Nonnberg nun monastery as a chaplain.[5] This was for him the opportunity to study sciences and to make himself known at the University.

In 1733, he became professor of mathematics at the University of Salzburg, and he applied his knowledge to construction.

In 1735, Stuart designed and had constructed the astronomical clock described in this chapter. This clock is now located in the Salzburg Residenz.[6] Another tall-case astronomical clock probably made around 1730/1740 is attributed to Stuart and Pentele and is also located in the Salzburg Residenz.[7] It carries many dials and has a similar appearance to the great astronomical clock by Frater David in 1769, or the clock made by Pater Aurelius and located in Innsbruck.

In 1735, Stuart also signified to the Prince-Archbishop Leopold that the country was not having enough wood and gave a plan to make the moor stretches arable by removing the peat and using it as combustible.[8] Fischer writes that Stuart drained a large bog near Salzburg after a labour of three years [5, p. 148-149].

For his work on the moor stretches, Stuart was promoted in 1736 by the Prince-Archbishop. According to Paricius in 1753, Stuart was made clerical adviser to the Prince-Archbishop on January 11, 1736 and on April 12 he was made director of the public constructions of the entire land [21, p. 334-336].

From then to 1741, he remained professor at the University, but without giving lectures.

According to Stetten, in 1739 Stuart constructed a theater for the pupils of the Jesuit seminary in Augsburg.[9] In Augsburg, he also built a strong embankment on the river Lech, which Fischer says could still be seen in 1902. [5, p. 148-149]. The Imperial Court of Vienna also employed him as Inspector of Fortresses in Swabia. [5, p. 148-149]. Stuart also built a missionary house in Schwarzach near Salzburg [12, p. 349].

In 1741 he left Salzburg following accusations about construction matters and became director of the constructions in Augsburg.[10]. In 1742, he travelled

[5]Esterl gives the list of chaplains at the Nonnberg nun monastery and writes that Stuart was chaplain from 1728 to 1733 [4, p. 250-251]. This account seems to be based on Gärtner's account [9]. In 1880, Lindner also wrote that Stuart became chaplain of the Nonnberg monastery in 1728 [15, p. 234-235]. Husty also writes that Stuart came to Salzburg in 1728 [12, p. 349].

[6]This clock was mentioned as early as 1792 by Hübner who also mentioned a sundial by Stuart [10]. Wurzbach [29] also mentioned both based on Hübner's account. However, it was sometimes incorrectly dated to 1731, for instance by Doblhoff [2, p. 148]. This may be the source used by Schulte when he also dated the clock to 1731 in 1902. He also mentioned that it is located in Vienna [23, p. 798].

[7][12, p. 350-351]

[8]This is also mentioned by Mayr [18, p. 12].

[9]This was repeated by Füssli in 1779 [8, p. 649]. In 1779, Stetten gave some more details [25, p. 106-107]. See also Lindner [15, p. 234-235] and Fischer [5, p. 148-149].

[10]Jakob Pentele, who built the clock described here, was named a witness in the conflict that opposed the adjunct court architect Johann Kleber and Stuart, see [12, p. 349-350]. Kleber's name appears on the engraving of the 1735 astronomical clock (figure 30.2).

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to Saint Petersburg where one of his brothers was a general in the Russian service.[11] There he was offered a 1000 ruble yearly salary if he could improve the teaching of mathematics in the Russian schools.

Because of the Nordic air, he returned to Germany. In 1743 he became abbot of his monastery in Regensburg[12] and the Archbishop of Mainz named him abbot of the Erfurt monastery.

In 1753, Paricius still mentioned Stuart as the current abbot of the Regensburg monastery and of Erfurt [21, p. 334-336]. Paricius also mentioned Stuart's uncle Maurus (1664-1720) who was an earlier abbot of the Scots monastery in Regensburg.

In 1755, because of his weak health, Stuart went to Ferrara. He called his brother Maurus Stuart who was prior in Würzburg, and they made their way to Rome together. Stuart died on his way to Rome.

Demonstrations of Stuart's knowledge are found in Salzburg, Augsburg and other places. He drew the plan of Schloss Leopoldskron.[13] Schloss Klessheim has also been improved by Stuart.[14] He has also proposed a new water pipe system from the Untersberg massif to Salzburg, but this project was not concretized.

Stuart did not publish a description of his astronomical clock, but he has published several issues of almanacs, at least for 1737 and 1738.[15]

Pentele (or Bentele or Bendele), on the other hand, was a Court clockmaker from the Allgäu area. From 1734 to 1740, he worked with Stuart and he also constructed a number of tower clocks [19]. Pentele also had a nephew who became clockmaker.[16]

It was first thought that the case was made by the cabinetmaker Simon Baldauf (1677-1753),[17] but it is now attributed to Thomas Ableuthner from Munich, as it bears its signature.[18]

30.2 Description and analysis of the clock

Stuart's clock is illustrated in figure 30.1. The clock was initially in Salzburg, then vanished at the time of the secularization. It resurfaced in Schloss Laxenburg near Vienna, and from there it went to the *Kunsthistorischesmuseum* in Vienna in 1879 [12, p. 348]. The clock is now exhibited in the Salzburg

[11] See also Fischer's account [5, p. 148-149].

[12] [5, p. 148-149]

[13] This is again mentioned by Lindner in 1880 [15, p. 234-235] and Fischer in 1902 [5, p. 148-149].

[14] Ilg comments on Stuart's improvement of Schloss Klessheim and gives a short biography of Stuart based on earlier accounts [13, p. 241].

[15] Lindner mentions Stuart's almanac for 1738 (*Ephemeris ecclesiastica, astronomica, astrologica, etc.*) [15, p. 234-235].

[16] [12, p. 353]

[17] [27]

[18] See especially [14] and [28].

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Residenz which for centuries was the residence of the Prince-Archbishops of Salzburg.[19]

It is a Boulle work, in that its case uses the Boulle marquetry process. Apart from the use of the Boulle process, the clock is also somewhat unusual in appearance, it is 117 cm tall table clock, with three dials in an equilateral triangle, topped by a celestial globe. It is also shown on a copper plate from 1735 (figure 30.2).



Figure 30.1: Bernard Stuart's astronomical clock (1735). (source: Wikipédia)

[19]The clock was described by Döry-Jobaháza [3], but I haven't seen his article. It was also shown at the 1989 Hahn exhibition [26, p. 66]. The case was described by Koller, Loicht and Zehetner [14]. Stuart's clock was described by Frieß [6] and Oechslin [20, p. 52-53, 218].

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Figure 30.2: Excerpt of a copper plate of Stuart's clock. (Salzburg Museum, Inv. 1137-96)

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30.2.1 The going work

The clock is spring-driven, with a barrel and fusee,[20] and is regulated by a pendulum. There is also a maintaining power device.[21] The fusee arbor 1 makes a turn in 4 hours and drives the arbor 2 carrying the minute hand which makes one turn in an hour:

$$\mathrm{V}_1^0 = 6 \tag{30.1}$$

$$\mathrm{P}_1^0 = \frac{1}{6} = 4 \text{ hours} \tag{30.2}$$

$$\mathrm{V}_2^0 = \mathrm{V}_1^0 \times \left(-\frac{64}{16}\right) = -24 \tag{30.3}$$

$$\mathrm{P}_2^0 = -\frac{1}{24} = -1 \text{ h} \tag{30.4}$$

The 30-teeth escape wheel is on arbor 4 and makes a turn in 30 seconds:

$$\mathrm{V}_4^0 = \mathrm{V}_2^0 \times \left(-\frac{70}{7}\right) \times \left(-\frac{72}{6}\right) = \mathrm{V}_2^0 \times 120 = -2880 \tag{30.5}$$

$$\mathrm{P}_4^0 = -\frac{1}{2880} \text{ days} = -30 \text{ s} \tag{30.6}$$

The pendulum makes a half oscillation in 0.5 s and is about 25 cm long.

30.2.2 The three dials and the globe

The three dials and the globe are driven by the arbor 2 which makes one turn in an hour. This arbor is first directly led to the small dial located between the three large dials. This small dials merely shows the minutes of time.

This motion is then transferred to the three other dials.

30.2.2.1 The lower left calendrical and astronomical dial

From the outside to the inside, this dial shows a annual calendar with a number of feasts, then a ring with the signs of the zodiac (also in counterclockwise order), then a ring for the positions of the lunar nodes, then a ring for the Sun, then a disk for the phase of the Moon.

[20] Excellent illustrations of these parts are provided by Frieß [6, p. 364].

[21] It is also illustrated by Frieß [6, p. 364].

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30.2.2.1.1 The motion of the Moon

The motion of arbor 2 is first used to move the hand of the Moon on the central arbor 11 of this dial. We have[22]

$$\mathrm{V}_{11}^0 = \mathrm{V}_2^0 \times \frac{1}{8} \times \left(-\frac{1}{82}\right) = \mathrm{V}_2^0 \times \left(-\frac{1}{656}\right) = 24 \times \frac{1}{656} = \frac{3}{82} \tag{30.7}$$

$$\mathrm{P}_{11}^0 = \frac{82}{3} = 27.3333\ldots \text{ days} \tag{30.8}$$

This is an approximation of the tropical month, but not a very good one. The same value is given by Oechslin. The arbor moves a hand counterclockwise and also a disk with a circular opening. This opening shows a part of a disk tied to the Sun, so that we see an approximation of the lunar phase.

30.2.2.1.2 The motion of the Sun

The motion of the Sun on tube 13 is also derived from that of arbor 2, via arbor 5. So, we first compute the motion of arbor 5, then that of tube 13:

$$\mathrm{V}_5^0 = \mathrm{V}_2^0 \times \left(-\frac{20}{80}\right) = \mathrm{V}_2^0 \times \left(-\frac{1}{4}\right) = 24 \times \frac{1}{4} = 6 \tag{30.9}$$

$$\mathrm{V}_{13}^0 = \mathrm{V}_5^0 \times \frac{1}{15} \times \frac{1}{146} = \mathrm{V}_5^0 \times \frac{1}{2190} = 6 \times \frac{1}{2190} = \frac{1}{365} \tag{30.10}$$

$$\mathrm{P}_{13}^0 = 365 \text{ days} \tag{30.11}$$

This is an approximation of the tropical year, but a very bad one. The Sun moves counterclockwise and carries a disk which is used to display an approximation of the lunar phase.

The tube 13 also carries a 108-teeth wheel which meshes with a similar wheel on arbor 14. This wheel is first used to drive arbor 15 which makes one turn in four years:

$$\mathrm{V}_{14}^0 = \mathrm{V}_{13}^0 \times \left(-\frac{108}{108}\right) = -\mathrm{V}_{13}^0 = -\frac{1}{365} \tag{30.12}$$

$$\mathrm{V}_{15}^0 = \mathrm{V}_{14}^0 \times \left(-\frac{24}{96}\right) = \mathrm{V}_{14}^0 \times \left(-\frac{1}{4}\right) \tag{30.13}$$

$$\mathrm{P}_{15}^0 = -4 \text{ years} \tag{30.14}$$

This arbor will be used to update the dominical letter and the other indications of the computus.

The 108-teeth on arbor 14 also moves a cam which is used to transmit an oscillating motion to the lower right dial for the indication of the lengths of the days.[23]

[22] See Frieß's drawing [6, p. 372].

[23] See Frieß's drawing [6, p. 370].

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30.2.2.1.3 The motion of the dragon

The motion of the dragon on tube 9 is also derived from that of arbor 2, via arbors 6 and 7. We compute all these velocities here, as they will be useful later.[24]

$$\mathrm{V}_6^0 = \mathrm{V}_5^0 \times \left(-\frac{32}{192}\right) = \mathrm{V}_5^0 \times \left(-\frac{1}{6}\right) = -1 \tag{30.15}$$

$$\mathrm{V}_7^0 = \mathrm{V}_6^0 \times \left(-\frac{24}{168}\right) = \mathrm{V}_6^0 \times \left(-\frac{1}{7}\right) = \frac{1}{7} \tag{30.16}$$

$$\mathrm{V}_9^0 = \mathrm{V}_7^0 \times \left(-\frac{1}{9}\right) \times \frac{1}{108} = \mathrm{V}_7^0 \times \left(-\frac{1}{972}\right) = -\frac{1}{7} \times \frac{1}{972} = -\frac{1}{6804} \tag{30.17}$$

$$\mathrm{P}_9^0 = -6804 \text{ days} \tag{30.18}$$

This is a (not very good) approximation of the period of precession of the lunar nodes. This period is negative, because the nodes are retrograding. The same value is given by Oechslin.

The tube 9 apparently moves a ring, but it isn't clear what the inscriptions of this ring are. There should at least be the positions of the ascending and descending lunar nodes.

30.2.2.2 The lower right dial with Babylonian and Italic hours

The large lower right dial shows the time, the Italic hours (number of hours since the last sunset) and the Babylonian hours (number of hours since the last sunrise) as well as the durations of the day and the night.

There is only one hand and this hand is on arbor 6, which makes one turn clockwise in 24 hours, as we have seen above.

$$\mathrm{V}_6^0 = -1 \tag{30.19}$$

The mean time can be read on the outer dial, but possibly also on the most inner dial (or these are perhaps the hours counted from midnight).

We have also seen that the motion of the arbor 6 is used to obtain the motion of arbor 7:

$$\mathrm{V}_7^0 = \frac{1}{7} \tag{30.20}$$

$$\mathrm{P}_7^0 = 7 \text{ days} \tag{30.21}$$

This arbor makes a turn in seven days and seems to carry figures for the day of the week in the opening between the two lower dials.

The Italic and Babylonian hours are shown by having mobile 24-hour rings. The ring with the red Roman numerals gives the Italic hours and we can see that it starts at sunset, at the boundary between the light upper sector

[24] See also Frieß's drawing [6, p. 370].

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(daytime) and the black lower sector (night). This ring has an oscillating motion as it is connected through a lever to the cam on arbor 14.

The Babylonian hours are given on the second ring from the inside. This ring must be connected to the sector for the daylight.

There are therefore only two motions, and in fact even only one. There is only one lever coming from the cam on arbor 14, and this lever moves symmetrically the sunrise and the sunset, and the two rings for the Italic and Babylonian hours move with the sunset and sunrise.

The only little difficulty is that the black and light sectors are usually longer than they appear. Part of the black sector is hidden by the light sector, and part of the light sector is hidden by the black one. These two sectors are probably not entirely flat, so that one can pass under the other. Similar mechanisms are found on other clocks.

30.2.2.3 The upper dial with the Sun and the Moon

This dial is simpler than the other two. It shows a fixed Northern planisphere, the motion of the Sun (tube 20), and the motion of the Moon (tube 21). The gears of this dial also produce the motion of the celestial sphere at the top of the clock, first through the motion of tube 17.

All these motions are derived from the motion of the central arbor 2. First, we obtain the motion of the intermediate arbor 16:

$$\mathrm{V}_{16}^0 = \mathrm{V}_2^0 \times \left(-\frac{20}{80}\right) = \mathrm{V}_2^0 \times \left(-\frac{1}{4}\right) = (-24) \times \left(-\frac{1}{4}\right) = 6 \tag{30.22}$$

$$\mathrm{P}_{16}^0 = \frac{1}{6} = 4 \text{ hours} \tag{30.23}$$

(Oechslin's drawing mistakenly puts a 60-teeth wheel on arbor 16, instead of the correct 80-teeth wheel.)

The arbor 16 carries a 61-teeth wheel which meshes both with the 366-teeth wheel for the Sun (tube 20) and the 365-teeth wheel for the sky (tube 17), and a 57-teeth wheel which meshes with a 354-teeth wheel for the Moon (tube 21).[25]

[25] These two wheels are illustrated by Husty [12, p. 366].

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We therefore easily obtain the motions of tubes 17, 20 and 21:

$$V^0_{17} = V^0_{16} \times \left(-\frac{61}{365}\right) = 6 \times \left(-\frac{61}{365}\right) = -\frac{366}{365} \tag{30.24}$$

$$P^0_{17} = -\frac{365}{366} = -23 \text{ h } 56 \text{ m } 3.9344\ldots \text{ s} \tag{30.25}$$

$$V^0_{20} = V^0_{16} \times \left(-\frac{61}{366}\right) = 6 \times \left(-\frac{1}{6}\right) = -1 \tag{30.26}$$

$$P^0_{20} = -1 \text{ day} \tag{30.27}$$

$$V^0_{21} = V^0_{16} \times \left(-\frac{57}{354}\right) = V^0_{16} \times \left(-\frac{19}{118}\right) = 6 \times \left(-\frac{19}{118}\right) = -\frac{57}{59} \tag{30.28}$$

$$P^0_{21} = -\frac{59}{57} = -24 \text{ h } 50 \text{ m } 31.5789\ldots \text{ s} \tag{30.29}$$

These are approximations of the sidereal day, the mean solar day and the lunar day. The same values are given by Oechslin.

According to Oechslin's drawing, the Moon is set on arbor 23 and the motion of arbor 23 replicates that of tube 20, which is that of the Sun.[26] The photograph of figure 30.1 doesn't show a figure of the Moon, although something seems to be protruding towards the right, between the hours VI and VII. A photograph in Frieß's article [6, p. 367] shows that there should be a disk with the face of the Moon, and it may have gone missing. Because of the motion of arbor 23, this figure should always keep the same orientation with respect to the Sun, and therefore show the phase of the Moon.

30.2.2.4 The computus indications

This clock also shows the dominical letter and golden number in small openings on the left, and the epact and Easter full Moon in small openings on the right. These indications are obtained through the motion of arbor 15 which makes one turn in four years. The 96-teeth wheel on this arbor carries a pin on one side, and four pins on the other.[27] These four pins act on a lever which does itself carry three pins, one that acts on a 7-pointed star wheel on arbor 27, another that acts on a 19-pointed star wheel on arbor 26, and a third one that acts on a 19-pointed star wheel on arbor 25. The pin above also acts on a lever that acts on another 7-pointed star wheel on arbor 27.

The arbor 25 carries the indications for the epacts and Easter full Moon, which are recurring after 19 years in the 18th century. However, these indications were no longer valid in the 19th century and should have been replaced by another series of epacts.

The arbor 26 carries the values of the golden number which repeat after 19 years and remain valid through the centuries.

[26] Frieß shows a 3-dimensional rendering of the involved gears [6, p. 366].

[27] See Frieß's drawing [6, p. 372].

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Finally, the arbor 27 carries the values of the dominical letter. These values do normally repeat after 28 years, with single dominical letters for common years and double dominical letters for leap years. There are therefore 14 different possibilities, and it isn't entirely clear from Oechlin's drawing how these 14 possibilities are obtained.

30.2.2.5 The globe at the top

The celestial globe at the top is carried by an inclined arbor 19 whose motion is merely derived from that of tube 17 seen above, using two 108-teeth wheels at right angle.[28] We have

$$\mathrm{V}_{19}^0 = \mathrm{V}_{17}^0 \times \left(-\frac{108}{108}\right) \times \left(-\frac{80}{80}\right) = \mathrm{V}_{17}^0 = -\frac{366}{365} \tag{30.30}$$

$$\mathrm{P}_{19}^0 = -\frac{365}{366} = -23\text{ h }56\text{ m }3.9344\ldots\text{ s} \tag{30.31}$$

The celestial globe rotates clockwise (from above) in one sidereal day, corresponding to the apparent motion of the stars.

[28] See Frieß's drawing [6, p. 366].

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Conclusion

The purpose of this work was mainly to compute exact ratios for all the astronomical periods involved in the clocks and mechanisms described by Ludwig Oechslin in 1996. Although my contribution to the analysis of the gears of these 29 clocks and mechanisms is limited, I believe that the various gear trains have been better explained, that gear ratios were better highlighted, and that some connections that Oechslin didn't make could be made. Perhaps most importantly, the new approach made it possible to check Oechslin's computations, and, in a few cases, to correct them.

My work has been limited to the astronomical motions, and does not cover the calendar parts in detail, except in some particular cases. This is essentially because I do not have sufficiently photographs of these parts, and I did not want to do some guess-work, although in his first volume Oechslin gives extensive descriptions of these elements. I have also intentionally omitted detailed descriptions of the principles of many mechanical constructions, and have not at all analyzed the dimensions, or compared the constructions used by various authors, or those of a same constructor. Oechslin went at great length over these topics, but I believe that several volumes like the present one would be needed for the exegesis of Oechslin's work.

It should in any case be clear that my work would not have been possible without Oechslin's work. And I therefore want to express my deepest gratitude to Ludwig Oechslin for his wonderful work.

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