Chapter 24

(Oechslin: 3.1)

Neßtfell's *Planetenmaschine* in Vienna (1745-1753)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

24.1 Introduction

The astronomical machine described here was constructed between 1745 and 1753 by Johann Georg Neßtfell (1694-1762).

Neßtfell was born in Alsfeld (Hessen) in a cabinetmaker family. He was trained as a cabinetmaker.[1] Because of a conflict with his corporation, he didn't become a master and offered his services to princes and the Church. In 1717, he was employed by the House of Schönborn and he made a number of works for Schloss Wiesentheid near Würzburg. He also made a number of book furnitures in monasteries, most of which do no longer exist. His specialty was marquetry in wood and precious materials such as tortoiseshell and mother-of-pearl.[2] Neßtfell was one of the most important ebenists at the service of the House of Schönborn.

Neßtfell's maternal grandfather was also working with watches or clocks,

[1] The biographical information given here is drawn from Hess [10], Henck [9], Sangl's summary [23] with additions from Stoehr [31] and Spenlé [29]. There are a number of other biographical mentions, such as by Stamminger [30, p. 249-250] and Maurice [15, v.1, p. 273-274]. The articles of Dotter [5] and Rübeling [22] have not been consulted. Weis's dissertation probably contains a lot of additional information, but I have not yet seen it [34]. A portrait of Neßtfell is shown by Hess and is reproduced here [10]. Spenlé [29, p. 145] and Dippold [4, p. 24] reproduce another portrait, which is also reproduced by Henck [9]. Some elements on Neßtfell are also given by Abeler [1] and by Mattl-Wurm [14, p. 39-42]. Bassermann-Jordan only illustrated the Vienna machine [2], [3].

[2] On Neßtfell's work in Schloss Wiesentheid, see Hess [10, p. 43-48]. See also Spenlé's article on a tortoiseshell casket made by Neßtfell in the 1720s [29].

and organs, and Neßtfell was certainly familiar with metal work from an early age. In the 1720s, he was also assigned to surveying and map making tasks.

Then, in Banz Abbey, Neßtfell began to study astronomy. He could study books in the library and built hydraulic machines and scientific instruments.[3] It seems that two celestial globes made in 1692 by Coronelli in Venice and bought by Banz Abbey in the 1730s aroused particular interest in Neßtfell. In 1745, Neßtfell made a wooden orrery which came to the attention of Emperor Francis I (1708-1765) who was crowned in Frankfurt am Main in 1745. When the Emperor saw the orrery, he gave him 200 ducats.[4] Neßtfell was then called for an audience in Vienna and the Emperor asked him to make such a machine in metal. For the computations, he was helped by the student Johann Ludwig Fricker (1729-1766). The machine, described here, was delivered in 1753 in Vienna.

Neßtfell then received the title of court mechanician and a yearly salary.

Between 1755 and 1761, Neßtfell constructed a second orrery which is now kept in Munich. This machine was decorated by figures made by Johann Peter Wagner.

In 1761, Neßtfell published a description of his machines [18]. He died in 1762 in Würzburg.

[3]For Neßtfell's activity in Banz Abbey, see especially Hess [10, p. 49-60] and Dippold [4].

[4]See Hess [10, p. 22] and Oechslin [20, p. 221]. On possible persons who helped Neßtfell in the construction of his machines, see Oechslin [20, p. 219-220].

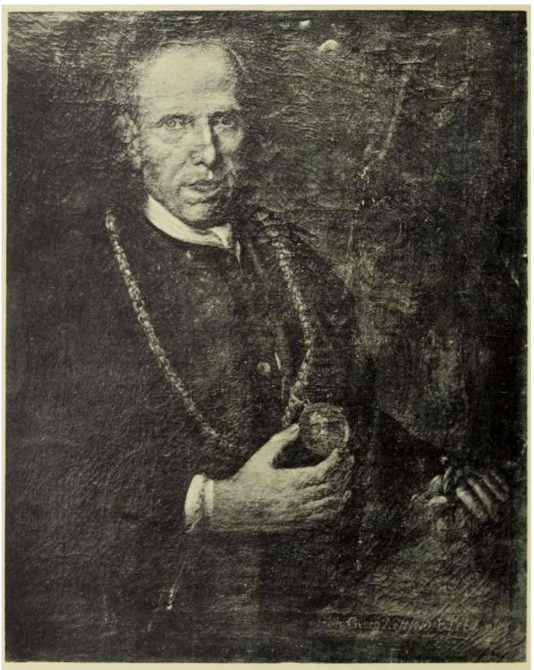

Figure 24.1: Johann Georg Neßtfell. (source: [10])

24.2 History and general description of the machine

Neßtfell's machine[5] was initially exhibited in the *k.-k. Hofbibliothek* in Vienna (now *Österreichische Nationalbibliothek*), then in 1768 in the *k.-k. Mechanisch-Physikalisches Kunstkabinett*,[6] then in 1797 in the *Thierkabinett*, then from 1806 to 1886 in the *k.-k. Physikalisch-Astronomisches Hofkabinett*. In 1868, Pater Mosheimer wrote a manuscript description of the machine [17]. Then, sometime after 1886 the machine was moved to the *Kunsthistorischesmuseum* in Vienna.[7] It is now exhibited in the *Naturhistorisches Museum* (museum of natural history) in Vienna.

After Neßtfell's death, Frater David a Sancto Cajetano restored his Vienna planetarium in the 1780s, and this may have brought him to develop his theory of epicyclic gears.[8]

[5]The best sources for Neßtfell's machines are Hess [10, p. 61-77] and Henck [9]. Besides Oechslin's work [20, p. 33, 36-37, 53-55, 199-203], see also King [11, p. 229-232] and Seelig's articles [24, 25, 28, 26, 27].

[6]See Weiskern's description [35, p. 58-64].

[7]See [13, p. 187]. Hauer [8, p. 3] seems to set the transfer in 1890. See also the less complete history given by Oechslin [20, p. 234]. This machine was part of the 1989 Hahn exhibition [32, p. 55, 57-59].

[8]See [6, p. 4] and [20, p. 227-228].

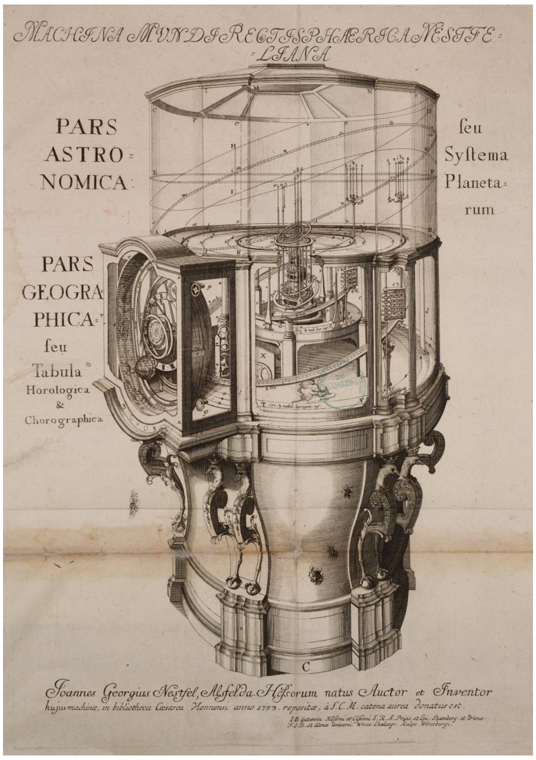

Figure 24.2: Engraving of Neßtfell's machine in Vienna, probably from 1754, given the year on the engraving showing the front dial. (Johann Balthasar Gutwein, source: [18])

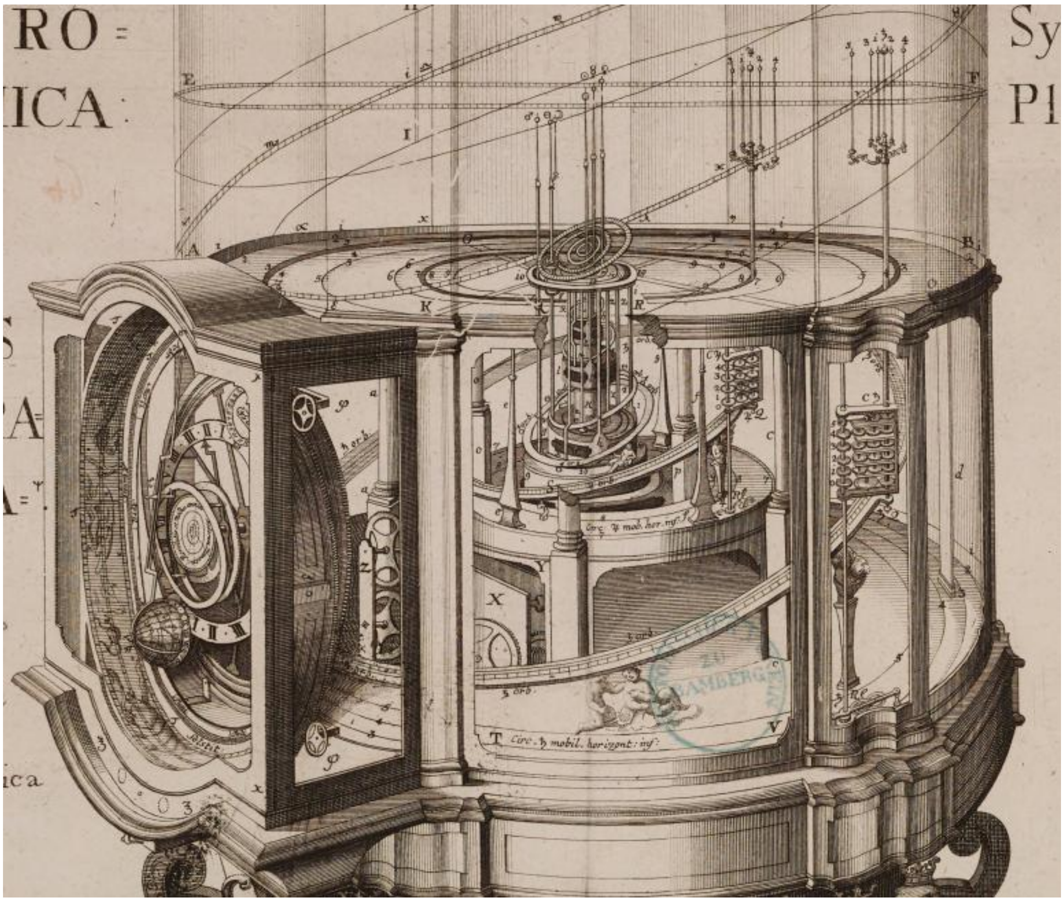

Figure 24.3: Detail of the previous engraving of Neßtfell's machine in Vienna, probably from 1754. (Johann Balthasar Gutwein, source: [18])

Figure 24.4: General view of Neßtfell's machine in in the *Naturhistorisches-museum* in Vienna. (photograph by the author)

Figure 24.5: Detail of Neßtfell's machine in Vienna. (photograph by the author)

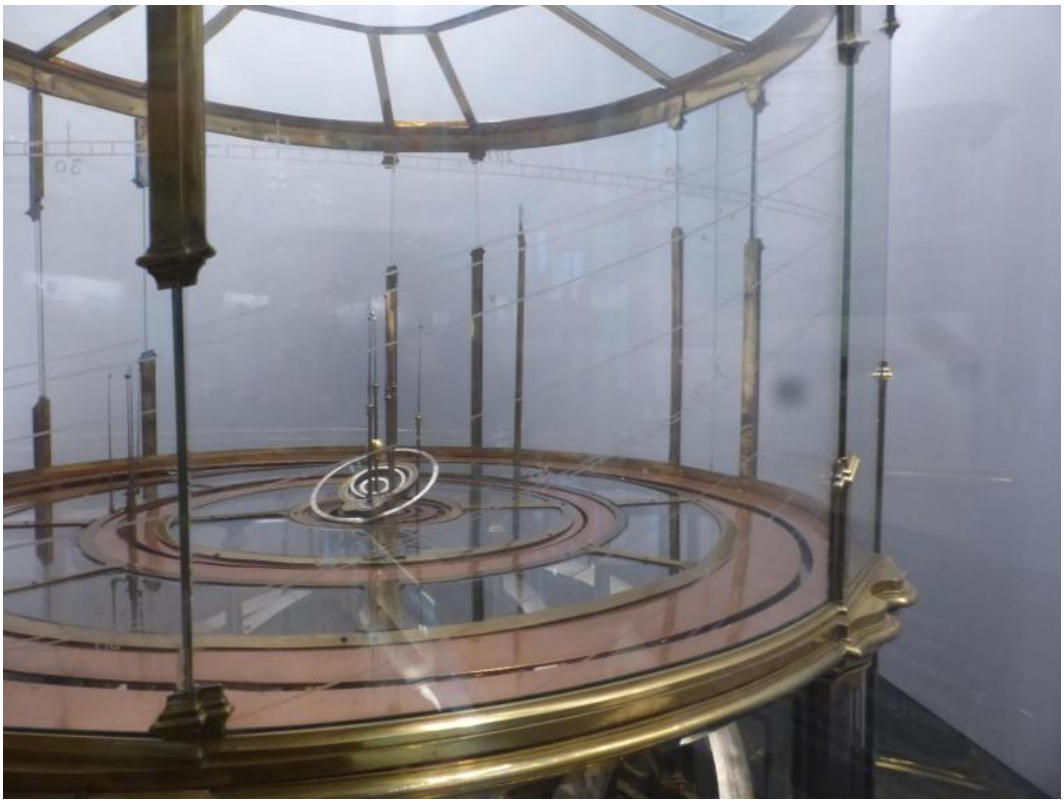

Figure 24.6: Detail of Neßtfell's machine in Vienna. (photograph by the author)

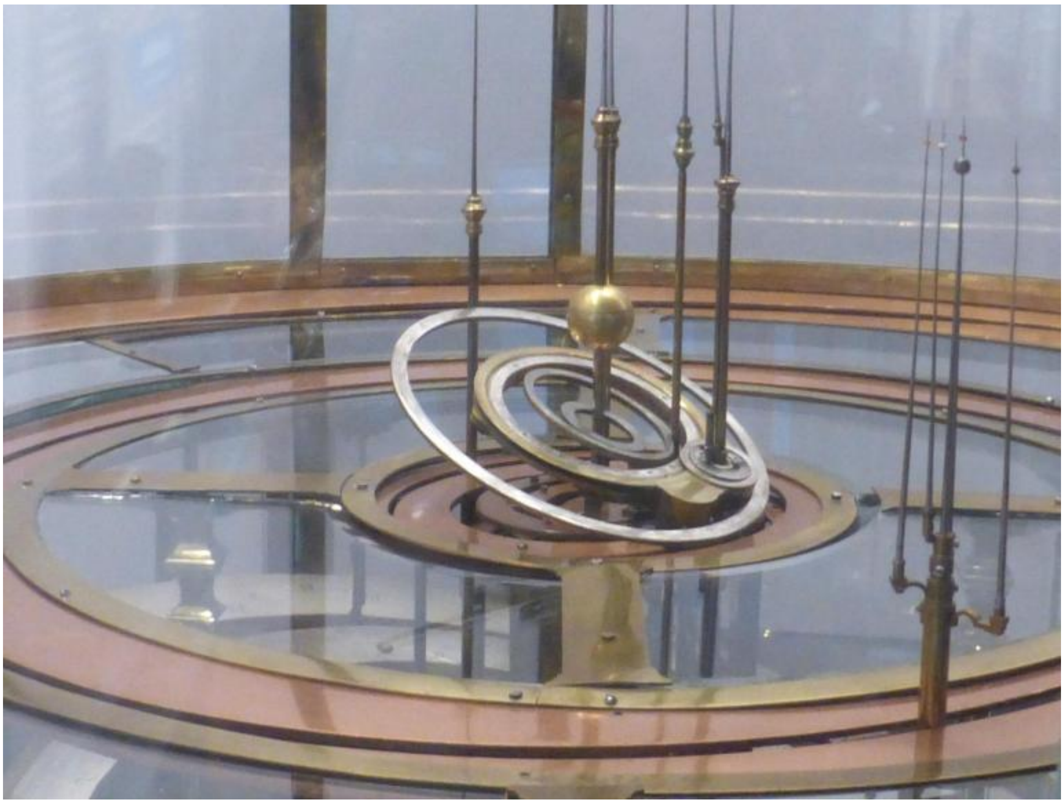

Figure 24.7: Detail of Neßtfell's machine in Vienna. The system of Jupiter's satellites is visible at the upper right. (photograph by the author)

It was also entirely restored in January-April 1980 for the exhibition *Maria Theresia und ihre Zeit* [12].

The machine is similar in appearance to Neßtfell's machine kept in Munich and which was constructed a few years later. The most obvious difference is that the base is decorated of simple columns in Munich, but of griffins in Vienna. There are also a number of sculpted figures in the Munich machine. However, there are also some differences in the gears. The descriptions of the two machines are very similar and overlap to a large extent, but that way they can be read in any order.

It should moreover be observed that Frater Fridericus's machine in Bamberg, made with the help of Johann Georg Fellwöck in 1772 (Oechslin 4.1), is really a simplified version of Neßtfell's machines. It has the same general cylindrical appearance, there is no orrery, and the entire front Earth-Mercury system is laid out horizontally.

Both of Neßtfell's machines are about 2 meters tall, they have a diameter of about 1 meter, and have a prismatic glass case containing the planetary system. Due to this construction, almost all the gears can be seen. In the front of the machine, there is a large dial representing the Earth-Mercury system.

These two machines are both very daunting, especially because it is rather difficult to understand them at first sight. I am going to split the machine in three parts: the going work, the orrery and the Earth-Mercury system.

24.3 The going work

The going work is set behind the Earth-Mercury system and it can be seen in Oechslin's figure 3.1.2 [20]. It is weight-driven and regulated by a pendulum. The going work contains two winding drums, that add up their strengths for a unique output on arbor 13 which makes one turn clockwise (seen from the front) in 24 hours. We have

$$\mathrm{T}^0_{13} = 1 \tag{24.1}$$

We can therefore deduce that the first drum (arbor 2) must have the velocity

$$\mathrm{T}^0_2 = \mathrm{T}^0_{13} \times \left(-\frac{54}{18}\right) \times \left(-\frac{18}{108}\right) = \mathrm{T}^0_{13} \times \frac{1}{2} = \frac{1}{2} \tag{24.2}$$

$$\mathrm{P}^0_2 = 2 \text{ days} \tag{24.3}$$

The second drum (arbor 8) must have the velocity

$$\mathrm{T}^0_8 = \mathrm{T}^0_{13} \times \left(-\frac{54}{86}\right) \times \left(-\frac{86}{18}\right) \times \left(-\frac{18}{108}\right) = \mathrm{T}^0_{13} \times \left(-\frac{1}{2}\right) \tag{24.4}$$

$$\mathrm{P}^0_8 = -2 \text{ days} \tag{24.5}$$

Each drum makes a turn in two days, but they turn in opposite directions.

These two drums can be rewound using the winding arbors 1 (for the first drum) and 7 (for the second drum).

A gear train connects the drums to the escape wheel on arbor 6. We have

$$T_6^0 = T_2^0 \times \left(-\frac{108}{18}\right) \times \left(-\frac{88}{11}\right) \times \left(-\frac{72}{9}\right) \times \left(-\frac{63}{7}\right) \quad (24.6)$$

$$= T_2^0 \times 3456 = 1728 \quad (24.7)$$

$$P_6^0 = \frac{1}{1728} \text{ days} = 50 \text{ seconds} \quad (24.8)$$

The escape wheel having 27 teeth, we deduce that the pendulum should make a half swing in $\frac{50}{2\times 27} = \frac{25}{27}$ seconds. Its length should be about 86 cm.

The only output of the going work is arbor 13 and it immediately drives the Earth-Mercury system in the front. Moreover, on arbor 13 there is a 96-teeth wheel which drives a train towards the orrery, but which can be put out of mesh using arbor 12. For now, I will merely assume that the going work directly drives the orrery.

24.4 The orrery

The most unusual feature of this machine is that the planets do not move in a horizontal plane, or near a horizontal plane. In almost every orrery, the planets move either in or near a horizontal plane, or in or near a vertical plane. In Neßtfell's machine, the ecliptic is tilted by an angle of 23.5° and the planets move in or near that tilted plane.[9] Consequently, the plane of the equator is a horizontal plane, although it doesn't have any particular significance here. For instance, no horizontal plane is materialized through the Earth.

Moreover, the planets only move indirectly in the ecliptic. In fact, there are horizontal base motions, and the planets are on vertical arms that glide along tilted guides. This all gives a very unusual appearance to this machine.

As mentioned above, the input to the orrery is a gear train starting with the horizontal arbor 13 making one turn in a day. This arbor is used to drive the gears for Saturn. Then, the motion of arbor 13 is transferred to arbor 65 which drives the other planets:

$$V_{65}^0 = V_{13}^0 \times \left(-\frac{96}{88}\right) \times \left(-\frac{88}{96}\right) = V_{13}^0 = -1 \quad (24.9)$$

Arbor 65 also makes one turn clockwise in a day.

All the planets are driven similarly, sitting on carriages which are stuck between moving wheels and wheels that are not moving. Neßtfell used large wheels which are about the size of the orbits, and Maurice and Oechslin assume that he took his inspiration from a British orrery by John Rowley which had been bought in 1723 in England by Prince Eugene Francis of Savoy-Carignano

[9] On this construction, see in particular [20, p. 146-147].

(1663-1736). Neßtfell repaired this orrery, probably during his first trip to Vienna in 1745 [10, p. 23-24], [36, p. 42], [15, v.1, p. 273]. This orrery may have been similar to the one by Adams described in this book [20, p. 154, 210, 226].

I am first considering the mean motions of the planets, then the rotations of the planets and the satellites of Jupiter and the Moon. Finally, I am considering the motion of the Moon.

24.4.1 The mean motions and rotations of the planets

I am starting with Saturn and going backwards to Mercury. These systems are in fact all independent and could be described in any order, but I believe it is easier to look at them from the front to the back, and also from the outside to the inside. The descriptions below are very similar, because the structures are similar. However, I prefer to repeat the descriptions, so that the motions can be studied in any order.

All the planets are given a motion of rotation around their axis, but I will only analyze the rotation of Mercury, Venus, the Earth and Mars after each analysis of the mean motions. The rotations of Jupiter and Saturn will be examined when I describe their two satellite systems.

As mentioned above, Neßtfell first produces horizontal motions, and tilted slopes ensure that these motions are transferred near the ecliptic. I assume that each slope is adapted to its planet and that different planets therefore have different slopes. It is also possible that some of the orbits have been made eccentric, but this should be checked. In addition, one should keep in mind that the transfer from the horizontal plane to the tilted plane is a projection and that it does alter the motion.[10] But Neßtfell did not implement irregular motions, other than possibly those resulting from offset orbits.

The mean motions of the planets are obtained through epicyclical gear trains. In each case, there is an input motion and the rotation of a set of gears on a mobile carriage, this rotation being determined by a constraint on another wheel, in this case a fixed wheel. These constructions often lead to periods whose rational expression contains large primes:

- for Mercury, the period involves the prime number 317;
- for Venus, the period involves the prime number 2411;
- for the Earth, the period involves the prime number 296983;
- for Mars, the period involves the prime numbers 101 and 173;
- for Jupiter, the period involves the prime numbers 131 and 389;
- for Saturn, the period involves the prime number 51817.

[10]This was also observed by Oechslin, see [20, p. 201-202].

Neßtfell's construction makes use of gears whose teeth numbers contain only small primes, the largest being 109. It is therefore tempting to view Neßtfell's construction as a way to avoid the use of large primes in gear trains.

However, I do not believe that Neßtfell chose to reach the exact ratios he finally produced. His objectif must have been much more to have the planets on moving carriages, and the complex ratios that these constructions entailed were rather side effects. This is therefore entirely different from Frater David's methods which aimed at obtaining specific ratios without resorting to too large primes. I am giving some details of Frater David's methods in the introduction of this book.

Neßtfell also gives the revolution periods of the planets in his description of the clock [18],[11] and the periods implemented on the clock are only approximations of these values:

Mercury	87d	23h	14m	40s		87.9685...
Venus	224d	17h	44m 11/12			224.7395...
Earth	365d	5h	48m	58s 1/4		365.2423...
Mars	686d	23h	31m	57s		686.9805...
Jupiter	4332d	14h	49m	31s	56‴	4332.6177...
Saturn	10759d	5h				10759.2083...

Neßtfell does not give the actual periods on his machine.

24.4.1.1 The motion of Saturn

On arbor 13, there is a 25-teeth wheel which meshes with a 897-teeth contrate wheel on frame 43. This wheel turns counterclockwise (from above) with the velocity

$$\mathrm{V}^0_{43} = \mathrm{V}^0_{13} \times \left(-\frac{25}{897}\right) = \frac{25}{897} \tag{24.10}$$

$$\mathrm{P}^0_{43} = \frac{897}{25} = 35.88 \text{ days} \tag{24.11}$$

Above the 897-teeth wheel, there is a 900-teeth wheel with interior gearing which meshes with a 25-teeth wheel which is part of a carriage 48 of which one pinion meshes with a fixed 690-teeth wheel.[12] This causes the entire carriage to move, and the carriage supports Saturn and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 44 on which the Saturn system moves up or down, as I will show later. The problem is to find the velocity of the carriage 48. I don't want to resort to some memorized formula. We need to be able to compute the motions without any cryptic magic. We can do so by setting ourself in the reference frame of the moving

[11] The number of days for Jupiter is mistakenly given as 4323, an obvious typo.

[12] There is a typo in the right part of Saturn's driving mechanism in Oechslin's drawing. 650 should be replaced by 690.

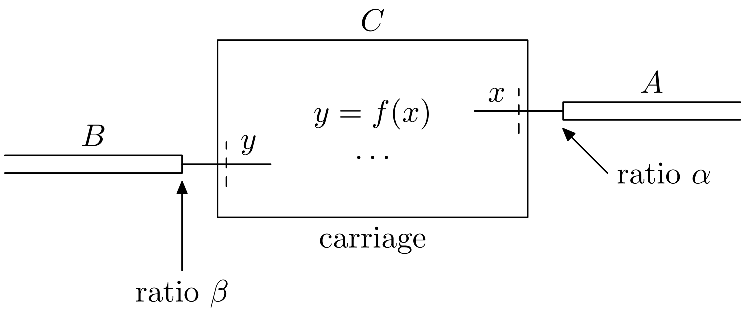

Figure 24.8: Principle of moving carriage C whose motion is determined by the motions of A and B.

carriage. We can merely compute the velocity of the 900-teeth wheel (frame 43) from that of the 690-teeth wheel (frame 47, fixed). We have

$$V_{43}^{48} = V_{47}^{48} \times \left(-\frac{690}{6}\right) \times \left(-\frac{109}{6}\right) \times \left(-\frac{103}{20}\right) \times \frac{25}{900} \tag{24.12}$$

$$= V_{47}^{48} \times \left(-\frac{258221}{864}\right) = V_{48}^{47} \times \frac{258221}{864} \tag{24.13}$$

It is important to note that the last ratio $\frac{25}{900}$ is positive, because the motion of arbor 44 is in the same direction as the frame 43, the 900-teeth wheel having an interior gearing.

Now, we have

$$V_{43}^{48} = V_{43}^{0} + V_{0}^{48} = V_{48}^{47} \times \frac{258221}{864} = V_{48}^{0} \times \frac{258221}{864} \tag{24.14}$$

Therefore

$$V_{43}^{0} = V_{48}^{0} \times \frac{258221}{864} + V_{48}^{0} = V_{48}^{0} \times \left(\frac{258221}{864} + 1\right) = V_{48}^{0} \times \frac{259085}{864} \tag{24.15}$$

And

$$V_{48}^{0} = V_{43}^{0} \times \frac{864}{259085} = \frac{25}{897} \times \frac{864}{259085} = \frac{1440}{15493283} \tag{24.16}$$

$$P_{48}^{0} = \frac{15493283}{1440} = 10759.2243\ldots \text{ days} \tag{24.17}$$

This is an approximation of the sidereal orbital period of Saturn. The same value is given by Oechslin.

The motion of Saturn around its axis is described below in the section of Saturn's satellites.

24.4.1.2 The motion of Jupiter

The mean motion of Jupiter is obtained like that of Saturn.

On arbor 65, there is a 30-teeth wheel which meshes with a 360-teeth contrate wheel on frame 66. This wheel turns counterclockwise (from above) with the velocity

$$V^0_{66} = V^0_{65} \times \left(-\frac{30}{360}\right) = \frac{30}{360} = \frac{1}{12} \quad (24.18)$$

$$P^0_{66} = 12 \text{ days} \quad (24.19)$$

Above the 360-teeth wheel, there is a 361-teeth wheel which meshes with a 30-teeth wheel which is part of a carriage 73 of which one pinion meshes with a fixed 306-teeth wheel. This causes the entire carriage to move, and the carriage supports Jupiter and its satellites. All these motions take place in a horizontal plane, but the carriage itself has an arbor 69 on which the Jupiter system moves up or down, as I will show later. The problem is to find the velocity of the carriage 73. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 361-teeth wheel (frame 66) from that of the 306-teeth wheel (frame 72, fixed). We have

$$V^{73}_{66} = V^{73}_{72} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \times \left(-\frac{14}{22}\right) \times \left(-\frac{22}{14}\right) \times \frac{30}{361} \quad (24.20)$$

$$= V^{73}_{72} \times \left(-\frac{355725}{988}\right) = V^{72}_{73} \times \frac{355725}{988} \quad (24.21)$$

The last ratio $\frac{30}{361}$ is positive for the same reason as in the case of Saturn. Now, we have

$$V^{73}_{66} = V^0_{66} + V^{73}_0 = V^{72}_{73} \times \frac{355725}{988} = V^0_{73} \times \frac{355725}{988} \quad (24.22)$$

Therefore

$$V^0_{66} = V^0_{73} \times \frac{355725}{988} + V^0_{73} = V^0_{73} \times \left(\frac{355725}{988} + 1\right) = V^0_{73} \times \frac{356713}{988} \quad (24.23)$$

And

$$V^0_{73} = V^0_{66} \times \frac{988}{356713} = \frac{1}{12} \times \frac{988}{356713} = \frac{247}{1070139} \quad (24.24)$$

$$P^0_{73} = \frac{1070139}{247} = 4332.5465\ldots \text{ days} \quad (24.25)$$

This is an approximation of the sidereal orbital period of Jupiter. The same value is given by Oechslin.

The motion of Jupiter around its axis is described below in the section of Jupiter's satellites.

24.4.1.3 The motion of Mars

The mean motion of Mars is obtained like those of Jupiter and Saturn.

On arbor 65, there is a 15-teeth wheel which meshes with a 101-teeth contrate wheel on frame 85. This wheel turns counterclockwise (from above) with the velocity

$$V_{85}^{0} = V_{65}^{0} \times \left(-\frac{15}{101}\right) = \frac{15}{101} \qquad (24.26)$$

$$P_{85}^{0} = \frac{101}{15} = 6.7333\ldots \text{ days} \qquad (24.27)$$

Above the 101-teeth wheel, there is a 102-teeth wheel which meshes with a 15-teeth wheel which is part of a carriage 90 of which one pinion meshes with a fixed 97-teeth wheel. This causes the entire carriage to move, and the carriage supports Mars. All these motions take place in a horizontal plane, but the carriage itself has an arbor 92 on which Mars moves up or down, as I will show later. The problem is to find the velocity of the carriage 90. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 102-teeth wheel (frame 85) from that of the 97-teeth wheel (frame 89, fixed). We have

$$V_{85}^{90} = V_{89}^{90} \times \left(-\frac{97}{4}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \frac{15}{102} \qquad (24.28)$$

$$= V_{89}^{90} \times \left(-\frac{879305}{8704}\right) = V_{90}^{89} \times \frac{879305}{8704} \qquad (24.29)$$

The last ratio $\frac{15}{102}$ is positive for the same reason as in the cases of Jupiter and Saturn.

Now, we have

$$V_{85}^{90} = V_{85}^{0} + V_{0}^{90} = V_{90}^{89} \times \frac{879305}{8704} = V_{90}^{0} \times \frac{879305}{8704} \qquad (24.30)$$

Therefore

$$V_{85}^{0} = V_{90}^{0} \times \frac{879305}{8704} + V_{90}^{0} = V_{90}^{0} \times \left(\frac{879305}{8704} + 1\right) = V_{90}^{0} \times \frac{888009}{8704} \qquad (24.31)$$

And

$$V_{90}^{0} = V_{85}^{0} \times \frac{8704}{888009} = \frac{15}{101} \times \frac{8704}{888009} = \frac{43520}{29896303} \qquad (24.32)$$

$$P_{90}^{0} = \frac{29896303}{43520} = 686.9554\ldots \text{ days} \qquad (24.33)$$

This is an approximation of the sidereal orbital period of Mars. The same value is given by Oechslin.

Mars also rotates around its axis. In the (synodic) reference frame 90, we have

$$V_{92}^{90} = V_{89}^{90} \times \left(-\frac{97}{4}\right) \times \left(-\frac{49}{8}\right) \times \left(-\frac{37}{8}\right) \times \left(-\frac{13}{21}\right) \times \left(-\frac{22}{14}\right) \quad (24.34)$$

$$= V_{89}^{90} \times \left(-\frac{513227}{768}\right) = V_{90}^{0} \times \frac{513227}{768} \quad (24.35)$$

$$= \frac{43520}{29896303} \times \frac{513227}{768} = \frac{87248590}{89688909} \quad (24.36)$$

And in the absolute frame:

$$V_{92}^{0} = V_{92}^{90} + V_{90}^{0} = V_{90}^{0} \times \left(\frac{513227}{768} + 1\right) = V_{90}^{0} \times \frac{513995}{768} \quad (24.37)$$

$$= \frac{43520}{29896303} \times \frac{513995}{768} = \frac{87379150}{89688909} \quad (24.38)$$

$$P_{92}^{0} = \frac{89688909}{87379150} = 1.0264\ldots \text{ days} = 24 \text{ h } 38 \text{ mn } 3.87\ldots \text{ s} \quad (24.39)$$

The same value is given by Oechslin. This period is close to the actual sidereal rotation of Mars.

24.4.1.4 The motion of the Earth

The mean motion of the Earth is obtained like those of Mars, Jupiter and Saturn.

On arbor 65, there is a 13-teeth wheel which meshes with a 170-teeth contrate wheel on frame 93. This wheel turns counterclockwise (from above) with the velocity

$$V_{93}^{0} = V_{65}^{0} \times \left(-\frac{13}{170}\right) = \frac{13}{170} \quad (24.40)$$

$$P_{93}^{0} = \frac{170}{13} = 13.0769\ldots \text{ days} \quad (24.41)$$

Above the 170-teeth wheel, there is a 217-teeth wheel which meshes with a 16-teeth wheel which is part of a carriage 98 of which one pinion meshes with a fixed 83-teeth wheel. This causes the entire carriage to move, and the carriage supports the Earth. All these motions take place in a horizontal plane, but the carriage itself has an arbor 100 on which the Earth moves up or down, as I will show later. The problem is to find the velocity of the carriage 98. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 217-teeth wheel (frame 93) from that of the 83-teeth wheel (frame 97, fixed). We have

$$V_{93}^{98} = V_{97}^{98} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \frac{16}{217} \quad (24.42)$$

$$= V_{97}^{98} \times \left(-\frac{286350}{10633}\right) = V_{98}^{97} \times \frac{286350}{10633} \quad (24.43)$$

The last ratio $\frac{16}{217}$ is positive for the same reason as in the cases of Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{93}^{98} = \mathrm{V}_{93}^{0} + \mathrm{V}_{0}^{98} = \mathrm{V}_{98}^{97} \times \frac{286350}{10633} = \mathrm{V}_{98}^{0} \times \frac{286350}{10633} \tag{24.44}$$

Therefore

$$\mathrm{V}_{93}^{0} = \mathrm{V}_{98}^{0} \times \frac{286350}{10633} + \mathrm{V}_{98}^{0} = \mathrm{V}_{98}^{0} \times \left(\frac{286350}{10633} + 1\right) = \mathrm{V}_{98}^{0} \times \frac{296983}{10633} \tag{24.45}$$

And

$$\mathrm{V}_{98}^{0} = \mathrm{V}_{93}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \tag{24.46}$$

$$\mathrm{P}_{98}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \tag{24.47}$$

This is an approximation of the sidereal year. The same value is given by Oechslin.

The Earth also rotates around its axis. In the (synodic) reference frame 98, we have

$$\mathrm{V}_{100}^{98} = \mathrm{V}_{97}^{98} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \left(-\frac{14}{56}\right) \times \left(-\frac{56}{14}\right) \tag{24.48}$$

$$= \mathrm{V}_{97}^{98} \times \left(-\frac{143175}{392}\right) = \mathrm{V}_{98}^{0} \times \frac{143175}{392} \tag{24.49}$$

$$= \frac{138229}{50487110} \times \frac{143175}{392} = \frac{80779335}{80779376} \tag{24.50}$$

$$\mathrm{P}_{100}^{98} = \frac{80779376}{80779335} = 1.0000005\ldots \text{ days} = 86400.0438\ldots \text{ seconds} \tag{24.51}$$

The Earth practically makes one turn around its axis in 24 hours with respect to the Sun. It should do so in exactly 24 hours, but it doesn't. In fact, Oechslin writes that the Earth makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}_{100}^{0} = \mathrm{V}_{100}^{98} + \mathrm{V}_{98}^{0} = \mathrm{V}_{98}^{0} \times \left(\frac{143175}{392} + 1\right) = \mathrm{V}_{98}^{0} \times \frac{143567}{392} \tag{24.52}$$

$$= \frac{138229}{50487110} \times \frac{143567}{392} = \frac{405002507}{403896880} \tag{24.53}$$

$$\mathrm{P}_{100}^{0} = \frac{403896880}{405002507} = 23 \text{ h } 56 \text{ mn } 4.1343\ldots \text{ s} \tag{24.54}$$

This period is close to the actual sidereal rotation of the Earth.

24.4.1.5 The motion of Venus

The mean motion of Venus is obtained like those of the Earth, Mars, Jupiter and Saturn.

On arbor 65, there is a 20-teeth wheel which meshes with a 96-teeth contrate wheel on frame 113. This wheel turns counterclockwise (from above) with the velocity

$$V_{113}^{0} = V_{65}^{0} \times \left(-\frac{20}{96}\right) = \frac{5}{24} \qquad (24.55)$$

$$P_{113}^{0} = \frac{24}{5} = 4.8 \text{ days} \qquad (24.56)$$

Above the 96-teeth wheel, there is a 103-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 118 of which one pinion meshes with a fixed 65-teeth wheel.[13] This causes the entire carriage to move, and the carriage supports Venus. All these motions take place in a horizontal plane, but the carriage itself has an arbor 120 on which Venus moves up or down, as I will show later. The problem is to find the velocity of the carriage 118. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 103-teeth wheel (frame 113) from that of the 65-teeth wheel (frame 117, fixed). We have

$$V_{113}^{118} = V_{117}^{118} \times \left(-\frac{65}{4}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{22}{10}\right) \times \frac{21}{103} \qquad (24.57)$$

$$= V_{117}^{118} \times \left(-\frac{4719}{103}\right) = V_{118}^{117} \times \frac{4719}{103} \qquad (24.58)$$

The last ratio $\frac{21}{103}$ is positive for the same reason as in the cases of the Earth, Mars, Jupiter and Saturn.

Now, we have

$$V_{113}^{118} = V_{113}^{0} + V_{0}^{118} = V_{118}^{117} \times \frac{4719}{103} = V_{118}^{0} \times \frac{4719}{103} \qquad (24.59)$$

Therefore

$$V_{113}^{0} = V_{118}^{0} \times \frac{4719}{103} + V_{118}^{0} = V_{118}^{0} \times \left(\frac{4719}{103} + 1\right) = V_{118}^{0} \times \frac{4822}{103} \qquad (24.60)$$

And

$$V_{118}^{0} = V_{113}^{0} \times \frac{103}{4822} = \frac{5}{24} \times \frac{103}{4822} = \frac{515}{115728} \qquad (24.61)$$

$$P_{118}^{0} = \frac{115728}{515} = 224.7145\ldots \text{ days} \qquad (24.62)$$

[13] However, Oechslin also writes "198/65" for the number of teeth, and it isn't clear what he meant. Perhaps there is an unused wheel of 198 teeth.

This is an approximation of the sidereal orbital period of Venus. The same value is given by Oechslin.

Venus also rotates around its axis. In the (synodic) reference frame 118, we have

$$V_{120}^{118} = V_{117}^{118} \times \left(-\frac{65}{4}\right) \times \left(-\frac{44}{7}\right) \times \left(-\frac{22}{10}\right) \times \left(-\frac{10}{20}\right) \times \left(-\frac{20}{10}\right) \quad (24.63)$$

$$= V_{117}^{118} \times \left(-\frac{1573}{7}\right) = V_{118}^{0} \times \frac{1573}{7} \quad (24.64)$$

$$= \frac{515}{115728} \times \frac{1573}{7} = \frac{810095}{810096} \quad (24.65)$$

$$P_{120}^{118} = \frac{810096}{810095} = 1.000001\ldots \text{ days} = 86400.1066\ldots \text{ seconds} \quad (24.66)$$

It would seem that Neßtfell attempted here too to have Venus make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. Like in the case of the Earth, Oechslin writes that Venus makes one turn with respect to the Sun in one day, but this is not true.

Incidentally, we can see that Nessfell uses a ratio $\frac{n+1}{n}$, and such ratios appear in several places, although there does not seem to be any good reason for it.

Now, in the absolute frame, we have:

$$V_{120}^{0} = V_{120}^{118} + V_{118}^{0} = V_{118}^{0} \times \left(\frac{1573}{7} + 1\right) = V_{118}^{0} \times \frac{1580}{7} \quad (24.67)$$

$$= \frac{515}{115728} \times \frac{1580}{7} = \frac{203425}{202524} \quad (24.68)$$

$$P_{120}^{0} = \frac{202524}{203425} = 23 \text{ h } 53 \text{ mn } 37.3213\ldots \text{ s} \quad (24.69)$$

The same value is given by Oechslin. This period is very inaccurate, because Venus rotates much more slowly. It makes one turn in 243 days. But in the 18th century, Venus's motion around its axis was still a mystery, and some thought it was rotating in 23 hours, others in 25 days.

24.4.1.6 The motion of Mercury

The mean motion of Mercury is obtained like those of Venus, the Earth, Mars, Jupiter and Saturn.

On arbor 65, there is a 21-teeth wheel which meshes with a 84-teeth contrate wheel on frame 121. This wheel turns counterclockwise (from above) with the velocity

$$V_{121}^{0} = V_{65}^{0} \times \left(-\frac{21}{84}\right) = \frac{1}{4} \quad (24.70)$$

$$P_{121}^{0} = 4 \text{ days} \quad (24.71)$$

Above the 84-teeth wheel, there is a 88-teeth wheel which meshes with a 21-teeth wheel which is part of a carriage 126 of which one pinion meshes with a fixed 60-teeth wheel. This causes the entire carriage to move, and the carriage supports Mercury. All these motions take place in a horizontal plane, but the carriage itself has an arbor 128 on which Mercury moves up or down, as I will show later. The problem is to find the velocity of the carriage 126. We can do so by setting ourself in the reference frame of the moving carriage. We can merely compute the velocity of the 88-teeth wheel (frame 121) from that of the 60-teeth wheel (frame 125, fixed). We have

$$\mathrm{V}_{121}^{126} = \mathrm{V}_{125}^{126} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{19}\right) \times \left(-\frac{30}{7}\right) \times \frac{21}{88} \tag{24.72}$$

$$= \mathrm{V}_{125}^{126} \times \left(-\frac{8775}{418}\right) = \mathrm{V}_{126}^{125} \times \frac{8775}{418} \tag{24.73}$$

The last ratio $\frac{21}{88}$ is positive for the same reason as in the cases of Venus, the Earth, Mars, Jupiter and Saturn.

Now, we have

$$\mathrm{V}_{121}^{126} = \mathrm{V}_{121}^{0} + \mathrm{V}_{0}^{126} = \mathrm{V}_{126}^{125} \times \frac{8775}{418} = \mathrm{V}_{126}^{0} \times \frac{8775}{418} \tag{24.74}$$

Therefore

$$\mathrm{V}_{121}^{0} = \mathrm{V}_{126}^{0} \times \frac{8775}{418} + \mathrm{V}_{126}^{0} = \mathrm{V}_{126}^{0} \times \left(\frac{8775}{418} + 1\right) = \mathrm{V}_{126}^{0} \times \frac{9193}{418} \tag{24.75}$$

And

$$\mathrm{V}_{126}^{0} = \mathrm{V}_{121}^{0} \times \frac{418}{9193} = \frac{1}{4} \times \frac{418}{9193} = \frac{209}{18386} \tag{24.76}$$

$$\mathrm{P}_{126}^{0} = \frac{18386}{209} = 87.9712\ldots \text{ days} \tag{24.77}$$

This is an approximation of the sidereal orbital period of Mercury. The same value is given by Oechslin.

Mercury also rotates around its axis. In the (synodic) reference frame 126, we have

$$\mathrm{V}_{128}^{126} = \mathrm{V}_{125}^{126} \times \left(-\frac{60}{6}\right) \times \left(-\frac{39}{19}\right) \times \left(-\frac{30}{7}\right) \times \left(-\frac{12}{18}\right) \times \left(-\frac{18}{12}\right) \tag{24.78}$$

$$= \mathrm{V}_{125}^{126} \times \left(-\frac{11700}{133}\right) = \mathrm{V}_{126}^{0} \times \frac{11700}{133} \tag{24.79}$$

$$= \frac{209}{18386} \times \frac{11700}{133} = \frac{64350}{64351} \tag{24.80}$$

$$\mathrm{P}_{128}^{126} = \frac{64351}{64350} = 1.00001\ldots \text{ days} = 86401.3426\ldots \text{ seconds} \tag{24.81}$$

Again, it seems that Neßtfell attempted here too to have Mercury make one turn around itself in 24 hours with respect to the Sun, although the basis of this assumption is not clear. And like in the case of the Earth and Venus, Oechslin writes that Mercury makes one turn with respect to the Sun in one day, but this is not true.

Now, in the absolute frame, we have:

$$\mathrm{V}_{128}^{0} = \mathrm{V}_{128}^{126} + \mathrm{V}_{126}^{0} = \mathrm{V}_{126}^{0} \times \left(\frac{11700}{133} + 1\right) = \mathrm{V}_{126}^{0} \times \frac{11833}{133} \tag{24.82}$$

$$= \frac{209}{18386} \times \frac{11833}{133} = \frac{130163}{128702} \tag{24.83}$$

$$\mathrm{P}_{128}^{0} = \frac{128702}{130163} = 23 \text{ h } 43 \text{ mn } 50.2128\ldots \text{ s} \tag{24.84}$$

Oechslin gives the same value, although there is a typo in his numerical evaluation.

This period is of course very inaccurate, because Mercury rotates much more slowly. It makes one turn in about 59 days. But in the 18th century, Mercury's motion around its axis was still a mystery.

Finally, there also seems to be a dial for the orbital rotation of Mercury. This dial is located on frame 131 which merely replicates the (fixed) frame 125:

$$\mathrm{V}_{131}^{126} = \mathrm{V}_{125}^{126} \times \left(-\frac{60}{6}\right) \times \left(-\frac{6}{60}\right) = \mathrm{V}_{125}^{126} \tag{24.85}$$

$$\mathrm{V}_{131}^{0} = \mathrm{V}_{131}^{126} + \mathrm{V}_{126}^{0} = \mathrm{V}_{125}^{126} + \mathrm{V}_{126}^{0} = \mathrm{V}_{125}^{0} \tag{24.86}$$

24.4.2 The motion of the Sun

The motion of the Sun on tube 130 is derived from that of Mercury:

$$\mathrm{V}_{130}^{126} = \mathrm{V}_{125}^{126} \times \left(-\frac{60}{6}\right) \times \left(-\frac{6}{10}\right) \times \left(-\frac{9}{22}\right) \tag{24.87}$$

$$= \mathrm{V}_{125}^{126} \times \left(-\frac{27}{11}\right) = \mathrm{V}_{126}^{125} \times \frac{27}{11} = \mathrm{V}_{126}^{0} \times \frac{27}{11} \tag{24.88}$$

$$\mathrm{V}_{130}^{0} = \mathrm{V}_{130}^{126} + \mathrm{V}_{126}^{0} = \mathrm{V}_{126}^{0} \times \left(\frac{27}{11} + 1\right) \tag{24.89}$$

$$= \mathrm{V}_{126}^{0} \times \frac{38}{11} = \frac{209}{18386} \times \frac{38}{11} = \frac{361}{9193} \tag{24.90}$$

$$\mathrm{P}_{130}^{0} = \frac{9193}{361} = 25.4653\ldots \text{ days} \tag{24.91}$$

The same value is given by Oechslin. This is a good approximation of the period of rotation of the Sun which is about 25 days at the equator. The motion is counterclockwise, as it should be. However, on the Munich machine, Neßtfell used a slightly different period, and had the Sun rotate clockwise (from above), which is wrong.[14]

[14] See also [20, p. 203].

24.4.3 The satellites of Jupiter and Saturn

24.4.3.1 The system of Jupiter

The gears for the rotation of Jupiter and its satellites are contained in frame 74 which is gliding along a tilted slope. This frame therefore does have a fixed orientation with respect to the Sun. The motion of frame 74 is therefore exactly that of the carriage 73 seen earlier:

$$V_{74}^{0} = V_{73}^{0} \tag{24.92}$$

The input to frame 74 is arbor 69 whose motion is

$$V_{69}^{73} = V_{72}^{73} \times \left(-\frac{306}{4}\right) \times \left(-\frac{95}{8}\right) \times \left(-\frac{62}{13}\right) \tag{24.93}$$

$$= V_{72}^{73} \times \left(-\frac{450585}{104}\right) = V_{73}^{72} \times \frac{450585}{104} = V_{73}^{0} \times \frac{450585}{104} \tag{24.94}$$

$$= \frac{247}{1070139} \times \frac{450585}{104} = \frac{2853705}{2853704} \tag{24.95}$$

We can then easily compute the velocities of Jupiter and its four satellites in the rotating frame. All these motions are directly obtained from arbor 69, and not one from the other.[15]

$$V_{76}^{74} = V_{69}^{74} \times \left(-\frac{29}{37}\right) \times \left(-\frac{37}{12}\right) = V_{69}^{74} \times \frac{29}{12} \text{ (Jupiter)} \tag{24.96}$$

$$V_{78}^{74} = V_{69}^{74} \times \left(-\frac{31}{32}\right) \times \left(-\frac{28}{48}\right) = V_{69}^{74} \times \frac{217}{384} \text{ (Io)} \tag{24.97}$$

$$V_{80}^{74} = V_{69}^{74} \times \left(-\frac{26}{42}\right) \times \left(-\frac{15}{33}\right) = V_{69}^{74} \times \frac{65}{231} \text{ (Europa)} \tag{24.98}$$

$$V_{82}^{74} = V_{69}^{74} \times \left(-\frac{20}{43}\right) \times \left(-\frac{12}{40}\right) = V_{69}^{74} \times \frac{6}{43} \text{ (Ganymede)} \tag{24.99}$$

$$V_{84}^{74} = V_{69}^{74} \times \left(-\frac{22}{43}\right) \times \left(-\frac{7}{60}\right) = V_{69}^{74} \times \frac{77}{1290} \text{ (Callisto)} \tag{24.100}$$

And in the absolute frame:

$$V_{76}^{0} = V_{76}^{74} + V_{74}^{0} = V_{69}^{74} \times \frac{29}{12} + V_{74}^{0} = \frac{2853705}{2853704} \times \frac{29}{12} + \frac{247}{1070139} \tag{24.101}$$

$$= \frac{82765349}{34244448} \tag{24.102}$$

$$P_{76}^{0} = \frac{34244448}{82765349} = 9.9300\ldots \text{ hours} = 9 \text{ h } 55 \text{ mn } 48.2973 \text{ s} \tag{24.103}$$

[15] In the gear train for Io, Oechslin mistakenly wrote 28 instead of 48.

The same value is given by Oechslin. This is an approximation of the rotation of Jupiter around its axis which is completed in about 9.8 hours.

$$V^0_{78} = V^{74}_{78} + V^0_{74} = V^{74}_{69} \times \frac{217}{384} + V^0_{74} = \frac{2853705}{2853704} \times \frac{217}{384} + \frac{247}{1070139} \tag{24.104}$$

$$= \frac{619506913}{1095822336} \tag{24.105}$$

$$P^0_{78} = \frac{1095822336}{619506913} = 1.7688\ldots \text{ days (Io)} \tag{24.106}$$

$$V^0_{80} = V^{74}_{80} + V^0_{74} = V^{74}_{69} \times \frac{65}{231} + V^0_{74} = \frac{2853705}{2853704} \times \frac{65}{231} + \frac{247}{1070139} \tag{24.107}$$

$$= \frac{185642977}{659205624} \tag{24.108}$$

$$P^0_{80} = \frac{659205624}{185642977} = 3.5509\ldots \text{ days (Europa)} \tag{24.109}$$

$$V^0_{82} = V^{74}_{82} + V^0_{74} = V^{74}_{69} \times \frac{6}{43} + V^0_{74} = \frac{2853705}{2853704} \times \frac{6}{43} + \frac{247}{1070139} \tag{24.110}$$

$$= \frac{25725829}{184063908} \tag{24.111}$$

$$P^0_{82} = \frac{184063908}{25725829} = 7.1548\ldots \text{ days (Ganymede)} \tag{24.112}$$

$$V^0_{84} = V^{74}_{84} + V^0_{74} = V^{74}_{69} \times \frac{77}{1290} + V^0_{74} = \frac{2853705}{2853704} \times \frac{77}{1290} + \frac{247}{1070139} \tag{24.113}$$

$$= \frac{44116993}{736255632} \tag{24.114}$$

$$P^0_{84} = \frac{736255632}{44116993} = 16.6887\ldots \text{ days (Callisto)} \tag{24.115}$$

The same values are given by Oechslin. The periods of the satellites are all good approximations of the actual ones.

24.4.3.2 The system of Saturn

The structure of the Saturn system is similar to that of Jupiter, except that it contains two worms for the slow motion of Saturn's fifth satellite, Iapetus.

Figure 24.9: Detail of Neßtfell's machine in Vienna. The gears for Saturn's satellites are visible in the back. The largest ring is for Saturn, the other one for Jupiter. (photograph by the author)

The gears for the rotation of Saturn and its satellites are contained in frame 49 which is gliding along a tilted slope. This frame therefore does have a fixed orientation with respect to the Sun. The motion of frame 49 is therefore exactly that of the carriage 48 seen earlier:

$$V_{49}^{0} = V_{48}^{0} \tag{24.116}$$

The input to frame 49 is arbor 44 whose motion is

$$V_{44}^{48} = V_{47}^{48} \times \left(-\frac{690}{6}\right) \times \left(-\frac{109}{6}\right) \times \left(-\frac{103}{20}\right) \tag{24.117}$$

$$= V_{47}^{48} \times \left(-\frac{258221}{24}\right) = V_{48}^{47} \times \frac{258221}{24} = V_{48}^{0} \times \frac{258221}{24} \tag{24.118}$$

$$= \frac{1440}{15493283} \times \frac{258221}{24} = \frac{673620}{673621} \tag{24.119}$$

Figure 24.10: Detail of Neßtfell's machine in Vienna. (photograph by the author)

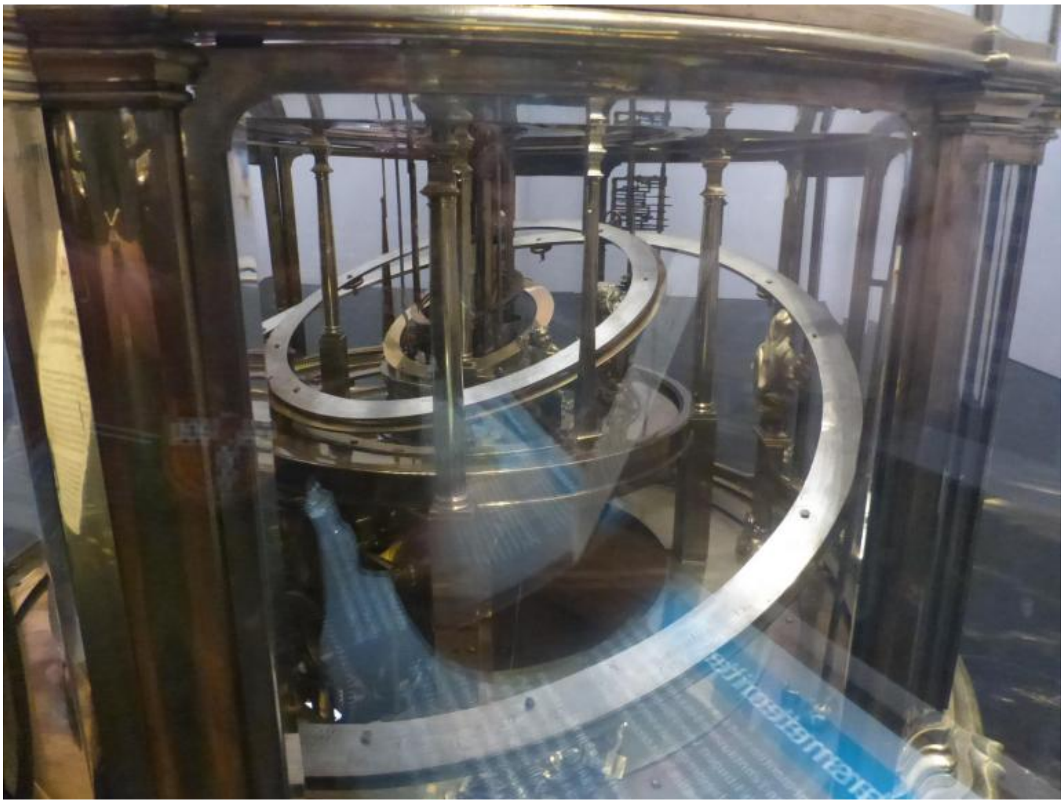

Figure 24.11: Detail of Neßtfell's machine in Vienna. (photograph by the author)

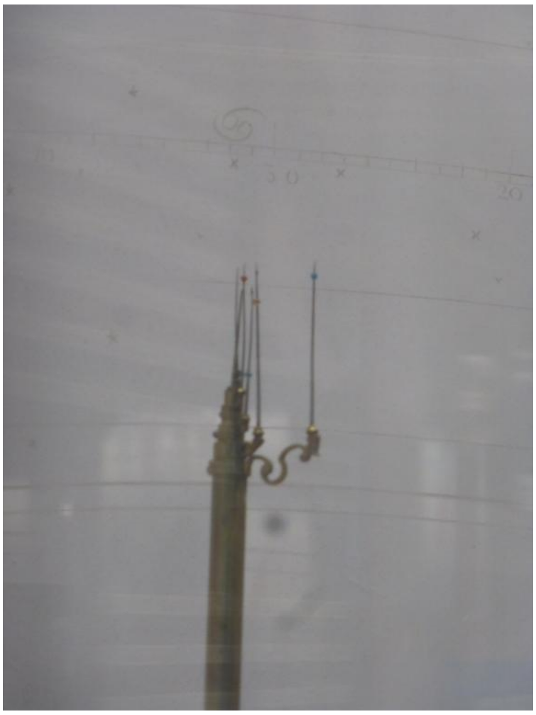

Figure 24.12: The satellites of Saturn on Neßtfell's machine in Vienna. (photograph by the author)

We can then easily compute the velocities of Saturn and its five satellites in the rotating frame. All these motions are directly obtained from arbor 44, and not one from the other.[16]

$$\mathrm{V}_{51}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{30}{40}\right) \times \left(-\frac{40}{10}\right) = \mathrm{V}_{44}^{49} \times 3 \text{ (Saturn)} \qquad (24.120)$$

$$\mathrm{V}_{53}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{24}{19}\right) \times \left(-\frac{13}{31}\right) = \mathrm{V}_{44}^{49} \times \frac{312}{589} \text{ (Tethys)} \qquad (24.121)$$

$$\mathrm{V}_{55}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{19}{29}\right) \times \left(-\frac{29}{52}\right) = \mathrm{V}_{44}^{49} \times \frac{19}{52} \text{ (Dione)} \qquad (24.122)$$

$$\mathrm{V}_{57}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{47}{30}\right) \times \left(-\frac{6}{43}\right) = \mathrm{V}_{44}^{49} \times \frac{47}{215} \text{ (Rhea)} \qquad (24.123)$$

$$\mathrm{V}_{59}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{29}{34}\right) \times \left(-\frac{5}{68}\right) = \mathrm{V}_{44}^{49} \times \frac{145}{2312} \text{ (Titan)} \qquad (24.124)$$

$$\mathrm{V}_{63}^{49} = \mathrm{V}_{44}^{49} \times \left(-\frac{2}{8}\right) \times \left(-\frac{1}{20}\right) \times \left(-\frac{53}{11}\right) \times \left(-\frac{9}{43}\right) \qquad (24.125)$$

$$= \mathrm{V}_{44}^{49} \times \frac{477}{37840} \text{ (Iapetus)} \qquad (24.126)$$

[16]In the case of Iapetus, Oechslin's drawing forgot to give the number of teeth 43 for the wheel on tube 63.

And in the absolute frame:

$$V_{51}^{0} = V_{51}^{49} + V_{49}^{0} = V_{44}^{49} \times 3 + V_{49}^{0} = \frac{673620}{673621} \times 3 + \frac{1440}{15493283} \quad (24.127)$$

$$= \frac{46481220}{15493283} \quad (24.128)$$

$$P_{51}^{0} = \frac{15493283}{46481220} = 7.9997\ldots \text{ hours (Saturn)} \quad (24.129)$$

$$7 \text{ h } 59 \text{ m } 59.1505\ldots \text{ s} \quad (24.130)$$

The same value is given by Oechslin. The actual period of rotation of Saturn is about 10.65 hours.

$$V_{53}^{0} = V_{53}^{49} + V_{49}^{0} = V_{44}^{49} \times \frac{312}{589} + V_{49}^{0} = \frac{673620}{673621} \times \frac{312}{589} + \frac{1440}{15493283} \quad (24.131)$$

$$= \frac{4834745280}{9125543687} \quad (24.132)$$

$$P_{53}^{0} = \frac{9125543687}{4834745280} = 1.8874\ldots \text{ days (Tethys)} \quad (24.133)$$

$$V_{55}^{0} = V_{55}^{49} + V_{49}^{0} = V_{44}^{49} \times \frac{19}{52} + V_{49}^{0} = \frac{673620}{673621} \times \frac{19}{52} + \frac{1440}{15493283} \quad (24.134)$$

$$= \frac{73611705}{201412679} \quad (24.135)$$

$$P_{55}^{0} = \frac{201412679}{73611705} = 2.7361\ldots \text{ days (Dione)} \quad (24.136)$$

$$V_{57}^{0} = V_{57}^{49} + V_{49}^{0} = V_{44}^{49} \times \frac{47}{215} + V_{49}^{0} = \frac{673620}{673621} \times \frac{47}{215} + \frac{1440}{15493283} \quad (24.137)$$

$$= \frac{145698564}{666211169} \quad (24.138)$$

$$P_{57}^{0} = \frac{666211169}{145698564} = 4.5725\ldots \text{ days (Rhea)} \quad (24.139)$$

$$V_{59}^{0} = V_{59}^{49} + V_{49}^{0} = V_{44}^{49} \times \frac{145}{2312} + V_{49}^{0} = \frac{673620}{673621} \times \frac{145}{2312} + \frac{1440}{15493283} \quad (24.140)$$

$$= \frac{562462995}{8955117574} \quad (24.141)$$

$$P_{59}^{0} = \frac{8955117574}{562462995} = 15.9212\ldots \text{ days (Titan)} \quad (24.142)$$

$$V_{63}^{0} = V_{63}^{49} + V_{49}^{0} = V_{44}^{49} \times \frac{477}{37840} + V_{49}^{0} \quad (24.143)$$

$$= \frac{673620}{673621} \times \frac{477}{37840} + \frac{1440}{15493283} \quad (24.144)$$

$$= \frac{372238731}{29313291436} \quad (24.145)$$

$$P_{63}^{0} = \frac{29313291436}{372238731} = 78.7486\ldots \text{ days (Iapetus)} \quad (24.146)$$

The same values are given by Oechslin. These period are good approximations of the actual sidereal periods of the satellites of Saturn.

24.4.4 The motion of the Moon

24.4.4.1 The mean motion

The main motion of the Moon is that of frame 104 which rotates around the axis of the Earth. Its motion is obtained through arbor 102 which is located on the Earth carriage 98. This arbor carries a 33-teeth wheel which meshes with a 20-teeth wheel at right angle, and whose arbor 103 carries a worm meshing with a 15-teeth wheel located on frame 104. We can compute the motion of frame 104 in frame 98:

$$V_{104}^{98} = V_{102}^{98} \times \left(-\frac{33}{20}\right) \times \left(-\frac{1}{15}\right) = V_{102}^{98} \times \frac{11}{100} \tag{24.147}$$

and

$$V_{102}^{98} = V_{97}^{98} \times \left(-\frac{83}{8}\right) \times \left(-\frac{69}{7}\right) \times \left(-\frac{50}{14}\right) \times \left(-\frac{17}{21}\right) \times \left(-\frac{27}{71}\right) \tag{24.148}$$

$$= V_{97}^{98} \times \left(-\frac{21905775}{194824}\right) \tag{24.149}$$

and therefore

$$V_{104}^{98} = V_{97}^{98} \times \left(-\frac{21905775}{194824}\right) \times \frac{11}{100} = V_{97}^{98} \times \left(-\frac{9638541}{779296}\right) \tag{24.150}$$

$$= V_{98}^{97} \times \frac{9638541}{779296} = V_{98}^{0} \times \frac{9638541}{779296} \tag{24.151}$$

$$= \frac{138229}{50487110} \times \frac{9638541}{779296} = \frac{228490119}{6747453760} \tag{24.152}$$

$$P_{104}^{98} = \frac{6747453760}{228490119} = 29.5306\ldots \text{ days} \tag{24.153}$$

The same value is given by Oechslin. This is the motion of the Moon with respect to the Sun, and an excellent approximation of the synodic month.

We can also compute the motion of the Moon in an absolute (sidereal) frame:

$$V_{104}^{0} = V_{104}^{98} + V_{98}^{0} = \frac{228490119}{6747453760} + \frac{138229}{50487110} = \frac{4198388311}{114706713920} \tag{24.154}$$

$$P_{104}^{0} = \frac{114706713920}{4198388311} = 27.3216\ldots \text{ days} \tag{24.155}$$

The same value is also given by Oechslin. This is an approximation of the sidereal month.

24.4.4.2 The nodes and the apsides

Neßtfell's construction for the nodes and apsides is at first sight very opaque (figure 24.13) and Oechslin's drawing (figure 24.14) does not help much to understand it. However, I believe that I have managed to understand it correctly. Here it goes.

(Copyrighted image not shown)

Figure 24.13: Neßtfell's construction for the motion of the nodes and apsides of the Moon.

(Copyrighted image not shown)

Figure 24.14: Oechslin's drawing of Neßtfell's construction.

There is a rotating frame 106 carrying a number of gears for the motions of the lunar nodes and apsides. This frame rotates with carriage 98, but remains in the plane of the ecliptic.

There is also a fixed frame 105 which carries a 138-teeth wheel. This wheel meshes with two different wheels. First it meshes with a 29-teeth wheel for the revolution of the nodes. The arbor of this wheel carries a 31-teeth wheel which meshes with a 7-teeth wheel whose arbor carries a worm. This worm meshes with a 20-teeth wheel which moves the nodes on arbor 109.

We can compute the motion of arbor 109 with respect to frame 106:

$$V_{109}^{106} = V_{105}^{106} \times \left(-\frac{138}{29}\right) \times \left(-\frac{31}{7}\right) \times \frac{1}{20} = V_{105}^{106} \times \frac{2139}{2030} \tag{24.156}$$

$$= V_{106}^{105} \times \left(-\frac{2139}{2030}\right) = V_{98}^{0} \times \left(-\frac{2139}{2030}\right) \tag{24.157}$$

$$= \frac{138229}{50487110} \times \left(-\frac{2139}{2030}\right) = -\frac{42238833}{14641261900} \tag{24.158}$$

$$P_{109}^{106} = -\frac{14641261900}{42238833} = -346.6303\ldots \text{ days} \tag{24.159}$$

This is the period of revolution of the nodes with respect to the Sun, the so-called eclipse year. I have assumed that the worm makes the 20-teeth wheel move in the correct direction, but I cannot check that this is actually the case.

We can then compute the motion of the nodes in the absolute frame:

$$V_{109}^{0} = V_{109}^{106} + V_{106}^{0} = V_{109}^{106} + V_{98}^{0} \tag{24.160}$$

$$= -\frac{42238833}{14641261900} + \frac{138229}{50487110} = -\frac{2152423}{14641261900} \tag{24.161}$$

$$P_{109}^{0} = -\frac{14641261900}{2152423} = -6802.2233\ldots \text{ days} \tag{24.162}$$

which is a good approximation of the actual value of 6798 days. This value is negative, because the lunar nodes are retrograding. The same value is given by Oechslin.

Second, the 138-teeth wheel of frame 105 meshes with a 11-teeth wheel for the revolution of the apsides. The arbor of this wheel carries a 2-threaded worm which meshes with a 6-teeth wheel. And the arbor of this wheel carries another worm which meshes with a 37-teeth wheel which moves the apsides on arbor 112.

We can compute the motion of arbor 112 with respect to frame 106:

$$V_{112}^{106} = V_{105}^{106} \times \left(-\frac{138}{11}\right) \times \left(-\frac{2}{6}\right) \times \left(-\frac{1}{37}\right) = V_{105}^{106} \times \left(-\frac{46}{407}\right) \tag{24.163}$$

$$= V_{106}^{105} \times \frac{46}{407} = V_{98}^{0} \times \frac{46}{407} \tag{24.164}$$

$$= \frac{138229}{50487110} \times \frac{46}{407} = \frac{3179267}{10274126885} \tag{24.165}$$

$$P_{112}^{106} = \frac{10274126885}{3179267} = 3231.6024\ldots \text{ days} \tag{24.166}$$

This is a good approximation of the revolution of the apsides, *but with respect to the absolute frame*. And therefore it is false.[17]

However, we should have found a period of about 411 days, which is the period of revolution of the apsides with respect to the Sun. It would then seem that Neßtfell mixed up his computations, and used a gear train meant for the absolute motion of the apsides in the context of a synodic motion of the apsides. The problem is not a mere typo or one wheel that could be easily fixed.

If we now compute the motion of the apsides in the absolute frame, we of course do not obtain anything meaningful:

$$\mathrm{V}_{112}^{0} = \mathrm{V}_{112}^{106} + \mathrm{V}_{106}^{0} = \mathrm{V}_{112}^{106} + \mathrm{V}_{98}^{0} \tag{24.167}$$

$$= \frac{3179267}{10274126885} + \frac{138229}{50487110} = \frac{62617737}{20548253770} \tag{24.168}$$

$$\mathrm{P}_{112}^{0} = \frac{20548253770}{62617737} = 328.1538\ldots \text{ days} \tag{24.169}$$

Interestingly, Oechslin has found a period of 411 days in the absolute frame using an incorrect computation, but dismissed it as unmeaningful.

[17] About the error in the motion of the apsides, see also [20, p. 203].

24.5 The Earth-Mercury system

The front part of the machine is the Earth-Mercury system and is focused on displaying the apparent motion of Mercury. Both the Earth and Mercury rotate around the central axis which represents the Sun. Such a main display with Mercury on an astronomical clock is unusual, but the choice of that display is certainly a consequence of the interest for the transits of Mercury in the 18th century. Neßtfell certainly did not choose to display the Earth-Venus system, because transits of Venus are much less frequent than those of Mercury. This representation also has a pedogogical function, it serves to highlight a particular part of the solar system, as stressed by Meier.[18] Finally, it is also a way to illustrate the retrogradations of the planets, as Neßtfell explained himself, and as Oechslin observed [20, p. 36].

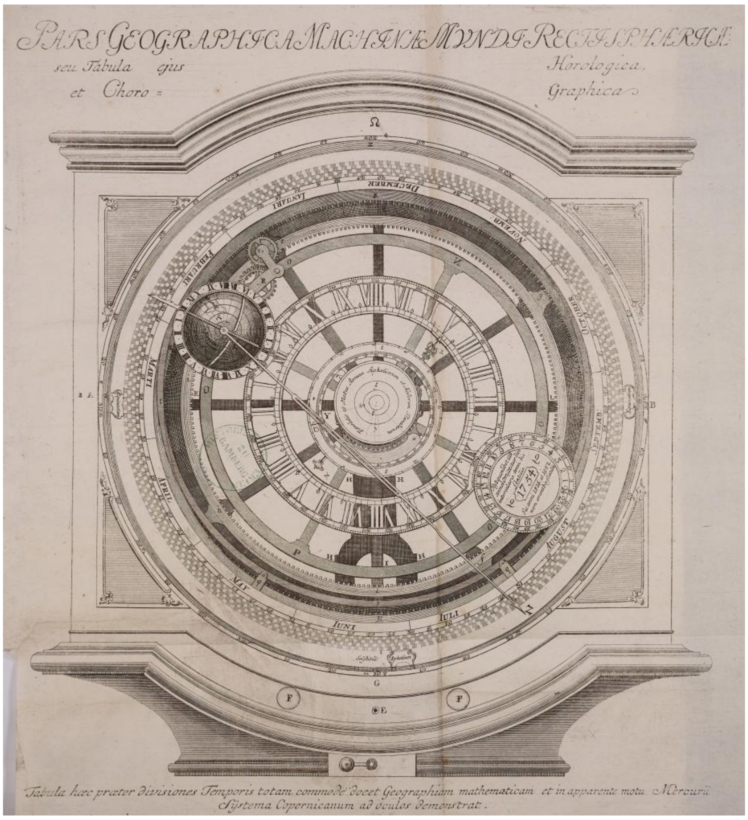

Figure 24.15: Engraving of the Earth-Mercury system on Neßtfell's machine in Vienna, probably from 1754, given the year on the engraving. (Johann Balthasar Gutwein, source: [18])

The central axis of the panel represents the position of the Sun and the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves around the Sun as I will describe

[18]Cf. [16, p. 44-45].

Figure 24.16: The front side of Neßtfell's machine in Vienna. (photograph by the author)

Figure 24.17: The 24-hour dial on the front side of Neßtfell's machine in Vienna. This dial gives also the year, but there is a mention that the year 1828 should not be a leap year. On the engraving of figure 24.15, it is 1826 that should not be a leap year. (photograph by the author)

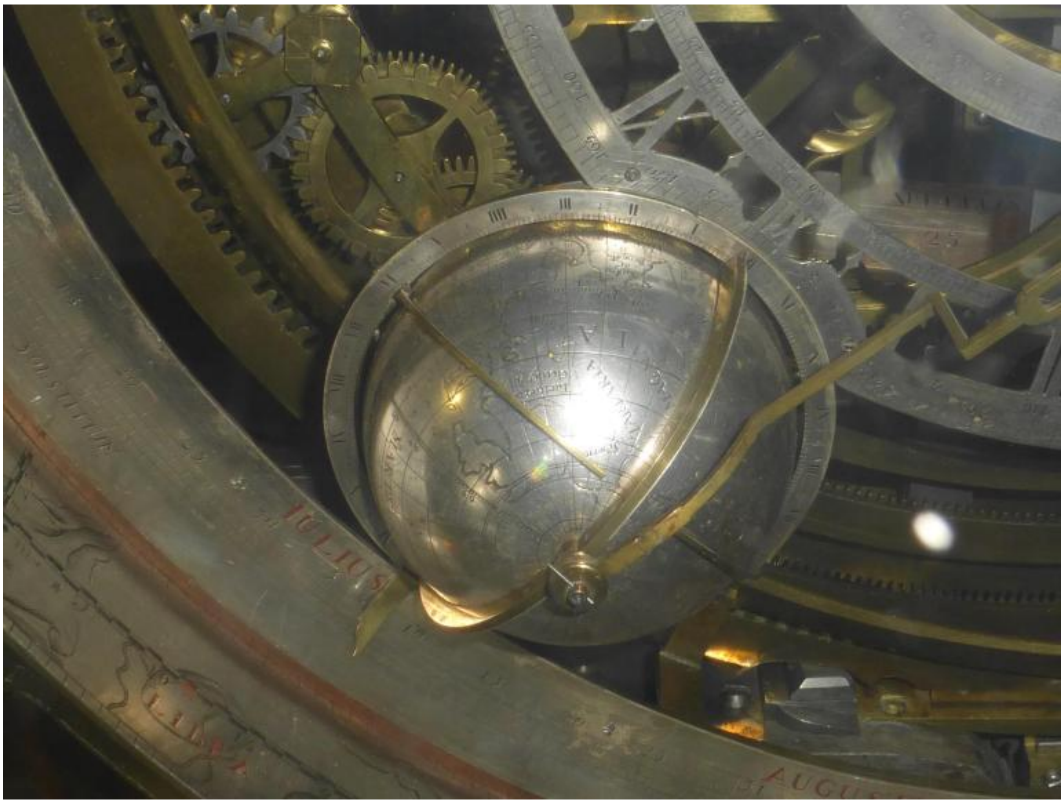

Figure 24.18: The Earth sphere on Neßtfell's machine in Vienna. (photograph by the author)

Figure 24.19: The central front dial of Neßtfell's machine in Vienna. Mercury is at the lower right. (photograph by the author)

Figure 24.20: The mention of the fall equinox on the calendar dial. (photograph by the author)

below.

The input to the Earth-Mercury system is arbor 13 which comes from the going work. This arbor makes one turn clockwise in one day:

$$\mathrm{V}_{13}^{0} = -\mathrm{T}_{13}^{0} = -1 \qquad (24.170)$$

This arbor is also used to derive several other auxiliary motions which will be described later.

24.5.1 The mean orbital motion of the Earth

Arbor 13 carries a 26-teeth wheel which meshes with a 340-teeth wheel on frame 14. The axis of this frame is the central axis of the front part. The velocity of frame 14 (seen from the front) is

$$\mathrm{V}_{14}^{0} = \mathrm{V}_{13}^{0} \times \left(-\frac{26}{340}\right) = \frac{13}{170} \qquad (24.171)$$

As I note below, it is possible that the distance between the axes of the 26-teeth wheel and the 340-teeth wheel varies, because of the motion of the line of apsides.

Frame 14 also carries an interior wheel of 434 teeth, which meshes with a 32-teeth wheel which is part of a train located on frame 20 and ending with a 32-teeth wheel on tube 22. This tube is not fixed, although one might expect it to be fixed. The frame 20 also carries the Earth's meridian as well as an arbor for the rotation of the Earth. We might therefore expect frame 20 to rotate in a tropical year.

The motion of frame 20 is derived from a train laid between the meridian's axis and a fixed 249-teeth wheel on frame 19. We can compute the motion of frame 20 by first considering the relative motion of frame 19 with respect to that of frame 14:

$$\mathrm{V}_{19}^{20} = \mathrm{V}_{14}^{20} \times \frac{434}{32} \times \left(-\frac{14}{50}\right) \times \left(-\frac{7}{69}\right) \times \left(-\frac{24}{19}\right) \times \frac{19}{249} \qquad (24.172)$$

$$= \mathrm{V}_{14}^{20} \times \left(-\frac{10633}{286350}\right) \qquad (24.173)$$

$$= \left(\mathrm{V}_{20}^{0} + \mathrm{V}_{0}^{14}\right) \times \frac{10633}{286350} \qquad (24.174)$$

Hence

$$\mathrm{V}_{14}^{0} \times \frac{10633}{286350} = \mathrm{V}_{20}^{0} \times \left(1 + \frac{10633}{286350}\right) = \mathrm{V}_{20}^{0} \times \frac{296983}{286350} \qquad (24.175)$$

And

$$\mathrm{V}_{20}^{0} = \mathrm{V}_{14}^{0} \times \frac{10633}{296983} = \frac{13}{170} \times \frac{10633}{296983} = \frac{138229}{50487110} \qquad (24.176)$$

$$\mathrm{P}_{20}^{0} = \frac{50487110}{138229} = 365.2425\ldots \text{ days} \qquad (24.177)$$

This is an approximation of the tropical year. The same value is given by Oechslin.

24.5.2 The rotation of the Earth around its axis

We can now also compute the velocity of the central tube 22:

$$\mathrm{V}_{22}^{20} = \mathrm{V}_{15}^{20} = \mathrm{V}_{14}^{20} \times \frac{434}{32} = \left(\mathrm{V}_{14}^{0} - \mathrm{V}_{20}^{0}\right) \times \frac{217}{16} \quad (24.178)$$

$$= \left(\frac{13}{170} - \frac{138229}{50487110}\right) \times \frac{217}{16} = \frac{372255}{5048711} \times \frac{217}{16} = \frac{80779335}{80779376} \quad (24.179)$$

$$\mathrm{V}_{22}^{0} = \mathrm{V}_{22}^{20} + \mathrm{V}_{20}^{0} = \frac{80779335}{80779376} + \frac{138229}{50487110} = \frac{405002507}{403896880} \quad (24.180)$$

$$\mathrm{P}_{22}^{0} = \frac{403896880}{405002507} = 0.9972\ldots \text{ days} = 86164.1343\ldots \text{ seconds} \quad (24.181)$$

The central tube 22 actually rotates with the velocity of the sidereal day.

Moreover, arbor 15 which carries the 32-teeth wheel meshing with the interior gearing also carries the frame for the Earth's meridian. Because of the two 32-teeth wheels, the one on the central fixed tube 22, and the one on tube 15, arbor 15 replicates the motion of the central tube. The Earth therefore rotates around a vertical axis (and not a tilted one) and makes one turn in one sidereal day in the absolute frame.

24.5.3 The line of apsides

As mentioned above, the Earth actually revolves around the Sun on an eccentric orbit. This eccentric orbit is centered on a point which moves like the line of the apsides. However, since the motion of the Earth is obtained through the 26-teeth wheel on arbor 13 meshing with the 340-teeth wheel on frame 14, this implies that there is a variable distance between the two axes. If this is the case, then the teeth are probably such that the meshing is maintained, even though the distance varies.[19]

The motion of the line of apsides is that of tube 33. The slow motion of tube 33 is obtained from that of arbor 13, with four intermediate worms:

$$\mathrm{V}_{33}^{0} = \mathrm{V}_{13}^{0} \times \frac{1}{34} \times \frac{1}{30} \times \frac{1}{15} \times \left(-\frac{1}{499}\right) = \mathrm{V}_{13}^{0} \times \left(-\frac{1}{7634700}\right) \quad (24.182)$$

$$= \frac{1}{7634700} \quad (24.183)$$

$$\mathrm{P}_{33}^{0} = 7634700 \text{ days} \approx 20900 \text{ years} \quad (24.184)$$

The same value is given by Oechslin. This is a good approximation of the period of tropical apsidal precession which is about 21000 years.

[19] See Oechslin's observations on this problem [20, p. 154-155, 199-200].

24.5.4 Two more front gears

The front panel carries two large rings, one with 499 teeth (frame 37) and the other with 650 teeth (frame 35). The velocities of these frames are:

$$\mathrm{V}^0_{35} = \mathrm{V}^0_{13} \times \frac{1}{34} \times \frac{1}{30} \times \left(-\frac{1}{14}\right) \times \left(-\frac{1}{650}\right) = \mathrm{V}^0_{13} \times \frac{1}{9282000} \quad (24.185)$$

$$= -\frac{1}{9282000} \quad (24.186)$$

$$\mathrm{P}^0_{35} = -9282000 \text{ days} \approx -25413 \text{ years} \quad (24.187)$$

The same value is given by Oechslin.

$$\mathrm{V}^0_{37} = \mathrm{V}^0_{13} \times \frac{1}{34} \times \frac{1}{30} \times \left(-\frac{1}{15}\right) \times \frac{1}{499} = \mathrm{V}^0_{33} \quad (24.188)$$

$$\mathrm{P}^0_{37} = \mathrm{P}^0_{33} \approx 20900 \text{ years} \quad (24.189)$$

Frame 35 corresponds to the period of precession of the equinoxes. The Munich machine uses a slightly different ratio for that period. The correct value should however have been closer to 26000 years.

However, given that the front panel has the Earth rotate in a tropical year, which is confirmed by the motion of the apsides (which otherwise would not take place in about 21000 years), the zodiac is actually fixed. The actual constellations should then move *counterclockwise* and this is presumably what Neßtfell attempted to display, in order to show where in the sky Mercury is located. But on Neßtfell's machine, frame 35 moves clockwise, as if he wanted to show a precession.

In any case, frame 35 is only used to position the actual axis of the Earth, if I understand it correctly.

Frame 37 is again the period of precession of the apsides with respect to the equinoxes. Frame 37 actually moves like frame 33 and seems to be used as a dial showing the position of the apsides.

24.5.5 The motion of Mercury

The main front reference frame for Mercury is frame 33. We have seen above that this frame moves with the motion of the line of apsides of the orbit of the Earth. The Mercury wheel is a wheel on arbor 24 which pivots eccentrically on frame 33. This will account for the elliptical orbit of Mercury, but it does assume that the line of apsides of Mercury coincides with that of the Earth, which is not the case.

24.5.5.1 The mean motion of Mercury

The mean motion of Mercury is given by the Mercury wheel on arbor 24. This motion is derived from that of tube 22. We have

$$V_{24}^{33} = V_{22}^{33} \times \left(-\frac{18}{68}\right) \times \left(-\frac{7}{163}\right) = V_{22}^{33} \times \frac{63}{5542} \tag{24.190}$$

$$V_{24}^{0} = V_{24}^{33} + V_{33}^{0} = V_{22}^{33} \times \frac{63}{5542} + V_{33}^{0} = \left(V_{22}^{0} - V_{33}^{0}\right) \times \frac{63}{5542} + V_{33}^{0} \tag{24.191}$$

$$= V_{22}^{0} \times \frac{63}{5542} + V_{33}^{0} \times \left(1 - \frac{63}{5542}\right) = V_{22}^{0} \times \frac{63}{5542} + V_{33}^{0} \times \frac{5479}{5542} \tag{24.192}$$

$$= \frac{405002507}{403896880} \times \frac{63}{5542} + \frac{1}{7634700} \times \frac{5479}{5542} \tag{24.193}$$

$$= \frac{572949380244583}{50263193608696800} \tag{24.194}$$

$$P_{24}^{0} = \frac{50263193608696800}{572949380244583} = 87.7271\ldots \text{ days} \tag{24.195}$$

The same value is given by Oechslin. This is a good approximation of the orbital period of Mercury. The same value is used in the Munich machine.

Of course, we could have ignored the term $\frac{1}{7634700}$, and this would only have slightly altered the orbital period of Mercury. But the exact ratio resulting from Neßtfell's construction is the one given above.

24.5.5.2 The inclination of Mercury's orbit

Neßtfell has Mercury move radially depending on its distance from its line of nodes. The orbit of Mercury is tilted by about 7 degrees, but on Neßtfell's panel, this angle is about 5 degrees, according to Oechslin. This tilted orbital plane is represented by frame 29 which carries a 259-teeth wheel. This is rotated using gears located on frame 33. We first compute the velocity of frame 29 with respect to frame 33:

$$V_{29}^{33} = V_{22}^{33} \times \left(-\frac{26}{96}\right) \times \left(-\frac{1}{14}\right) \times \left(-\frac{1}{26}\right) \times \frac{1}{16} \times \left(-\frac{1}{259}\right) \qquad (24.196)$$

$$= V_{22}^{33} \times \frac{1}{5569536} \qquad (24.197)$$

$$V_{29}^{0} = V_{29}^{33} + V_{33}^{0} = V_{22}^{33} \times \frac{1}{5569536} + V_{33}^{0} \qquad (24.198)$$

$$= \left(V_{22}^{0} - V_{33}^{0}\right) \times \frac{1}{5569536} + V_{33}^{0} \qquad (24.199)$$

$$= V_{22}^{0} \times \frac{1}{5569536} + V_{33}^{0} \times \left(1 - \frac{1}{5569536}\right) \qquad (24.200)$$

$$= V_{22}^{0} \times \frac{1}{5569536} + V_{33}^{0} \times \frac{5569535}{5569536} \qquad (24.201)$$

$$= \frac{405002507}{403896880} \times \frac{1}{5569536} + \frac{1}{7634700} \times \frac{5569535}{5569536} \qquad (24.202)$$

$$= \frac{3142112029261}{10102586296593530880} \qquad (24.203)$$

$$P_{29}^{0} = \frac{10102586296593530880}{3142112029261} = 3215221.5460\ldots \text{ days} \approx 8800 \text{ years} \qquad (24.204)$$

The same value is given by Oechslin. This is the period of the revolution of the nodes resulting from Neßtfell's construction, but it is possible that Neßtfell aimed at having the nodes fixed and that his gears merely provide a very long period. The period of the revolution of the nodes of Mercury was actually not well known in the 18th century.

Incidentally, even though Oechslin didn't work out the exact fractions, he still provided the period 3215221.546060 days, which he could only have obtained by taking accurate values of each term. However, the exact evaluation of the above ratio is 3215221.546053..., and this shows that even with computations to many decimal places, it may still not yield an entirely correct result.

24.5.5.3 The viewpoint from the Earth

Finally, there is a guide linking the Earth to Mercury, and showing the backdrop of the sky where Mercury is located. Neßtfell did however not attempt to show the transits of Mercury.

24.5.6 The calendar part

The front panel also has a calendar display, which I am not describing here. It shows the year in an opening and it has a mention about the year 1828 not being a leap year. However, Gutwein's engraving (figure 24.15) mentions the year 1826. I don't know if a similar mention appears on the Munich machine.[20]

Neßtfell's description of the clock also mentions that a day should be removed every 126 years [18, p. 54]. After that time, the excess of the Julian year to the tropical year accumulates to about a day, but it is not totally clear how Neßtfell arrived at that value. In any case, on Gutwein's engraving from 1754, this is indicated by the two years 1826 and 1952.

24.6 Conclusion

Neßtfell's clock follows a very original construction, and one can only be in awe of his imagination, and also in admiration of Oechslin's efforts to render the structure of the machine in a plan, which is certainly not easy.

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