Chapter 22 **(Oechslin: 2.3)**

Klein's Tychonic clock in Prague (1751)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

22.1 Introduction

The clock described here was completed in 1751 by Jan Klein (1684-1762).[1] This clock displays the Tychonic system and belongs to a group of three similar clocks kept in the Clementinum in Prague.

This clock was first described by Stepling in 1751 [17], mentioned by Pelcl [13], and described in detail by Böhm in 1863 [1, 2]. Böhm wrote about the "very complex mechanism of this clock" [1, p. 7], but this clock is in fact a rather simple clock. It is a table clock with two sides, one for the time and calendar indications, the other one for the Tychonic system.

Oechslin did not have the opportunity to disassemble the clock and based his description only on that published by Böhm.[2]

[1]For biographical information on Klein, see the chapter on the geographic clock in Dresden.

[2]This clock was shown at the 1989 Hahn exhibition [20, p. 65]. See also Oechslin [10, p. 32-33, 49-50] and Michal [9].

Figure 22.1: Left: Portrait of Jan Klein with his geographic clock from 1753/1754. (source: [1]) Right: portrait in Pelcl's biographical notice [13].

22.2 The going work

The going work and the striking work are described neither by Böhm [1] nor by Oechslin, but we know that the going work drives the central tube 2, which is the common input to the two faces of the clock. I will measure the motion of this tube from the Tychonic side only. On that side, tube 2 makes a turn clockwise in 24 hours:

$$V_2^0 = -1 \tag{22.1}$$
$$P_2^0 = -1 \text{ day} \tag{22.2}$$

22.3 The front side of the clock

This side of the clock (figure 22.2) shows the time on a 12-hour dial, as well as the digits of the year (in two parts), the dominical letter, the golden number, the solar cycle, the epact, the indiction, the Easter full Moon, the day with which the year ends, and data for the current month (fixed feasts, month, signs of the zodiac, lengths of days and nights, and sunrises and sunsets).

This side of the clock takes its input from tube 2 which provides the base motion of the two sides of the clock. Using two worms, this motion is used to obtain the motion of arbor 20 (all motions measured from this side of the clock, except for V_2^0):

$$V_{20}^0 = V_2^0 \times \left(-\frac{1}{5}\right) \times \left(-\frac{1}{73}\right) = -\frac{1}{365} \tag{22.3}$$
$$P_{20}^0 = -365 \text{ days} \tag{22.4}$$

This arbor makes a turn clockwise in a year (but with a crude approximation). This arbor also carries a finger which acts on a lever, causing the arbor 22 which carries a 6-pointed star to make a sixth of a turn. This arbor also carries a 12-leaves pinion which meshes with a 200-teeth wheel on arbor 23 which carries a disk with the year, the dominical letter and the day of the week with which the year ends. We thus have

$$V_{22}^0 = -\frac{1}{365} \times \frac{1}{6} \tag{22.5}$$
$$P_{22}^0 = -6 \text{ years} \tag{22.6}$$
$$V_{23}^0 = V_{22}^0 \times \left(-\frac{12}{200}\right) = V_{22}^0 \times \left(-\frac{6}{100}\right) = \frac{1}{365} \times \frac{1}{100} \tag{22.7}$$
$$P_{23}^0 = 100 \text{ years} \tag{22.8}$$

Finally, the arbor 22 also carries a 24-teeth pinion which meshes with a

Figure 22.2: The front side of Klein's clock. (source: [1])

Figure 22.3: The front side of Klein's clock. (source: [17])

Figure 22.4: The front side of Klein's clock. (source: [18])

76-teeth wheel on arbor 24. We therefore have

$$V_{24}^0 = V_{22}^0 \times \left(-\frac{24}{76}\right) = V_{22}^0 \times \left(-\frac{6}{19}\right) \tag{22.9}$$

$$P_{24}^0 = P_{22}^0 \times \left(-\frac{19}{6}\right) = 19 \text{ years} \tag{22.10}$$

This arbor carries the figures for the golden numbers.

The motion of arbor 24 is also transferred to the arbors 25, 26 and 27:

$$V_{25}^0 = V_{24}^0 \times \left(-\frac{76}{60}\right) = V_{24}^0 \times \left(-\frac{19}{15}\right) \tag{22.11}$$

$$P_{25}^0 = P_{24}^0 \times \left(-\frac{15}{19}\right) = -15 \text{ years} \tag{22.12}$$

$$V_{26}^0 = V_{25}^0 \times \left(-\frac{60}{76}\right) = V_{25}^0 \times \left(-\frac{15}{19}\right) \tag{22.13}$$

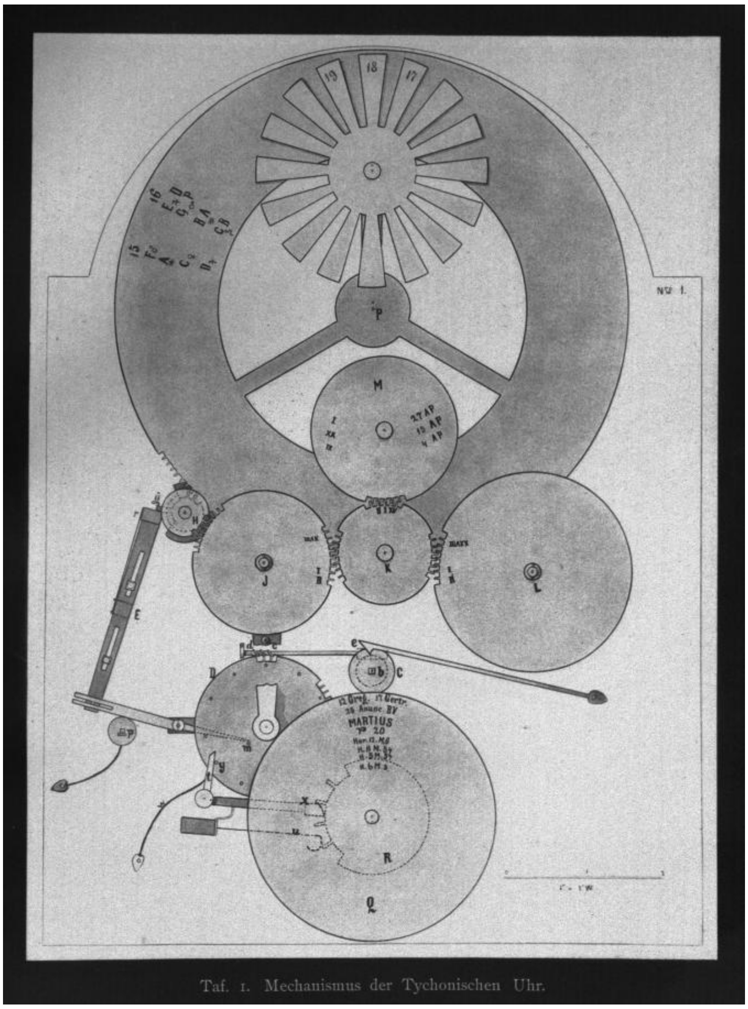
$$P_{26}^0 = P_{25}^0 \times \left(-\frac{19}{15}\right) = 19 \text{ years} \tag{22.14}$$

$$V_{27}^0 = V_{25}^0 \times \left(-\frac{60}{112}\right) = V_{25}^0 \times \left(-\frac{15}{28}\right) \tag{22.15}$$

$$P_{27}^0 = P_{25}^0 \times \left(-\frac{28}{15}\right) = 28 \text{ years} \tag{22.16}$$

These arbors carry the figures for the indiction (15 year cycle), the epact and Easter full Moons (19 year cycle) and the solar cycle (28 year cycle).

Finally, the arbor 20 also carries a disk with 12 pins (see figure 22.5), which acts on a lever which moves the arbor 21 by a twelfth of a turn each year. These twelve pins are shown as a wheel by Oechslin. This arbor carries the data for each month, in particular the signs of the zodiac, the times of sunrise and sunset, and the durations of the day and night.



Taf. 1. Mechanismus der Tychonischen Uhr.

Figure 22.5: The gears for the computus and calendar elements in Klein's clock. (source: [1])

22.4 The Tychonic system

This side of the clock shows the Tychonic system of the world, with the Earth at the center, the Moon around the Earth, and further the Sun, and all the planets rotating around the Sun. The Sun, with all the planets, rotates around the Earth. This system was published by Tycho Brahe in 1588 (figure 22.6).

There is also a hand for the lunar nodes, but which rotates around the Sun.

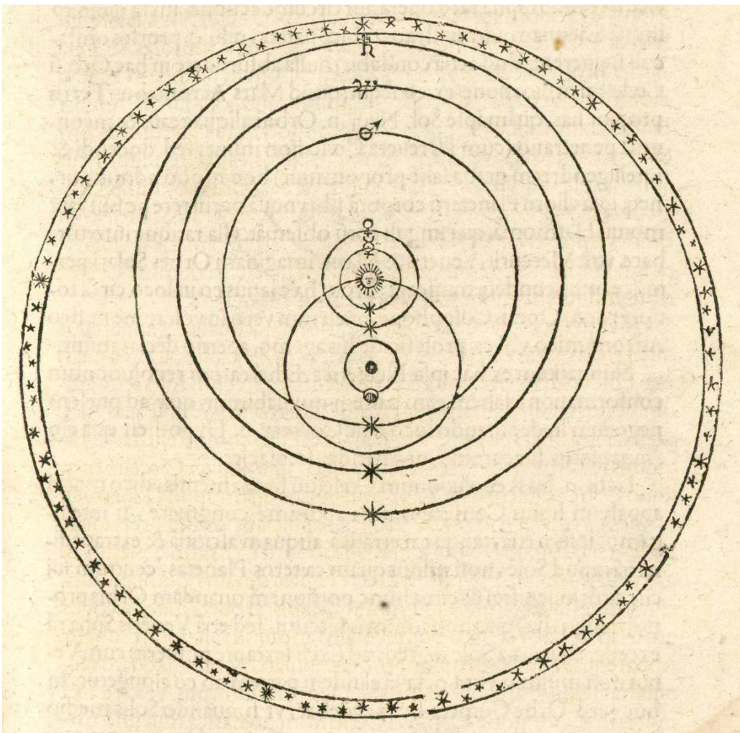


Figure 22.6: The Tychonic system (1588) (source: [3])

22.4.1 The general diurnal motion

As I have mentioned above, tube 2 makes a turn in 24 hours:

$$V_2^0 = -1 \tag{22.17}$$

$$P_2^0 = -1 \text{ day} \tag{22.18}$$

This tube carries an entire frame which rotates clockwise in one day. This frame carries the system around the Sun, so that the Sun rotates around the Earth (center) in 24 hours.

On the central axis, there are two arbors. The arbor in the front is the one carrying the figure of the Earth in the center of the dial. Another arbor 3 is located in the back, on the same axis, but is fixed. The latter carries a 6-teeth

Figure 22.7: The Tychonic system on Klein's clock. (source: [1])

Figure 22.8: The Tychonic system on Klein's clock. (source: [17])

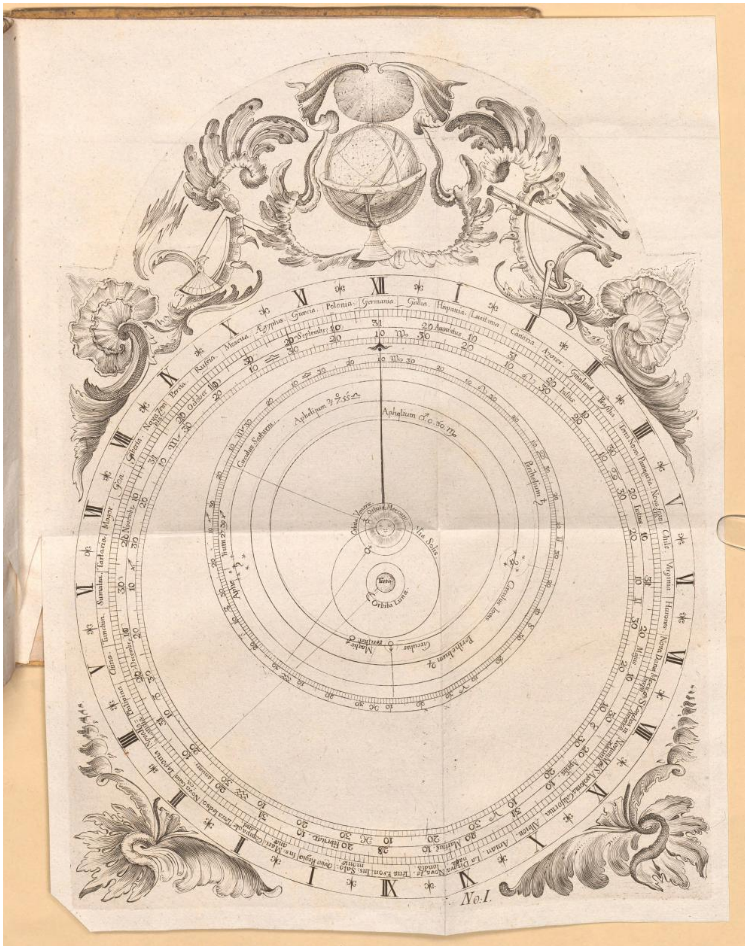


Figure 22.9: The Tychonic system on Klein's clock. (source: [18])

pinion and it is the rotation of the frame 2 which introduces motions inside this frame, using this fixed pinion.

The frame 2 can be thought of the synodic frame of the Earth. The motions of the planets will all be relative to the Sun, and we can expect to find the synodic periods of the planets.

22.4.2 The motion of the Earth

The Earth is carried by the arbor 6 mentioned earlier and we can compute the motion of this arbor with respect to the general moving frame 2 (see also figure 22.10):

$$\mathrm{V}_3^2 = -\mathrm{V}_2^3 = -\mathrm{V}_2^0 = 1 \qquad (22.19)$$

$$\mathrm{V}_6^2 = \mathrm{V}_3^2 \times \left(-\frac{6}{60}\right) \times \left(-\frac{15}{60}\right) \times \left(-\frac{8}{73}\right) = \mathrm{V}_3^2 \times \left(-\frac{1}{365}\right) = -\frac{1}{365} \qquad (22.20)$$

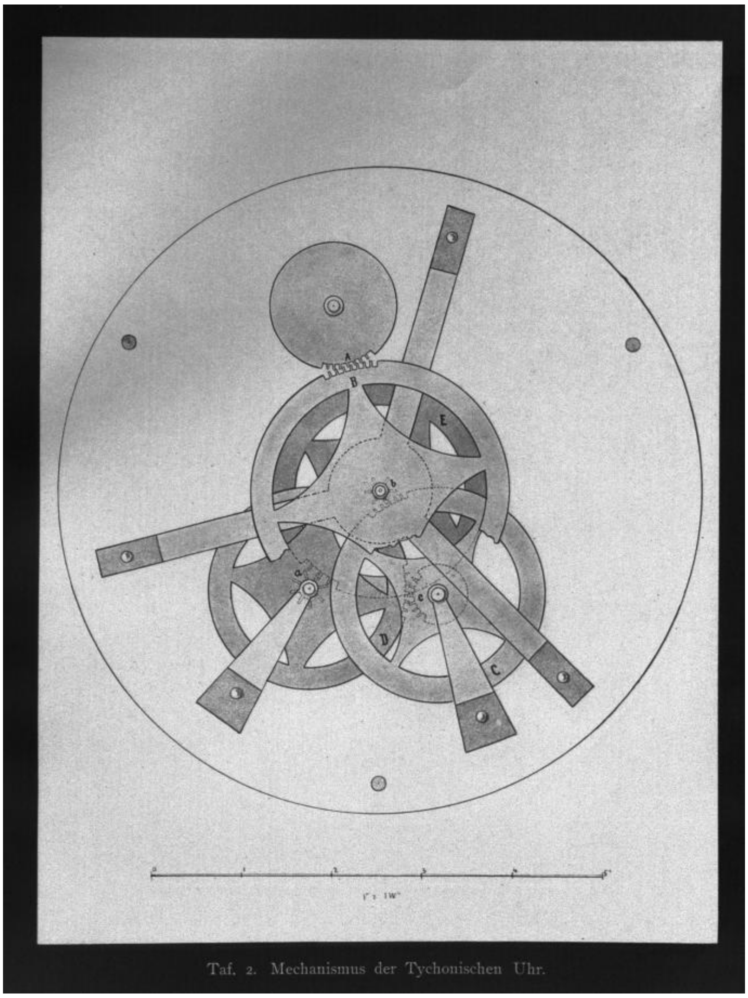
In the absolute frame, we have

$$\mathrm{V}_6^0 = \mathrm{V}_6^2 + \mathrm{V}_2^0 = -\frac{1}{365} - 1 = -\frac{366}{365} \qquad (22.21)$$

$$\mathrm{P}_6^0 = -\frac{365}{366} = -23 \text{ h } 56 \text{ m } 3.9344\ldots \text{ s} \qquad (22.22)$$

It isn't clear from the photograph of the clock if the figure of the Earth shows a planisphere. If it does, then the Earth actually shouldn't be moving and be still, with only the Sun rotating clockwise around the Earth.

In any case, the motion of arbor 6 will serve as an input for the motion of all the other planets, for the line of nodes and for the Moon.



Taf. 2. Mechanismus der Tychonischen Uhr.

Figure 22.10: The gears for the motion of arbor 6 (center) in Klein's clock. (source: [1])

22.4.3 The motions of the planets

22.4.3.1 The theoretical motions

Before examining how the planets move on Klein's clock, it is useful to study first how they should move, and to see what are the properties of the motions. As mentioned above, in the Tychonic system the Earth is at the center and the Sun revolves around the Earth, in the same time the Earth makes a turn around the Sun in the Copernican system. Moreover, the other planets rotate around the Sun. There is in fact a simple relation between the Copernican system and the Tychonic system.

In kinematics, we can describe the motion $z(t)$ of a point around a circle as a function of the time using Euler's notation:

$$z(t) = re^{i\omega t} = r\cos(\omega t) + ir\sin(\omega t) \qquad (22.23)$$

where t is the time (for instance expressed in days), r is the radius of the circle and ω is the angular velocity, expressed in radians per unit of time. If P is the period of rotation, then $\omega = 2\pi/P$.

In the Copernican system, there is such a relation for each planet. For the Earth:

$$z_E(t) = r_E e^{i\omega_E t} \qquad (22.24)$$

where r_E is the radius of the orbit of the Earth and $\omega_E = 2\pi/P_E$, P_E being the tropical year.

For another planet P:

$$z_P(t) = r_P e^{i\omega_P t} \qquad (22.25)$$

with corresponding definitions for r_P and ω_P.

The Tychonic system is merely obtained by a translation of $\tau(t) = -z_E(t)$. This translation varies with the time t. If we denote the positions of the Earth and the planets in the Tychonic system with z', we have

$$z'_E(t) = z_E(t) + \tau(t) = 0 \qquad (22.26)$$
$$z'_P(t) = z_P(t) + \tau(t) = r_P e^{i\omega_P t} - r_E e^{i\omega_E t} \qquad (22.27)$$

Consequently, the motion of each planet (including the Earth, as a matter of fact) is the sum of two circular motions, one around the Earth, and one around the Sun. Incidentally, we can choose the order of rotation, and a planet could be thought of moving on epicycles around a point moving around the Earth, with exactly the same positions.

In the Tychonic system, the Sun is located at $\tau(t)$ and we can look at the synodic motions of the planets by making the Sun still, that is by rotating all the motions by $-\omega_E t$, which corresponds to a multiplication by $e^{-i\omega_E t}$. Denoting the synodic positions with z'', we have

$$z''_P(t) = (z_P(t) + \tau(t))e^{-i\omega_E t} = r_P e^{i\omega_P t} e^{-i\omega_E t} + r_E e^{i\omega_E t} e^{-i\omega_E t} \tag{22.28}$$

$$= r_P e^{i(\omega_P - \omega_E)t} + r_E \tag{22.29}$$

This is a circular motion of period

$$S_P = \frac{2\pi}{\omega_P - \omega_E} = \frac{2\pi}{2\pi/P_P - 2\pi/P_E} = \frac{1}{1/P_P - 1/P_E} = \frac{P_P P_E}{P_E - P_P} \tag{22.30}$$

We thus have a simple formula for the computation of the synodic periods, and also of the tropical period of a planet from the synodic one, because

$$P_P = \frac{S_P P_E}{S_P + P_E} \tag{22.31}$$

and finally we can also see that in the synodic frame the motion of a planet is counterclockwise if $P_E > P_P$ (Mercury and Venus) and clockwise if $P_E < P_P$ (the superior planets).

We can now examine how this is implemented on Klein's clock.

22.4.3.2 The motions on the clock

The planets, as well as the lunar nodes, are all moving around the Sun (see figure 22.11). We first compute these motions in the moving frame 2. For Mercury, we have

$$\mathrm{V}_7^2 = \mathrm{V}_6^2 \times \left(-\frac{137}{33}\right) = -\frac{1}{365} \times \left(-\frac{137}{33}\right) = \frac{137}{12045} \tag{22.32}$$

$$\mathrm{P}_7^2 = \frac{12045}{137} = 87.9197\ldots \text{ days} \tag{22.33}$$

This in fact an approximation of the orbital period of Mercury, but one would have expected the synodic period (about 116 days). The direction of rotation (counterclockwise) is however correct, since $P_E > P_P$.

For Venus, we have

$$\mathrm{V}_8^2 = \mathrm{V}_6^2 \times \left(-\frac{39}{24}\right) = -\frac{1}{365} \times \left(-\frac{39}{24}\right) = \frac{13}{2920} \tag{22.34}$$

$$\mathrm{P}_8^2 = \frac{2920}{13} = 224.6153\ldots \text{ days} \tag{22.35}$$

This in fact an approximation of the orbital period of Venus, but one would have expected the synodic period (about 584 days). The direction of rotation (counterclockwise) is however correct, since $P_E > P_P$.

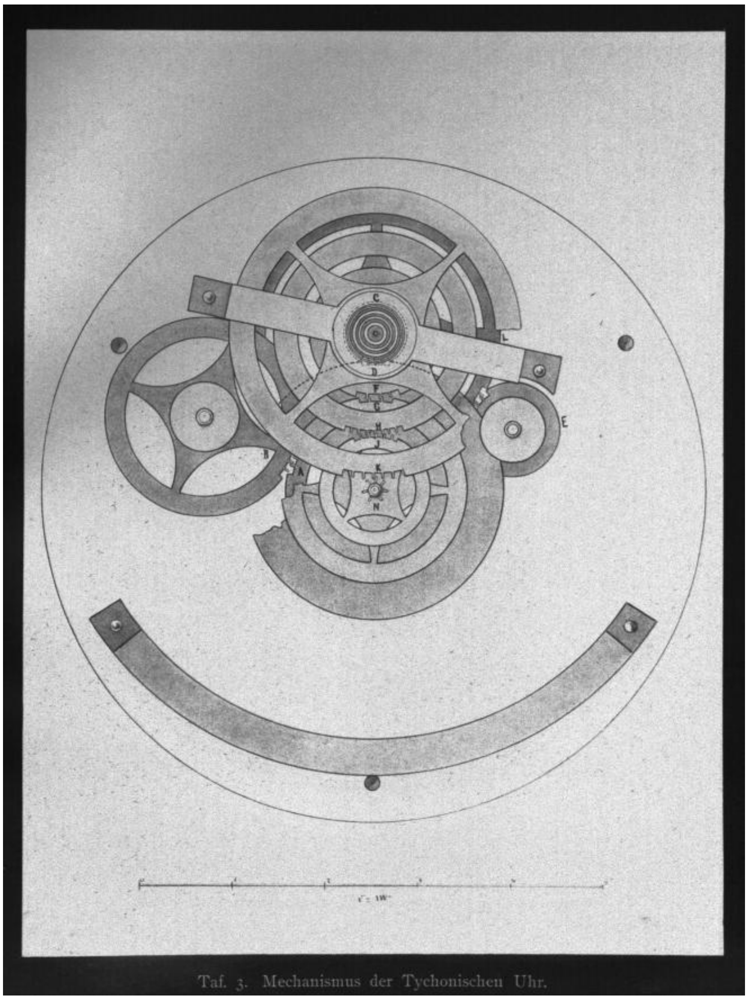


Figure 22.11: General view of the gears for the motions of the planets in Klein's clock. The axis of the Earth is at the center with the 7-leaves pinion on arbor 6 in the front. The group of wheels in the upper part are centered on the Sun. (source: [1])

For Mars, we have

$$\mathrm{V}_9^2 = \mathrm{V}_6^2 \times \left(-\frac{42}{79}\right) = -\frac{1}{365} \times \left(-\frac{42}{79}\right) = \frac{42}{28835} \qquad (22.36)$$

$$\mathrm{P}_9^2 = \frac{28835}{42} = 686.5476\ldots \text{ days} \qquad (22.37)$$

This in fact an approximation of the orbital period of Mars, but one would have expected the synodic period (about 780 days). Moreover, the direction of rotation (counterclockwise) is wrong, since $P_E < P_P$.

For Jupiter, we have

$$\mathrm{V}_{10}^2 = \mathrm{V}_6^2 \times \left(-\frac{7}{83}\right) = -\frac{1}{365} \times \left(-\frac{7}{83}\right) = \frac{7}{30295} \qquad (22.38)$$

$$\mathrm{P}_{10}^2 = \frac{30295}{7} = 4327.8571\ldots \text{ days} \qquad (22.39)$$

This in fact an approximation of the orbital period of Jupiter, but one would have expected the synodic period (about 399 days). Moreover, the direction of rotation (counterclockwise) is wrong, since $P_E < P_P$.

For Saturn, we have

$$\mathrm{V}_{11}^2 = \mathrm{V}_6^2 \times \left(-\frac{7}{206}\right) = -\frac{1}{365} \times \left(-\frac{7}{206}\right) = \frac{7}{75190} \qquad (22.40)$$

$$\mathrm{P}_{11}^2 = \frac{75190}{7} = 10741.4285\ldots \text{ days} \qquad (22.41)$$

This in fact an approximation of the orbital period of Saturn, but one would have expected the synodic period (about 378 days). Moreover, the direction of rotation (counterclockwise) is wrong, since $P_E < P_P$.

For the line of nodes (see figure 22.12), we have

$$\mathrm{V}_{13}^2 = \mathrm{V}_6^2 \times \left(-\frac{72}{72}\right) \times \left(-\frac{60}{57}\right) = -\frac{1}{365} \times \frac{20}{19} = -\frac{4}{1387} \qquad (22.42)$$

$$\mathrm{P}_{13}^2 = -\frac{1387}{4} = -346.75 \text{ days} \qquad (22.43)$$

This is an approximation of the so-called eclipse year, the period of revolution of the nodes with respect to the Sun. This value is correct. Incidentally, Klein could also have made the nodes move around the Earth, both are possible.

Oechslin has given the same values, and he made the same observations about the incorrect revolution periods and rotation directions of the planets [10, p. 197, 237-238].

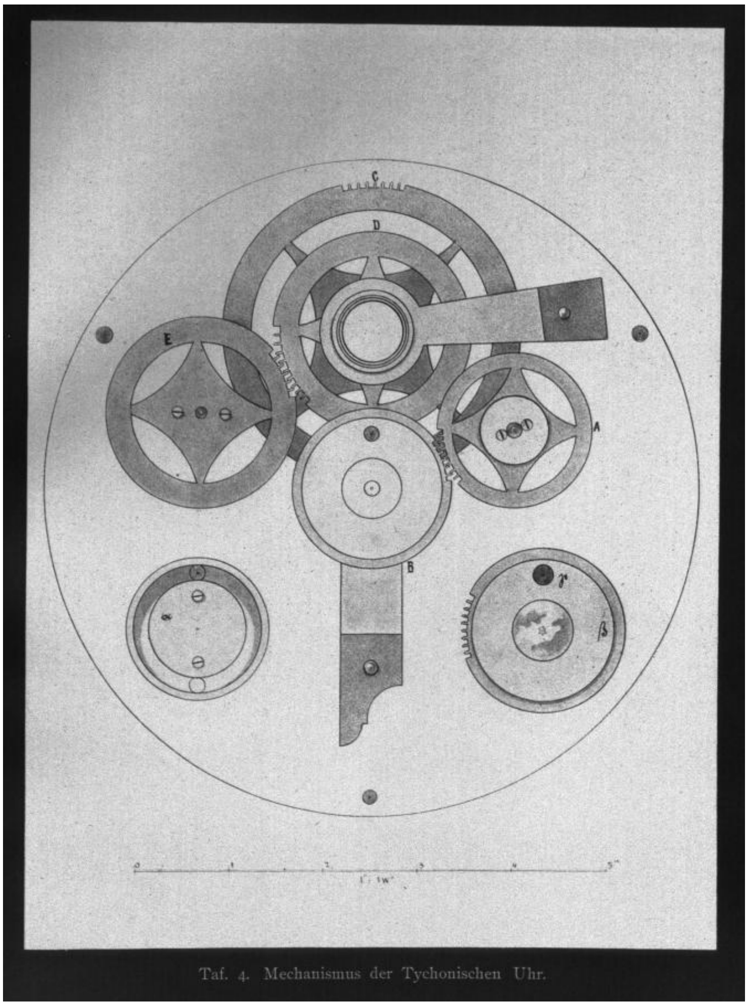


Figure 22.12: The motion of the lunar nodes in Klein's clock. (source: [1])

22.4.4 The motion of the Moon

The Moon is carried by tube 15 and rotates around the Earth. The relative motion of tube 15 is derived from the motion of tube 7:

$$V_{15}^2 = V_7^2 \times \left(-\frac{78}{26}\right) \times \left(-\frac{60}{60}\right) = V_7^2 \times 3 = \frac{137}{12045} \times 3 = \frac{137}{4015} \tag{22.44}$$

$$P_{15}^2 = \frac{4015}{137} = 29.3065\ldots \text{ days} \tag{22.45}$$

This is an approximation of the synodic month, but a rather bad one. The same value is given by Oechslin.

22.4.5 The motion of the ecliptic

Between the 24-hour dial and the celestial disc, there is a ring 18 corresponding to the ecliptic. This ring carries a 365-teeth wheel which meshes with a 6-leaves pinion on the arbor 17 which is part of frame 2. This pinion advances by one tooth for every turn of frame 2, as a consequence of a finger on the fixed 24-hour dial (frame 16). Therefore, in the reference frame 2, we have

$$V_{18}^2 = V_{16}^2 \times \frac{1}{6} \times \left(-\frac{6}{365}\right) = -V_{16}^2 \times \frac{1}{365} \tag{22.46}$$

$$= V_2^{16} \times \frac{1}{365} = V_2^0 \times \frac{1}{365} = -\frac{1}{365} \tag{22.47}$$

$$V_{18}^0 = V_{18}^2 + V_2^0 = -\frac{1}{365} - 1 = -\frac{366}{365} \tag{22.48}$$

$$P_{18}^0 = -\frac{365}{366} = -23 \text{ h } 56 \text{ m } 3.9344\ldots \text{ s} \tag{22.49}$$

The ecliptic ring 18 rotates clockwise in one sidereal day.

22.4.6 Summary of the motions

The Sun makes a turn clockwise in 24 hours and therefore goes through the midday line once a day. The ecliptic moves clockwise in one sidereal day, which corresponds to the apparent motion of the stars. The Moon does also rotate around the Earth in a synodic month with respect to the Sun. These motions are all correct.

However, the motions of all the planets appear to be incorrect, as Klein has given them tropical motions when synodic motions should have been used.

22.5 References

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