Chapter 7

(Oechslin: 8.1)

Hahn's *Weltmaschine* in Stuttgart (1769)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

7.1 Introduction

The "world machine", or *Weltmaschine*, described in this chapter was constructed under the supervision of Philipp Matthäus Hahn (1739-1790) in 1768/69 when he was pastor in Onstmettingen. Of all the *Priestermechaniker* mentioned in this book, Hahn is the one who made the most clocks, the one on which the most has been written, and also the one who left the most written traces. I will therefore not attempt to give a complete biography of Hahn and I will only sketch the main events in his life and direct the interested reader to more comprehensive sources.[1]

Hahn was born in Scharnhausen, near Stuttgart. His father Georg Gottfried Hahn (1705-1766) was a pastor who taught him Greek, Latin and Hebrew when he was only four years old. He was also early on interested in astronomy

[1] A number of very good accounts of Hahn's life have been published, in particular by Engelmann [8], Roeßle [34], Munz [27, 29], Bertsch [7] and King [21, p. 232-242]. Hahn himself has left journals which have been published [11, 12], workshop journals [15, 16, 17, 18] and a description of some of his works [10, 13, 14]. And in 1989, two great volumes have been published for the Hahn exhibition that was then organized [38, 39]. Some references to that work are given later, but there are many other interesting parts not cited here, for instance on the cases of Hahn's clocks. Zinner also gave an interesting summary of Hahn's life [44, p. 351-355], largely based on Engelmann's monography. Maurice also gave a short summary of Hahn's clocks with a number of illustrations [26, v.1, p. 269-273]. The *Allgemeine deutsche Biographie* has a very short notice by Hartmann [19]. The *Neue deutsche Biographie* has a more comprehensive notice by Freytag-Löringhoff [9]. See also Bassermann-Jordan [2] and Abeler's notice [1]. Besides Oechslin's work [33], there are many other short accounts of Hahn's life, such as the one by Roloff [36].

Figure 7.1: Portait of Philipp Matthäus Hahn by Johann Philipp Weisbrod. (source: Wikimedia, Historisches Museum Basel, Inv. 1913.94.1.)

and constructed sundials when he was eight years old. He went to the nearby school in Esslingen then to the school in Nürtingen, not far from Scharnhausen. There he became under the influence of pietism.

In 1756 his father was transferred to Onstmettingen (now part of Albstadt) and he became acquainted with the teacher Philipp Gottfried Schaudt (1739-1809) who had been to an horology school.[2] He made a sundial for the church of Balingen and later other sundials in order to earn some money. At that time, he also read Wolff's *Anfangs-Gründe aller mathematischen Wissenschaften* and made excerpts of it.[3] From 1757 to 1759, Hahn studied theology at the University of Tübingen. After the end of his studies, Hahn occupied several positions, and was vicar in several parishes.

In 1763 he built his first "*Öhrsonnenuhr*," a portable sundial that could be used to set a clock (figure 7.3).[4] This is a sundial where a pinhole (*Öhr*) projects the image of the sun, instead of the Sun projecting the shadow of a gnomon. In other words, the time is read from the inverted shadow. However, the image of the sun is not read on a scale. Instead, the user must set the time on the dial until the image of the sun is located at a certain position.[5]

[2]On Schaudt, see especially Alfred Munz's chapter for the 1989 Hahn exhibition [39, p. 380-390].

[3]Among the interesting parts, see the one on gears [41]. Engelmann illustrates a page copied from Wolf [8, fig. 8]. See also the 1989 Hahn exhibition catalogue [38, pl .4].

[4]See Engelmann [8, p. 128-132, fig. 28-29], Zinner [44, p. 352], Syndram [37, p. 188-190] and especially the 1989 Hahn exhibition catalogue [38, p. 367-373].

[5]Hahn was not the first to use such devices. Sundials with alidades and hour/minute dials did exist before him, see for instance Johann Michael Bergauer's sundial made in Innsbruck

In 1764, he became pastor in Onstmettingen, as successor to his father. In addition to his work as a pastor, he maintained from then on a precision mechanical workshop in the presbytery, which involved a number of workers and later some of his sons.[6] There, with the help of Schaudt, he invented a pendulum scale (figure 7.2)[7] and had constructed other machines and instruments, without constructing them himself. This workshop is considered as the germ cell of the contemporary precision industry in Württemberg.[8]

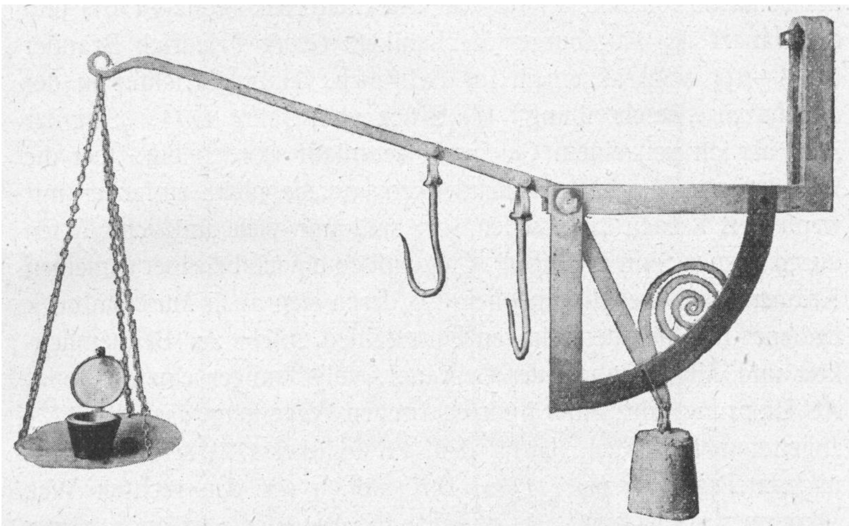

Figure 7.2: Hahn's pendulum scale. (source: [8])

After 1764, he commissioned a weaver to build a complex wooden clock, and then an orrery. Then, he let Schaudt make an astronomical clock in wood for Charles Eugene, Duke of Württemberg (1728-1793), for which he received 300 gulden in 1767.[9]

Hahn then received an order from the duke for an astronomical clock in metal, which is the machine described in this chapter, and which was completed in 1769.[10] The machine was sold to the duke for 8000 gulden and Hahn obtained the wooden clock back.

As a result of this new machine, Hahn obtained in 1770 the parish of Kornwestheim. He was also offered a position of professor of mathematics in Tübingen, but didn't accept it. In Kornwestheim, Hahn also had a workshop

before 1717 and kept at the *Germanisches Nationalmuseum* in Nuremberg (Inv. Nr. WI 1216).

[6] On Hahn's workshop, see especially Ulrike Zubal's chapter for the 1989 Hahn exhibition [39, p. 391-401]. See also the part on Hahn's collaborators [38, p. 44-52].

[7] See [20].

[8] On the work of Hahn on scales and the development of the scales industry, see especially Hans Jenemann's chapters for the 1989 Hahn exhibition [38, p. 150-187, 357-366], [39, p. 479-499].

[9] [33, p. 221]

[10] See Engelmann [8, p. 142-148] and Zinner [44, p. 352].

Figure 7.3: Hahn's "Öhrsonnenuhr" at the *Landesmuseum für Technik und Arbeit*, Mannheim. (source: Wikimedia, photograph by H. Zell)

which involved a number of people who constructed instruments according to his plans.

For his astronomical clocks, Hahn constructed five or six rotary calculating machines, of which two survive. These machines use Leibniz' principle of the stepped drum in Leupold circular layout. Hahn's calculating machine was the first really usable such machine. There are also several reconstructions of these machines.[11]

Hahn received more and more orders and besides Schaudt, Hahn employed Johann Gottfried Ewald Sechting (1749-1818), his brothers Georg David and Gottfried, and later also his children.

Hahn also made tall-case clocks, so-called "Dielenuhren."[12]

In 1781, he became pastor of Echterdingen, not far from his birthplace. There the workshop mostly made pocket watches with improved cylinder escapements.[13]

Hahn was married twice and had a number of children. He published several books on theology and died in 1790 in Echterdingen.

After his death, Hahn's sons continued the manufacture of watches and clocks. His portable sundial was also copied by Georg Matthias Burger (1750-1825) in Nurnberg,[14] Hollrich in Mainz[15] and J. J. Sauter in Stockholm and St Petersburg.[16]

Among Hahn's sons, there was Christoph Matthäus (1767-1833) who helped his father, was court mechanician in Stuttgart, and made watches and clocks after his father's death. Christian Gottfried was also court mechanician in Stuttgart, Berlin and then went to America. Johann Georg also made watches and was court mechanician in Stuttgart from 1807 to 1813.

[11]On Hahn's calculating machines and their context, see especially Erhard Anthes's chapter for the 1989 Hahn exhibition [38, p. 463-466], [39, p. 456-478].

[12]Cf. [8, fig. 32].

[13]See [8, fig. 24-27] and [23, 22]. See especially Günther Oestmann's chapters on Hahn's watches for the 1989 catalogue [38, p. 467-514], [39, p. 500-510].

[14]See Zinner [44, p. 276-277].

[15]See Zinner [44, p. 387].

[16]See Zinner [44, p. 498].

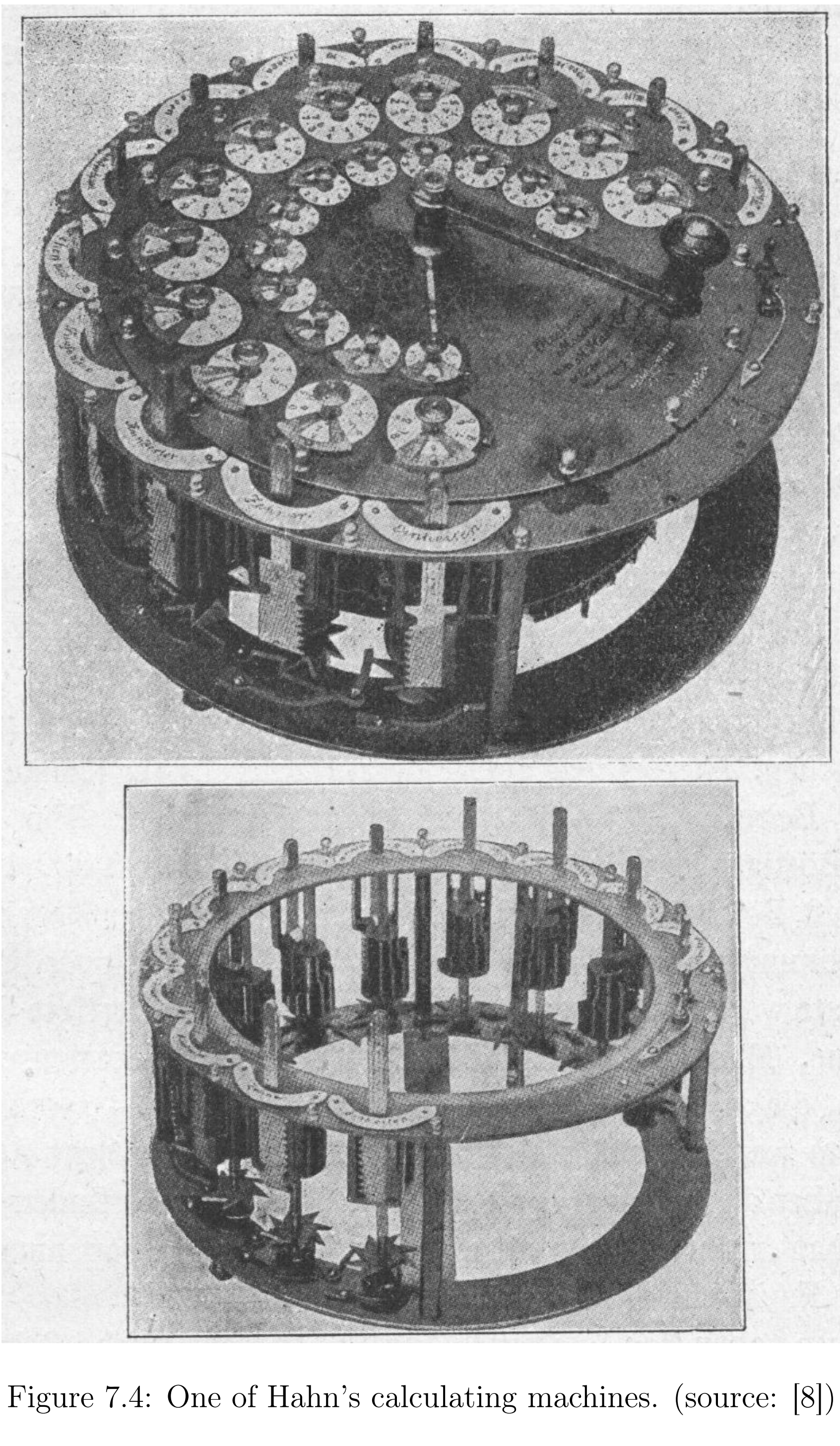

Figure 7.4: One of Hahn's calculating machines. (source: [8])

7.2 History and general description

For the machine described in this chapter, Hahn was helped by the teacher Gottfried Schaudt (1739-1809).[17] This machine is Hahn's first large *Weltmaschine* and, as mentioned above, it was an order from Charles Eugene, duke of Württemberg (1728-1793), who installed it in the library of his castle in Ludwigsburg. This machine is therefore sometimes called the *Ludwigsburger Weltmaschine*.[18]

It is now exhibited in the *Landesmuseum Württemberg* (Württemberg State Museum) in Stuttgart.[19]

Hahn's machine is made of three parts (figure 7.5). The central part shows the time and the calendar, the left part shows the solar system in a heliocentric representation, and the right part shows a geocentric representation as a celestial sphere. These parts are connected through the base of the furniture. The current appearance of the furniture is not the original one (figure 7.8).[20]

[17] [33, p. 221]

[18] Descriptions of the machine were given by Vischer [40], Hahn [10] and Oechslin [30]. See especially the 1989 Hahn exhibition catalogue [38, p. 375-382]. See also Oechslin [33, p. 55-57].

[19] For details on the history of the machine, see Oechslin [33, p. 234-235]. In 2008, a movie accompanying the machine was made by Christoph Prenosil and Steffen Schönbrunn as part of their final project in the *Hochschule für Medien* Stuttgart, under the direction of Prof. Dr. Bernd Eberhardt. This five minute movie is shown next to the machine, but it does not describe the clock in detail. The heliocentric system has been modelled in 3D, at least for a global rendering, but without any animation. The gears of the celestial sphere are not shown in the movie, and have probably not been modelled in 3D. The sphere itself is shown, probably using a 3D textured sphere, together with the various hands. (movie seen on April 11, 2025)

[20] The Hahn museum in Onstmettingen shows a 1/4 model of the original Ludwigsburg machine. This model is presumably non functional. The original engraving of the Ludwigsburg machine was presumably the one published in 1774 [10].

(Copyrighted image not shown)

Figure 7.5: Hahn's "Weltmaschine" in Stuttgart out of its glass window. (source: `https://bawue.museum-digital.de/object/81`)

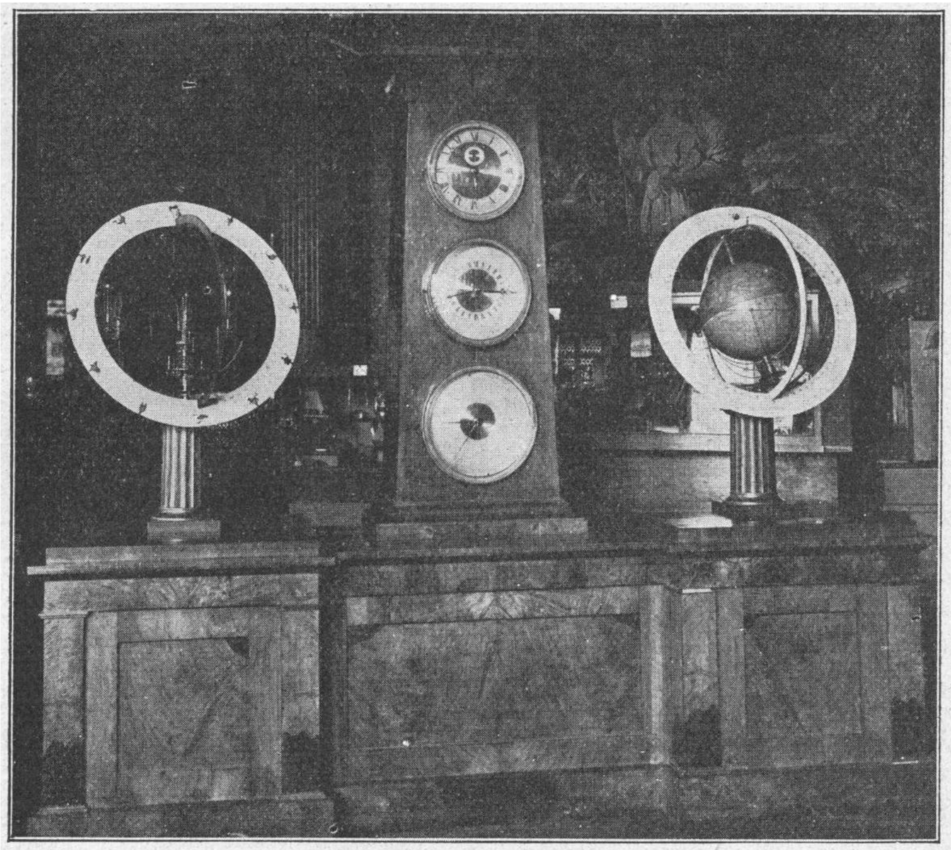

Figure 7.6: Hahn's *Weltmaschine* as it appeared in 1923. (source: [8])

Figure 7.7: General view of Hahn's *Weltmaschine*. It is very difficult to photograph the machine in its entirety, because of the windows and lighting. (photograph by the author, 2025)

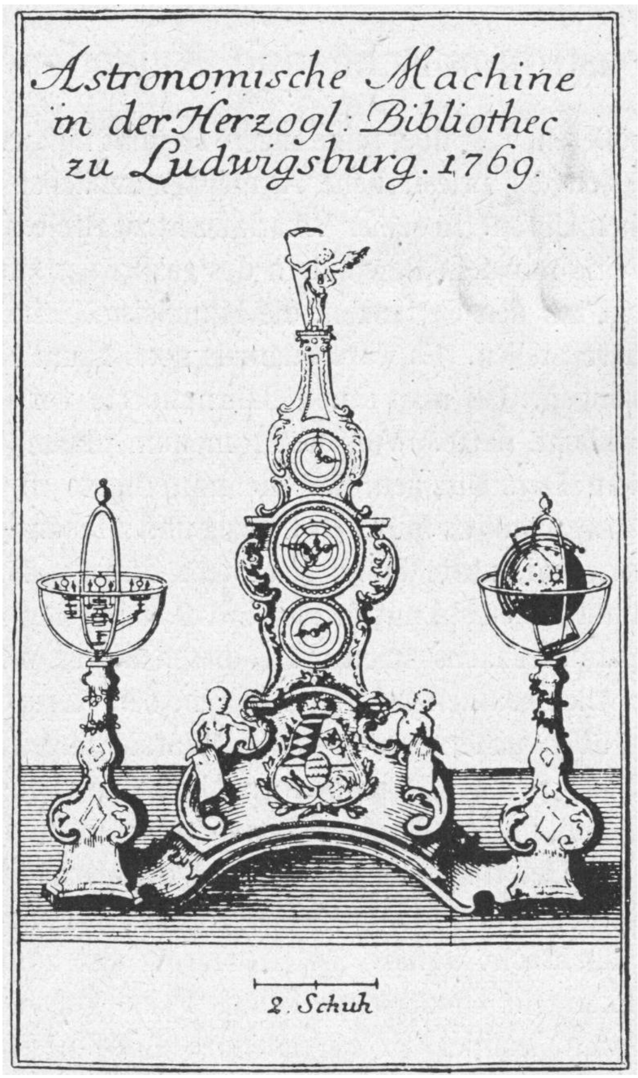

Figure 7.8: The original appearance of Hahn's *Weltmaschine*. (source: [8], from [10]) A reduced-scale non functional model of this case is exhibited in the Onstmettingen museum.

The central part contains a clockwork with a 30-teeth escape wheel making one turn in one minute and leading to an arbor making one turn in an hour. Using four wheels, the hour hand cannon is made to make one turn in twelve hours. This cannon carries a 24-teeth wheel which meshes with a 48-teeth wheel which is making one turn in a day. This is the input motion for the other parts of the machine.

The first vertical arbor of that part is arbor 8 and it moves counterclockwise as seen from above (I assume that the motion of vertical arbors is measured from above):

$$\mathrm{V}_8^0 = 1 \tag{7.1}$$

This motion is transfered to the vertical arbor 31, but the direction of the motion is reversed (details of this reversal are given in the section on the calendar part):

$$\mathrm{V}_{31}^0 = -1 \tag{7.2}$$

This motion is again transfered to the left to the orrery part through the horizontal arbor 33 and to the right to the globe through the horizontal arbor 141. Both arbors make one turn in a day counterclockwise when looked from the right:

$$\mathrm{V}_{33}^0 = \mathrm{V}_{141}^0 = 1 \tag{7.3}$$

7.3 The celestial globe

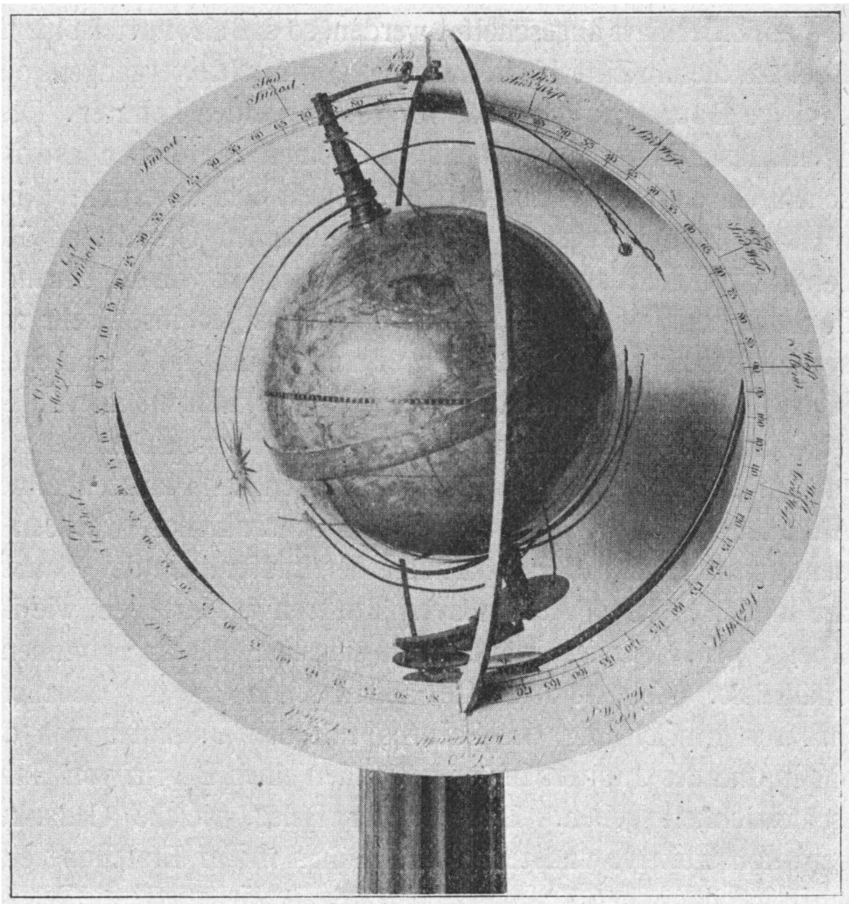

Figure 7.9: The celestial globe at the right of Hahn's *Weltmaschine*. (source: [8])

Figure 7.10: Hahn's *Weltmaschine*: the celestial globe. (photograph by the author, 2025)

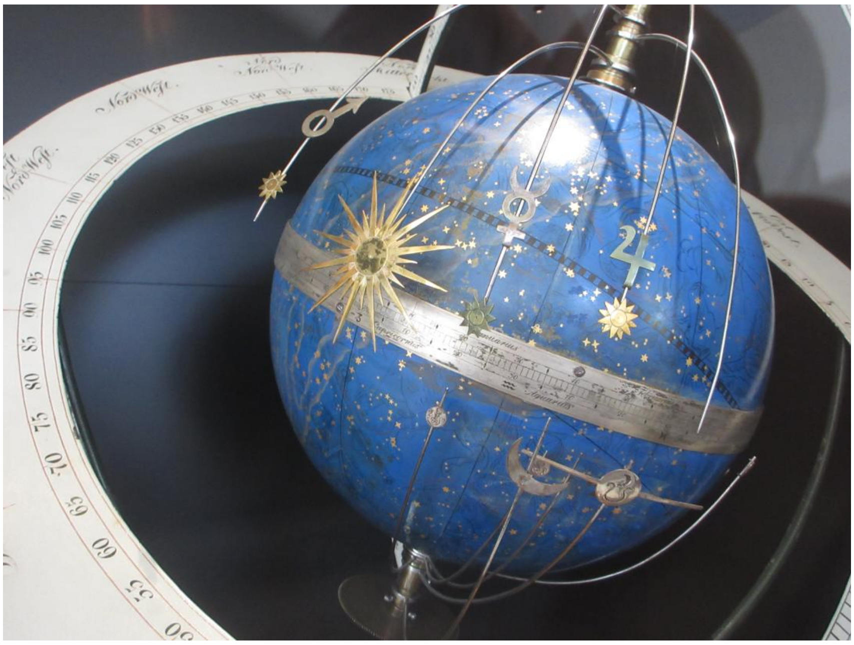

Figure 7.11: Hahn's *Weltmaschine*: the celestial globe. (photograph by the author, 2025)

Figure 7.12: Hahn's *Weltmaschine*: detail of the celestial globe. (photograph by the author, 2025)

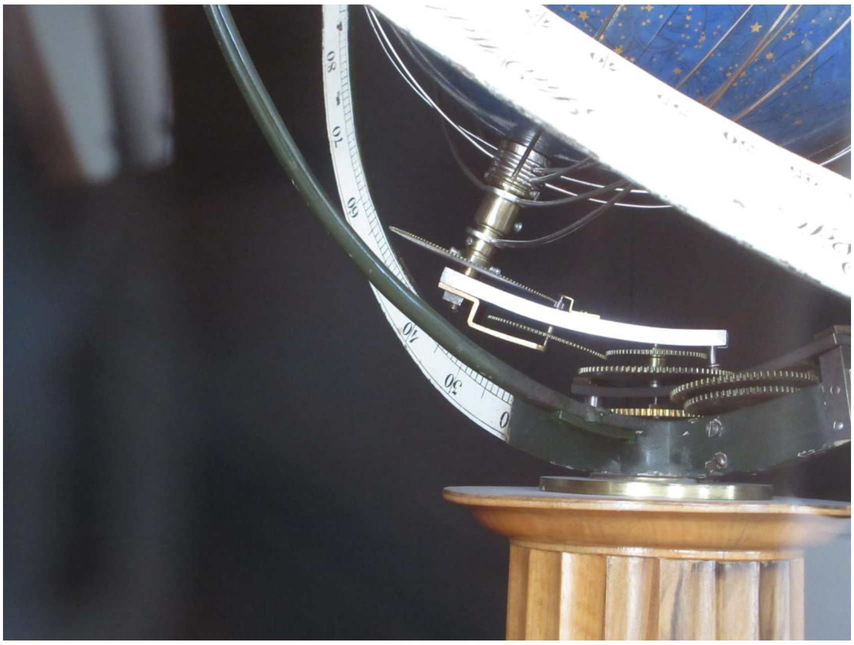

Figure 7.13: Hahn's *Weltmaschine*: detail of the transmission to the celestial globe. (photograph by the author, 2025)

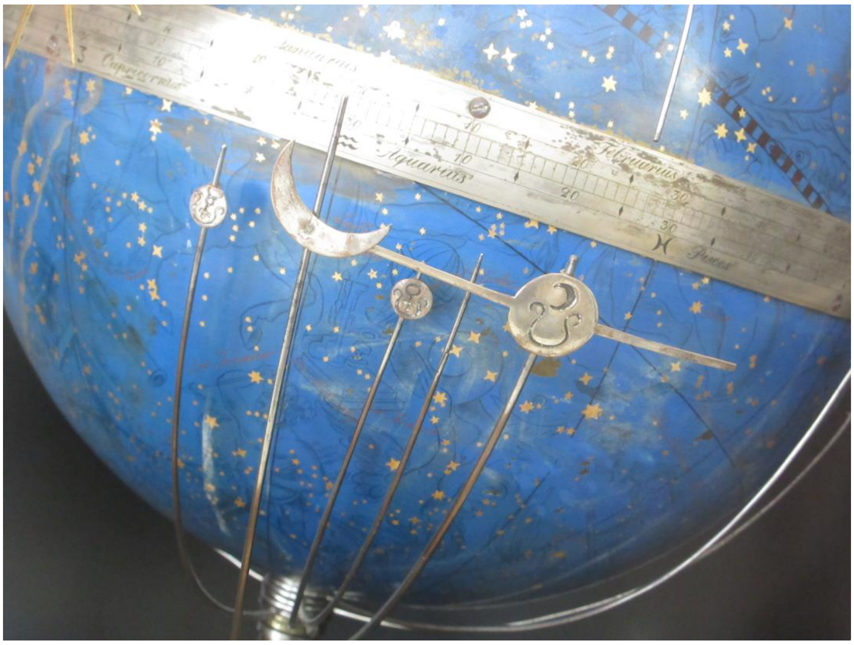

Figure 7.14: Hahn's *Weltmaschine*: detail of the celestial globe. From left to right, the figures of Mercury, the Moon, Venus, and the descending node of the Moon. It is possible that some of the figures are attached to the wrong hands. (photograph by the author, 2025)

The globe in the right part of the machine can be thought of as a geocentric representation of the universe, with the Earth at its center.[21] This representation complements the heliocentric view of the left part.

The globe not only shows the apparent motion of the stars, but also represents the geocentric motion of the Moon, the Sun and the planets. The vertical axis represents the axis of the Earth, and the globe rotates around this axis. However, the planets, the Sun and the Moon rotate nearly in the plane of the ecliptic which is inclined by 23.5 degrees with the axis of the Earth. The main axis of the globe is therefore that of the planets, but this axis does therefore also rotate around the vertical axis.

A comparison with the gears of this globe and the globes of the *Weltmaschine* of Gotha (Oechslin 8.3) and the Aschaffenburg globe clock (Oechslin 8.4) shows that the structures of these three globes are very similar, and there are mainly small differences in teeth counts, but without altering the ratios.

On this globe, there are according to Oechslin's drawing two hands driven at the bottom of the globe (the corrected Moon and the lunar nodes), and six hands driven at the top of the globe (the Sun and the five planets). On the photographs, we can indeed see six hands fixed at the top of the globe, but also seven (?) at the bottom of the globe. I believe therefore that some of the hands are fixed at both ends of the globe, but only driven either at the top or the bottom, something that Oechslin did not indicate.

There are probably two problems in the mounting of the globe. First, some of the upper parts do not seem to bet connected to their corresponding lower parts. Second, some of the figures of the planets do not seem to be on the tube with which they should move. For instance, the figure of the Sun seems to be on the tube corresponding to Jupiter, the figure of Mercury seems to be on the tube of Venus, the figure of Jupiter seems to be on the tube for Mercury, and there may be other anomalies of that sort, which should be sorted out. In the sequel, I will only use Oechslin's drawing as a basis, but future work should look into this matter and clarify it.

Consequently, we can divide the motions in two parts. First, there is the apparent motion of the sky, then there is the geocentric motion of the other bodies with respect to the globe. The stars, moreover, must be thought of having fixed coordinates, in other words the precession of equinoxes is not taken into account by Hahn.

The motion of the globe originates from the horizontal arbor 141 mentioned earlier. This arbor makes one turn in a day and carries a 24-teeth wheel meshing with another 24-teeth wheel on a vertical arbor 142. This arbor therefore makes in one day one turn counterclockwise as seen from above:

$$V^0_{142} = 1 \tag{7.4}$$

[21] Besides the previous photographs, the globe is also pictured in the 1989 Hahn exhibition catalogue [38, pl .12].

(Copyrighted image not shown)

Figure 7.15: Detail of the globe. It is possible that some figures are attached to the wrong hands. (source: `https://www.leo-bw.de`)

The globe is contained in a fixed meridian ring. The arbor 142 carries a 57-teeth wheel meshing with a 58-teeth wheel on arbor 143. This motion is transfered to the arbor 144 and eventually to the tube 145. This tube has the following velocity:

$$V^0_{145} = V^0_{142} \times \left(-\frac{57}{58}\right) \times \left(-\frac{70}{66}\right) \times \left(-\frac{76}{79}\right) \tag{7.5}$$

$$= -\frac{25270}{25201} \tag{7.6}$$

which corresponds to the period

$$P^0_{145} = -\frac{25201}{25270} \text{ days } = -\ 23\text{h } 56\text{mn } 4.0838\ldots \text{ s} \tag{7.7}$$

The rotation of tube 145 is therefore clockwise. The motion of tube 145 is transfered to an inclined arbor 156 which does therefore rotate in one sidereal day around the vertical axis. This arbor 156 carries the globe.

Within the globe, all the motions are obtained from tube 148 whose motion is obtained from a 92-teeth wheel rotating on a fixed 71-teeth wheel bound to the meridian ring. As a consequence, in the reference frame of the globe (156), this 71-teeth wheel on arbor 146 is making one turn counterclockwise in one sidereal day:

$$V_{146}^{156} = -V_{145}^{0} = \frac{25270}{25201} \tag{7.8}$$

Therefore

$$V_{148}^{156} = V_{146}^{156} \times \left(-\frac{71}{92}\right) \times \left(-\frac{7}{148}\right) \tag{7.9}$$

$$= \frac{25270}{25201} \times \frac{497}{13616} = \frac{6279595}{171568408} \tag{7.10}$$

and

$$P_{148}^{156} = \frac{171568408}{6279595} = 27.3215\ldots \text{ days} \tag{7.11}$$

This is an excellent approximation of the tropical month. It is the revolution of the Moon with respect to the zodiac. This motion is counterclockwise. Oechslin gives the same value, also in sidereal days.

The inside of the globe is organized in five compartments. The first compartment at the bottom takes care of the motions of the Moon. The second compartment takes care of the motions of Mercury and Venus. The third compartment takes care of the motions of Mars and the Sun. The fourth compartment takes care of the motion of Jupiter and the last compartment takes care of the motion of Saturn.

7.3.1 The motions of the Moon

The tube 148 carries a 112-teeth wheel which meshes with a 53-teeth wheel on arbor 149. This arbor carries a 4-leaves pinion meshing with a tube 150 carrying a 113-teeth wheel in the second compartment. This tube therefore has the velocity

$$V_{150}^{156} = V_{148}^{156} \times \left(-\frac{112}{53}\right) \times \left(-\frac{4}{113}\right) \tag{7.12}$$

$$= \frac{6279595}{171568408} \times \frac{448}{5989} = \frac{351657320}{128440399439} \tag{7.13}$$

$$P_{150}^{156} = \frac{128440399439}{351657320} = 365.2430\ldots \text{ days} \tag{7.14}$$

The same value is given by Oechslin, also in sidereal days. This is a (not so good) approximation of the tropical year, which should be about 365.2422 days. The tube 150 therefore represents the mean longitude of the Sun.

This motion is then used to produce the motion of the lunar nodes on tube 155 in the first compartment, via the arbor 151:

$$V_{151}^{156} = V_{150}^{156} \times \left(-\frac{28}{78}\right) = -\frac{4923202480}{5009175578121} \tag{7.15}$$

$$V_{155}^{156} = V_{151}^{156} \times \left(-\frac{25}{33}\right) \times \left(-\frac{17}{86}\right) \tag{7.16}$$

$$= \left(-\frac{4923202480}{5009175578121}\right) \times \frac{425}{2838} = -\frac{1046180527000}{7108020145353699} \tag{7.17}$$

$$P_{155}^{156} = -\frac{7108020145353699}{1046180527000} = -6794.2577\ldots \text{ days} \tag{7.18}$$

This is an excellent approximation of the period of precession of the nodes which is about 6793 days. This precession is clockwise, hence the negative sign. Oechslin gives the same value, also in sidereal days.

The motion of tube 151 is also used to produce the motion of tube 152:

$$V_{152}^{156} = V_{151}^{156} \times \left(-\frac{23}{73}\right) = \left(-\frac{4923202480}{5009175578121}\right) \times \left(-\frac{23}{73}\right) \tag{7.19}$$

$$= \frac{4923202480}{15898687704471} \tag{7.20}$$

$$P_{152}^{156} = \frac{15898687704471}{4923202480} = 3229.3385\ldots \text{ days} \tag{7.21}$$

Oechslin gives the same value, also in sidereal days.

Now, the 112-teeth wheel on tube 148 carries an eccentric arbor 153 with a 60-teeth wheel meshing with another 60-teeth wheel on tube 152. We can compute the period of arbor 153 with respect to tube 148 as follows:

$$V_{153}^{148} = -V_{152}^{148} = -\left(V_{152}^{156} - V_{148}^{156}\right) = V_{148}^{156} - V_{152}^{156} \tag{7.22}$$

$$= \frac{6279595}{171568408} - \frac{4923202480}{15898687704471} \tag{7.23}$$

$$= \frac{106165514457065}{2925358537622664} \tag{7.24}$$

$$P_{153}^{148} = \frac{2925358537622664}{106165514457065} = 27.5546\ldots \text{ days} \tag{7.25}$$

This is an excellent approximation of the anomalistic month which is about 27.55455 days. It is not given by Oechslin. This motion is used to move an eccentric pin and create the correction known as the anomaly to the mean motion of the moon.

The motion of the nodes and the corrected motion of the moon are shown below the globe.

7.3.2 The motions of the Sun, Mercury and Venus

The second compartment the globe is mainly focused on the motions of Mercury and Venus. Hahn's purpose was to show the geocentric motion of the planets and in that perspective the inner planets Mercury and Venus oscillate around the Sun. Their mean geocentric motion is the mean motion of the Sun, which has a period of a tropical year with respect to the zodiac.

The periods of oscillations are those of the synodic revolutions of the planets, and so should be 115.88 days for Mercury and 583.92 days for Venus.

Let's see how Hahn tried to achieve this. We have already mentioned the tube 150 which corresponds to the tropical year. As a first step, the true motion of the Sun is obtained. In order to do so, the mean position of the Sun is corrected by the equation of center. This is done as follows. The 113-teeth wheel on tube 150 carries an arbor 158 whose motion is derived from a fixed 36-teeth wheel on the globe arbor 156. Since the train only contains 36-teeth wheels, the period of the wheel on arbor 158 is the same as that of tube 150 with respect to the globe, except that the motion of 158 is clockwise with respect to tube 150. Consequently the period of 158 with respect to tube 150 is the tropical year, but the wheel on arbor 158 is in fact still with respect to the globe.

Now, the wheel on arbor 158 carries an eccentric pin which makes the entire structure for Mercury and Venus oscillate with the period of the tropical year. However, the period of the equation of center is not the tropical year (365.2422 days), but the anomalistic year (about 365.2596 days), and so tube 150 should actually have been designed with that motion in mind. Moreover, as we have seen above, the mean motion of the Sun is not as accurate as it could have been. In any case, this still provides a good approximation of the longitude of the Sun. All that remains to be seen is how the actual motions of Mercury and Venus are obtained.

Now, Oechslin numbers the frame with the true motion of the Sun also 150, but I will name it 150′. This frame carries a number of wheels which derive their motion from two fixed wheels on the globe's central arbor. The velocity of these two wheels (on arbor 156) with respect to the frame 150′ is

$$V_{156}^{150'} \approx V_{156}^{150} = -V_{150}^{156} = -\frac{351657320}{128440399439} \tag{7.26}$$

The oscillation of the Venus hand therefore occurs with velocity

$$\begin{aligned} V_{168}^{150'} &= V_{156}^{150'} \times \left(-\frac{62}{40}\right) \times \left(-\frac{52}{44}\right) \times \left(-\frac{28}{82}\right) && (7.27)\\ &= V_{156}^{150'} \times \left(-\frac{2821}{4510}\right) && (7.28)\\ &\approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{2821}{4510}\right) && (7.29)\\ &\approx \frac{99202529972}{57926620146989} && (7.30)\\ P_{168}^{150'} &\approx \frac{57926620146989}{99202529972} = 583.92\ldots \text{ days} && (7.31) \end{aligned}$$

Venus having a near circular orbit, Hahn did not take its eccentricity into account. But he did so for Mercury. Its motion is obtained likewise from the true longitude of the Sun by adding an oscillation of period the synodic period of Mercury, but in addition Hahn adds a small term to account for Mercury's eccentric orbit.

We can also compute the motion of Venus with respect to the globe:

$$V_{168}^{156} = V_{168}^{150'} + V_{150'}^{156} \approx V_{168}^{150'} + V_{150}^{156} \tag{7.32}$$

$$\approx \frac{99202529972}{57926620146989} + \frac{351657320}{128440399439} = \frac{257799981292}{57926620146989} \tag{7.33}$$

$$P_{168}^{156} \approx \frac{57926620146989}{257799981292} = 224.6959\ldots \text{ days} \tag{7.34}$$

The same value is given by Oechslin.

The oscillation of the Mercury hand due to motion of the Earth therefore occurs with velocity

$$V_{163}^{150'} = V_{156}^{150'} \times \left(-\frac{58}{39}\right) \times \left(-\frac{58}{34}\right) \times \left(-\frac{41}{33}\right) \tag{7.35}$$

$$= V_{156}^{150'} \times \left(-\frac{68962}{21879}\right) \tag{7.36}$$

$$\approx \left(-\frac{351657320}{128440399439}\right) \times \left(-\frac{68962}{21879}\right) \tag{7.37}$$

$$\approx \frac{836241106960}{96901637907789} \tag{7.38}$$

$$P_{163}^{150'} \approx \frac{96901637907789}{836241106960} = 115.8776\ldots \text{ days} \tag{7.39}$$

See also Oechslin's additional observations [33, p. 135-136].

We can also compute the motion of Mercury with respect to the globe:

$$V_{163}^{156} = V_{163}^{150'} + V_{150'}^{156} \approx V_{163}^{150'} + V_{150}^{156} \tag{7.40}$$

$$\approx \frac{836241106960}{96901637907789} + \frac{351657320}{128440399439} = \frac{31944902606120}{2810147499325881} \tag{7.41}$$

$$P_{168}^{156} \approx \frac{2810147499325881}{31944902606120} = 87.9685\ldots \text{ days} \tag{7.42}$$

The same value is given by Oechslin.

We can observe that Hahn obtained very good approximations of the periods of the main oscillations of Mercury and Venus.

The eccentricity of Mercury's orbit is taken into account as follows. Hahn used a similar device as for the eccentricity of the Earth. He put the eccentric pin moving the Mercury hand on a wheel (arbor 165) which is still with respect to the globe frame. This means, like for the Sun, that a specific direction is defined in the sky and the motion of Mercury will be accelerated or decelerated according to the closeness to this direction. This will account for Mercury's equation of center.

The motion of the frame 150′ is also transfered to the third compartment, using two parallel trains of 116 teeth (arbors 150′ and 177), 22 teeth (arbors 169 and 176) and again 22 teeth (arbor 170). On Oechslin's plan, it seems that arbor 170 goes through the frames of Mars and Jupiter, but perhaps Oechslin's plan is not entirely faithful to the actual layout and the wheels of Mars and Jupiter are smaller than shown. In any case, it is through this transfer that the motion of the solar hand is obtained on top of the globe and this hand moves with the true longitude of the Sun.

7.3.3 The motions of Mars, Jupiter and Saturn

The motions of Mars, Jupiter and Saturn are obtained in very similar ways, all from the true motion of the Sun. First Hahn obtains the tropical orbit periods.

Then he adds a correction for the equation of center. This is done using merely two identical wheels and an eccentric pin. The deviation due to the pin is positive during half of the tropical orbit period, and negative during the other half.

Finally, Hahn adds another correction to take into account the retrogradation due to the motion of the Earth and which has the period of the synodic revolution of the planet. This is also done in a very simple way, in that in all three cases, Hahn puts a wheel having the motion of the true sun (the mean sun with the equation of center) on the frame which has taken the equation of center of the planet into account. This wheel has a pin which puts the hand of the planet in its final position. Since the position of the pin depends of the relative angle of the motion of the Sun and the mean position of the planet, it does naturally have the period of the synodic revolution of the planet.

7.3.3.1 The motion of Mars

First, the mean motion of Mars is obtained from the true motion of the Sun. This motion corresponds to tube 172 whose velocity is

$$V_{172}^{156} = V_{150'}^{156} \times \left(-\frac{116}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{12}{30}\right) \times \left(-\frac{30}{119}\right) \tag{7.43}$$

$$= V_{150'}^{156} \times \frac{696}{1309} \tag{7.44}$$

$$\approx V_{150}^{156} \times \frac{696}{1309} \tag{7.45}$$

$$\approx \frac{351657320}{128440399439} \times \frac{696}{1309} \tag{7.46}$$

$$\approx \frac{1205682240}{828219127417} \tag{7.47}$$

$$P_{172}^{156} \approx \frac{828219127417}{1205682240} = 686.9298\ldots \text{ days} \tag{7.48}$$

This is a good approximation of the tropical orbit period of Mars which is about 686.972 days. The same value is given by Oechslin, also in sidereal days.

The synodic revolution of Mars can be computed as follows. We compute the difference of velocities between the tube for the mean motion of the Sun and the tropical period of Mars:

$$\mathrm{V}_{150}^{172} = \mathrm{V}_{150}^{156} - \mathrm{V}_{172}^{156} \approx \frac{351657320}{128440399439} - \frac{1205682240}{828219127417} \tag{7.49}$$

$$\approx \frac{30795133880}{24018354695093} \tag{7.50}$$

$$\mathrm{P}_{150}^{172} \approx \frac{24018354695093}{30795133880} = 779.9399\ldots \text{ days} \tag{7.51}$$

This is very close to the actual value of 779.94 days. This value is not given by Oechslin.

7.3.3.2 The motion of Jupiter

Like for Mars, the mean motion of Jupiter is obtained from the true motion of the Sun. This motion corresponds to tube 181 whose velocity is

$$\mathrm{V}_{181}^{156} = \mathrm{V}_{172}^{156} \times \left(-\frac{119}{30}\right) \times \left(-\frac{30}{31}\right) \times \left(-\frac{5}{34}\right) \times \left(-\frac{34}{121}\right) \tag{7.52}$$

$$= \mathrm{V}_{172}^{156} \times \frac{595}{3751} \tag{7.53}$$

$$\approx \frac{1205682240}{828219127417} \times \frac{595}{3751} \tag{7.54}$$

$$\approx \frac{42198878400}{182744114525951} \tag{7.55}$$

$$\mathrm{P}_{181}^{156} \approx \frac{182744114525951}{42198878400} = 4330.5443\ldots \text{ days} \tag{7.56}$$

This is a good approximation of the tropical orbit period of Jupiter which is about 4330.595 days. The same value is given by Oechslin, also in sidereal days.

We could similarly also compute the synodic revolution of Jupiter.

7.3.3.3 The motion of Saturn

Like for Mars and Jupiter, the mean motion of Jupiter is obtained from the true motion of the Sun. This motion corresponds to tube 187 whose velocity is

$$\begin{aligned}
V^{156}_{187} &= V^{156}_{181} \times \left(-\frac{121}{34}\right) \times \left(-\frac{24}{212}\right) && (7.57)\\
&= V^{156}_{181} \times \frac{363}{901} && (7.58)\\
&\approx \frac{42198878400}{182744114525951} \times \frac{363}{901} && (7.59)\\
&\approx \frac{126596635200}{1360764026346131} && (7.60)\\
P^{156}_{187} &\approx \frac{1360764026346131}{126596635200} = 10748.8166\ldots \text{ days} && (7.61)
\end{aligned}$$

This is a good approximation of the tropical orbit period of Saturn which is about 10746.94 days. The same value is given by Oechslin, also in sidereal days.

We could similarly also compute the synodic revolution of Saturn.

7.4 The planetary system or orrery

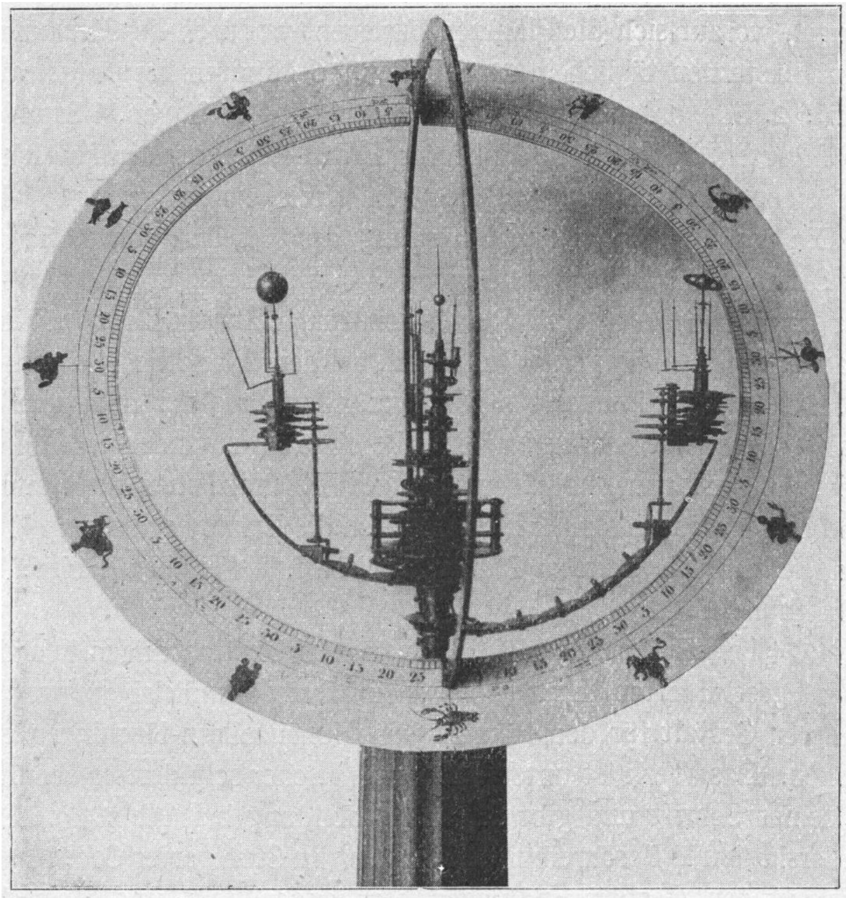

Figure 7.16: The orrery at the left of Hahn's *Weltmaschine*. (source: [8])

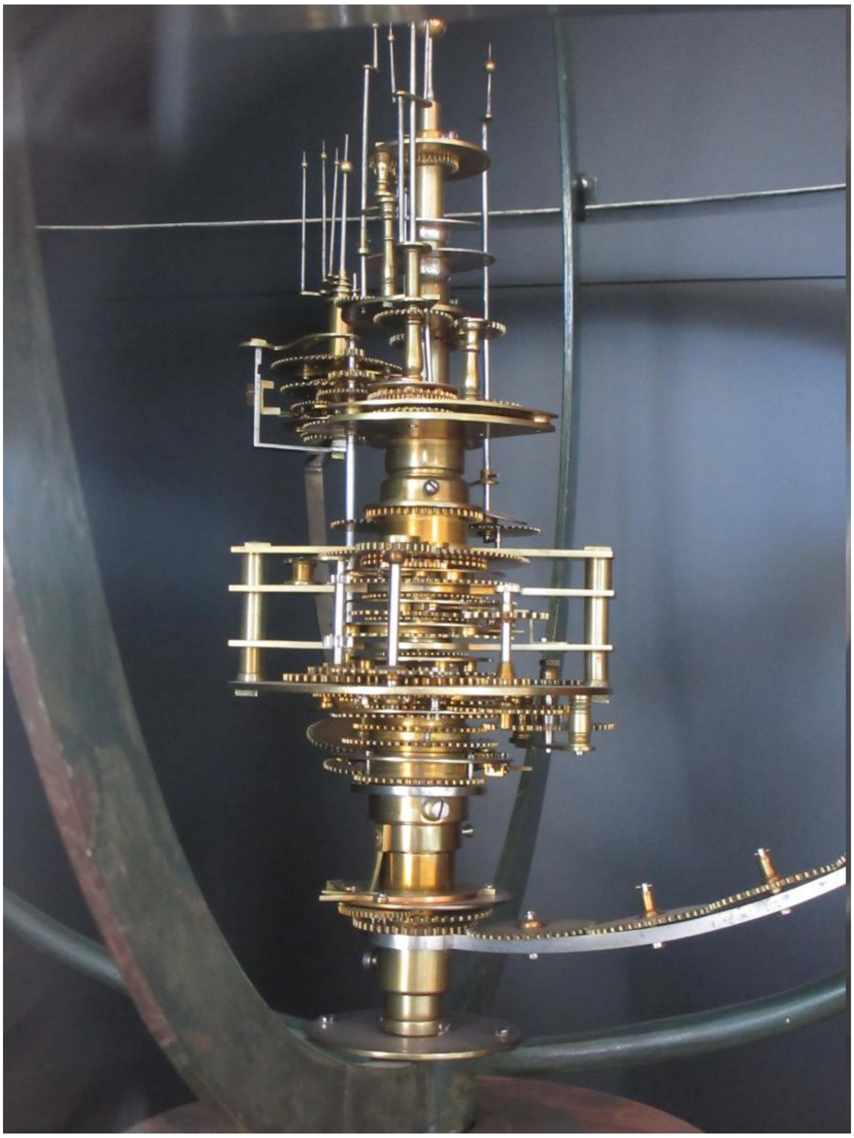

Figure 7.17: The central part of the orrery of Hahn's *Weltmaschine*. (photograph by the author, 2025)

The orrery on the left of the machine shows the heliocentric motions of the planets Mercury to Saturn, as well as the Moon, the four then known satellites of Jupiter and the five then known satellites of Saturn. Mercury, Venus, Mars and the Moon are moreover given a motion in tilted planes.

As mentioned above, the orrery is driven by the horizontal arbor 33 coming from the central part. This arbor makes one turn counterclockwise in one day when when looked from the right:

$$\mathrm{V}_{33}^{0} = 1 \tag{7.62}$$

When reaching the orrery part, this arbor carries a 24-teeth wheel which meshes with a similar wheel on the vertical arbor 34. This arbor makes one turn clockwise in a day:

$$\mathrm{V}_{34}^{0} = -1 \tag{7.63}$$

7.4.1 The Sun

The central vertical arbor 34 actually goes through the entire planetary system and carries the Sun which then makes one turn in a day. As the Sun has presumably no features, this is in fact of no consequences, merely a side-effect of the construction.

7.4.2 The fixed cage

The central vertical arbor 34 goes through a fixed cage 54 which contains a number of gears that are used to produce the mean motions of Mercury, Venus, the Moon, the Earth and Mars. All these motions are derived from a single 24-teeth wheel located on that arbor 34.

Moreover, this motion is also transfered below the cage to the arms of Jupiter and Saturn, and eventually to the satellites of Jupiter and Saturn.

7.4.3 The mean motions of Mercury, Venus, the Moon, the Earth and Mars

The input motion of arbor 34 is first used to obtain the motion of a number of intermediary arbors 37, 39, 42, 43 and 45. The velocities of these arbors are the following:

$$V_{37}^{54} = V_{34}^{54} \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) \times \left(-\frac{9}{45}\right) = \frac{1}{5} \tag{7.64}$$

$$V_{39}^{54} = V_{37}^{54} \times \left(-\frac{15}{42}\right) = \frac{1}{5} \times \left(-\frac{15}{42}\right) = -\frac{1}{14} \tag{7.65}$$

$$V_{42}^{54} = V_{39}^{54} \times \left(-\frac{14}{28}\right) = \left(-\frac{1}{14}\right) \times \left(-\frac{14}{28}\right) = \frac{1}{28} \tag{7.66}$$

$$V_{43}^{54} = V_{42}^{54} \times \left(-\frac{4}{32}\right) = \frac{1}{28} \times \left(-\frac{4}{32}\right) = -\frac{1}{224} \tag{7.67}$$

$$V_{45}^{54} = V_{42}^{54} \times \left(-\frac{28}{30}\right) = \frac{1}{28} \times \left(-\frac{28}{30}\right) = -\frac{1}{30} \tag{7.68}$$

Then, these arbors are used to obtain a number of "base motions." The base motion of Mercury is given by tube 46 surrounding the central vertical arbor 34. This tube has the velocity

$$V_{46}^{54} = V_{45}^{54} \times \left(-\frac{15}{44}\right) = \left(-\frac{1}{30}\right) \times \left(-\frac{15}{44}\right) = \frac{1}{88} \tag{7.69}$$

$$P_{46}^{54} = 88 \text{ days} \tag{7.70}$$

The base motion of Venus is given by tube 47. This tube has the velocity

$$V_{47}^{54} = V_{45}^{54} \times \left(-\frac{6}{45}\right) = \left(-\frac{1}{30}\right) \times \left(-\frac{6}{45}\right) = \frac{1}{225} \tag{7.71}$$

$$P_{47}^{54} = 225 \text{ days} \tag{7.72}$$

The base motion of the Moon is given by tube 40. This tube has the velocity

$$V_{40}^{54} = V_{39}^{54} \times \left(-\frac{20}{39}\right) = \left(-\frac{1}{14}\right) \times \left(-\frac{20}{39}\right) = \frac{10}{273} \tag{7.73}$$

$$P_{40}^{54} = 27.3 \text{ days} \tag{7.74}$$

The base motion of the Earth is given by tube 48. This tube has the velocity

$$V_{48}^{54} = V_{45}^{54} \times \left(-\frac{6}{73}\right) = \left(-\frac{1}{30}\right) \times \left(-\frac{6}{73}\right) = \frac{1}{365} \tag{7.75}$$

$$P_{48}^{54} = 365 \text{ days} \tag{7.76}$$

The motion of Mars is given by tube 44. It is actually not the base motion, but the mean motion of Mars. Tube 44 has the velocity

$$V_{44}^{54} = V_{43}^{54} \times \left(-\frac{30}{92}\right) = \left(-\frac{1}{224}\right) \times \left(-\frac{30}{92}\right) = \frac{15}{10304} \tag{7.77}$$

$$P_{44}^{54} = \frac{10304}{15} = 686.9333\ldots \text{ days} \tag{7.78}$$

The same value is given by Oechslin.

Each of the base wheels, except the one for Mars, carries a small pinion which meshes with the actual wheel corresponding to the mean motion of the planet or the Moon. These pinions are made to rotate slowly and thus slightly change the velocity of the base wheels. Similar constructions are also used for the satellites of Jupiter and Venus.

More precisely, this is done as follows. Mercury's base wheel 46 carries a train of wheels whose last element is a 6-pointed star wheel. This star wheel is advanced by one tooth for every turn of the base wheel. The tube 58 whose mean motion is the mean motion of Mercury makes x_1 turns with respect to the base wheel when the base wheel makes one turn. We have

$$x_1 = \left(-\frac{1}{6}\right) \times \left(-\frac{6}{18}\right) \times \left(-\frac{3}{27}\right) \times \left(-\frac{3}{51}\right) = \frac{1}{2754} \tag{7.79}$$

and

$$\mathrm{V}_{58}^{46} = x_1 \times \mathrm{V}_{46}^{54} \tag{7.80}$$

Consequently

$$\mathrm{V}_{58}^{54} = \mathrm{V}_{58}^{46} + \mathrm{V}_{46}^{54} = \mathrm{V}_{46}^{54} \times (1 + x_1) \tag{7.81}$$

$$= \frac{1}{88} \times \frac{2755}{2754} = \frac{2755}{242352} \tag{7.82}$$

and

$$\mathrm{P}_{58}^{54} = \frac{242352}{2755} = 87.9680\ldots \text{ days} \tag{7.83}$$

This is the (mean) period of tube 58 and it is very close to the actual orbital period of Mercury which is about 87.9691 days. Oechslin gives a slightly incorrect value of the period, because he mistakenly used the ratio $\frac{1}{9}$ instead of $\frac{1}{6}$ in the computation of x_1.

It is such a construction that enabled Hahn to obtain accurate periods without resorting to many-numbered teeth counts. The greatest number of teeth for a wheel in the orrery is 92.

In the case of Venus, we have

$$x_2 = \left(-\frac{1}{6}\right) \times \left(-\frac{3}{24}\right) \times \left(-\frac{24}{27}\right) \times \left(-\frac{3}{43}\right) = \frac{1}{774} \tag{7.84}$$

and

$$\mathrm{V}_{64}^{47} = x_2 \times \mathrm{V}_{47}^{54} \tag{7.85}$$

Consequently

$$V_{64}^{54} = V_{64}^{47} + V_{47}^{54} = V_{47}^{54} \times (1 + x_2) \tag{7.86}$$
$$= \frac{1}{225} \times \frac{775}{774} = \frac{31}{6966} \tag{7.87}$$

and

$$P_{64}^{54} = \frac{6966}{31} = 224.7096\ldots \text{ days} \tag{7.88}$$

This is the (mean) period of tube 64 and it is again an excellent approximation of the actual orbital period of Venus. Oechslin gives the same value.

In the case of the Moon, we have

$$x_3 = \left(-\frac{1}{9}\right) \times \left(-\frac{3}{23}\right) \times \left(-\frac{3}{55}\right) = -\frac{1}{1265} \tag{7.89}$$

and

$$V_{69}^{40} = x_3 \times V_{40}^{54} \tag{7.90}$$

Consequently

$$V_{69}^{54} = V_{69}^{40} + V_{40}^{54} = V_{40}^{54} \times (1 + x_3) \tag{7.91}$$
$$= \frac{10}{273} \times \frac{1264}{1265} = \frac{2528}{69069} \tag{7.92}$$

and

$$P_{69}^{54} = \frac{69069}{2528} = 27.3215\ldots \text{ days} \tag{7.93}$$

This is the (mean) period of tube 69 and it is again an excellent approximation of the actual tropical month. Oechslin gives the same value.

In the case of the Earth, we have

$$x_4 = \left(-\frac{1}{9}\right) \times \left(-\frac{3}{36}\right) \times \left(-\frac{3}{43}\right) = -\frac{1}{1548} \tag{7.94}$$

and

$$V_{78}^{48} = x_4 \times V_{48}^{54} \tag{7.95}$$

Consequently

$$V_{78}^{54} = V_{78}^{48} + V_{48}^{54} = V_{48}^{54} \times (1 + x_4) \tag{7.96}$$
$$= \frac{1}{365} \times \frac{1547}{1548} = \frac{1547}{565020} \tag{7.97}$$

and

$$P_{78}^{54} = \frac{565020}{1547} = 365.2359\ldots \text{ days} \tag{7.98}$$

This is the (mean) period of tube 78 and it should be equal to the tropical year which is about 365.2422 days. Oechslin gives the same value, but there is an important discrepancy here, and it is to be noted that in Oechslin's figure the arbor with the 36-teeth wheel and the 3-leaves pinion has its teeth counts parenthesized, perhaps meaning that this arbor was missing. If this was the case, it appears that replacing 36 by 35 would give a more accurate value for the tropical year, namely $\frac{549325}{1504} = 365.24268\ldots$ days.

7.4.4 The upper part of the orrery

All the motions that I have described are transfered to the upper part of the orrery. From the central arbor to the outer tube fixed on the cage, we have in that order 1) the central arbor carrying the Sun, 2) the mean motion of Mercury, 3) a tube fixed the cage, 4) the mean motion of Venus, 5) the mean motion of the Moon, 6) another fixed tube, 7) the mean motion of the Earth, 8) the mean motion of Mars, and 9) another fixed tube.

I will now go again from the inside to the outside, except for the Earth and the Moon which I will consider last. After the central arbor moving the Sun we have the tube 58 for the mean motion of Mercury. However, Mercury is given two additional motions. First, the axis of Mercury is actually sliding on a tilted plane in order to have Mercury move on a tilted orbit. This tilted plane is fixed on the fixed tube surrounding the Mercury tube.

The second additional motion of Mercury is to account for Mercury's eccentric orbit. Hahn put Mercury slightly off its axis and Mercury's axis rotates. The elliptic motion of Mercury is therefore approximated by an epicycle. In order to keep the perihelion in a fixed direction, Hahn rotates Mercury's axis in such a way that it is only moving in translation with respect to frame 54. This can be observed by computing the velocity of Mercury's axis 60 with respect to the fixed frame 54:

$$V_{60}^{54} = V_{60}^{58} + V_{58}^{54} \tag{7.99}$$

$$= V_{54}^{58} \times \left(-\frac{36}{18}\right) \times \left(-\frac{12}{24}\right) + V_{58}^{54} \tag{7.100}$$

$$= V_{54}^{58} + V_{58}^{54} = 0 \tag{7.101}$$

The same is done with Venus, but with only three 36-teeth wheels, and also with Mars, but with three 42-teeth wheels. In all three cases, the trains of three wheels starts with a fixed wheel. In the case of Mercury and Venus, the tilted planes are fixed to a fixed tube, but in the case of Mars, the tilted plane is fixed to the wheel which replicates the motion of a fixed central tube.

It now remains to see how the motions of the Earth and the Moon are obtained. We have seen above that the motion of tube 78 is that of the tropical year. This tube carries the axis of the Earth. Like for Mercury, Venus and Mars, the Earth is offset and moves on an epicycle, with the perihelion always in the same direction, again with three 36-teeth wheels, as for Venus. The motion of the Earth occurs in the plane of the ecliptic, and there is therefore no tilt.

Hahn next has the Moon move around the Earth and he achieves this by replicating the motion of a central tube like for the epicycles for the elliptic motions of Mercury, Venus, the Earth and Mars, but this time the central tube is not fixed. It is instead the tube whose motion is the tropical month. Consequently, the Moon revolves around the Earth, with a motion with respect to the fixed frame equal to that of the tropical month.

Finally, Hahn takes the tilt of the lunar orbit into account. But this tilt is not fixed in time. For the other planets Hahn considered the orientation of the orbital planes to be fixed, but in the case of the Moon, there is a greater precession of the nodes which needs to be taken into account if one wishes to display the solar and lunar eclipses, for instance. In order to better appreciate the motion of this plane, Hahn also shows the plane of the ecliptic around the lunar plane. The ecliptic is moved by three wheels again mimicking the motion of a fixed central tube.

Now, we can compute the motion of the lunar plane as follows. The lunar plane is fixed on tube 74. Let us compute the velocity of this tube with respect to the fixed frame 54:

$$\begin{aligned}
\mathrm{V}_{74}^{54} &= \mathrm{V}_{74}^{78} + \mathrm{V}_{78}^{54} && (7.102)\\
&= \mathrm{V}_{54}^{78} \times \left(-\frac{36}{36}\right) \times \left(-\frac{59}{56}\right) + \mathrm{V}_{78}^{54} && (7.103)\\
&= \mathrm{V}_{54}^{78} \times \frac{59}{56} + \mathrm{V}_{78}^{54} && (7.104)\\
&= \mathrm{V}_{54}^{78} \times \left(\frac{59}{56} - 1\right) && (7.105)\\
&= \mathrm{V}_{54}^{78} \times \frac{3}{56} = \left(-\frac{1547}{565020}\right) \times \frac{3}{56} && (7.106)\\
&= -\frac{221}{1506720} && (7.107)
\end{aligned}$$

The sign is negative, because this is a precession.

$$\mathrm{P}_{74}^{54} = -\frac{1506720}{221} = -6817.737\ldots \text{ days} \qquad (7.108)$$

Oechslin gives the same value.

This is a somewhat less accurate value than the one used in the globe, which was 6794.257 days, the correct period being about 6793 days. However, given that Hahn has only used the pair 59/56, this is in fact quite remarkable.

7.4.5 The lower part of the orrery

The lower part of the orrery is concerned with the motions of Jupiter, Saturn and their satellites. Things are a bit simpler here, in that there are no eccentric motions, and no tilted planes. Moreover, the motions are obtained like for Mercury and other bodies, namely by first creating a base motion, the correcting it at regular intervals.

But first, from the fixed cage, Hahn obtains the motions of four tubes around the central vertical arbor of the orrery, tubes 51 (Jupiter), 38 (Jupiter's satellites), 53 (Saturn) and 41 (Saturn's satellites). These motions are themselves first obtained from those of arbors 50, 37, 52 and 39. We have already found above that

$$V_{37}^{54} = \frac{1}{5} \tag{7.109}$$

$$V_{39}^{54} = -\frac{1}{14} \tag{7.110}$$

and we have

$$V_{49}^{54} = V_{43}^{54} \times \left(-\frac{16}{16}\right) = \frac{1}{224} \tag{7.111}$$

$$V_{52}^{54} = V_{49}^{54} \times \left(-\frac{16}{49}\right) = \frac{1}{224} \times \left(-\frac{16}{49}\right) = -\frac{1}{686} \tag{7.112}$$

$$V_{50}^{54} = V_{49}^{54} \times \left(-\frac{16}{16}\right) = -\frac{1}{224} \tag{7.113}$$

Now

$$V_{51}^{54} = V_{50}^{54} \times \left(-\frac{3}{58}\right) = \left(-\frac{1}{224}\right) \times \left(-\frac{3}{58}\right) = \frac{3}{12992} \tag{7.114}$$

$$P_{51}^{54} = \frac{12992}{3} = 4330.6666\ldots \text{ days} \tag{7.115}$$

$$V_{38}^{54} = V_{37}^{54} \times \left(-\frac{45}{45}\right) = -\frac{1}{5} \tag{7.116}$$

$$V_{53}^{54} = V_{52}^{54} \times \left(-\frac{3}{47}\right) = \left(-\frac{1}{686}\right) \times \left(-\frac{3}{47}\right) = \frac{3}{32242} \tag{7.117}$$

$$P_{53}^{54} = \frac{32242}{3} = 10747.3333\ldots \text{ days} \tag{7.118}$$

$$V_{41}^{54} = V_{39}^{54} \times \left(-\frac{42}{42}\right) = \frac{1}{14} \tag{7.119}$$

We can see that the motions of tubes 51 and 53 are those of the orbital periods of Jupiter and Saturn. Oechslin gives the same values.

The motions of tubes 38 and 41 are used to obtain the motions of tubes 83 and 109, which are the base motions for the satellites of Jupiter and Saturn.

The motion of tube 83 is obtained like the corrective motions of Mercury, Venus, etc. When tube 38 makes one turn, tube 83 makes an additional rotation of x_5 turns

$$x_5 = \left(-\frac{1}{7}\right) \times \left(-\frac{5}{32}\right) \times \left(-\frac{3}{58}\right) = -\frac{15}{12992} \tag{7.120}$$

and

$$\mathrm{V}_{83}^{38} = x_5 \times \mathrm{V}_{38}^{54} \tag{7.121}$$

Consequently

$$\mathrm{V}_{83}^{54} = \mathrm{V}_{83}^{38} + \mathrm{V}_{38}^{54} = \mathrm{V}_{38}^{54} \times (1 + x_5) \tag{7.122}$$

$$= \left(-\frac{1}{5}\right) \times \frac{12977}{12992} = -\frac{12977}{64960} \tag{7.123}$$

$$\mathrm{P}_{83}^{54} = -\frac{64960}{12977} = -5.00577\ldots \text{ days} \tag{7.124}$$

It should be noted that Oechslin shows the direction of rotation of tube 83 in two ways: above, it shows the motion as counterclockwise, that is the opposite direction to tube 38 (hence the negative sign of x_5), but below it shows it clockwise which corresponds to the negative sign of V_{83}^{54}.

The motion of tube 109 is obtained similarly:

$$x_6 = \left(-\frac{1}{6}\right) \times \left(-\frac{3}{49}\right) \times \left(-\frac{3}{47}\right) = -\frac{3}{4606} \tag{7.125}$$

and

$$\mathrm{V}_{109}^{41} = x_6 \times \mathrm{V}_{41}^{54} \tag{7.126}$$

Consequently

$$\mathrm{V}_{109}^{54} = \mathrm{V}_{109}^{41} + \mathrm{V}_{41}^{54} = \mathrm{V}_{41}^{54} \times (1 + x_6) \tag{7.127}$$

$$= \frac{1}{14} \times \frac{4603}{4606} = \frac{4603}{64484} \tag{7.128}$$

$$\mathrm{P}_{109}^{54} = \frac{64484}{4603} = 14.00912\ldots \text{ days} \tag{7.129}$$

Oechslin also shows the direction of rotation of tube 109 in two ways: above, it shows the motion as clockwise, that is the opposite direction to tube 41 (hence the negative sign of x_6), but below it shows it counterclockwise which corresponds to the positive sign of V_{109}^{54}.

7.4.5.1 Jupiter and its satellites

The Jupiter system is located on an arm fixed on tube 51 whose motion is the mean motion of Jupiter. Jupiter is not given any eccentric motion, although Hahn took it into account in the Furtwangen orrery (Oechslin 8.12) in 1774, in the Gotha *Weltmaschine* (Oechslin 8.3) in 1780, and in the Nuremberg *Weltmaschine* (Oechslin 8.2) in the 1780s.

I am first going to compute the velocities and periods in the Jupiter arm reference frame, then compute the absolute velocities and periods.

Figure 7.18: Hahn's *Weltmaschine*: the central part of the orrery and the system of Jupiter. (photograph by the author, 2025)

The input motion on the arm is the relative motion of tube 83 with respect to tube 51.

$$\mathrm{V}_{83}^{51} = \mathrm{V}_{83}^{54} - \mathrm{V}_{51}^{54} = -\frac{12977}{64960} - \frac{3}{12992} = -\frac{1}{5} \tag{7.130}$$

On the Jupiter arm, there is a first train leading to arbor 87. This arbor carries wheels which are driving the four base wheels of the satellites. Arbor

Figure 7.19: Hahn's *Weltmaschine*: the system of Jupiter. (photograph by the author, 2025)

87 has the following velocity:

$$V_{87}^{51} = V_{83}^{51} \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{48}\right) \times \left(-\frac{48}{24}\right) \times \left(-\frac{24}{24}\right) \quad (7.131)$$

$$= V_{83}^{51} \times 2 = -\frac{2}{5} \quad (7.132)$$

and the base wheels have these motions:

$$\mathrm{V}_{92}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{48}{34}\right) = \frac{48}{85} \tag{7.133}$$

$$\mathrm{P}_{92}^{51} = \frac{85}{48} = 1.7708\ldots \text{ days (Io)} \tag{7.134}$$

$$\mathrm{V}_{93}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{45}{64}\right) = \frac{9}{32} \tag{7.135}$$

$$\mathrm{P}_{93}^{51} = \frac{32}{9} = 3.5555\ldots \text{ days (Europa)} \tag{7.136}$$

$$\mathrm{V}_{98}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{15}{43}\right) = \frac{6}{43} \tag{7.137}$$

$$\mathrm{P}_{98}^{51} = \frac{43}{6} = 7.1666\ldots \text{ days (Ganymede)} \tag{7.138}$$

$$\mathrm{V}_{103}^{51} = \mathrm{V}_{87}^{51} \times \left(-\frac{10}{67}\right) = \frac{4}{67} \tag{7.139}$$

$$\mathrm{P}_{103}^{51} = \frac{67}{4} = 16.75 \text{ days (Callisto)} \tag{7.140}$$

The base wheels have heliocentric periods which are very close to the actual orbital periods. The absolute velocity of Io's base wheel, for instance, is

$$\mathrm{V}_{92}^{54} = \mathrm{V}_{92}^{51} + \mathrm{V}_{51}^{54} = \frac{48}{85} + \frac{3}{12992} = \frac{623871}{1104320} \tag{7.141}$$

$$\mathrm{P}_{92}^{54} = \frac{1104320}{623871} = 1.7701\ldots \text{ days} \tag{7.142}$$

and this differs little from the heliocentric period, because the satellites move much faster around Jupiter than Jupiter around the Sun.

The figure given by Oechslin for the motion of Io is not clear and is probably erroneous. I will assume below that Io's base wheel of 34 teeth carries an eccentric arbor on which there is a 10-teeth wheel and a 4-leaves pinion. I assume that this pinion meshes with the actual wheel of Io which has 52 teeth. On Oechslin's drawing, the 52-teeth wheel is bound to the base wheel and the actual Io wheel is not toothed.

Assuming that this is correct, the 4-leaves pinion then makes a small angular motion for each rotation of the base wheel, using a similar construction than that described earlier for Mercury and other planets. Eventually, the tube 88 carrying Io makes an additional fraction of a turn x_7 when the base wheel makes one turn. We have:

$$x_7 = \left(-\frac{1}{42}\right) \times \left(-\frac{3}{10}\right) \times \left(-\frac{4}{52}\right) = -\frac{1}{1820} \tag{7.143}$$

Then

$$\mathrm{V}_{88}^{92} = x_7 \times \mathrm{V}_{92}^{51} \tag{7.144}$$

Consequently

$$\mathrm{V}_{88}^{51} = \mathrm{V}_{88}^{92} + \mathrm{V}_{92}^{51} = \mathrm{V}_{92}^{51} \times (1 + x_7) \tag{7.145}$$

$$= \frac{48}{85} \times \frac{1819}{1820} = \frac{1284}{2275} \tag{7.146}$$

$$\mathrm{P}_{88}^{51} = \frac{2275}{1284} = 1.7718\ldots \text{ days} \tag{7.147}$$

Europa's base wheel of 64 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Europa's actual satellite wheel (30 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 97 carrying Europa makes an additional fraction of a turn x_8 when the base wheel makes one turn. We have:

$$x_8 = \left(-\frac{1}{8}\right) \times \left(-\frac{1}{30}\right) \times \left(-\frac{30}{30}\right) \times \left(-\frac{3}{30}\right) = \frac{1}{2400} \tag{7.148}$$

Then

$$\mathrm{V}_{97}^{93} = x_8 \times \mathrm{V}_{93}^{51} \tag{7.149}$$

Consequently

$$\mathrm{V}_{97}^{51} = \mathrm{V}_{97}^{93} + \mathrm{V}_{93}^{51} = \mathrm{V}_{93}^{51} \times (1 + x_8) \tag{7.150}$$

$$= \frac{9}{32} \times \frac{2401}{2400} = \frac{7203}{25600} \tag{7.151}$$

$$\mathrm{P}_{97}^{51} = \frac{25600}{7203} = 3.5540\ldots \text{ days} \tag{7.152}$$

The 3-leaves pinion and the 30-teeth wheel on its arbor are parenthesized by Oechslin, and I assume that they were missing.

Ganymede's base wheel of 43 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Ganymede's actual satellite wheel (43 teeth). Eventually, the tube 102 carrying Ganymede makes an additional fraction of a turn x_9 when the base wheel makes one turn. We have:

$$x_9 = \left(-\frac{1}{8}\right) \times \left(-\frac{1}{27}\right) \times \left(-\frac{3}{30}\right) \times \left(-\frac{3}{43}\right) = \frac{1}{30960} \tag{7.153}$$

Then

$$\mathrm{V}_{102}^{98} = x_9 \times \mathrm{V}_{98}^{51} \tag{7.154}$$

Consequently

$$\mathrm{V}_{102}^{51} = \mathrm{V}_{102}^{98} + \mathrm{V}_{98}^{51} = \mathrm{V}_{98}^{51} \times (1 + x_9) \tag{7.155}$$

$$= \frac{6}{43} \times \frac{30961}{30960} = \frac{30961}{221880} \tag{7.156}$$

$$\mathrm{P}_{102}^{51} = \frac{221880}{30961} = 7.1664\ldots \text{ days} \tag{7.157}$$

Callisto's base wheel of 67 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Callisto's actual satellite wheel (55 teeth). Eventually, the tube 106 carrying Callisto makes an additional fraction of a turn x_{10} when the base wheel makes one turn. We have:

$$x_{10} = \left(-\frac{1}{6}\right) \times \left(-\frac{1}{43}\right) \times \left(-\frac{3}{55}\right) = -\frac{1}{4730} \tag{7.158}$$

Then

$$\mathrm{V}_{106}^{103} = x_{10} \times \mathrm{V}_{103}^{51} \tag{7.159}$$

Consequently

$$\mathrm{V}_{106}^{51} = \mathrm{V}_{106}^{103} + \mathrm{V}_{103}^{51} = \mathrm{V}_{103}^{51} \times (1 + x_{10}) \tag{7.160}$$

$$= \frac{4}{67} \times \frac{4729}{4730} = \frac{9458}{158455} \tag{7.161}$$

$$\mathrm{P}_{106}^{51} = \frac{158455}{9458} = 16.7535\ldots \text{ days} \tag{7.162}$$

We can also compute the absolute motions:

$$\mathrm{V}_{88}^{54} = \mathrm{V}_{88}^{51} + \mathrm{V}_{51}^{54} = \frac{1284}{2275} + \frac{3}{12992} = \frac{2384079}{4222400} \tag{7.163}$$

$$\mathrm{P}_{88}^{54} = \frac{4222400}{2384079} = 1.7710\ldots \text{ days (Io)} \tag{7.164}$$

$$\mathrm{V}_{97}^{54} = \mathrm{V}_{97}^{51} + \mathrm{V}_{51}^{54} = \frac{7203}{25600} + \frac{3}{12992} = \frac{1463409}{5196800} \tag{7.165}$$

$$\mathrm{P}_{97}^{54} = \frac{5196800}{1463409} = 3.5511\ldots \text{ days (Europa)} \tag{7.166}$$

$$\mathrm{V}_{102}^{54} = \mathrm{V}_{102}^{51} + \mathrm{V}_{51}^{54} = \frac{30961}{221880} + \frac{3}{12992} = \frac{50363869}{360333120} \tag{7.167}$$

$$\mathrm{P}_{102}^{54} = \frac{360333120}{50363869} = 7.1545\ldots \text{ days (Ganymede)} \tag{7.168}$$

$$\mathrm{V}_{106}^{54} = \mathrm{V}_{106}^{51} + \mathrm{V}_{51}^{54} = \frac{9458}{158455} + \frac{3}{12992} = \frac{123353701}{2058647360} \tag{7.169}$$

$$\mathrm{P}_{106}^{54} = \frac{2058647360}{123353701} = 16.6889\ldots \text{ days (Callisto)} \tag{7.170}$$

Oechslin gives the same values for the periods of Ganymede and Callisto, but slightly different values for Io and Europa. I believe that he made small errors in his computations.

The actual values are 1.7627 days, 3.5255 days, 7.1556 days and 16.690 days.

7.4.5.2 Saturn and its satellites

The Saturn system is located on an arm fixed on tube 53 whose motion is the mean motion of Saturn. It is very similar to the Jupiter system, but there is a fifth satellite. And like in the case of Jupiter, Saturn is not given any eccentric motion, although Hahn took it into account in the Furtwangen orrery (Oechslin 8.12) in 1774, in the Gotha *Weltmaschine* (Oechslin 8.3) in 1780, and in the Nuremberg *Weltmaschine* (Oechslin 8.2) in the 1780s.

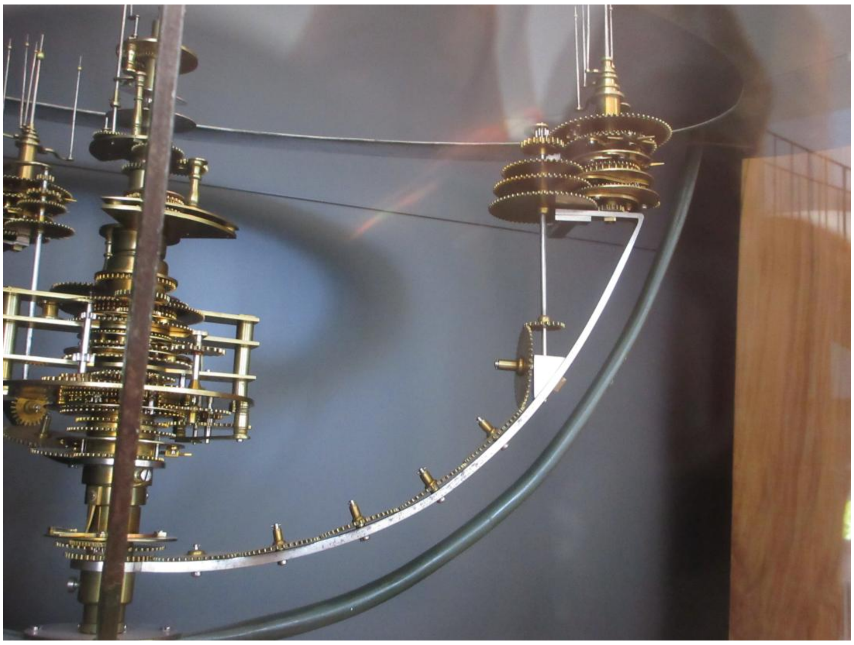

Figure 7.20: Hahn's *Weltmaschine*: the transmission to the system of Saturn in the orrery. (photograph by the author, 2025)

The input motion on the arm is the relative motion of tube 109 with respect to tube 53.

$$\mathrm{V}_{109}^{53} = \mathrm{V}_{109}^{54} - \mathrm{V}_{53}^{54} = \frac{4603}{64484} - \frac{3}{32242} = \frac{4597}{64484} \tag{7.171}$$

On the Saturn arm, there is a first train leading to arbor 116. This arbor carries wheels which are driving the five base wheels of the satellites. Arbor 116 has the following velocity:

$$\mathrm{V}_{116}^{53} = \mathrm{V}_{109}^{53} \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \tag{7.172}$$

$$\times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{42}\right) \times \left(-\frac{42}{21}\right) \tag{7.173}$$

$$= \mathrm{V}_{109}^{53} \times (-2) = -\frac{4597}{32242} \tag{7.174}$$

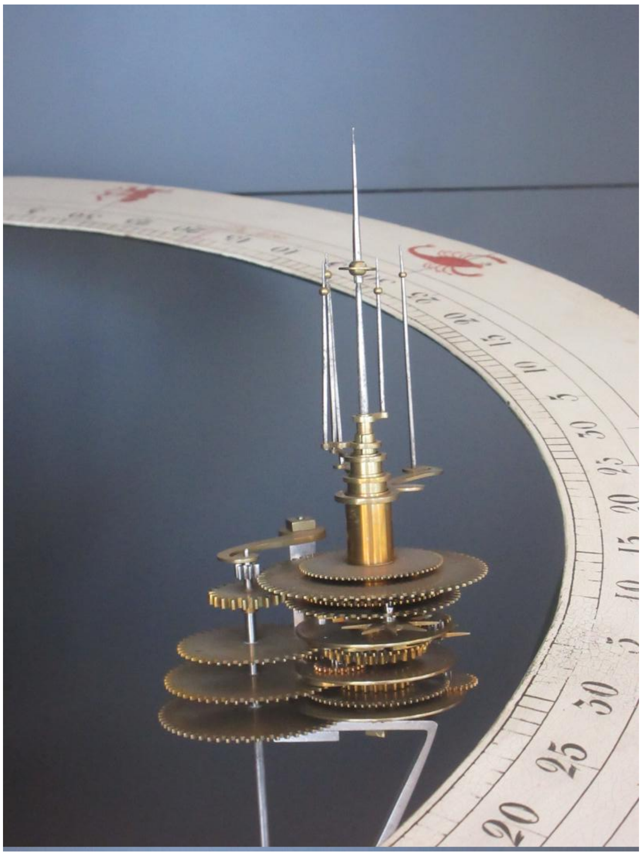

Figure 7.21: Hahn's *Weltmaschine*: detail of the system of Saturn. (photograph by the author, 2025)

and the base wheels have these motions:

$$\mathrm{V}_{117}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{63}{17}\right) = \left(-\frac{4597}{32242}\right) \times \left(-\frac{63}{17}\right) = \frac{41373}{78302} \tag{7.175}$$

$$\mathrm{P}_{117}^{53} = \frac{78302}{41373} = 1.8925\ldots \text{ days (Tethys)} \tag{7.176}$$

$$\mathrm{V}_{123}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{56}{22}\right) = \frac{9194}{25333} \tag{7.177}$$

$$\mathrm{P}_{123}^{53} = \frac{25333}{9194} = 2.7553\ldots \text{ days (Dione)} \tag{7.178}$$

$$\mathrm{V}_{132}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{42}{27}\right) = \frac{4597}{20727} \tag{7.179}$$

$$\mathrm{P}_{132}^{53} = \frac{20727}{4597} = 4.5088\ldots \text{ days (Rhea)} \tag{7.180}$$

$$\mathrm{V}_{133}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{21}{48}\right) = \frac{4597}{73696} \tag{7.181}$$

$$\mathrm{P}_{133}^{53} = \frac{73696}{4597} = 16.0313\ldots \text{ days (Titan)} \tag{7.182}$$

$$\mathrm{V}_{136}^{53} = \mathrm{V}_{116}^{53} \times \left(-\frac{6}{68}\right) = \frac{13791}{1096228} \tag{7.183}$$

$$\mathrm{P}_{136}^{53} = \frac{1096228}{13791} = 79.4886\ldots \text{ days (Iapetus)} \tag{7.184}$$

Tethys' base wheel of 17 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Tethys' actual satellite wheel (65 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 122 carrying Tethys makes an additional fraction of a turn x_{11} when the base wheel makes one turn. We have:

$$x_{11} = \left(-\frac{1}{9}\right) \times \left(-\frac{3}{27}\right) \times \left(-\frac{27}{27}\right) \times \left(-\frac{3}{65}\right) = \frac{1}{1755} \tag{7.185}$$

Then

$$\mathrm{V}_{122}^{117} = x_{11} \times \mathrm{V}_{117}^{53} \tag{7.186}$$

Consequently

$$\mathrm{V}_{122}^{53} = \mathrm{V}_{122}^{117} + \mathrm{V}_{117}^{53} = \mathrm{V}_{117}^{53} \times (1 + x_{11}) \tag{7.187}$$

$$= \frac{41373}{78302} \times \frac{1756}{1755} = \frac{4036166}{7634445} \tag{7.188}$$

$$\mathrm{P}_{122}^{53} = \frac{7634445}{4036166} = 1.8915\ldots \text{ days} \tag{7.189}$$

The 3-leaves pinion and the 27-teeth wheel on its arbor are parenthesized by Oechslin, and I assume that they were missing.

Dione's base wheel of 22 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Dione's actual satellite wheel (23 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 127 carrying Dione makes an additional fraction of a turn x_{12} when the base wheel makes one turn. We have:

$$x_{12} = \left(-\frac{1}{6}\right) \times \left(-\frac{3}{22}\right) \times \left(-\frac{22}{22}\right) \times \left(-\frac{3}{23}\right) = \frac{3}{1012} \tag{7.190}$$

This ratio was erroneously given as $\frac{3}{572}$ by Oechslin who mistakenly replaced $\frac{3}{23}$ by $\frac{3}{13}$ in his computations.

Then

$$\mathrm{V}_{127}^{123} = x_{12} \times \mathrm{V}_{123}^{53} \tag{7.191}$$

Consequently

$$\mathrm{V}_{127}^{53} = \mathrm{V}_{127}^{123} + \mathrm{V}_{123}^{53} = \mathrm{V}_{123}^{53} \times (1 + x_{12}) \tag{7.192}$$

$$= \frac{9194}{25333} \times \frac{1015}{1012} = \frac{666565}{1831214} \tag{7.193}$$

$$\mathrm{P}_{127}^{53} = \frac{1831214}{666565} = 2.7472\ldots \text{ days} \tag{7.194}$$

The 3-leaves pinion and the 22-teeth wheel on its arbor are parenthesized by Oechslin, and I assume that they were missing.

The structure of the gears driving the satellite Rhea is also confuse in Oechslin's figure. The problem is similar to the one observed for Io, the first satellite of Jupiter. Here, I am assuming that Rhea's base wheel has 27 teeth and that it carries an arbor with a 3-leaves pinion meshing with Rhea's actual satellite wheel (43 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 128 carrying Rhea makes an additional fraction of a turn x_{13} when the base wheel makes one turn. We have:

$$x_{13} = \left(-\frac{1}{5}\right) \times \left(-\frac{5}{18}\right) \times \left(-\frac{18}{18}\right) \times \left(-\frac{3}{43}\right) = \frac{1}{258} \tag{7.195}$$

The same ratio is given by Oechslin.

Then

$$\mathrm{V}_{128}^{132} = x_{13} \times \mathrm{V}_{132}^{53} \tag{7.196}$$

Consequently

$$\mathrm{V}_{128}^{53} = \mathrm{V}_{128}^{132} + \mathrm{V}_{132}^{53} = \mathrm{V}_{132}^{53} \times (1 + x_{13}) \tag{7.197}$$

$$= \frac{4597}{20727} \times \frac{259}{258} = \frac{170089}{763938} \tag{7.198}$$

$$\mathrm{P}_{128}^{53} = \frac{763938}{170089} = 4.4914\ldots \text{ days} \tag{7.199}$$

Titan's base wheel of 48 teeth carries an eccentric arbor with a 2-leaves pinion meshing with Titan's actual satellite wheel (65 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 135 carrying Titan makes an additional fraction of a turn x_{14} when the base wheel makes one turn. We have:

$$x_{14} = \left(-\frac{1}{9}\right) \times \left(-\frac{2}{65}\right) = \frac{2}{585} \tag{7.200}$$

The same ratio is given by Oechslin.
Then

$$\mathrm{V}_{135}^{133} = x_{14} \times \mathrm{V}_{133}^{53} \tag{7.201}$$

Consequently

$$\mathrm{V}_{135}^{53} = \mathrm{V}_{135}^{133} + \mathrm{V}_{133}^{53} = \mathrm{V}_{133}^{53} \times (1 + x_{14}) \tag{7.202}$$

$$= \frac{4597}{73696} \times \frac{587}{585} = \frac{2698439}{43112160} \tag{7.203}$$

$$\mathrm{P}_{135}^{53} = \frac{43112160}{2698439} = 15.9767\ldots \text{ days} \tag{7.204}$$

Iapetus' base wheel of 68 teeth carries an eccentric arbor with a 3-leaves pinion meshing with Iapetus' actual satellite wheel (70 teeth). This pinion makes a little angular motion for each rotation of the base wheel. Eventually, the tube 140 carrying Iapetus makes an additional fraction of a turn x_{15} when the base wheel makes one turn. We have:

$$x_{15} = \left(-\frac{1}{8}\right) \times \left(-\frac{3}{18}\right) \times \left(-\frac{4}{34}\right) \times \left(-\frac{3}{70}\right) = \frac{1}{9520} \tag{7.205}$$

The same ratio is given by Oechslin.
Then

$$\mathrm{V}_{140}^{136} = x_{15} \times \mathrm{V}_{136}^{53} \tag{7.206}$$

Consequently

$$\mathrm{V}_{140}^{53} = \mathrm{V}_{140}^{136} + \mathrm{V}_{136}^{53} = \mathrm{V}_{136}^{53} \times (1 + x_{15}) \tag{7.207}$$

$$= \frac{13791}{1096228} \times \frac{9521}{9520} = \frac{131304111}{10436090560} \tag{7.208}$$

$$\mathrm{P}_{140}^{53} = \frac{10436090560}{131304111} = 79.4803\ldots \text{ days} \tag{7.209}$$

We can also compute the absolute motions:

$$V_{122}^{54} = V_{122}^{53} + V_{53}^{54} = \frac{4036166}{7634445} + \frac{3}{32242} = \frac{56516269}{106882230} \quad (7.210)$$

$$P_{122}^{54} = \frac{106882230}{56516269} = 1.8911\ldots \text{ days} \quad (7.211)$$

$$V_{127}^{54} = V_{127}^{53} + V_{53}^{54} = \frac{666565}{1831214} + \frac{3}{32242} = \frac{16335017}{44864743} \quad (7.212)$$

$$P_{127}^{54} = \frac{44864743}{16335017} = 2.7465\ldots \text{ days} \quad (7.213)$$

$$V_{128}^{54} = V_{128}^{53} + V_{53}^{54} = \frac{170089}{763938} + \frac{3}{32242} = \frac{4168922}{18716481} \quad (7.214)$$

$$P_{128}^{54} = \frac{18716481}{4168922} = 4.4895\ldots \text{ days} \quad (7.215)$$

$$V_{135}^{54} = V_{135}^{53} + V_{53}^{54} = \frac{2698439}{43112160} + \frac{3}{32242} = \frac{18917153}{301785120} \quad (7.216)$$

$$P_{135}^{54} = \frac{301785120}{18917153} = 15.9529\ldots \text{ days} \quad (7.217)$$

$$V_{140}^{54} = V_{140}^{53} + V_{53}^{54} = \frac{131304111}{10436090560} + \frac{3}{32242} = \frac{132275151}{10436090560} \quad (7.218)$$

$$P_{140}^{54} = \frac{10436090560}{132275151} = 78.8968\ldots \text{ days} \quad (7.219)$$

Oechslin obtains slightly different values, apart from the above error on x_{12}, and I am inclined to think that some errors have crept in his computations. Moreover, his values (1.83, 2.65, 4.38, 15.45 and 76.42) differ more from the actual ones (1.9 days, 2.7 days, 4.5 days, 16 days and 79 days) than the ones I have obtained. The discrepancies are greater than for Jupiter's satellites.

7.5 The calendar part (summary)

As mentioned earlier, the calendar part is driven by arbor 8 which makes one rotation counterclockwise (seen from above) in a day. This arbor carries a 50-teeth wheel which meshes with another 50-teeth wheel on tube 9 which drives a hand making one turn clockwise in a day:

$$\mathrm{T}_9^0 = 1 \tag{7.220}$$

A coupling pairs the tube 9 with the central arbor 10 which then has the same motion:

$$\mathrm{T}_{10}^0 = 1 \tag{7.221}$$

A 24-teeth wheel on that arbor meshes with a similar wheel on arbor 11, which in turn meshes with another similar wheel on arbor 31, which goes down vertically towards the orrery and globe.

I will not describe the calendar gears in more detail, partly because I want to focus on the astronomical gears, and partly because I do not have sufficient details in the intermittent motions and cams involved in this part of the machine.

I am merely giving some information on the dials. The upper dial of the middle part of the machine shows the hours, minutes and seconds, as mentioned earlier. The middle dial shows the day of the month, the day of the week and the month. This part of the clock does take into account the lengths of the months, probably also the leap years, but perhaps not the common years such as 1800, 1900, 2100, etc. One of the arbors of that part of the clock makes a turn in 10 years and leads to the lower dial. This dial is a counter for the years and has two hands, one making a turn in 100 years and another in 8000 years. This is used to indicate the chronology of the world as described by Johann Albrecht Bengel.[22]

[22] See in particular Bengel's works [3, 4, 5, 6], Wilfried Veeser's chapter in 1989 [39, p. 327-339], and Marini [25]. This dial is also illustrated in the 1989 Hahn exhibition catalogue [38, p. 374].

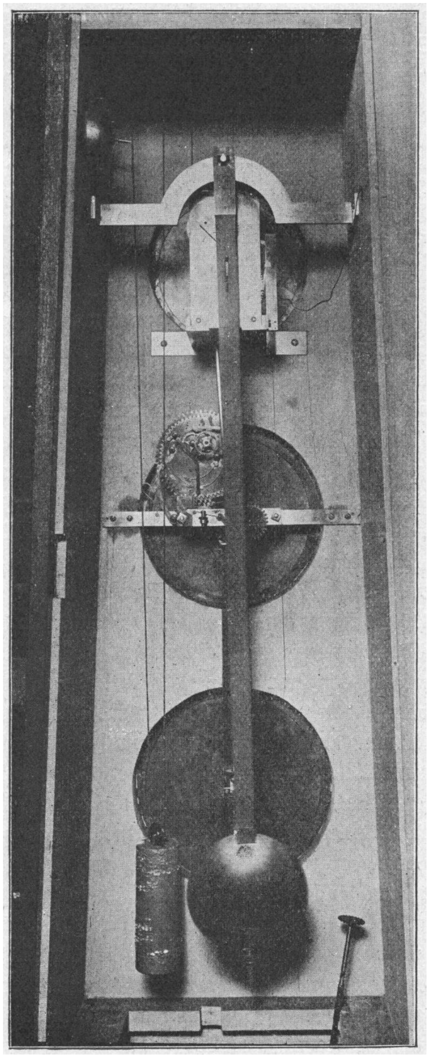

Figure 7.22: Hahn's *Weltmaschine*: the back of the central dials with the pendulum and the weight. (source: [8])

Figure 7.23: Hahn's *Weltmaschine*: the upper central dial giving the time. (photograph by the author, 2025)

Figure 7.24: Hahn's *Weltmaschine*: the middle central dial giving the day of the week and the month. (photograph by the author, 2025)

Figure 7.25: Hahn's *Weltmaschine*: detail of the middle central dial giving the day of the week and the month. (photograph by the author, 2025)

Figure 7.26: Hahn's *Weltmaschine*: the lower central dial giving the years, following Bengel's chronology. (photograph by the author, 2025)

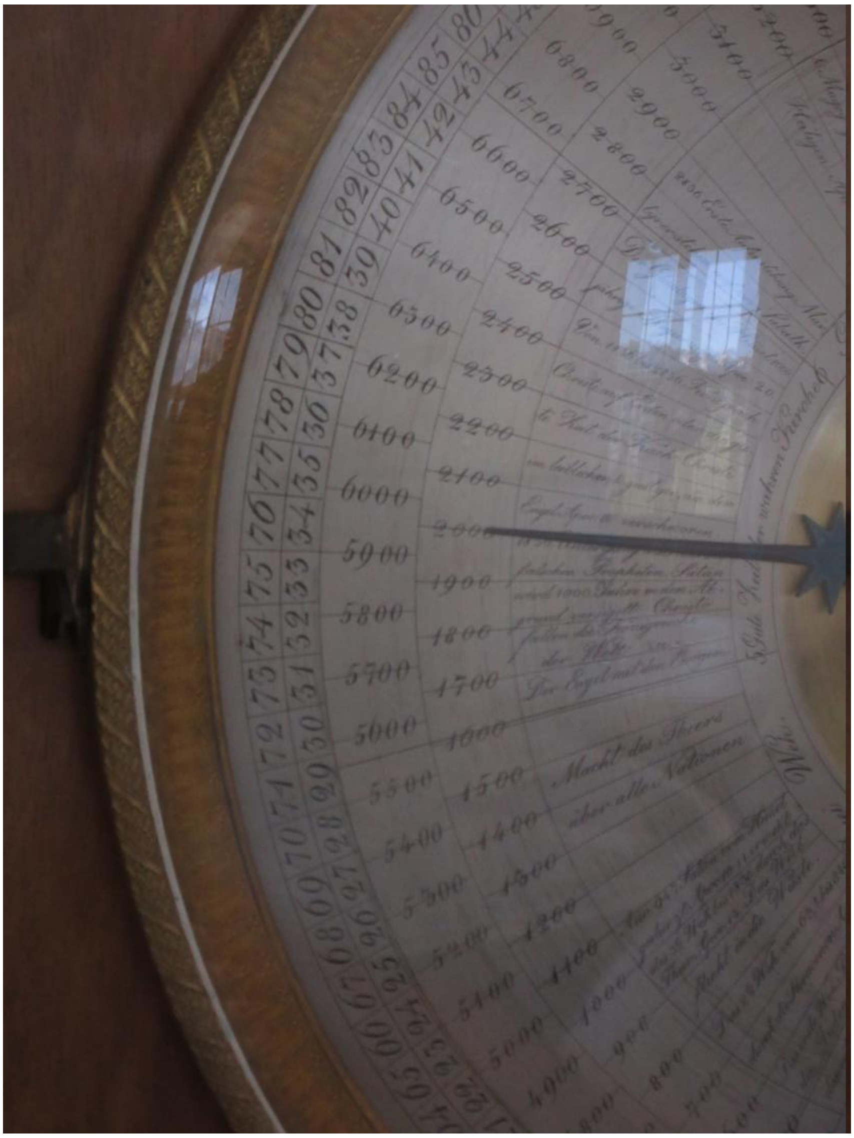

Figure 7.27: Hahn's *Weltmaschine*: detail of the lower central dial giving the years. (photograph by the author, 2025)

Figure 7.28: Hahn's *Weltmaschine*: detail of the lower central dial giving the years. We can for instance see the birth of Christ (*Jesus Christus geboren*). (photograph by the author, 2025)

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