Chapter 4 **(Oechslin: 6.1)**

Frater David's first clock in Vienna (1769)

This chapter is a work in progress and is not yet finalized. See the details in the introduction. It can be read independently from the other chapters, but for the notations, the general introduction should be read first. Newer versions will be put online from time to time.

4.1 Introduction

The clock described here was constructed in 1769 by David Ruetschmann, who went by the name Frater David a Sancto Cajetano (1726-1796). I am giving here a summary of his life, of which we don't know very much.[1] He was born in Lembach, in the Black Forest (Germany). When he was 12 years old, he became apprentice to a cabinetmaker. When he was 20 years old, he travelled as a journeyman to Vienna. He worked there as a cabinetmaker for 8 years. In 1754 he made the religious vows in the order of the Augustinians in the Mariabrunn monastery near Vienna. In 1760, he was moved to Vienna, so that he could have better opportunities for his scientific development. There he studied mathematics and astronomy. He became close to the mathematics professors Joseph Walcher and Wilhelm Bauer and this started his inventive activity.[2]

[1]This account is mostly based on Kunitsch's account [37, p. 71-75] and lost writings by Joseph Rendler and told by Engelmann [21]. See also Czermak [10, 11], Bassermann-Jordan [7], Kaftan [31], Bertele [8, 9], Lloyd [38], Killian [35, 36], Maurice [47, v.1, p. 275, 278-279], Abeler [1], and the collective "*Himmlisches Räderwerk*" [46]. There are many other mentions and short biographies of Frater David, and I am not making any attempt at exhaustivity. See the references at the end of this chapter. In his *Cornelius Nepos der Uhrmacher* [20], Dietzschold compared Frater David to Antide Janvier in France, but this comparison seems excessive. An account of Frater David's encounter with the Emperor Joseph II is given in [4, 5] and by Killian [35, p. 27-33].

[2]Kunitsch writes that Frater David attended Walcher's Sunday lectures on mechanics for craftsmen and artists and consequently developed new mathematical theories [37, p. 74-75]. Kunitsch, however, does not mention the construction of any clock.

The present clock is Frater David's first great clock and he built several more, including the one for the Prince of Schwarzenberg (Oechslin 6.2).[3] Around 1770, Frater David also restored the astronomical clock made by Christoph Schenner (or Schöner?) or Carl Graff from Augsburg in 1702. This clock was almost 6 meters tall and was presumably destroyed in 1848.[4] In 1772, he has also repaired the clock of St. Stephen's Cathedral in Vienna [30].

In the 1780s, Frater David restored Neßtfell's planetarium in Vienna.[5] This may have brought him to develop his theory of epicyclic gears.[6]

Frater David also had a nephew Joseph Ruetschmann (1755-1801) who was clockmaker in Vienna.[7] He constructed the clock designed by his uncle for the Palais Schwarzenberg around 1793 (Oechslin 6.2), as well as a clock with a small orrery on top of it.[8]

The clock described in this chapter was located from 1769 to 1835 in the Vienna Augustine monastery.[9] Then, it was sold to someone in Hungary, the city of Vienna having failed to acquire it. It was bought back in 1860 by the Viennese clockmaker Ignaz Lutz who restored it in 1862 and sold it in 1866 to abbot Augustin Steininger in the Zwettl abbey for 1200 gulden. This abbot was also a mechanician and built several clocks. Lutz died around 1880 and Frater David's clock was again restored in 1886 and 1901 by Mathias Römer,

[3] Bertele ([8, p. 24-25], [9, p. 110-115]) lists three clocks, those described in this book, and a clock made in 1793 by Ignaz Berlinger after Frater David's design, which only slightly differs from the Schwarzenberg clock. Kaltenböck mentions four clocks, those mentioned by Bertele, and a clock made in 1777 [33, p. 38-41]. In the 1996 exhibition catalogue, only three clocks are shown, not the one from 1777 [46].

[4] See the accounts of Fidler [22, p. 204-208], Scheiger [56, p. 60-65], Czermak [11] and especially Maurice [47, v.1, p. 188-192] and Killian [35] who show engravings of the clock. An engraving of the clock is also shown by Mattl-Wurm [46, p. 36].

[5] See [22, p. 4] and [52, p. 227]. Czermak also mentions the restoration of Neßtfell's planetarium, but around 1773 [11], and he does not seem to know that this clock is not lost.

[6] See Frater David's books [12, 13, 14] and the series of articles published shortly before his death [15, 16, 17, 18, 19].

[7] Some accounts have listed Joseph Ruetschmann as Frater David's brother [8, p. 24], but this is incorrect.

[8] See [29], [33, p. 43] and [25, p. 127-128]. This orrery shows the planets up to Uranus and was made very compact. It uses a number of interior gearings [29]. An anonymous notice in 1798 mentions another orrery-clock by Joseph Rutschmann, made in 1798, but which seems different from the one kept in Francfort [2]. It is possibly the same orrery-clock as the one mentioned in the Salzburger Zeitung in 1884 [6]. The author of the note moreover assumed that the Salzburg orrery was made by David Ruetschmann. When Hügin described the Francfort orrery clock, he didn't mention Salzburg [29]. Another tall-case clock by Joseph Ruetschmann is illustrated by Lunardi [44, p. 68]. See also [45, p. 434-435].

[9] There are many mentions, reproductions and sometimes detailed descriptions of this clock. It was first described by Joseph Rendler in 1771 [53]. Other descriptions are those of Schulte [57, p. 104-109], Frischholz [24, p. 198-200] and Kames [34, p. 415]. The most comprehensive description is that of the "*Himmlisches Räderwerk*" volume [46]. Detailed descriptions and analysis of the gears were also published by Lunardi [42, 43, 44, 45] and White [60, 61, 62] who focused on Frater David's new techniques for gear trains, especially for the Schwarzenberg clock. See also the 1989 exhibition catalogue [59, p. 55-56] and the c1989 catalogue from the *Uhrenmuseum* [55, p. 57-58].

Lutz's successor. During that time, the clock was located in the library of the Zwettl abbot. The clock was finally bought back from this abbey in 1928 and is now located in the Vienna clock museum.[10]

The clock is housed in a case 265 cm tall, 77 cm wide and 49 cm deep. It has dials on the front and rear sides. The front side (figure 4.3) has a number of dials and carries the date and signature of Fr. David a. S. Cajetano.

Although the clock looks very complicated, this is mainly an illusion due to the great number of dials. The clock is in fact rather simple, as we will see. However, making this simplicity visible has been difficult, because of Oechslin's drawing which splits the gears in three parts, with many overlaps. The difficulty is the intrication of the gear trains, not the trains themselves.

I will distinguish the front and the rear of the clock.

The front side of the clock is made of two parts. The lower part has a central dial showing the time, and it is surrounded by twelve smaller dials of the same size and showing various astronomical or calendrical indications.

These dials are in turn surrounded by a large ring which is the annual calendar. A number of hands rotate around the center of this part and they will be described later.

The upper part of the front side shows the motion of the Sun, the Moon and the lunar nodes, pictured as a dragon. It also shows the italic hours counted from last sunset.

The rear side of the clock gives the day of the month as well as the true solar time.

4.2 The going work

The clock is driven by a 26 kg lead weight and regulated by a Graham escapement. The pendulum is a seconds pendulum, that is it makes a half oscillation in one second. Once rewound, the clock can work for about 32 days.

The arbor 6 is the arbor driving the dials on the rear side and it makes one turn couterclockwise in one hour as seen from the back:

$$\mathrm{V}_6^0 = 24 \tag{4.1}$$

From this, we derive the velocity of arbor 5, but this time measured from the front side. (This may seem a bit annoying, but I do not want to measure the velocities of an arbor from two different vantage points, and one motion goes to the front, whereas the other one goes to the rear side.) We have

$$\mathrm{V}_5^0 = \mathrm{V}_6^0 \times \frac{12}{96} = 24 \times \frac{1}{8} = 3 \tag{4.2}$$

[10]Details of the history of the clock are given by Rössler [54], Czermak [11], Loeske [39], Dietzschold [20], Kaftan [31], and Oechslin [52, p. 233]. However the older accounts (for instance Czermak's) are partly inaccurate. Loeske also mentions the "*Oesterreichisch-Ungarische Uhrmacher-Zeiting*" which I haven't seen.

Figure 4.1: Frater David's clock. (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (https://sammlung.wienmuseum.at/objekt/382414))

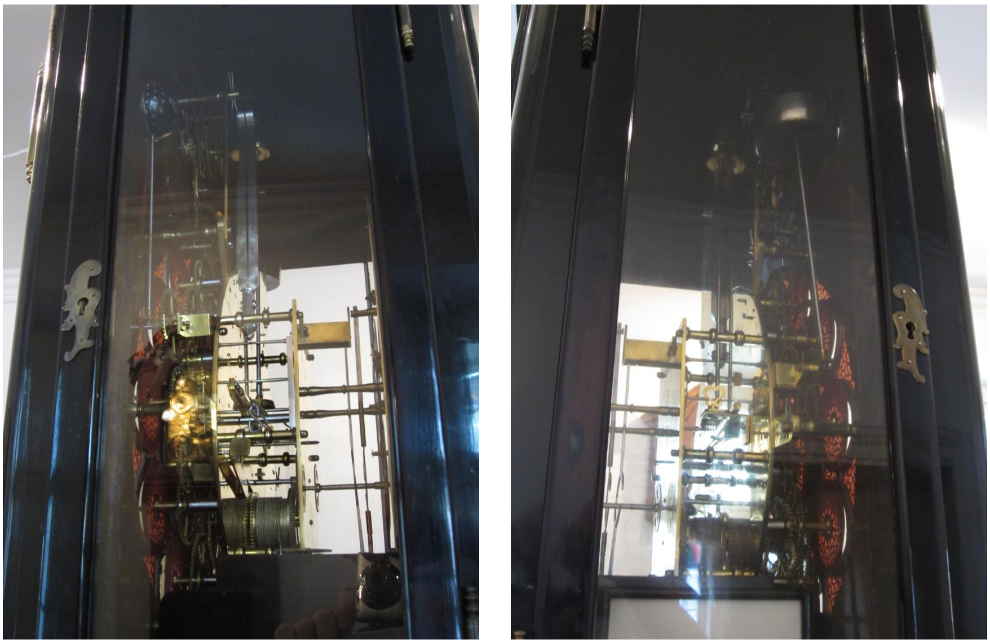

Figure 4.2: The mechanism of Frater David's clock seen from the sides. (photograph by the author)

The drum is on arbor 4 and we also measure it from the front:

$$V_4^0 = V_5^0 \times \left(-\frac{12}{96}\right) = 3 \times \left(-\frac{1}{8}\right) = -\frac{3}{8} \tag{4.3}$$

$$P_4^0 = -\frac{8}{3} \text{ days} = 64 \text{ hours} \tag{4.4}$$

Arbor 1 is the winding and it is located below the bottom hour dial, just outside of the calendar squares. If the arbor 1 were to turn with the drum, we would have

$$V_1^0 = V_4^0 \times \left(-\frac{50}{35}\right) \times \left(-\frac{35}{35}\right) \times \left(-\frac{35}{35}\right) = V_4^0 \times \left(-\frac{10}{7}\right) \tag{4.5}$$

$$= \left(-\frac{3}{8}\right) \times \left(-\frac{10}{7}\right) = \frac{15}{28} \tag{4.6}$$

$$P_1^0 = \frac{28}{15} \text{ days } = 44 \text{ h } 48 \text{ m} \tag{4.7}$$

So, one turn of the crank makes the clock work for almost 45 hours.

Finally, the escape wheel is on arbor 10 and its velocity (measured from the rear) is:

$$V_{10}^0 = V_6^0 \times \left(-\frac{80}{10}\right) \times \left(-\frac{75}{10}\right) = V_6^0 \times 60 = 24 \times 60 = 1440 \tag{4.8}$$

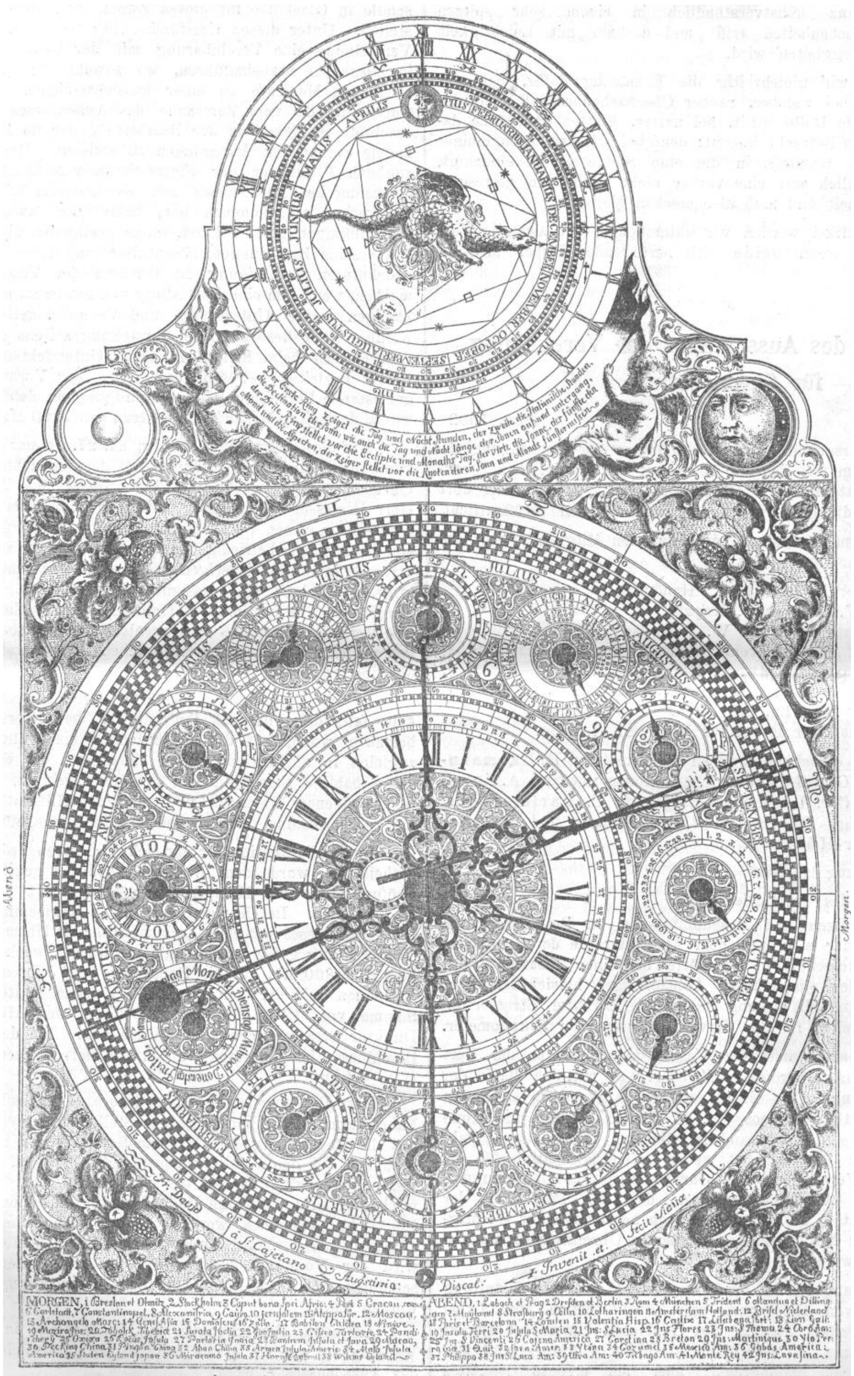

Figure 4.3: The front side of Frater David's clock, in Loeske's reproduction [39] of the engraving published by Rendler in 1771 [53].

The escape wheel therefore makes a turn counterclockwise (from the rear) in one minute. It carries 30 teeth and the pendulum therefore must make a half-oscillation in one second. Its length is about 1 meter.

Next, we describe the rear side, which is the simplest part.

4.3 The rear side

The rear side of the clock shows a dial for the time, topped by a semi-circulair dial for the day of the month.

The input for the rear side is arbor 6 which makes one turn counterclockwise (as seen from the rear) in one hour. All the motions on this side are measured from this side and this will be implicit from now on.

Figure 4.4: The rear side of Frater David's clock (from the museum page). (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (`https://sammlung.wienmuseum.at/objekt/382414`))

From arbor 6, we obtain the motion of the central arbor 9 of the time dial:

$$\mathrm{V}_9^0 = \mathrm{V}_6^0 \times \left(-\frac{60}{60}\right) = -24 \tag{4.9}$$

This arbor makes one turn clockwise in an hour and carries the hand of the minutes of mean time.

4.3.1 The day of the month

The day of the month is shown on a semi-circular and goes from 1 to 31, then springs again back to 1.

The motion of arbor 9 is first used to obtain the motion of arbor 21. We have:

$$\mathrm{V}_{21}^{0} = \mathrm{V}_{9}^{0} \times \left(-\frac{16}{48}\right) \times \left(-\frac{20}{80}\right) \times \left(-\frac{36}{72}\right) = \mathrm{V}_{9}^{0} \times \left(-\frac{1}{24}\right) = 1 \tag{4.10}$$

This arbor thus makes one turn counterclockise in a day. It is used to update the hand for the days, but I will not describe this part in detail. The lengths of the months, as well as the ordinary leap years, are taken into account, so that the hand sometimes jumps from 28 to 1, from 29 to 1, or from 30 to 1.

4.3.2 The time dial

The central arbor 9 actually moves an entire frame containing a number of gears. The rotation of this frame causes the motion of the internal gears, as a consequence of the fixed tube 10. The last arbor 14 of the internal train carries a 10-teeth wheel meshing with a 180-teeth wheel on tube 15 which does itself carry the equation cam. We only compute the motion of tube 15 in the rotating frame 9:

$$\mathrm{V}_{10}^{9} = -\mathrm{V}_{9}^{10} = -\mathrm{V}_{9}^{0} = 24 \tag{4.11}$$

$$\mathrm{V}_{15}^{9} = \mathrm{V}_{10}^{9} \times \left(-\frac{24}{40}\right) \times \left(-\frac{9}{48}\right) \times \left(-\frac{8}{48}\right) \times \left(-\frac{8}{73}\right) \times \left(-\frac{10}{180}\right) \tag{4.12}$$

$$= \mathrm{V}_{10}^{9} \times \left(-\frac{1}{8760}\right) = 24 \times \left(-\frac{1}{8760}\right) = -\frac{1}{365} \tag{4.13}$$

It would not be meaningful to compute the motion of tube 15 in the absolute frame, because Frater David has the feeler and rack of the equation mechanism fixed on the moving frame 9. It is therefore important that the cam makes a turn in a year with respect to the moving frame, which it does. Actually, Frater David could have done better, and it is surprising that he contented himself with such an approximation (365 days) of the tropical year.

The rack linked to the feeler meshes with a 24-teeth wheel on tube 16 which carries the hand for the minutes of true time. In addition the hours of true time are obtained with a train fixed on the fixed frame 10.

4.4 The front side

The front side of the clock is made of two parts. The lower part has a central dial showing the time, and it is surrounded by twelve smaller dials of the same size and showing various astronomical or calendrical indications. Starting at the bottom and going clockwise, we have 1) the time, with hour and minute hands, 2) the tropical motion of Mercury, 3) the day of the week, 4) the draconic month, 5) the tropical motion of Jupiter, 6) the epacts, the golden number and the indiction, 7) the tropical motion of Saturn as well as the prograde motion of the Earth's perihelion, 8) the dominical letter and the solar cycle, 9) the tropical motion of Mars, 10) the age of the Moon, 11) the anomalistic month (angle from perigee), and 12) the tropical motion of Venus.

These dials have been one source of inspiration for Seige's 1791 clock (Oechslin 10.1), but Seige arranged the dials on two opposite faces, and also added the motion of Uranus.

These dials are in turn surrounded by a large ring which is the annual calendar [52, p. 44]. A number of hands rotate around the center of this part and they will be described later.

The upper part of the front side shows the motion of the Sun, the Moon and the lunar nodes, pictured as a dragon. It also shows the italic hours counted from last sunset.

There are also two openings on the sides of this dial and showing the size of a lunar eclipse and the lunar phase.

The figures of the year are shown through for small apertures.

Figure 4.5: The front side of Frater David's clock (from the museum page). (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (https://sammlung.wienmuseum.at/objekt/382414))

Figure 4.6: The gears behind the dials of the front side of Frater David's clock (from the museum page). (David Cajetano (Uhrmacher), Astronomische Bodenstanduhr von David a Santo Cajetano, Wien, 1762-1769, 1762-1769, Wien Museum Inv.-Nr. U 435, CC BY 4.0, Foto: Paul Kolp, Wien Museum (https://sammlung.wienmuseum.at/objekt/382414))

I will first analyze the yearly motions (motion of the Sun and computus), then the dials of the time and the day of the week, then those of the planets, then the motion of the line of the apsides (on the Saturn dial), then those of the Moon, including the display of the lunar phases and the eclipse window. Afterwards, I will consider the motion of the other central hands on the main dial, then the upper part of the clock. All of these motions take their source in the motion of arbor 5 from the going work, whose velocity (measured from the front) was computed above and is

$$V_5^0 = 3 \tag{4.14}$$
$$P_5^0 = 8 \text{ hours} \tag{4.15}$$

All of the other velocities on the front side will implicitely be measured from the front.

4.4.1 The yearly motions

4.4.1.1 The annual motion of the Sun

The Sun is pictured by a hand on the central dial. It is carried by tube 34. The motion of this tube is obtained through a train in which we also explicit the motions of tube 26, 30 and 33 which will be useful later. The last wheel has 269 teeth, but these teeth have been unevenly cut [52, p. 137]. We have:

$$V_{26}^0 = V_5^0 \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{72}\right) = V_5^0 \times \frac{1}{3} = 1 \tag{4.16}$$
$$P_{26}^0 = 1 \text{ day} \tag{4.17}$$
$$V_{30}^0 = V_{26}^0 \times \left(-\frac{24}{78}\right) = -\frac{4}{13} \tag{4.18}$$
$$V_{33}^0 = V_{30}^0 \times \left(-\frac{24}{94}\right) \times \left(-\frac{15}{15}\right) \times \left(-\frac{15}{64}\right) \tag{4.19}$$
$$= V_{30}^0 \times \left(-\frac{45}{752}\right) = \left(-\frac{4}{13}\right) \times \left(-\frac{45}{752}\right) = \frac{45}{2444} \tag{4.20}$$

and finally

$$V_{34}^0 = V_{33}^0 \times \left(-\frac{40}{269}\right) = \frac{45}{2444} \times \left(-\frac{40}{269}\right) = -\frac{450}{164359} \tag{4.21}$$
$$P_{34}^0 = -\frac{164359}{450} = -365.2422\ldots \text{ days} - 365 \text{ d } 5 \text{ h } 48 \text{ m } 48 \text{ s} \tag{4.22}$$

The same value is given by Oechslin. This is an approximation of the tropical year.[11] The solar hand moves clockwise along the signs of the zodiac, but also along the calendar rings [52, p. 44].

[11] Incidentally, this value for the tropical year is exactly the one used by Jean-Baptiste Schwilgué (1776-1856) in the Strasbourg astronomical clock [58].

However, because the 269-teeth wheel has unevenly cut teeth (Oechslin counted 132 teeth on one half and 137 on the other), its motion is also uneven, and the solar hand has an uneven motion. This is meant to account for the equation of center, and it is the only mechanism in this book with such features.

The 269-teeth wheel also carries a finger or pin which is used to update the indication of the year using a carry mechanism.

4.4.1.2 The epacts, the golden number and the indiction

This dial shows the golden number (from 1 to 19), the indiction (from 1 to 15) and two series of epacts. The outer series is that valid for the 18th and 19th centuries, the inner series is valid for the 20th, 21st and 22nd centuries. The same dial is found in Seige's 1791 clock (Oechslin 10.1) who obviously copied it.

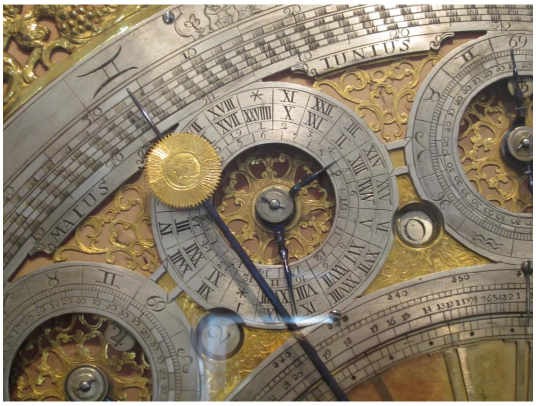

Figure 4.7: The dial for the epacts, the golden number and the indiction on Frater David's clock. (photograph by the author)

The motions of the hands for the epacts, the golden number and the indiction are derived from the motion of tube 34 seen above. We first compute the motion of arbor 99, isolating arbor 66 which is useful later:

$$V^0_{66} = V^0_{34} \times \left(-\frac{18}{72}\right) \times \left(-\frac{24}{70}\right) = V^0_{34} \times \frac{3}{35} \tag{4.23}$$

$$V^0_{99} = V^0_{66} \times \left(-\frac{70}{90}\right) = V^0_{34} \times \left(-\frac{1}{15}\right) \tag{4.24}$$

and then

$$V_{100}^0 = V_{99}^0 \times \left(-\frac{90}{90}\right) = -\frac{450}{164359} \times \left(-\frac{1}{15}\right) \tag{4.25}$$

$$P_{100}^0 = -\frac{164359}{450} \times 15 = -15 \text{ tropical years} \tag{4.26}$$

$$V_{101}^0 = V_{99}^0 \times \left(-\frac{45}{57}\right) = V_{34}^0 \times \left(-\frac{1}{15}\right) \times \left(-\frac{45}{57}\right) = V_{34}^0 \times \frac{1}{19} \tag{4.27}$$

$$= -\frac{450}{164359} \times \frac{1}{19} \tag{4.28}$$

$$P_{101}^0 = -\frac{164359}{450} \times 19 = -19 \text{ tropical years} \tag{4.29}$$

So, the central arbor 100 of this dial makes a turn clockwise in 15 tropical years and it carries the indiction hand. The tube 101 makes a turn clockwise in 19 years and carries the hand indicating the epacts and the golden number. (Note that Oechslin's drawing incorrectly swaps the indiction and the epacts.)

It is of course surprising that Frater David doesn't switch the epacts and the indiction at the New Year, but has them gradually change throughout the year. This is far from adequate.

Although this motion is derived from that of tube 34 which is uneven, this is of no consequences, because the uneveness of tube 34 has a period of one year. So, after one year, the tube 34 will always have made one turn.

4.4.1.3 The dominical letter and the solar cycle

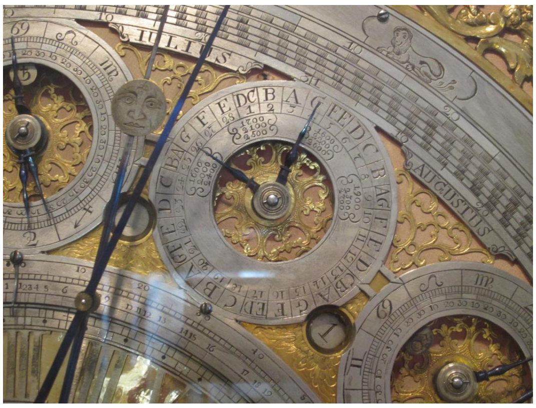

Figure 4.8: The dial for the dominical letter and the solar cycle on Frater David's clock. (photograph by the author)

The hand of the dominical letter and solar cycle is on arbor 68 and its motion is derived from that of arbor 66 (the motion of arbor 67 is used below for the motion of the apsides on the dial of Saturn):

$$V^0_{67} = V^0_{66} \times \left(-\frac{30}{72}\right) = V^0_{66} \times \left(-\frac{5}{12}\right) = V^0_{34} \times \frac{3}{35} \times \left(-\frac{5}{12}\right) \quad (4.30)$$

$$= V^0_{34} \times \left(-\frac{1}{28}\right) \quad (4.31)$$

$$V^0_{68} = V^0_{67} \times \left(-\frac{72}{72}\right) = V^0_{34} \times \frac{1}{28} = -\frac{450}{164359} \times \frac{1}{28} \quad (4.32)$$

$$P^0_{68} = -\frac{164359}{450} \times 28 = -28 \text{ tropical years} \quad (4.33)$$

This hand makes a turn clockwise in 28 years. It is also moving gradually throughout the year.

The correspondence between the dominical letter and the solar cycle is only valid in the 18th century.

4.4.2 The time of the day and the day of the week

4.4.2.1 The time dial

Figure 4.9: The dial for the time on Frater David's clock. (photograph by the author)

This dial gives the minutes and hours of mean time. The minute hand is carried by arbor 114 and the hour hand by tube 116. Both are derived from the motion of arbor 5, but the motion of the hours is not derived as usual from the motion of the minutes. We first derive the motion of arbor 112:

$$V^0_{112} = V^0_{5} \times \left(-\frac{96}{24}\right) = 3 \times (-4) = -12 \quad (4.34)$$

$$P^0_{112} = -2 \text{ hours} \quad (4.35)$$

Then we have

$$V^0_{114} = V^0_{112} \times \left(-\frac{60}{30}\right) \times \left(-\frac{60}{60}\right) = V^0_{112} \times 2 = -24 \qquad (4.36)$$

$$P^0_{114} = -1 \text{ hour} \qquad (4.37)$$

$$V^0_{116} = V^0_{112} \times \left(-\frac{60}{60}\right) \times \left(-\frac{12}{72}\right) = V^0_{112} \times \frac{1}{6} = -2 \qquad (4.38)$$

$$P^0_{116} = -12 \text{ hours} \qquad (4.39)$$

Hence, arbor 114 makes a turn clockwise in one hour, and tube 116 makes a turn clockwise in twelve hours.

4.4.2.2 The day of the week

Figure 4.10: The dial for the day of the week on Frater David's clock. (photograph by the author)

The day of the week is given by the hand carried by arbor 92. The motion of this arbor is derived from that of arbor 30 seen above. We have

$$V^0_{92} = V^0_{30} \times \left(-\frac{78}{56}\right) \times \left(-\frac{24}{72}\right) = V^0_{30} \times \frac{13}{28} = -\frac{4}{13} \times \frac{13}{28} = -\frac{1}{7} \qquad (4.40)$$

$$P^0_{92} = -7 \text{ days} \qquad (4.41)$$

Arbor 92 makes a turn clockwise in seven days.

Figure 4.11: The dial for the tropical motion of Mercury on Frater David's clock. (photograph by the author)

4.4.3 The motion of the planets

4.4.3.1 The tropical motion of Mercury

The motion of Mercury is given by the arbor 94 and is derived from arbor 92. We have

$$\mathrm{V}_{94}^{0} = \mathrm{V}_{92}^{0} \times \left(-\frac{16}{58}\right) \times \left(-\frac{15}{52}\right) = \mathrm{V}_{92}^{0} \times \frac{30}{377} = -\frac{1}{7} \times \frac{30}{377} \tag{4.42}$$

$$= -\frac{30}{2639} \tag{4.43}$$

$$\mathrm{P}_{94}^{0} = -\frac{2639}{30} = -87.9666\ldots \text{ days} = -87 \text{ d } 23 \text{ h } 12 \text{ m} \tag{4.44}$$

The same value is given by Oechslin. This is an approximation of Mercury's tropical orbital period. The velocity is negative, because the signs of the zodiac are marked clockwise.

Incidentally, the 2010 museum guide states that Mercury rotates with its sidereal period, and that the period is 87 days 23 hours 14 minutes and 4 seconds [23]. It isn't clear where this value was taken, but Oechslin finds the same value as I do. I see no reason to assume that the motions of the planets on the clock are not the tropical ones.

Figure 4.12: The dial for the tropical motion of Venus on Frater David's clock. (photograph by the author)

4.4.3.2 The tropical motion of Venus

The motion of Venus is given by the arbor 86 and is derived from tube 26. We have:

$$V^0_{74} = V^0_{26} \times \left(-\frac{24}{75}\right) = -\frac{8}{25} \tag{4.45}$$

$$V^0_{86} = \left(-\frac{8}{25}\right) \times \left(-\frac{10}{10}\right) \times \left(-\frac{10}{53}\right) \times \left(-\frac{18}{66}\right) \times \left(-\frac{20}{74}\right) \tag{4.46}$$

$$= -\frac{96}{21571} \tag{4.47}$$

$$P^0_{86} = -\frac{21571}{96} = -224.6979\ldots \text{ days} = -224 \text{ d } 16 \text{ h } 45 \text{ m} \tag{4.48}$$

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Venus. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin, but, again, the 2010 museum guide gives a different period of 224 days 16 hours 48 minutes, without giving any source [23].

4.4.3.3 The tropical motion of Mars

The hand of Mars is located on arbor 89. This motion is derived from that of arbor 80. We first obtain the motion of arbor 75, which is derived from that of arbor 74 seen above. We have

$$V^0_{75} = V^0_{74} \times \left(-\frac{75}{48}\right) = \left(-\frac{8}{25}\right) \times \left(-\frac{75}{48}\right) = \frac{1}{2} \tag{4.49}$$

$$V^0_{80} = V^0_{75} \times \left(-\frac{12}{31}\right) \times \left(-\frac{16}{31}\right) = V^0_{75} \times \frac{192}{961} = \frac{1}{2} \times \frac{192}{961} = \frac{96}{961} \tag{4.50}$$

Figure 4.13: The dial for the tropical motion of Mars on Frater David's clock. (photograph by the author)

and finally

$$V_{89}^0 = V_{80}^0 \times \left(-\frac{20}{61}\right) \times \left(-\frac{9}{45}\right) \times \left(-\frac{12}{54}\right) = V_{80}^0 \times \left(-\frac{8}{549}\right) \quad (4.51)$$

$$= \frac{96}{961} \times \left(-\frac{8}{549}\right) = -\frac{256}{175863} \quad (4.52)$$

$$P_{89}^0 = -\frac{175863}{256} = -686.9648\ldots \text{ days} = -686 \text{ d } 23 \text{ h } 9 \text{ m } 22.5 \text{ s} \quad (4.53)$$

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Mars. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin, but, again, the 2010 museum guide gives a different period of 1y 321d 23h 31m 56s, without giving any source [23]. Moreover, if we subtract the value found above for the tropical year to Mars' orbital period, we find 321 d 17 h 20 m 34.5 s, and not the value stated in the guide.

4.4.3.4 The tropical motion of Jupiter

The hand of Jupiter is located on arbor 60. Its motion is derived from that of arbor 33 seen earlier. We have

$$V_{60}^0 = V_{33}^0 \times \left(-\frac{18}{48}\right) \times \left(-\frac{16}{82}\right) \times \left(-\frac{12}{70}\right) = V_{33}^0 \times \left(-\frac{18}{1435}\right) \quad (4.54)$$

$$= \frac{45}{2444} \times \left(-\frac{18}{1435}\right) = -\frac{81}{350714} \quad (4.55)$$

$$P_{60}^0 = -\frac{350714}{81} = -4329.8024\ldots \text{ days} \quad (4.56)$$

$$= -4329 \text{ d } 19 \text{ h } 15 \text{ m } 33.3333\ldots \text{ s} \quad (4.57)$$

Figure 4.14: The dial for the tropical motion of Jupiter on Frater David's clock. (photograph by the author)

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Jupiter. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin, although he did not correctly convert it to hours, minutes and seconds. If we assume the tropical year of 365d 5h 48m 48s, this is then 11 years and 312 days 3 hours 18 minutes and 45.3333... s, but the 2010 museum guide gives a different period of 11 years 314d 22h, without giving any source [23].

4.4.3.5 The tropical motion of Saturn

Figure 4.15: The dial for the tropical motion of Saturn as well as the prograde motion of the Earth's perihelion on Frater David's clock. (photograph by the author)

The hand of Saturn is carried by the arbor 64. Its motion is derived from the motion of Jupiter. We have

$$V^0_{64} = V^0_{60} \times \left(-\frac{54}{54}\right) \times \left(-\frac{54}{54}\right) \times \left(-\frac{54}{54}\right) \times \left(-\frac{29}{72}\right) = V^0_{60} \times \frac{29}{72} \quad (4.58)$$

$$= -\frac{81}{350714} \times \frac{29}{72} = -\frac{261}{2805712} \quad (4.59)$$

$$P^0_{64} = -\frac{2805712}{261} = -10749.8544\ldots \text{ days} \quad (4.60)$$

$$= -10749 \text{ d } 20 \text{ h } 30 \text{ m } 20.6896\ldots \text{ s} \quad (4.61)$$

The same value is given by Oechslin. This is an approximation of the tropical orbital period of Saturn. The velocity is negative, because the signs of the zodiac are marked clockwise. The same value was found by Oechslin. If we assume the tropical year of 365d 5h 48m 48s, this is then 29 years and 157 days 19 hours 55 minutes and 8.6896... s, but the 2010 museum guide gives a different period of 29 years 167d 22h, without giving any source [23].

4.4.4 The line of apsides

On the dial for Saturn, there is another hand carried by tube 73. This hand shows the motion of the Earth's perihelion which moves around the zodiac in about 21000 years. The motion is derived form arbor 67 which is also used for the solar cycle.

$$V^0_{73} = V^0_{67} \times \left(-\frac{14}{14}\right) \times \left(-\frac{14}{52}\right) \times \left(-\frac{12}{67}\right) \times \left(-\frac{12}{72}\right) \times \left(-\frac{13}{78}\right) \quad (4.62)$$

$$= V^0_{67} \times \left(-\frac{7}{5226}\right) \quad (4.63)$$

$$= V^0_{34} \times \left(-\frac{1}{28}\right) \times \left(-\frac{7}{5226}\right) = V^0_{34} \times \frac{1}{20904} \quad (4.64)$$

$$= -\frac{450}{164359} \times \frac{1}{20904} \quad (4.65)$$

$$P^0_{73} = -\frac{164359}{450} \times 20904 = -20904 \text{ years} \quad (4.66)$$

This is an approximation of the tropical period of precession of the Earth's apsides. The value is negative, because the signs are given clockwise. However, Oechslin's calculations use the ratio 13/76 instead of 13/78 and arrive at a period of 20368 tropical years. This is certainly a typo.

The same value of 20904 years is given in the 2010 museum guide, albeit without any source [23], and claiming incorrectly that this dial gives the period of precession of the equinoxes.

The period of about 21000 years is in fact due to the difference between the anomalistic and the tropical year. The anomalistic year is about 365 days 6 hours and 14 minutes. This was already known at Frater David's time, and it does for instance appear in Ferguson's *Astronomy explained* published in

1764. The difference between the anomalistic and the tropical years is about 25 minutes, and it will take about 21000 years to accumulate to a full revolution.

4.4.5 The motion of the Moon

4.4.5.1 The draconic month

Figure 4.16: The dial for the draconic month on Frater David's clock. (photograph by the author)

The hand on arbor 96 shows the number of days elapsed since the mean Moon was at its ascending node, which may be called the draconic age, or dial of the draconic month. This motion is derived from that of arbor 92. We have

$$V^0_{96} = V^0_{92} \times \left(-\frac{25}{41}\right) \times \left(-\frac{27}{64}\right) = V^0_{92} \times \frac{675}{2624} \quad (4.67)$$

$$= -\frac{1}{7} \times \frac{675}{2624} = -\frac{675}{18368} \quad (4.68)$$

$$P^0_{96} = -\frac{18368}{675} = -27.2118\ldots \text{ days} \quad (4.69)$$

$$= -27 \text{ d } 5 \text{ h } 5 \text{ m } 4 \text{ s} \quad (4.70)$$

This is an approximation of the draconic month. The same value was found by Oechslin. The 2010 museum guide doesn't give a value for this period [23].

4.4.5.2 The age of the Moon

The hand of the age of the Moon is located on arbor 90. This motion is derived from that of arbor 80, like the motion of Mars. We have:

$$V^0_{90} = V^0_{80} \times \left(-\frac{20}{59}\right) = \frac{96}{961} \times \left(-\frac{20}{59}\right) = -\frac{1920}{56699} \quad (4.71)$$

$$P^0_{90} = -\frac{56699}{1920} = -29.5307\ldots \text{ days} - 29 \text{ d } 12 \text{ h } 44 \text{ m } 15 \text{ s} \quad (4.72)$$

Figure 4.17: The dial for the age of the Moon on Frater David's clock. (photograph by the author)

This is an approximation of the synodic month. The same value was found by Oechslin, but, again, the 2010 museum guide gives the truncated period of 29d 12h 44m without giving any source [23].

4.4.5.3 The mean anomaly of the Moon

Figure 4.18: The dial for the anomalistic month on Frater David's clock. (photograph by the author)

The hand on arbor 78 shows the motion of the mean Moon with respect to its perigee. This motion is derived from that of arbor 75:

$$V^0_{78} = V^0_{75} \times \left(-\frac{12}{37}\right) \times \left(-\frac{47}{51}\right) \times \left(-\frac{17}{70}\right) = V^0_{75} \times \left(-\frac{94}{1295}\right) \quad (4.73)$$

$$= \frac{1}{2} \times \left(-\frac{94}{1295}\right) = -\frac{47}{1295} \quad (4.74)$$

$$P^0_{78} = -\frac{1295}{47} = -27.5531\ldots \text{ days} \quad (4.75)$$

$$= -27 \text{ d } 13 \text{ h } 16 \text{ m } 35.7446 \text{ s} \quad (4.76)$$

This is an approximation of the anomalistic month. The same value was given by Oechslin. However, the 2010 museum guide gives a slightly different period of 27d 13h 16m 35s without giving any source [23].

4.4.5.4 The Moon phases

The phase of the Moon is shown on a lunar sphere on arbor 82. The velocity of this arbor is

$$V^0_{82} = V^0_{75} \times \left(-\frac{12}{31}\right) \times \left(-\frac{16}{31}\right) \times \left(-\frac{20}{59}\right) \times \frac{12}{12} \quad (4.77)$$

$$= V^0_{75} \times \left(-\frac{3840}{56699}\right) = \frac{1}{2} \times \left(-\frac{3840}{56699}\right) = -\frac{1920}{56699} \quad (4.78)$$

$$P^0_{82} = -\frac{56699}{1920} = -29.5307\ldots \text{ days} \quad (4.79)$$

The same value is given by Oechslin. This is an approximation of the synodic month, using the same ratio as the one used for the age of the Moon.

4.4.5.5 The eclipse "window"

The arbor 98 is used to show the eclipse "window," that is, when an eclipse can occur [52, p. 31]. The velocity of this arbor is computed as follows:

$$V^0_{98} = V^0_{96} \times \left(-\frac{64}{32}\right) \times \frac{16}{16} = \frac{675}{9184} \quad (4.80)$$

$$P^0_{98} = \frac{9184}{675} = 13.6059\ldots \text{ days} \quad (4.81)$$

The same value is given by Oechslin. This is the period on the display of the eclipses, and it is actually a "window" for possible eclipses. Every 13.6 days, the Moon goes through a node, and if the longitude of the Sun is close to that of the Moon, an eclipse may take place.

4.4.6 The main dial

The main dial shows the motion of the Sun (with a hand carrying a golden Sun), the Moon (with a hand carrying the Moon on one side, and a black disk

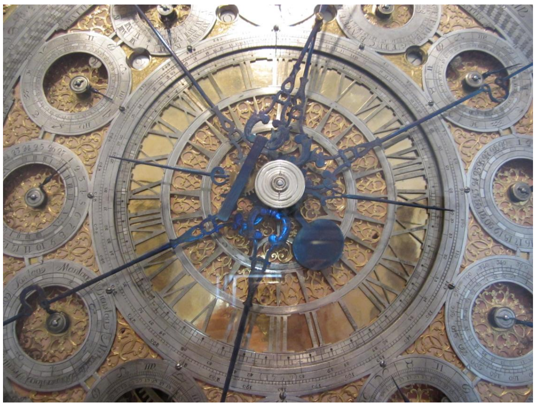

Figure 4.19: The center of the front dials of Frater David's clock. (photograph by the author)

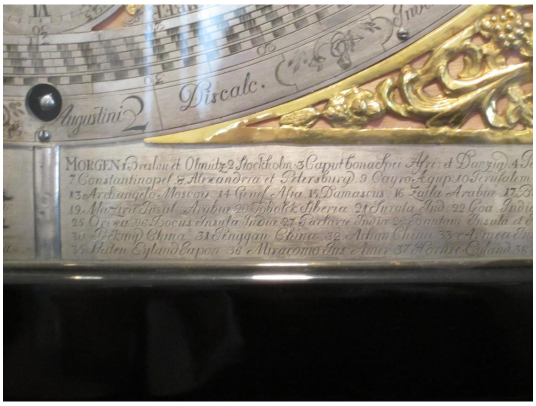

Figure 4.20: Excerpt of the list of places on Frater David's clock. This corresponds to the numbers written outside the 24-hours dial. When the Sun is over a given value, the table below tells where the Sun is culminating at this moment. (photograph by the author)

on the other), the lunar nodes (with a hand carrying the signs for ascending and descending nodes) and the lunar apsides (a smaller double-hand slightly extending beyond the hours). The position of the apsides was used to alter the position of the mean Moon and an additional hand is given for this purpose.

First, I will consider the display of time, which is actually absent from the main dial.

4.4.6.1 The 24-hour wheel

The main dial contains a 24-hour dial, and one would expect to be able to read the time, at least the hours. Indeed, the central tube 43 makes one turn in 24 hours. The motion of this tube is derived from arbor 5. We have

$$V_{43}^0 = V_5^0 \times \left(-\frac{24}{24}\right) \times \left(-\frac{24}{24}\right) \times \left(-\frac{12}{36}\right) \times \left(-\frac{90}{90}\right) \tag{4.82}$$

$$= V_5^0 \times \frac{1}{3} = 3 \times \frac{1}{3} = 1 \tag{4.83}$$

$$P_{43}^0 = 1 \text{ day} \tag{4.84}$$

However, the motion of this tube is counterclockwise, and the dial with the hours runs clockwise. No hand is carried by this tube. This is likely some unfinished part of the clock.

4.4.6.2 The motion of the Sun

The motion of the Sun is given by the hand on tube 34 and was computed earlier. We found that

$$P_{34}^0 = -\frac{164359}{450} = -365.2422\ldots \text{ days} \tag{4.85}$$

This is actually the mean motion of the Sun and Frater David did not take into account any correction for the elliptical orbit of the Earth.

4.4.6.3 The mean motion of the Moon

The mean motion of the Moon is given by the hand on tube 29 and this motion is derived from that of tube 26. We have

$$V_{29}^0 = V_{26}^0 \times \left(-\frac{20}{61}\right) \times \left(-\frac{19}{69}\right) \times \left(-\frac{30}{74}\right) = V_{26}^0 \times \left(-\frac{1900}{51911}\right) \tag{4.86}$$

$$= -\frac{1900}{51911} \tag{4.87}$$

$$P_{29}^0 = -\frac{51911}{1900} = -27.3215\ldots \text{ days} \tag{4.88}$$

The same value is given by Oechslin. This is an approximation of the tropical month.

This motion is later altered by the motion of the lunar apsides.

4.4.6.4 The lunar nodes

The hand of the lunar nodes, the dragon, is carried by tube 39. Its motion is derived from that of tube 34. We have

$$V^0_{39} = V^0_{34} \times \left(-\frac{18}{67}\right) \times \left(-\frac{16}{16}\right) \times \left(-\frac{16}{80}\right) = V^0_{34} \times \left(-\frac{18}{335}\right) \tag{4.89}$$

$$= \left(-\frac{450}{164359}\right) \times \left(-\frac{18}{335}\right) = \frac{1620}{11012053} \tag{4.90}$$

$$P^0_{39} = \frac{11012053}{1620} = 6797.5635\ldots \text{ days} \tag{4.91}$$

The same value is given by Oechslin, also in years. This is an approximation of the period of precession of the lunar nodes. The value is positive, because the lunar nodes are retrograding (which corresponds to a direct motion here, because the signs of the zodiac are given clockwise).

4.4.6.5 The lunar apsides

The hand of the lunar apsides is carried by tube 36. Its motion is also derived from that of tube 34. We have

$$V^0_{36} = V^0_{34} \times \left(-\frac{13}{45}\right) \times \left(-\frac{20}{51}\right) = V^0_{34} \times \frac{52}{459} = \left(-\frac{450}{164359}\right) \times \frac{52}{459} \tag{4.92}$$

$$= -\frac{200}{644793} \tag{4.93}$$

$$P^0_{36} = -\frac{644793}{200} = -3223.965 \text{ days} \approx -8.83 \text{ years} \tag{4.94}$$

The same value is given by Oechslin, also in years. This is an approximation of the period of precession of the lunar apsides.

It should be noted that the motion of the lunar apsides is thus slightly irregular, because the motion of tube 34 is uneven, as we have seen above.

4.4.6.6 The correction of the Moon

The actual hand of the Moon pivots first on the hand of the mean Moon, and then on a pin rotating around the center of the dial on arbor 118. Both are visible on photographs of the dial. It isn't clear if arbor 118 is still rotating, as Oechslin's drawing seems to imply that a fixed 31-teeth on the central arbor is missing. And this wheel would have meshed with a 25-teeth wheel on arbor 118. This arbor moves on the apsides hand and therefore it moves around the center in the period of precession of the lunar apsides. But the pin is actually slightly offset from arbor 118, so that an additional correction is introduced, and this correction depends on the ratio between the central wheel (31 teeth) and the satellite wheel (25 teeth).

With respect to tube 36 (the apsides hand), arbor 118 has the velocity:

$$V_{118}^{36} = V_{117}^{36} \times \left(-\frac{31}{25}\right) \tag{4.95}$$

where 117 is the central fixed arbor.

$$= -V_{36}^{0} \times \left(-\frac{31}{25}\right) = \frac{200}{644793} \times \left(-\frac{31}{25}\right) \tag{4.96}$$

$$= -\frac{248}{644793} \tag{4.97}$$

$$P_{118}^{36} = -\frac{644793}{248} = -2599.9717\ldots \text{ days} \tag{4.98}$$

$$V_{118}^{0} = V_{118}^{36} + V_{36}^{0} = -\frac{248}{644793} - \frac{200}{644793} = -\frac{448}{644793} \tag{4.99}$$

$$P_{118}^{0} = -\frac{644793}{448} = -1439.2700\ldots \text{ days} \approx= -3.94 \text{ years} \tag{4.100}$$

It isn't clear what Frater David had in mind here, but Oechslin obtains the same value.[12] Frater David may have tried to obtain the motion of the lunar evection, and a simple way to obtain such a motion would have been to put a 28-teeth wheel on tube 29, and having it mesh with a 32-teeth wheel on arbor 118 *under* the apsides hand. The relative motion of the wheel on tube 29 (the mean Moon) to the apsides hand is the velocity of the anomalistic month and the period of arbor 118 with respect to tube 36 would have been about 31.5 days. Then, adding this motion to the motion of the apsides would have resulted in a motion whose period is about 31.8 days, the period of the evection.

4.4.7 The upper dial

The upper dial is a relatively conventional astrolabe dial, showing the motions of the zodiac, the Sun, the Moon, and the dragon (line of nodes). Moreover, this dial also shows the italic hours, that is the number of hours since the last sunset, and which are derived from the annual rotation of a cam.

4.4.7.1 The italic hours

The italic hours are shown on a mobile 24-hour ring just inside the fixed 24-hour ring. Since the difference between the hours of mean time and the italic hours oscillates and is about 6 hours on average, this ring has a back-and-forth motion controlled by a cam. This cam makes a turn in one year and is located

[12] See [52, p. 208-209] on this problem.

Figure 4.21: The upper dial of the front side of Frater David's clock. (photograph by the author)

on arbor 104. The motion of this cam is derived from that of tube 34 which also makes a turn in a year:

$$V^0_{104} = V^0_{34} \times \left(-\frac{18}{72}\right) \times \left(-\frac{72}{18}\right) \times \frac{16}{16} \times \left(-\frac{18}{18}\right) = -V^0_{34} \tag{4.101}$$

The cam on arbor 104 moves a lever which moves another toothed segment. This segment meshes with a 14-teeth wheel on arbor 106, which carries a 30-teeth wheel meshing with a 32-teeth circular segment on the italic hours ring. Since I do not have the dimensions of the cam and the lever, I will not go further into this description, but the shape of the cam could be recomputed and the motion of the ring could be obtained from the dimensions of the various parts. However, this also raises the question of whether the actual cam takes the equation of time into account.

The italic hours are also shown on the 1793 clock made by Frater David for the Prince of Schwarzenberg (Oechslin 6.2). They are also shown on Pater Aurelius' clock in Munich (Oechslin 5.1).

4.4.7.2 The motion of the Sun

The main input to the upper dial is provided by arbor 44, whose motion is derived from tube 26. We have

$$V^0_{44} = V^0_{26} \times \left(-\frac{72}{36}\right) = V^0_{26} \times (-2) = -2 \tag{4.102}$$

This motion is then transferred to tube 45:

$$V^0_{45} = V^0_{44} \times \frac{40}{80} = -1 \tag{4.103}$$

$$P^0_{45} = -1 \text{ day} \tag{4.104}$$

This tube, or frame, which carries a number of gears, makes one turn clockwise in one day. This is also the motion of the Sun. The Sun is pictured by a golden disk and thin hand, and this hand passes through a narrow groove between the disk of the Moon and the ring of the ecliptic or zodiac.

The motions of the various internal gears of frame 45 are obtained from two fixed wheels on the central arbor 46 of this dial.

4.4.7.3 The motion of the Moon

The Moon is located on tube 50 but corresponds to an entire disk on the front of the dial. This disk shows the position of the Moon, and also the various aspects that can occur between the Sun and the Moon (sextile, quartile, trine, opposition, etc.). The motion of this tube is derived from the fixed 16-teeth wheel on the central arbor 46. We have

$$\mathrm{V}_{50}^{45} = \mathrm{V}_{46}^{45} \times \left(-\frac{16}{59}\right) \times \left(-\frac{15}{15}\right) \times \left(-\frac{15}{62}\right) \times \left(-\frac{32}{62}\right) \tag{4.105}$$

$$= \mathrm{V}_{46}^{45} \times \frac{1920}{56699} = -\mathrm{V}_{45}^{0} \times \frac{1920}{56699} = \frac{1920}{56699} \tag{4.106}$$

$$\mathrm{P}_{50}^{45} = \frac{56699}{1920} = 29.5307\ldots \text{ days} \tag{4.107}$$

The same value is given by Oechslin. This is an approximation of the synodic month, using the same ratio as the one used earlier.

We can also compute the motion of the Moon with respect to the fixed frame:

$$\mathrm{V}_{50}^{0} = \mathrm{V}_{50}^{45} + \mathrm{V}_{45}^{0} = \frac{1920}{56699} - 1 = -\frac{54779}{56699} \tag{4.108}$$

$$\mathrm{P}_{50}^{0} = -\frac{56699}{54779} = -1.0350\ldots \text{ days} \tag{4.109}$$

$$= -24 \text{ h } 50 \text{ m } 28.3137 \text{ s} \tag{4.110}$$

The same value is given by Oechslin. The disk of the Moon rotates clockwise.

4.4.7.4 The motion of the zodiac

The zodiac is located on tube 55. It is pictured by a mobile ring on the dial which carries the names of the months. Its motion is derived from the fixed 15-teeth wheel on the central arbor 46. We have

$$\mathrm{V}_{55}^{45} = \mathrm{V}_{46}^{45} \times \left(-\frac{15}{15}\right) \times \left(-\frac{15}{39}\right) \times \left(-\frac{12}{47}\right) \times \left(-\frac{12}{64}\right) \times \left(-\frac{40}{269}\right) \tag{4.111}$$

$$= \mathrm{V}_{46}^{45} \times \left(-\frac{450}{164359}\right) = -\mathrm{V}_{45}^{0} \times \left(-\frac{450}{164359}\right) \tag{4.112}$$

$$= -\frac{450}{164359} \tag{4.113}$$

$$\mathrm{P}_{55}^{45} = -\frac{164359}{450} = -365.2422\ldots \text{ days} \tag{4.114}$$

The same value is given by Oechslin.

We can also compute the motion of the zodiac with respect to the fixed frame:

$$\mathrm{V}_{55}^{0} = \mathrm{V}_{55}^{45} + \mathrm{V}_{45}^{0} = -\frac{450}{164359} - 1 = -\frac{164809}{164359} \tag{4.115}$$

$$\mathrm{P}_{55}^{0} = -\frac{164359}{164809} = -0.9972\ldots \text{ days} \tag{4.116}$$

$$= -23 \text{ h } 56 \text{ m } 4.0905\ldots \text{ s} \tag{4.117}$$

The same value is given by Oechslin. The zodiac rotates clockwise.

4.4.7.5 The motion of the dragon

The dragon is located on tube 57. This motion is derived from that of the ecliptic tube 55. We have

$$\mathrm{V}_{57}^{45} = \mathrm{V}_{55}^{45} \times \left(-\frac{30}{73}\right) \times \left(-\frac{72}{28}\right) = \mathrm{V}_{55}^{45} \times \frac{540}{511} \tag{4.118}$$

$$= -\frac{450}{164359} \times \frac{540}{511} = -\frac{243000}{83987449} \tag{4.119}$$

$$\mathrm{P}_{57}^{45} = -\frac{83987449}{243000} = -345.6273\ldots \text{ days} \tag{4.120}$$

The same value is given by Oechslin. This is an approximation of the eclipse year.

We can also compute the motion of the dragon with respect to the fixed frame:

$$\mathrm{V}_{57}^{0} = \mathrm{V}_{57}^{45} + \mathrm{V}_{45}^{0} = -\frac{243000}{83987449} - 1 = -\frac{84230449}{83987449} \tag{4.121}$$

$$\mathrm{P}_{57}^{0} = -\frac{83987449}{84230449} = 0.9971\ldots \text{ days} \tag{4.122}$$

$$= -23 \text{ h } 55 \text{ m } 50.7409 \text{ s} \tag{4.123}$$

The same value is given by Oechslin. The dragon moves clockwise.

4.5 References

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