# How to measure the corrections of the eyes without an ophtalmologist or an optician*

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When we need new glasses, we usually go to the ophtalmologist and get a prescription. Our eyes are then measured and the appropriate glasses are defined, depending on our sight. This usually involves elaborate machinery, testing various lenses, and so on, often conducted by an optometrist.

It is of course important to get one's eyes checked, especially for diseases such as ARMD (Age-Related Macular Degeneration) and glaucoma, but for the purely optical investigation, it is in fact possible to find the necessary corrections of one's eyes without an ophtalmologist or an optician. One way to do it is to have a set of glasses with varying vergences, and to find which one is most appropriate for each eye at a given distance.

Another way is to measure the basic features of our crystalline lenses using our capacities to distinguish between a sharp and blurred image. This is what I am showing here. I am however restricting myself to the correction of myopia and hypermetropia, and in cases where there is already (still) a range of sharp visibility. I am giving hints at further developments at the end of this note.

## 1 Basic optics

Figure 1 illustrates a lens centered at $O$, with $A'$ being an object, and $A$ its image, or conversely. We have the following conjugation relation, where $f$ is the focal distance of the lens:

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f} \tag{1}$$

$f > 0$ if the lens is convergent (figure 1), and $f < 0$ if it is divergent (figure 2).

*This work was done in 2014, but was only made public in 2025.

This relation is valid for thin lenses and with the paraxial approximation. It follows from the above relation that if $O$ and $A$ are fixed, a decrease of $f$ leads to a decrease of $\overline{OA'}$.

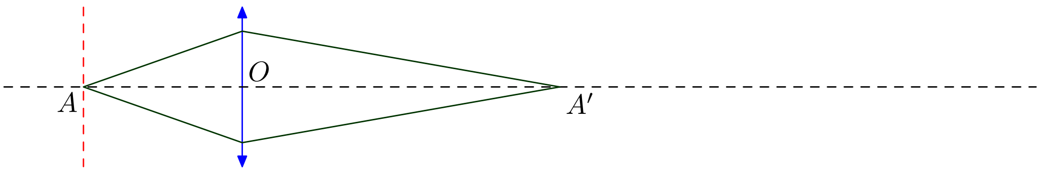


Figure 1: A convergent lens ($f > 0$).

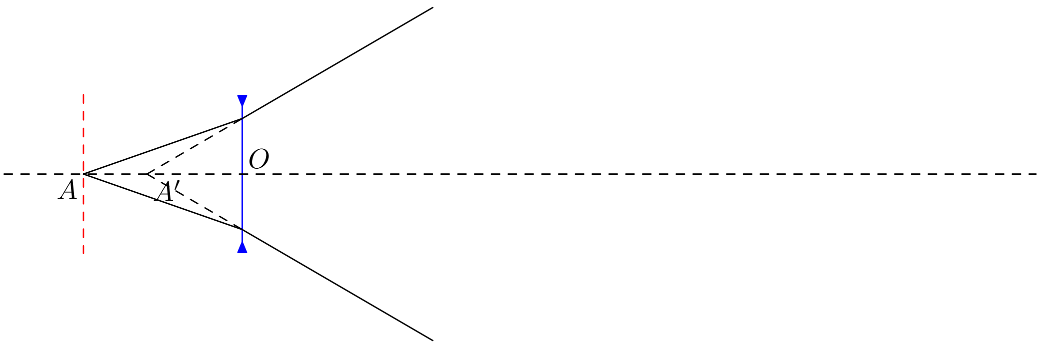


Figure 2: A divergent lens ($f < 0$). When the incoming rays are parallel, the outcoming rays virtually converge at the right of the lens.

## 2 The normal eye

The human eye (figure 3) is made of a number of parts, and for my purpose I only consider two: the crystalline lens and the retina.[1] The light goes through the crystalline lens and the rays are concentrated on the retina. In figures 1 and 2, the lens at $O$ will represent the crystalline lens, and the plane at $A$ will represent the retina.

I call the distance between the center of the crystalline lens and the retina $d$ and I will assume that it is constant.[2] The crystalline lens is not a rigid lens,

[1]For a comprehensive treatment of the physiology of the eye, see for instance Adler's textbook [3].

[2]This is of course a much simplified model, where the retina is assumed to be a plane, and where $d$ is constant. The actual retina is not flat and different persons may have eyes with different values of $d$.

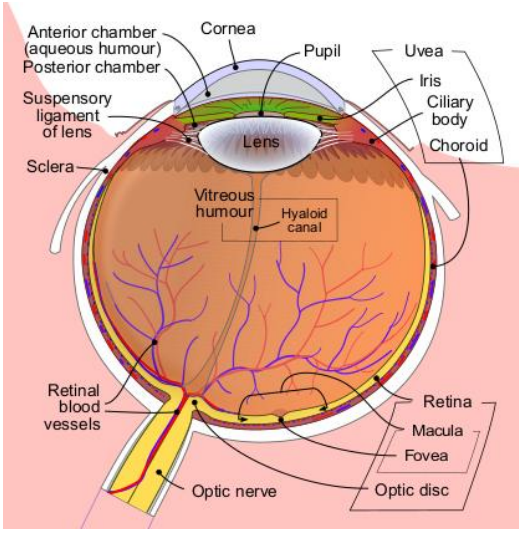


Figure 3: The anatomy of the eye
(source: Wikimedia/`Schematic_diagram_of_the_human_eye_en.svg`).

but a lens whose curvature depends on muscles on its periphery. When the lens is taut, the curvature of its surface is smallest (the radius of curvature is greatest).

For a normal eye, when the crystalline lens is taut, we have a clear vision at infinity. The focal distance of the taut lens is therefore $f_0 = d$. This corresponds to figure 4, with a vision at infinity.

When the crystalline lens is contracted, it becomes less taut and its curvature increases. The focal distance therefore decreases, and for a same distance $OA$ the viewing distance $OA'$ is made smaller. The closest distance that the eye can accomodate is called the *punctum proximum* (PP) (figure 5).

With rays coming from a far distance, if the eye contracts, the rays will converge before reaching the retina, thus between the retina and the lens. Contracting the eye will always move the convergence point ($A$) towards the right. (This corresponds to fixing a closer point than the far distance.)

On the other hand, if we try to fix a point closer than the *punctum proximum*, the convergence point will also be moved to the left, after the retina, and the sight will be blurred.

In all our experiments, I assume that the distance between the crystalline lens and the retina remains constant.

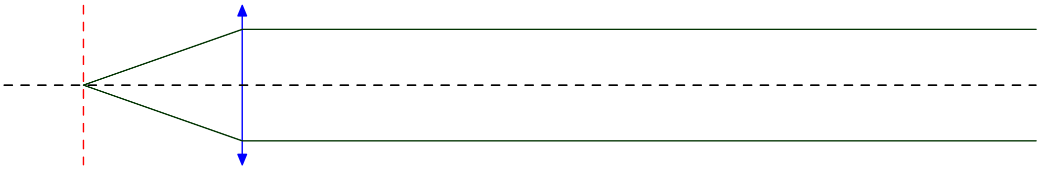

Figure 4: The normal eye looking at a great distance. The eye is relaxed and the crystalline lens is taut. The red line at the left represents the retina.

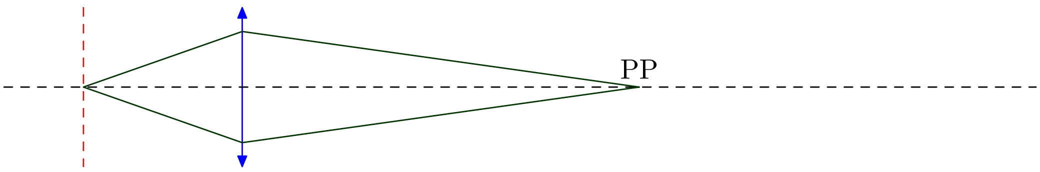


Figure 5: The normal eye looking at the *punctum proximum*. If the lens had the same focal distance as in figure 4, the image would be left of the retina. But here, the eye is contracted and the crystalline lens is not taut. The focal distance of the crystalline lens is therefore smallest.

# 3 The defect eye

In the defect eye, the focal distance of the taut crystalline lens is too large or too small, compared to the distance between the crystalline lens and the retina. Of course, we could also say that the latter is too small or too large with respect to the former. In any case, when the focal distance is too small, the eye can not see clearly at an infinite distance and in fact, it can not see clearly beyond a certain distance. This is what is called myopia. When the object gets closer, its image comes closer to the retina, until we reach the *punctum remotum* (PR). This is the greatest distance where we can see clearly. This is shown in figure 6. In that figure, the crystalline lens is taut. When the object continues to come closer, the image goes beyond the retina, so that we contract the eye, change its focal distance, in order to bring the image back on the retina. This can be done only up to the *punctum proximum* (PP).

On the other hand, when the focal distance of the taut crystalline lens is too large, the eye can usually accomodate for an infinite distance (*punctum remotum* $= \infty$) but the closest sharp distance is further away than the average *punctum proximum*. This condition is called hypermetropia.

The interval between the *punctum proximum* and the *punctum remotum* is the interval of distances where the eye can see clearly, and it does also correspond to a range of focal distances for the crystalline lens. The ideal principle of corrective glasses is to transform this range $[pp, pr]$ so that it covers the range $[npp, \infty]$, where $npp$ is the normal *punctum proximum*, that is, about 25 cm. This transformation is done with glasses and results in a function $F$. The new range is $[F(pp), F(pr)]$.

Unfortunately, after a certain age, the function $F$ can not be such that $F(pp) \leq npp$ and $F(pr) = \infty$, and two functions $F_1$ and $F_2$ are introduced, one such that $F_1(pp) = npp$ and the other such that $F_2(pr) = \infty$.

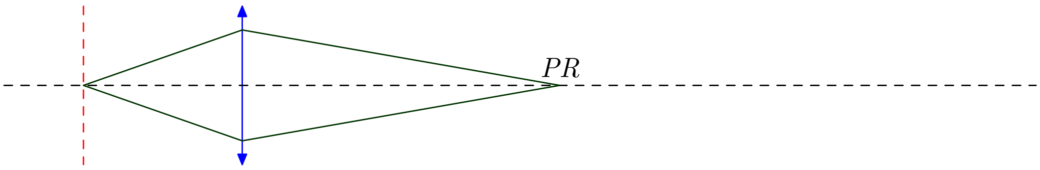


Figure 6: *Punctum remotum* too close (myopia). The normal *punctum remotum* should be at infinity.

Equation (1) can be used to compute the range $[f_p, f_r]$ of focal distances permitted by an eye, hence the elasticity of the crystalline lens. In equation (1), $AO = d$, which I assume to be about 17 mm. We then have

$$f = f_1(OA') = \frac{d \times \overline{OA'}}{d + \overline{OA'}} \tag{2}$$

The measure of the extreme values of $OA'$ ($pp$ and $pr$) then gives the extreme values of the focal distance:

We obtain $f_p = f_1(pp)$ and $f_r = f_1(pr)$.

For instance, if $OA' = pp = 50\,\text{cm}$, $f_p = 1.64\,\text{cm}$. If $OA' = pr = 200\,\text{cm}$, $f_r = 1.69\,\text{cm}$.

# 4 The corrected eye

Figure 7 shows how the *punctum remotum*, which was too close (myopia), was removed further using a divergent lens (in green). The purpose is of course to remove the *punctum remotum* to an infinite distance.

Figure 8 corresponds to a too distant *punctum proximum* (hypermetropia) which is brought closer using a convergent lens (in green). The purpose is to bring the *punctum proximum* back to about 25 cm.

Correcting the vision amounts to find the appropriate convergent or divergent lens to put in front of the crystalline lens.

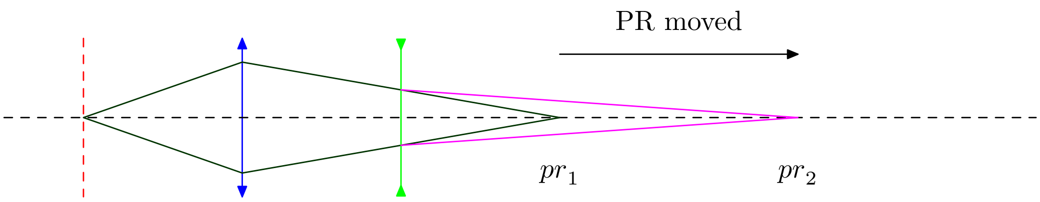


Figure 7: *Punctum remotum* too close, corrected with a divergent lens.

I assume that we know in both cases the (extreme) focal distances of the eye.

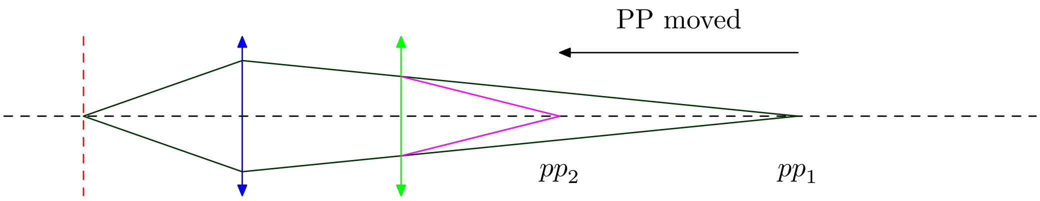


Figure 8: *Punctum proximum* too distant, corrected with a convergent lens.

Figure 9 shows the general problem. We have two lenses, one is the crystalline lens at $O$ and I will consider the two extreme focal distances measured earlier. The second lens is the corrective lens, located at $O'$. The two lenses are at a distance $e$ which is a parameter. The focal distance of the corrective lens will be computed in order to put $A''$ either at the desired *punctum proximum* (when the crystalline lens is not taut) or at the desired *punctum remotum* (when the crystalline lens is taut).

We have the two conjugation relations (valid for thin lenses):

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f_1} \qquad\qquad \frac{1}{\overline{O'A''}} - \frac{1}{\overline{O'A'}} = \frac{1}{f_2} \tag{3}$$

as well as

$$\overline{OA'} + \overline{A'O'} = e \tag{4}$$

$f_1$ is variable ($f_{1p}$,$f_{1r}$) and $f_2$ is fixed.

We want to compute $f_2$. We can express $\overline{OA'}$ as a function of $f_1$ and $f_2$

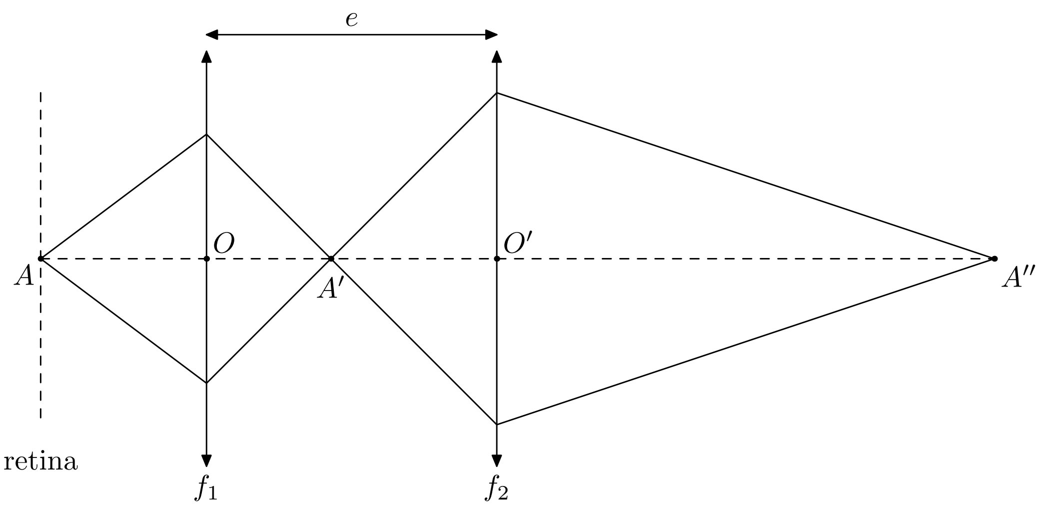


Figure 9:

as a function of $\overline{OA'}$ and $\overline{O'A''}$:

$$\overline{OA'} = \frac{f_1 \cdot \overline{OA}}{f_1 + \overline{OA}} \tag{5}$$

$$f_2 = \frac{\overline{O'A'} \cdot \overline{O'A''}}{\overline{O'A'} - \overline{O'A''}} \tag{6}$$

$$= \frac{\overline{O'A'} \cdot \overline{O'A''}}{\overline{O'A'} + \overline{A''O'}} \tag{7}$$

$$= \frac{(\overline{OA'} - e) \cdot \overline{O'A''}}{\overline{OA'} - e + \overline{A''O'}} \tag{8}$$

Setting $p = \overline{O'A''}$ and $\overline{AO} = d$, we have

$$\overline{OA'} = -\frac{f_1 \cdot d}{f_1 - d} \tag{9}$$

and

$$f_2(f_1, p) = \frac{\left(\frac{f_1 \cdot d}{f_1 - d} + e\right) \cdot p}{\frac{f_1 \cdot d}{f_1 - d} + e + p} \tag{10}$$

$$= \frac{[f_1 \cdot d - e(d - f_1)] \cdot p}{(p + e)(f_1 - d) + f_1 \cdot d} \tag{11}$$

The latter equation will be used to compute the focal distance of the corrective lens, given the focal distance $f_1$ of the crystalline lens, the distance

$d$ between the crystalline lens and the retina, the distance $e$ between the two lenses and the distance $p$ to the desired *punctum proximum* or *punctum remotum*.

In summary, $f_1$ is a function of the actual *punctum proximum* or *punctum remotum*:

$$f_{1p} = f_1(pp) \qquad\qquad f_{1r} = f_1(pr) \tag{12}$$

Hence

$$f_{2p} = f_2(f_1(pp), 25\,\text{cm}) \tag{13}$$

$$f_{2r} = f_2(f_1(pr), \infty) \tag{14}$$

Now, $f_2$ being given, we can again compute the range of clearness from the two extreme values of $f_1$:

$$\overline{O'A''} = \frac{1}{\frac{1}{f_2} + \frac{1}{\overline{O'A'}}} = \frac{\overline{O'A'} \cdot f_2}{\overline{O'A'} + f_2} = \frac{\left(\overline{OA'} - e\right) f_2}{\overline{OA'} - e + f_2} \tag{15}$$

$$\overline{OA'} = \frac{\overline{OA} \cdot f_1}{\overline{OA} + f_1} = \frac{-df_1}{f_1 - d} \tag{16}$$

Therefore

$$\overline{O'A''} = -\frac{\left(e + \frac{df_1}{f_1 - d}\right) f_2}{f_2 - e - \frac{df_1}{f_1 - d}} = -\frac{\left[e(f_1 - d) + df_1\right] f_2}{f_2(f_1 - d) - e(f_1 - d) - df_1} \tag{17}$$

Replacing $f_1$ by $f_{1p}$ and $f_{1r}$, we obtain:

$$\overline{O'A''_p} = -\frac{\left[e(f_{1p} - d) + df_{1p}\right] f_2}{f_2(f_{1p} - d) - e(f_{1p} - d) - df_{1p}} \tag{18}$$

$$\overline{O'A''_r} = -\frac{\left[e(f_{1r} - d) + df_{1r}\right] f_2}{f_2(f_{1r} - d) - e(f_{1r} - d) - df_{1r}} \tag{19}$$

and we therefore have the range of clearness resulting from wearing the eyeglasses of focal length $f_2$.

# 5 Measuring the PP and PR

For each eye, we can usually measure the *punctum proximum* and the *punctum remotum*. We can take a book or some document with large letters, and we can measure with a ruler the approximate closest and farthest distance for which the text is sharp. This is not always possible, either because the eye has no range of sharp sight, or because of important astigmatism. In the other cases, the measurements should be possible and approximate ranges suffice for our purposes.

Let us consider two examples. In each case, we take $e = 3\,\mathrm{cm}$, $d = 1.7\,\mathrm{cm}$, we compute $f_2$ with a desired *punctum proximum* of 25 cm ($p = 25\,\mathrm{cm} - e$), and a desired *punctum remotum* of 1000 cm ($p = 1000\,\mathrm{cm} - e$). We therefore obtain two values of $f_2$ in each example.

**Example 1: PP = 29 cm, PR = 60 cm** In this case, we find $f_{1p} \approx 1.606$ (*punctum proximum*) and $f_{1r} \approx 1.653$ (*punctum remotum*). Then, we compute two lenses:

- $f_{2p} = 143\,\mathrm{cm}$ (0.7 dioptries) which results in a vision range from 25 cm to 44 cm; this is the glass for near-vision;
- $f_{2r} = -60\,\mathrm{cm}$ ($-1.65$ dioptries) which results in a vision range from 49 cm to 10 m; this is the glass for far-vision.

There is a slight gap between the two ranges, but it doesn't require an additional glass.

**Example 2: PP = 46 cm, PR > 3m** In this case, we find $f_{1p} \approx 1.64$ (*punctum proximum*) and $f_{1r} \approx 1.7$ (*punctum remotum*). Then, we compute two lenses:

- $f_{2p} = 45\,\mathrm{cm}$ (2.2 dioptries) which results in a vision range from 25 cm to 48 cm; this is the glass for near-vision;
- $f_{2r} = 1.3\,\mathrm{m}$ (0.08 dioptries) which results in a vision range from 45 cm to 10 m; this is the glass for far-vision.

Since these two ranges overlap, there is no need for an additional glass.

The features of the lenses then make it possible to order glasses online or elsewhere. This is what I have been doing for more than ten years. When ordering glasses, one should also give the pupillary distance which can easily be measured.

However, the above calculations have assumed a distance of 3 cm between the crystalline lens and the glasses, and this may need to be adapted. I have also assumed that the distance between the crystalline lens and the retina is 17 mm.

# 6 Special cases

## 6.1 The case of no clear vision

It may happen that a person has no clear vision, whatever the distance. This happens if the taut crystalline lens is not enough convergent, hence if its focal distance is too large. In that case, the eye cannot be sufficiently contracted for the rays to reach the retina at infinity, and also not at a closer distance. We can then not directly measure the *punctum proximum* and *punctum remotum*.

What can then be done is to wear correcting glasses which allow a clear vision in some range (figure 10). Such correcting glasses can be found in various shops and their refracting power is given in dioptries. For instance, a glass of 2 dioptries corresponds to a focal distance of 0.5 meters. It matters little which refracting power is chosen, as long as it is strong enough to have some clear vision. One can then measure the *punctum proximum* and *punctum remotum*. Using these measures, we can then compute the range of focal distances of the crystalline lens, and then return to the formulæ of section 4. I might detail this in a separate article.

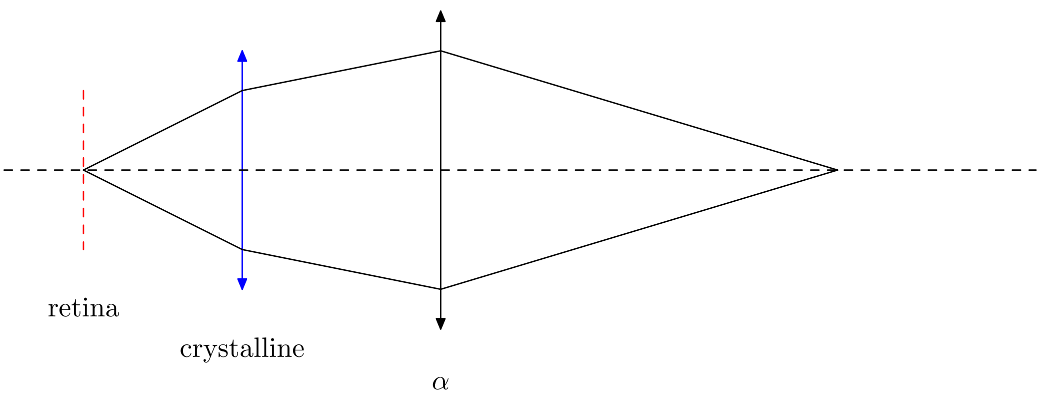


Figure 10: An eye with no clear vision, corrected with a glass $\alpha$.

### 6.2 Astigmatism

It is also possible to measure the required correction for astigmatism, and this was first described by Airy [1, 4, 2]. This might also be detailed in a separate article.

## References

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[2] John R. Levene. Sir George Biddell Airy, F.R.S. (1801-1892) and the discovery and correction of astigmatism. *Notes and Records of the Royal Society of London*, 21(2):180–199, December 1966.

[3] Leonard A. Levin, Paul L. Kaufman, and Mary Elizabeth Hartnett. *Adler's physiology of the eye.* Philadelphia: Elsevier, 2025.

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